THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 61, No. 7, September 1982
Printed in U.S.A.

Equalization of Multimode Optical Fiber Systems

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(Manuscript received December 31, 1981)

Decision feedback equalization (DFE) may provide a simple way to dramatically increase the bit rate on multimode optical systems. Computer simulations of digital multimode systems with tapped delay-line linear equalization and/or DFE are described. Results indicate that with relatively simple hardware it may be possible to increase by many times the usable bit rate on dispersion-limited optical channels, with a power penalty less than that for multilevel transmission and without the latter's complexity. For example, one could double the bit rate of a two-level system by paying an equalization power penalty of about 3 dB with DFE, whereas doubling the bit rate by going to four levels requires a penalty of 4.8 dB.

I. INTRODUCTION

It has been recognized for some time[1] that conventional linear equalization is of limited benefit in improving the performance of multimode optical fiber channels. The major reason is the rapid falloff in the equivalent baseband frequency response of the optical channel, which results in considerable noise enhancement when linear equalization is attempted using a filter which emphasizes the high frequencies.

However, for data rates above 100 Mb/s, multimode systems are generally modal dispersion rather than loss-limited[2] and some form of equalization is highly desirable. This is particularly true considering that index-grading imperfections in presently manufactured fibers produce a wide spread in fiber bandwidths and that equalization could improve the yield of usable fibers.

Decision feedback equalization (DFE) is known to be superior to linear equalization for channels which exhibit amplitude distortion.[3,4] Previous theoretical investigations[5,6] of DFE for fiber transmission systems have shown that power penalties are significantly lower than those using linear equalization, with the improvement becoming greater as the amount of dispersion increases.

This paper gives the results of computer simulation of fiber optic channels for cases having no equalization, linear equalization via transversal filtering, and DFE both with and without a preceding stage of transversal equalization. We show that for pulses with either Gaussian or cosine-squared dispersion, decision feedback alone does a remarkably good job. A single DFE tap is adequate over a wide range of fiber bandwidths. A discussion of the effect of timing phase on DFE performance is included. The use of a transversal filter with minimum mean-square error (mse) tap weights preceding the DFE stage is found to produce little improvement in performance.

II. SYSTEM MODEL

To investigate the performance of various equalizers, the model shown in Fig. 1 was chosen. The input and output parameters of the model are shown and are explained below.

2.1 Transmitter

The input data sequence, $\{a_n\}$, consists of elements which are assumed to be independent identically distributed discrete random variables. These discrete amplitudes modulate the power of the transmitted pulse $p_t(t)$ at a rate $B = 1/T$ to produce the transmitted signal. Optical detectors produce an output current proportional to received power that is always positive; therefore, $p_t(t)$ which defines the pulse shape and the elements of $\{a_n\}$ which determine the pulse amplitudes must always be greater than or equal to zero. Input data sequence,

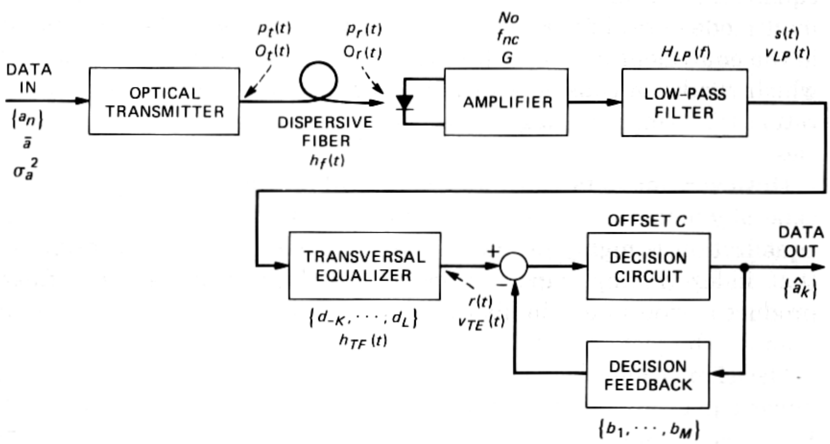

Fig. 1—Block diagram of optical system model showing important parameters.

$\{a_n\}$, has an expected value $\bar{a}$ and a variance σ_a^2. For the binary case, $a_n = 0$ or 1, we have $\bar{a} = 0.5$ and $\sigma_a^2 = 0.25$.

The transmitted optical signal is thus

$$O_t(t) = \sum_{n=-\infty}^{\infty} a_n p_t(t - nT),$$

where $O_t(t)$ is in units of optical power. The average transmitted power is

$$\langle O_t(t) \rangle = \bar{a} \int_{-\infty}^{\infty} p_t(t) dt.$$

The optical signal can be constrained by a limit in either average power or peak power.

2.2 Channel

The optical fiber has an impulse response $h_f(t)$ such that the received optical signal power is

$$O_r(t) = \int_{-\infty}^{\infty} O_t(\tau) h_f(t - \tau) d\tau.$$

The average received power will be

$$\begin{aligned}
\langle O_r(t) \rangle &= \int_{-\infty}^{\infty} O_r(t) dt \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} O_t(\tau) h_f(t - \tau) dt d\tau \\
&= \int_{-\infty}^{\infty} O_t(\tau) \int_{-\infty}^{\infty} h_f(t - \tau) dt d\tau \\
&= \int_{-\infty}^{\infty} O_t(\tau) d\tau \int_{-\infty}^{\infty} h_f(t) dt \\
&= \langle O_t(t) \rangle \cdot H_f(0),
\end{aligned}$$

where $H_f(0) = \int_{-\infty}^{\infty} h_f(t) dt$ is the dc or steady-state fiber loss.

The shape of an isolated received pulse will be

$$p_r(t) = p_t(t) * h_f(t),$$

where $*$ denotes convolution, hence the received signal can be expressed as

$$O_r(t) = \sum_{n=-\infty}^{\infty} a_n p_r(t - nT).$$

2.3 Detector and amplifier

The detector and amplifier together produce an output voltage $v_d(t)$ proportional to the received optical power, plus added noise $n(t)$,

$$v_d(t) = G[O_r(t) + n(t)],$$

where G is a gain constant.

In general,[7] the noise spectrum of receivers referred to the input can be characterized by a noise corner frequency, f_{nc}, above which the noise power increases at 6 dB/octave. Hence

$$N(f) = N_0[1 + (f/f_{nc})^2],$$

where N_0 is the low-frequency noise spectral density. This characterization is a good approximation for receivers employing either avalanche photodiode or p-i-n detectors, and either high impedance (integrating) or transimpedance amplifiers. It is assumed that $1/f$ noise can be neglected for bit rates of 100 Mb/s or more.

2.4 Low-pass filter

The amplifier in Fig. 1 is followed by a low-pass filter $H_{LP}(f)$ which attempts to maximize the s/n by eliminating high-frequency noise and by performing phase equalization on the received pulses. If linear equalization or DFE is employed following this filter, its optimum form[4] is known to be a whitening matched filter for the noise spectrum $N(f)$ and received pulse shape $p_r(t)$. Such a filter produces perfect phase equalization but does not perform amplitude equalization, hence intersymbol interference (ISI) will generally be present between pulses at its output, if the channel exhibits amplitude distortion. If additional equalization does not follow, then the low-pass filter should be used to reduce ISI through amplitude, as well as phase equalization. The desired goal is often a raised-cosine pulse which has zero-crossings at all other sampling instants. However, amplitude equalization results in the enhancement of noise at frequencies which are "bumped up," limiting the attainable s/n improvement.

In a practical repeater, $H_{LP}(f)$ will generally be some nonoptimum fixed filter. If subsequent equalization is included, the filter will be near-optimum as long as it passes frequencies where $p_r(t)$ has significant energy and rejects noise at frequencies where $p_r(t)$ has little energy. Phase equalization, if necessary, should be done here or by a following transversal equalizer. If phase equalization is not attempted, $H_{LP}(f)$ should have a linear phase characteristic to prevent the addition of phase distortion.

In this paper, no attempt is made to optimize $H_{LP}(f)$. A fixed filter is chosen with the optimization being left to later equalization stages.

2.5 Transversal equalizer

The transversal equalizer is illustrated in Fig. 2. The output $v_{TE}(t)$ consists of the weighted sum of delayed versions of the input $v_{LP}(t)$. The weighting coefficients are $\{d_k\}$ and the delay interval is T. Hence

$$v_{TE}(t) = \sum_{k=-K}^{L} d_k v_{LP}(t + kT).$$

A number of papers have been written about transversal equalizers, and their characteristics are well known. The design problem consists of choosing the number of leading and trailing taps L and K, and the weighting coefficients $\{d_k\}$. If $H_{LP}(f)$ is a whitening matched filter, then a synchronous equalizer with tap spacing, T, is known to be optimum.[4] If $H_{LP}(f)$ is suboptimum, then a fractionally spaced equalizer[8] with more frequent taps can be used to advantage. The extra degrees of freedom provided by more taps are used to approach the matched filter condition.

If decision feedback is used following the transversal equalizer, then it has been shown[4] that trailing taps which correspond to decision feedback taps can be eliminated. With enough DFE taps to cancel all pulse postcursors, only leading transversal taps are necessary. The coefficients $\{d_k\}$ are commonly chosen to either minimize peak distortion (the zero-forcing equalizer) or mse.[9]

The transversal equalizer can be viewed as an extension of the lowpass filter $H_{LP}(f)$. To eliminate ISI, it must shape the pulse spectrum to that of a Nyquist pulse. Frequencies attenuated by the channel, usually the higher frequencies, must be enhanced with the consequent penalty of increased noise. The advantage of using a transversal structure to accomplish this filtering is ease of adjustability, especially when the equalizer must be adapted to an originally unknown or slowly time-varying channel characteristic. However, as mentioned in the introduction, linear equalization of this type is known to be of little benefit for optical channels because of exorbitant noise enhancement.

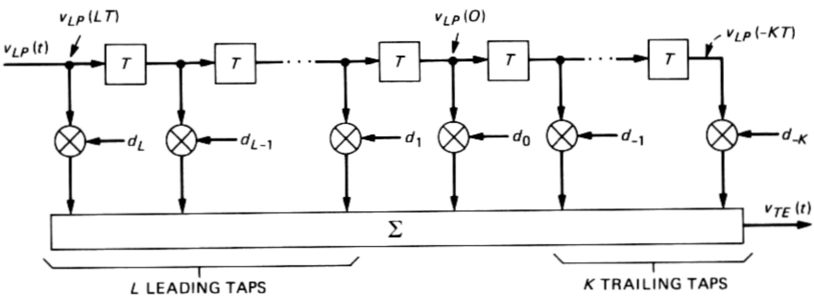

Fig. 2—Block diagram of transversal equalizer stage.

2.6 Decision feedback

The decision feedback stage is diagrammed in Fig. 3. A decision circuit or comparator produces estimates $\{\hat{a}_k\}$ of the transmitted data. A weighted sum of past decisions is subtracted from the incoming signal, with the weights $\{b_m\}$ being exactly equal to the amplitude of corresponding pulse postcursors at each sampling instant. Thus, if the decisions $\{\hat{a}_k\}$ are correct, then ISI from already-detected pulses is completely removed with no enhancement whatsoever of the noise.

An offset C is included to allow optimization of the decision threshold.

III. COMPUTER SIMULATIONS

The objective of the computer simulations is to compare the power penalties of different equalizers for various amounts of fiber dispersion. Inputs to the model include the number of DFE taps M, the number of leading and trailing transversal filter taps L and K, the low-pass filter response $H_{LP}(f)$, the receiver noise corner frequency f_{nc}, and the received pulse shape $p_r(t)$. Outputs consist of the receiver low-frequency noise level N_0 for an error probability of 10^{-9}, the optimum transversal tap weights $\{d_k\}$ and DFE tap weights $\{b_k\}$, and the equalized pulse shape $r(t)$.

3.1 Error probability bounds

The calculation of exact error probabilities in the presence of ISI plus noise is very complex, hence one generally resorts to various bounds. One common worst-case bound compares eye opening for the most adverse message sequence to the noise standard deviation. The criterion Q is defined as

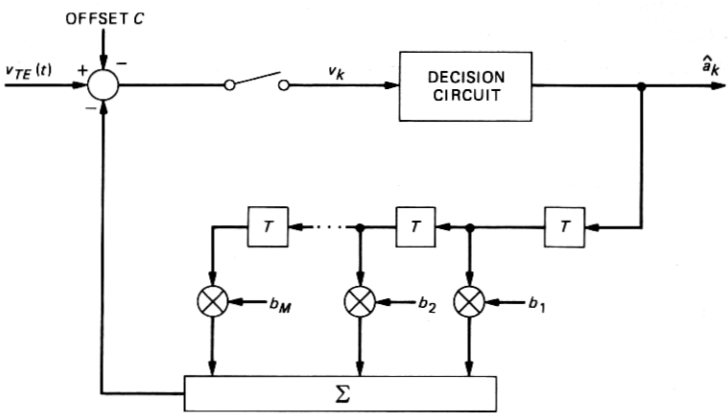

Fig. 3—Block diagram of decision feedback stage.

$$Q = \frac{0.5\left(r_o - \sum_{\substack{n \notin S \\ n \neq 0}} |r_n|\right)}{\sigma},$$

where

r_o = pulse height at sampling instant

r_n = pulse height (ISI) in other time slots

S = set of time slots equalized by decision feedback

σ = noise standard deviation.

This bound is relatively good if the ISI is large relative to the noise and is limited to only a few symbols, but it is pessimistic if ISI is small and extended over time, as the worst-case bit sequence will then occur only rarely. For this case, a better bound is obtained by approximating ISI by a normal distribution and adding its variance to the noise variance, giving the usual mse bound.[4,5]

Tighter bounds than either of these can be found at the expense of various degrees of computational complexity.[10]

The impulse response of optical fibers is typically found to resemble a Gaussian or a cosine-squared pulse.[11] Therefore, the ISI is limited to only a few adjacent symbols. In this situation, the worst-case eye opening or Q bound is a reasonably good choice.

3.2 Offset and decision feedback optimization

The optimum mse DFE for fiber optic systems has been analyzed by Messerschmitt.[5] Let the shape of an isolated pulse at the transversal filter output be denoted by $r(t)$:

$$r(t) = G[p_r(t) * h_{LP}(t) * h_{TF}(t)],$$

where

G = amplifier gain

$h_{LP}(t)$ = low-pass filter impulse response

$$h_{TF}(t) = \text{transversal filter impulse response} = \sum_{k=-K}^{L} d_k \delta(t + kT).$$

The decision threshold can be optimized by an offset C given by[5]

$$C = \lambda_0 G H_{LP}(0) H_{TF}(0) + \bar{a}\left[\sum_{n=-\infty}^{\infty} r_n - \sum_{m \in S} b_m - 1\right],$$

where λ_0 is the detector dark current and S is the set of DFE taps 1, $\cdots$, M. Also, the decision feedback tap weights are optimized by setting

$$b_m = r_m,\ m \in S.$$

The values r_m are sampled versions of $r(t)$, with r_o corresponding to the instant at which the pulse is sampled for detection. With the above choice of DFE tap weights, we have

$$C = \lambda_0 G H_{LP}(0) H_{TF}(0) + \bar{a}\left[\sum_{n \notin S} r_n - 1\right].$$

With the above choices of C and $\{b_m\}$, and assuming correct decisions, the mse is shown to be[5]

$$\begin{aligned} \text{mse} &= E[(v_k - \hat{a}_k)^2] \\ &= \sigma_a^2\left[\sum_{n \notin S} r_n^2 - 2r_o + 1\right] + \sigma^2, \end{aligned}$$

where

$$\begin{aligned} \sigma^2 &= \text{noise variance} \\ &= E\big[[Gn(t) * h_{LP}(t) * h_{TF}(t)]^2\big]. \end{aligned}$$

As before, $n(t)$ is the receiver noise (thermal plus shot noise) referred to the receiver input. Note that only terms r_n, $n \in S$ (i.e., no corresponding DFE taps) contribute to mse.

3.3 Transversal filter optimization

The problem of determining optimum transversal filter tap weights $\{d_k\}$ when decision feedback is used was first solved by Austin.[12] In matrix form, the solution can be expressed as

$$\mathbf{D} = \mathbf{A}^{-1}\boldsymbol{\alpha},$$

where

$$\begin{aligned} \mathbf{D} &= \text{vector of tap weights} \\ &= [d_{-K}, d_{-K+1}, \cdots, d_L]^T \\ \mathbf{A} &= \psi + \frac{1}{\sigma_a^2}\phi \\ \phi &= \text{noise covariance matrix of terms } \phi_{km} \end{aligned}$$

$$\phi_{km} = \phi_{k-m} = \int_{-\infty}^{\infty} N(f) H_{LP}(f) e^{-j(k-m)2\pi f T} df$$

$$\psi = \text{modified signal autocorrelation matrix of terms } \psi_{km}$$

$$\psi_{km} = \sum_{n \notin S} s_{n+k} s_{n+m}$$

$$\boldsymbol{\alpha} = [s_{-K}, s_{-K+1}, \cdots, s_L]^T$$

$$s_k = \text{sample of isolated pulse at low-pass filter output}$$

$$= s(kT)$$

$$s(t) = G[p_r(t) * h_{LP}(t)].$$

The matrix

$$\mathbf{A} = \psi + \frac{1}{\sigma_a^2} \phi$$

is symmetric and positive semidefinite, hence it is invertible, except in extreme cases unlikely to be encountered in practice. The above solution for the tap weight vector **D** minimizes mse. This criterion may not produce the best bound on the probability of error, but is used here because it is the only bound for which optimum tap weights are known to have been formulated.

3.4 Power penalty calculation

Equalization power penalties are computed by assuming that the received pulse $p_r(t)$ has unit area (i.e., unit energy) and solving for the receiver noise level N_0 which will produce an error probability of 10^{-9}. Because the transversal filter coefficients **D** are dependent upon the noise covariance matrix ϕ, which is itself proportional to N_0, an iterative solution is necessary. The simulation program allows the operator to enter an initial value for N_0, and Newton-Raphson iteration is then used to converge to a final solution for both N_0 and **D**.

An error probability of less than 10^{-9} is assured by making Q equal to 5.99781. The eye opening E in the presence of decision feedback is calculated as

$$E = r_o - \sum_{\substack{n \notin S \\ n \neq 0}} |r_n|,$$

while the noise standard deviation is

$$\sigma = [N_0 \mathbf{D}^T \phi \mathbf{D}]^{1/2}.$$

Iteration proceeds by assuming a value for N_0 and calculating the corresponding tap weights **D**. The eye opening and σ, plus their derivatives with respect to N_0, can then be found, allowing new values of N_0 to be calculated until the ratio $Q = 0.5E/\sigma$ converges to 5.99781.

3.5 Timing phase optimization

When decision feedback with no transversal equalizer was analyzed, or when only leading or anticausal transversal taps were specified, the power penalties were found to depend heavily upon timing phase. Optimization was carried out by simply advancing the sampling time (i.e., delaying the pulse) and finding the delay which resulted in the lowest power penalty.

IV. RESULTS

4.1 Gaussian dispersion and equalizing filter

Fibers with perfect mode mixing exhibit Gaussian dispersion and have an impulse response given by

$$h_f(t) = \frac{1}{\sqrt{2\pi}\alpha T} e^{-[t^2/2(\alpha T)^2]}.$$

The baseband frequency response is

$$H_f(f) = e^{-[(2\pi\alpha T f)^2/2]},$$

and the 6-dB electrical bandwidth is

$$f_{6\,\mathrm{dB}} = \frac{1}{2\pi\alpha T}\sqrt{2\ln 2}$$

$$= 0.1874/\alpha T.$$

Figures 4, 5, and 6 show power penalties for rectangular nonreturn to zero (NRZ) pulses transmitted over such a fiber. An NRZ pulse is defined by

$$x(t) = \begin{cases} 1, & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0, & \text{elsewhere} \end{cases}$$

$$X(f) = \frac{1}{\pi f}\sin \pi f T.$$

The low-pass filter $H_{LP}(f)$ is chosen to produce a Nyquist output pulse with a raised cosine characteristic[7] and an excess bandwidth of $\beta = 0.5$ for an NRZ input pulse. The output pulse spectrum is

$$Y(f) = \begin{cases} 1, \ 0 \le |f| \le \frac{B}{2}(1-\beta) \\ \frac{1}{2}\left[1 - \sin\frac{\pi}{\beta}\left(\frac{f}{B} - \frac{1}{2}\right)\right], \ \frac{B}{2}(1-\beta) < |f| < \frac{B}{2}(1+\beta) \\ 0, \ \text{elsewhere}, \end{cases}$$

hence the required low-pass filter characteristic is

$$H_{LP}(f) = \begin{cases} \dfrac{Y(f)}{X(f)}, \; |f| < \dfrac{B}{2}(1+\beta) \\ 0, \text{ elsewhere}. \end{cases}$$

For zero fiber dispersion, this low-pass filter produces a Nyquist output pulse with no ISI. The ISI will be produced by fiber dispersion and will increase as the fiber bandwidth decreases.

Power penalties in the following results are calculated relative to ideal matched-filter detection of an isolated NRZ pulse with zero dispersion.

4.1.1 White noise ($f_{nc} > B$)

An actual optical receiver will not generally have a white noise spectrum. With a high-gain APD, for example, the noise is proportional to the received signal and so is time dependent. However, as a worst-case situation, one may assume stationary white noise with a level equal to that for the most adverse message sequence.

Figure 4 shows power penalties for a white receiver noise spectrum. The four curves are for A—no additional equalization; B—11-tap transversal equalizer; C—five DFE taps plus 11-tap transversal equalizer; and D—five taps of decision feedback with optimized timing

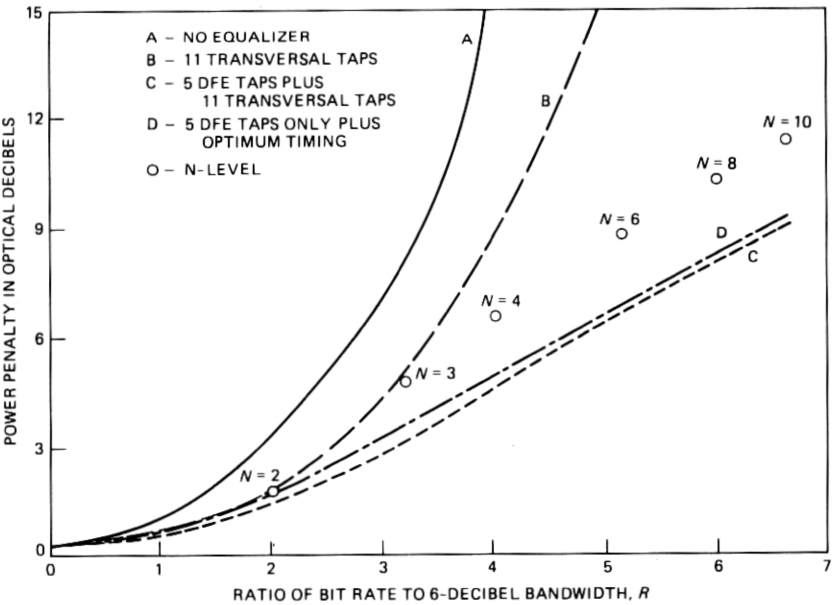

Fig. 4—Power penalties versus ratio of bit rate to fiber bandwidth for Gaussian dispersion, equalizing low-pass filter, and white noise.

phase but no transversal equalizer. Also shown are power penalties for multilevel signaling which are discussed in Section 4.5.

The horizontal axis values are in terms of a variable R given by

$$R = \frac{\text{bit rate } B}{\text{6-dB electrical bandwidth (3-dB optical)}}.$$

As the 6-dB channel bandwidth becomes less than the Nyquist frequency (i.e., $R > 2$), DFE plus linear equalization (curve C) begins to perform much better than linear equalization alone (curve B). Most notable, however, is that DFE only (curve D) does nearly as well as DFE plus linear, and beyond $R = 4.5$ remains within 0.2 dB. The addition of a transversal equalizer stage prior to DFE produces almost no improvement. The reason for this is that the transversal tap weights have been chosen to minimize mse, whereas the criterion of goodness is the ratio of eye opening to noise standard deviation. The problem is that almost all of the ISI is due to one large immediate precursor. The mse algorithm seeks to minimize the sum of the square of this precursor and the noise variance. To reduce the ISI, it must allow the noise to increase, which it does to such a degree that according to the eye opening criterion very little has been gained.

Prior to adoption of the eye opening criterion, an attempt was made to use mse as the power penalty measure. However, this criterion proved to be extremely pessimistic in the absence of a transversal equalizer. The mse is dominated by a few large ISI values and indicated very high-power penalties. However, the eye opening criterion showed the actual penalties to be much smaller.

4.1.2 Colored noise ($f_{nc} = B/3$)

The curves in Fig. 5 were obtained assuming a receiver noise spectrum which increases with frequency. The noise corner frequency is chosen as $f_{nc} = B/3$, and is based upon parameters of a typical p-i-n/FET receiver operating at 100 Mb/s as described in the Appendix.

In comparing Figs. 5 and 4, note the difference of 1.12 dB in the 0-dB reference levels (i.e., matched-filter detection is 1.12 dB worse for colored noise than for white noise). Disregarding this baseline shift, curves A and D in both figures are identical, as should be expected, because with no transversal equalizer the error rate depends only on the noise variance and not on the shape of the noise spectrum. The addition of a transversal equalizer does not help as much as in the white noise case. The reason is that a linear equalizer works by enhancing high frequencies which now contain more noise than they did for white noise.

Comparing curves C and D, we see that the addition of a transversal

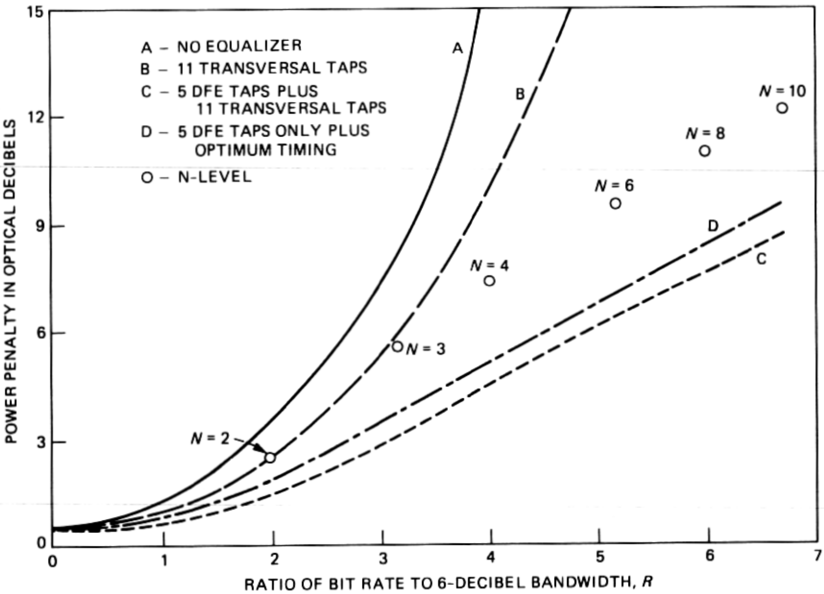

Fig. 5—Power penalties for Gaussian dispersion, equalizing low-pass filter and colored noise with $f_{nc} = B/3$. Receiver noise spectrum corresponds to a p-i-n FET receiver as described in the Appendix.

equalizer improves the power penalty by about 1 dB over DFE alone. As DFE removes much of the ISI, the transversal stage is able to decrease the mse by acting to reduce high-frequency noise. It can be expected that the choice of a fixed low-pass filter $H_{LP}(f)$ with a lower 3-dB frequency than $B/2$ would produce similar power penalty improvements for DFE only. A narrower $H_{LP}(f)$ would not affect ISI significantly if the channel bandwidth is already less than the low-pass filter bandwidth.

For cases where f_{nc} is lower than $B/3$ as assuméd here, the choice of a minimum $H_{LP}(f)$ bandwidth becomes more crucial because the amount of high-frequency noise increases. Decision feedback equalization is very important in such cases as it provides a way of noiselessly eliminating the ISI introduced by a narrow low-pass filter.

4.1.3 One decision feedback tap

The curves in Fig. 6 are for white noise with five DFE taps plus optimum timing phase and one DFE tap plus optimum timing phase. Up to a bit rate of 3.5 times the 6-dB bandwidth, the curves remain within 0.4 dB of one another. Hence, a single DFE tap with no transversal equalizer, which in hardware would require one flip-flop, a summer and a weighting network, does a remarkably good job of

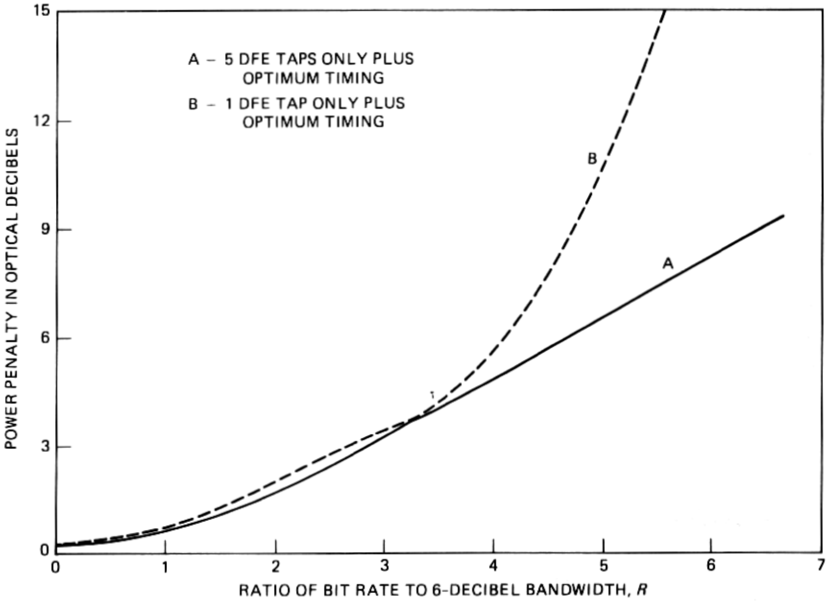

Fig. 6—Power penalties for decision feedback alone. Gaussian dispersion, equalizing low-pass filter and white noise.

equalization for Gaussian dispersion. The reason for this success is that up to $R = 3.5$, almost all of the ISI is due to one large postcursor (provided sampling is done before the pulse peak), and one DFE tap can remove this ISI completely.

4.2 Gaussian dispersion and Bessel filter

The raised cosine low-pass filter used in the previous cases could be difficult to design in practice. Therefore, it is of interest to see what happens with an ordinary realizable filter, in this case a 5th order maximally linear phase or Bessel filter, with a 3-dB bandwidth of $B/2$. The exact low-pass filter response is

$$H_{LP}(f) = \frac{1}{\left[1 - 10.47\left(\frac{f}{B}\right)^2 + 8.832\left(\frac{f}{B}\right)^4\right] + j\frac{f}{B}\left[4.855 - 12.70\left(\frac{f}{B}\right)^2 + 2.859\left(\frac{f}{B}\right)^4\right]}.$$

Results for white receiver noise and Gaussian dispersion are shown in Fig. 7.

Compared to Fig. 4, the power penalty for no equalization is higher

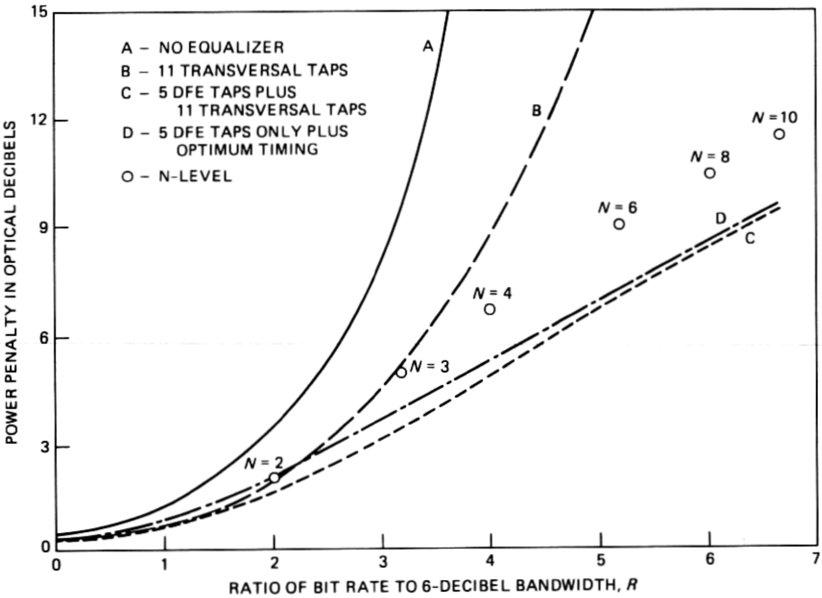

Fig. 7—Power penalties for Gaussian dispersion, Bessel low-pass filter and white noise. The low-pass filter is a 5th order maximally linear phase Bessel filter with a 3-dB bandwidth of $B/2$.

after $R = 2.5$ as expected because the Bessel filter does a poorer job of equalization than the previous low-pass filter. The linear equalization curves are identical because the transversal filter can compensate for a poorer choice of $H_{LP}(f)$. Both decision feedback curves show penalties less than 0.3 dB greater than those in Fig. 4, which would indicate that the choice of low-pass filter is not too critical.

4.3 Cosine-squared dispersion and equalizing filter

Measurements of actual fibers indicate that a cosine-squared pulse shape is often more representative of the actual impulse response than a Gaussian.[11] Power penalties for NRZ pulses transmitted over a fiber with cosine-squared dispersion are shown in Fig. 8. The receiver noise spectrum is white and the low-pass filter produces $\beta = 0.5$ raised cosine equalization for an NRZ pulse with zero dispersion.

Compared to Fig. 4, we see that linear equalization is much less effective for cosine-squared dispersion than for Gaussian dispersion. The reason for this is found by considering the channel's bandpass characteristic. If its impulse response $h(t)$ is given by

$$h_f(t) = \begin{cases} 2/\tau \cos^2 \dfrac{\pi t}{\tau}, \; -\tau/2 \le t \le \tau/2 \\ 0, \text{ elsewhere}, \end{cases}$$

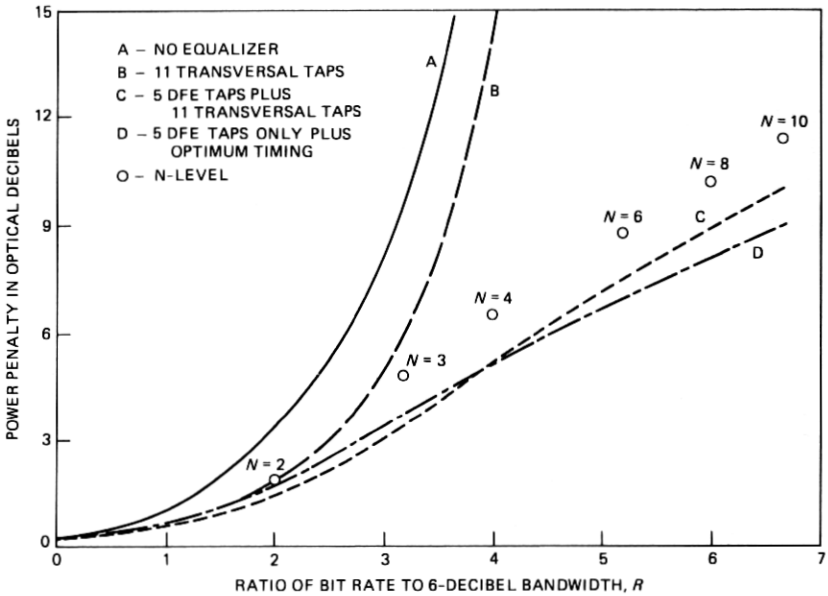

Fig. 8—Power penalties for cosine-squared dispersion, equalizing filter, and white noise.

then the baseband frequency response is

$$H_f(f) = \frac{\sin \pi f \tau}{\pi f \tau}\left[\frac{1}{1 - f^2\tau^2}\right].$$

The channel will have nulls wherever $\sin \pi f \tau = 0$ (except at $f = 0$ and $f = 1/\tau$). As is well known, a linear equalizer would need infinite gain to correct for such nulls, resulting in infinite noise enhancement.

The DFE alone is able to perform equally as well for cosine-squared dispersion as for Gaussian. However, DFE plus linear equalization is somewhat worse than DFE alone above $R = 4.0$. The choice of minimum mse tap weights appears to be particularly bad for cosine-squared dispersion.

4.4 Dependence on timing offset

To increase the effectiveness of decision feedback in eliminating ISI, one should arrange to have as much of the ISI as possible come from pulses which have already been received (postcursors) and for as little as possible to come from future pulses (precursors). The simplest way to do this is to advance the sampling instant such that decisions are made on the leading edge of the pulse rather than on the center of the pulse. Because optical pulses are generally quite limited in duration

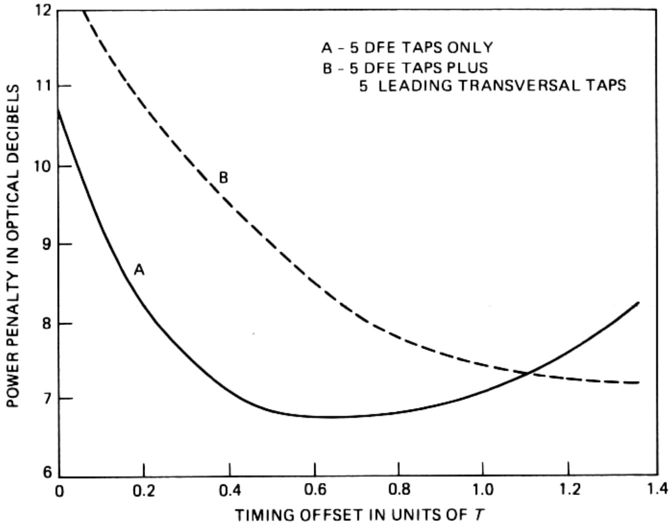

Fig. 9—Power penalties versus sampling time advance for cosine-squared dispersion. Ratio of bit rate to fiber 6-dB bandwidth is $R = 5.0$. Equalizing low-pass filter and white noise are assumed.

and do not have extensive post- and precursors, this simple technique combined with DFE may offer a substantial improvement in performance.

Figs. 9 and 10 show calculated power penalties versus timing offset for cosine-squared and Gaussian dispersion when $R = 5.0$. The receiver noise is white, and an equalizing low-pass filter is assumed. The solid curves are for five DFE taps only, whereas the dashed curves are for five DFE taps plus five leading transversal taps.

For DFE only, advancing the sampling time by 0.6 of a bit interval improves the power penalty by 4.0 and 2.6 dB in these two cases. Improvements are also produced for DFE plus linear equalization, with the optimum timing advance being greater than one bit interval.

To illustrate the effect of offset sampling, consider the example in Fig. 11. The pulses are produced by convolving a rectangle of width T with a Gaussian having a standard deviation of 0.937 T, equivalent to a channel bandwidth of $B/5$. With DFE, only precursor ISI is important. The eye openings for (a) a centrally sampled pulse and (b) a pulse-sampled 0.5 T earlier will be

(a) $$E = r_0 - \sum_{\substack{n \notin s \\ n \neq 0}} |r_n|$$

$$= 1.0 - 0.58 - 0.10 = 0.32$$

(b) $$E = 0.87 - 0.29 - 0.02 = 0.56.$$

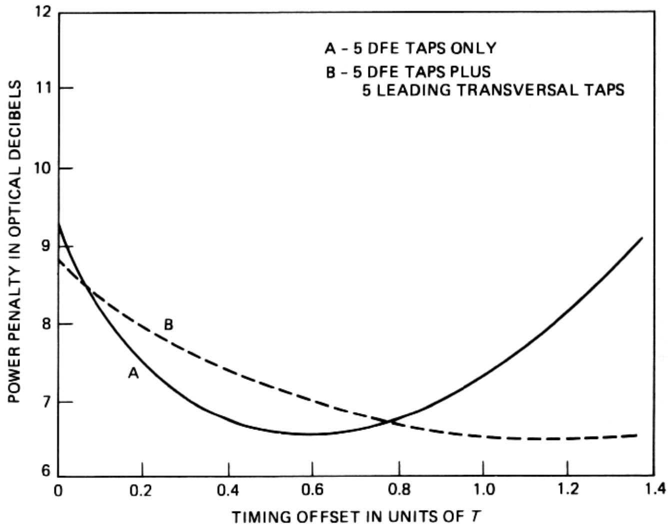

Fig. 10—Power penalty versus sampling time advance for Gaussian dispersion.

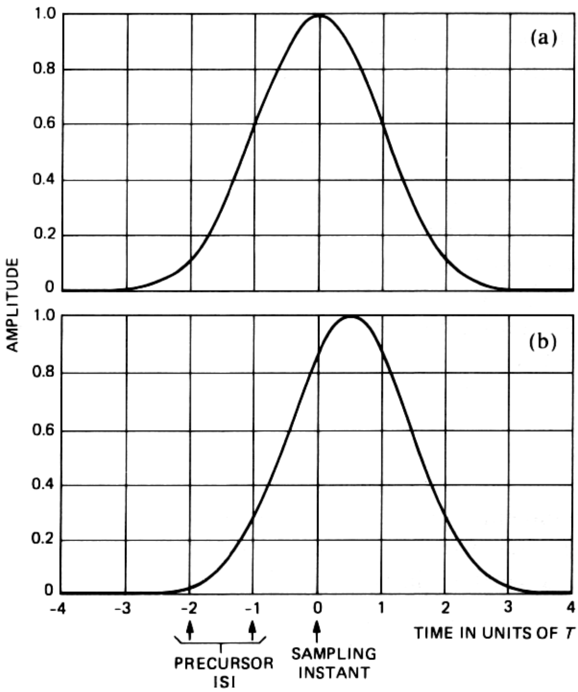

Fig. 11—Pulse waveforms with Gaussian dispersion to illustrate the advantages of sampling prior to the center of the pulse. (a) Central sampling. (b) Sampling advanced by 0.5 T.

The larger eye opening with advanced sampling produces a 2.4 dB-optical power penalty improvement, as σ is the same in both cases.

One disadvantage of advancing the sampling time is that it will increase the likelihood of DFE error propagation. This likelihood is dependent upon the DFE tap weights, and will increase with timing advance because the postcursors become both larger in amplitude and more extended in duration. Therefore, the improvement in power penalty will not be as large in practice as indicated above. However, the gains can be expected to be much greater than the losses, because doubling the size of the eye opening through DFE can reduce the probability of error by perhaps four orders of magnitude, whereas the increase in the error rate because of error propagation cannot be more than 2^M where M is the number of DFE taps.[13]

A most interesting point is that theoretical derivations of the optimum transversal filter for use with decision feedback have always concluded that a one-sided transversal filter is optimal and that no taps following the peak of a matched-filter pulse are necessary.[3,4,14] This implies a timing phase such that sampling coincides with the peak of the pulse. However, the simulation results herein show that advanced sampling leads to better performance. One way to obtain the necessary optimal timing phase is to have a few trailing transversal taps which can delay the pulse relative to the sampling instant. Hence, 11 transversal taps with five leading and five trailing were found to perform much better than five leading taps only.

4.5 Comparison of DFE and multilevel signaling

The use of DFE more closely approximates the use of partial-response[9] or multilevel signaling than the use of traditional linear equalization. Binary signals transmitted at a rate greater than twice the optical-channel, 6-dB bandwidth will always contain ISI when received. The received level, however, is a predictable function of the transmitted message sequence. Decision feedback provides a simple way of decoding the multiple received levels back into the transmitted binary digits.

It is useful to compare the previous DFE power penalties to those which one could expect for multilevel signaling. Consider the case of Gaussian dispersion and white receiver noise. For two-level transmission, one might operate at a symbol rate equal to twice the channel's 6-dB bandwidth (i.e., $R = 2$). Very good linear equalization would be needed to prevent ISI, hence from curve B of Fig. 4 the power penalty for two-level transmission would be 1.8 dB. For N-level transmission, there is an additional power penalty of $10 \log_{10}(N - 1)$ dB, with a factor of $\log_2 N$ increase in R. Total power penalties for multilevel transmission for the various combinations of receiver noise spectra, low-pass filters, and fiber impulse responses considered previously are included in Figs. 4, 5, 7, and 8. In all cases, both DFE plus transversal equalization and DFE only show better performance than multilevel signaling at the same bit rate. For example, by using DFE doubling R from 2 to 4 results in a penalty of about 3 dB in all cases, whereas doubling R by using 4-level signaling results in a penalty of 4.8 dB. For tripling to $R = 6$, DFE is consistently better than 8-level signaling by at least 2 dB.

Hence, DFE may offer a way of obtaining the performance of multilevel optical transmission without many of the latter's disadvantages. For instance, only two-level (on-off) transmitter modulation is needed. Also, complex linear equalization to prevent ISI is unnecessary. The hardware to implement DFE is extremely simple.

V. CONCLUSIONS

The main conclusion is that DFE offers substantial benefits for multimode optical fiber systems. Channels which are presently dispersion-limited could potentially be operated at many times the existing bit rate with a power penalty less than that for multilevel signaling. The hardware required to implement the necessary equalization can be extremely simple as transversal filters are not a necessary component and no special filtering need be included. In addition, the characteristics of an optical channel are very stable, making circuitry for continuous adaptability unnecessary.

A good deal of theoretical work remains. For transversal equalizers, a new technique of selecting tap weights besides the traditional zero-forcing or minimum mse algorithms must be developed. For decision feedback, further investigation of error propagation is needed. Attention should be given to optimizing the sampling time and to its effects on the probability of error propagation. Sensitivity to errors in the sampling time should also be considered.

Experimental confirmation of the simulation results described above is also very desirable, and work along these lines is now proceeding.

APPENDIX

P-i-n FET Receiver Parameters and Noise Corner Frequency

According to Smith and Personick,[7] the noise corner frequency is given by

$$f_{nc} = \frac{1}{2\pi C_T}\left[\frac{d\langle i^2(\omega)\rangle_{\text{eq}}/df}{d\langle e_a^2(\omega)\rangle/df} + \frac{1}{R_{\text{in}}^2}\right]^{1/2}.$$

For a p-i-n FET receiver, we have

$$\frac{d}{df}\langle i^2(\omega)\rangle_{\text{eq}} = \text{equivalent input shunt current noise source} = \frac{4kT}{R_L} + 2qI_{\text{gate}} + 2qI_{\text{dark}}$$

$$\frac{d}{df}\langle e_a^2(\omega)(\rangle = \text{equivalent input series voltage noise source} = \frac{4kT\Gamma}{g_m}.$$

Some typical parameter values for a good p-i-n FET receiver at a bit rate of $B = 100$ Mb/s are

$$R_{\text{in}} = R_L = 1\ \text{M}\Omega$$

$$C_T = 1\ \text{pF}$$

$$g_m = 50\ \text{ms}$$

$$\Gamma = 1.75$$

$$I_{\text{gate}} = 10\ \text{nA}$$

$$I_{\text{dark}} = 10\ \text{nA}.$$

With the above values, one obtains a noise corner frequency of $f_{nc} = 31.7\ \text{MHz} \simeq B/3$.

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