THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 61, No. 10, December 1982
Printed in U.S.A.

Multitone Frequency-Hopped MFSK System for Mobile Radio

By UZI TIMOR

(Manuscript received October 12, 1981)

A digital spread spectrum technique employing frequency hopping and multilevel frequency-shift keying has recently been examined for possible application in mobile radiotelephony. Two system parameters, the number of bits per message and the number of tones in the frequency-hopping sequence, are determined by the available bandwidth and the data rate of each user. These parameters in turn determine the tone duration, which strongly influences the vulnerability of the system to transmission distortions. In this paper we describe a generalization of the MFSK scheme that allows users to transmit more than one tone simultaneously. Multitone transmission makes it possible for designers to increase the duration of each tone by increasing the total number of system frequencies. This flexibility slightly increases maximum efficiency and makes the system less vulnerable to multipath delay spread. It could also have implementation advantages.

I. INTRODUCTION

A multiple access modulation technique that uses multilevel frequency shift keying (MFSK) to modulate frequency-hopped spread spectrum carriers has been examined for possible applications in satellite communication[1] and in digital mobile radiotelephony.[2] In every time frame, each user communicates a K-bit message by frequency-shifting a tone sequence that is unique to the user. System efficiency depends strongly on the length of the sequence (L tones per message) and on the size of the tone alphabet. The total available bandwidth and the transmission rate of each user determine an optimum K, L pair; unless the system design parameters are very near this optimum, efficiency is prohibitively low.

As Refs. 1 and 2 report, K, L, and the user bit rate determine the duration of each tone. An important transmission impairment in mobile radio is the multipath delay spread (the difference in delays between the various simultaneous propagation paths). This delay spread makes it difficult to communicate with short tones and, in fact, the 13-μs tone duration prescribed by the optimum K and L derived in Ref. 2 is uncomfortably close to delay spreads observed in practice.

The purpose of this paper is to propose a modification of the modulation scheme that increases design flexibility. By increasing the size of the tone alphabet and allowing each user to transmit more than one tone simultaneously, a designer can increase the duration of each tone, thereby making the system less vulnerable to multipath delay spread without degrading overall efficiency. In fact, there is a slight improvement (10 to 20 percent) in the maximum number of simultaneous users.

For example, with a total (one-way) bandwidth of 20 MHz and users' rate of 32 kb/s, a conventional system can be designed with $K = 8$ bits per message and $L = 19$ tones per message. The tone duration is 13 μs and up to 209 users can operate simultaneously with a bit-error probability less than 10^{-3}. If the bit rate is 31.96 kb/s, $K = 9$, $L = 11$ is possible with a tone duration of 25 μs and 216 simultaneous users. With two-tone operation and $K = 9$, $L = 11$, the tone duration is 26 μs and 225 users can share the bandwidth. Other possibilities are $K = 10$, $L = 6$ (237 users, 52 μs time slots) and $K = 11$, $L = 3$ (224 users, 115 μs time slots).

II. SYSTEM DESCRIPTION[1,2]

2.1 Principle of operation

The elementary signals are a set of 2^K tones, each of duration τ seconds. Each link (between a mobile user and the base station) is identified by an address that is a sequence of L K-bit words. A new K-bit message is transmitted every $T = L\tau$ seconds as a sequence of L tones determined by the sum (modulo 2^K) of the K-bit message and the K-bit address words.

The received signal is a composite of the tone sequences of M users. Every τ seconds the receiver performs a spectral analysis of the received signal and decides which of the 2^K frequency cells contain energy. Thus, after T seconds the $2^K x L$ frequency-time received energy matrix (A) is generated. The decoded matrix (A_m) of user m is obtained by subtracting (modulo 2^K) the address words from A (see Fig. 1). The message X_m will appear as a complete row in A_m, at row number X_m. Ambiguous decoding occurs when transmissions by other users combine to form other complete rows in A_m.

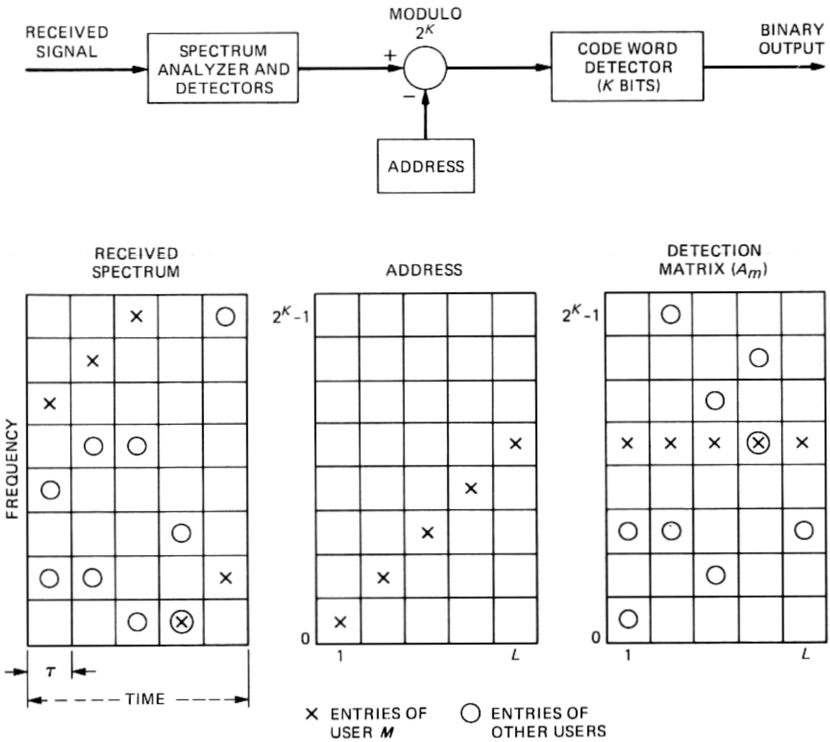

Fig. 1—Receiver block diagram and signal matrices. The matrices show the received spectrum, and the address and decoded matrix of user m, with his entries (X) and those of other users (0).

2.2 System performance

The one-way bandwidth W necessary to support 2^K tones that are mutually orthogonal over $\tau = T/L$ seconds is

$$W = \frac{2^K}{\tau} = \frac{2^K L}{T} \text{ Hz.} \tag{1}$$

The users' data rate is

$$R = \frac{K}{T} \text{ b/s.} \tag{2}$$

For given W, R, and K the length of the sequence L is determined by

$$L = rK2^{-K}, \tag{3}$$

where

$$r \triangleq \frac{W}{R}. \tag{4}$$

Let M be the maximum number of users that can simultaneously share the system at a given error probability. The efficiency η of the system is defined to be the total rate transmitted through the system per unit bandwidth

$$\eta = \frac{MR}{W} = \frac{M}{r}. \tag{5}$$

It has been shown[1] that the bit-error probability owing to mutual interference between M simultaneous users is upperbounded by

$$P_b < 2^{K-2}p^L, \tag{6}$$

where

$$p = 1 - (1 - 2^{-K})^{M-1}. \tag{7}$$

Thus, even without channel impairments the efficiency of the system (or equivalently the number of users that the system can accommodate at a given error probability) is interference limited.

Noisy reception affects the decoded matrix by causing additional entries owing to false alarms and missing entries owing to deletions. Thus, a majority decoder has to be used, i.e., the row with the maximum number of entries is decoded as the message. The performance of the system with Gaussian noise and multipath fading was analyzed[2] and shown to degrade gracefully with increasing noise.

2.3 System design

In designing a spread spectrum system the total available bandwidth and the users' rate (or equivalently their ratio r) constrain the choice of the number of frequencies (2^K) and the sequence length (L) via relation (3). For example, if $r = 626$ the following pairs of (K, L) are possible: (7, 34), (8, 19), (9, 11), (10, 6), and (11, 3).

Substituting (3) into the expression for the bit-error probability [(6) for the noiseless case and a similar expression for the noisy case] yields the optimum pair, i.e., the one that maximizes the number of users that can operate at a given error probability. In the above example, the optimum pair for bit-error probability of 10^{-3} (noiseless case) is $K = 9$ (512 frequencies) and $L = 11$, allowing 216 users.

When a system is being designed, there might be other considerations that would favor changing these parameters. For example, there is a chip synchronization error owing to multipath, and the performance depends on the relative synchronization error (with respect to the chip duration). Decreasing the number of chips (i.e., increasing chip duration) reduces this relative error. Thus, flexibility in the choice of system parameters is desirable. However, a significant deviation from the optimum parameters sharply reduces the efficiency. In the above example, with $K = 10$ ($L = 6$) the number of users is reduced from 216 to 138.

We will show that by allowing each user to transmit two or more frequencies per time slot we obtain this flexibility and thereby improve system performance. We will consider two cases:

(*i*) The frequency band is divided into several subbands and each user transmits one frequency per subband per chip.

(*ii*) Each user transmits several frequencies per chip without any segmentation.

III. MULTITONE SYSTEM WITH SUBDIVIDED FREQUENCY BAND

Consider a conventional scheme with 2^K frequencies and L chips, where $L = 2L_1$ is even. Let us divide the total frequency band into two, each half containing 2^K frequency slots (the number of frequencies is doubled). Suppose each user transmits a sequence of length L by simultaneously transmitting the first L_1 chips on the lower band, and the remaining L_1 chips on the upper band. In the receiver, the two subbands are detected and combined to yield the original $2^K xL$ received matrix (Fig. 2).

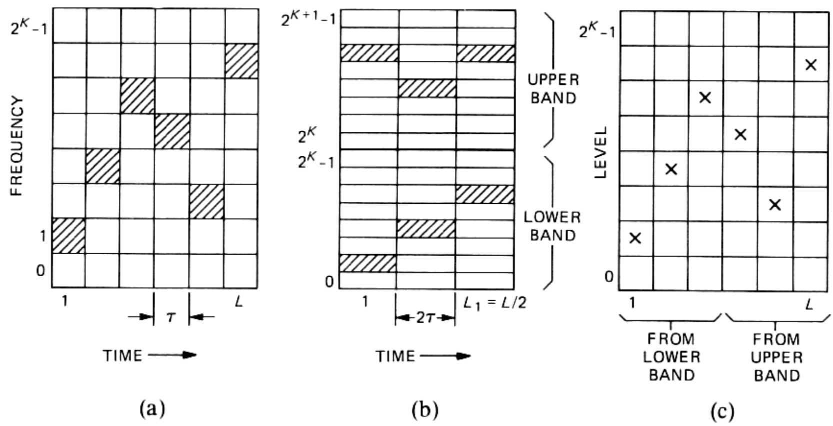

Fig. 2—Transmission of a sequence of L (out of possible 2^K) tones using $L/2$ time slots and 2^{K+1} frequency slots. (a) Generated sequence. (b) Transmitted matrix. (c) Received matrix.

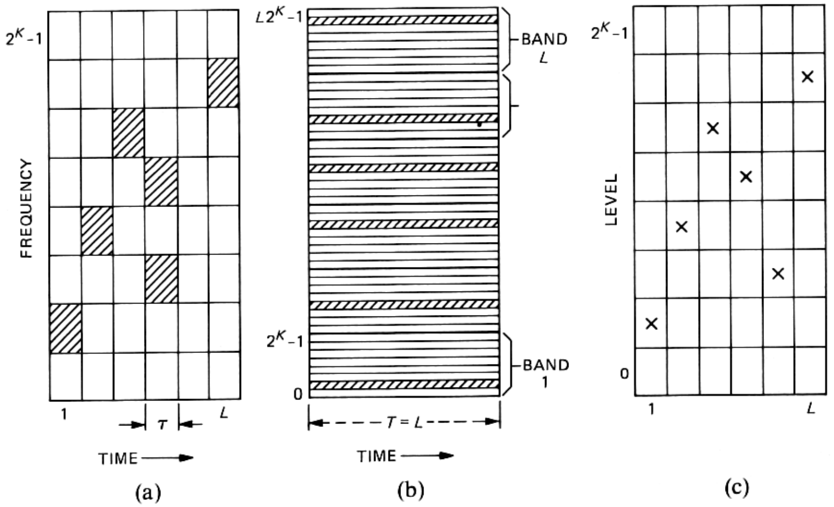

Fig. 3—Simultaneous transmission of a sequence of L (out of possible 2^K) tones using $L2^K$ frequency slots. (a) Generated sequence. (b) Transmitted matrix. (c) Received matrix.

The result is a system with twice the number of frequencies, half the number of chips (the duration of each chip is doubled), and two frequencies per chip.

Similarly, if $L = jL_1$ we can divide the frequency band into j subbands and let each user transmit one frequency per subband. The total number of frequency slots is $j2^K$ and the number of chips is L/j.

In the extreme case we will have one chip (of duration T) and $L2^K$ frequencies divided into L subbands. Each user transmits L frequencies simultaneously, one in each subband. The receiver performs one spectral analysis to determine which of the $L2^K$ frequency cells contain energy. From this, the original $2^K xL$ received matrix is generated (see Fig. 3).

Under the same assumptions, the analysis[1,2] of the conventional scheme for the noiseless and for the noisy and multipath fading channel is valid for the multitone scheme, resulting in a performance that is identical in both schemes.

IV. MULTITONE SYSTEM WITH NONSEGMENTED FREQUENCY BAND

Let each user generate a sequence of length nL, using an address of nL K-bit words. The user then transmits the sequence as follows: At the first time slot, terms $1, L + 1, \cdots, (n - 1)L + 1$ are sent; at the second time slot, terms $2, L + 2, \cdots, (n - 1)L + 2$ are sent, and so on.

Thus, each user simultaneously transmits n frequencies at each time slot for a total of nL frequencies (see Fig. 4).

At the receiver, a spectral analysis is performed every τ seconds to generate the $2^K xL$ received matrix. The matrix is then expanded by repeating it n times, i.e., the first column is also entered as the $L + 1$, $2L + 1, \cdots, (n - 1)L + 1$ columns, and so on, resulting in a $2^K xnL$ matrix (Fig. 4). The expanded matrix is then decoded using the address of length nL.

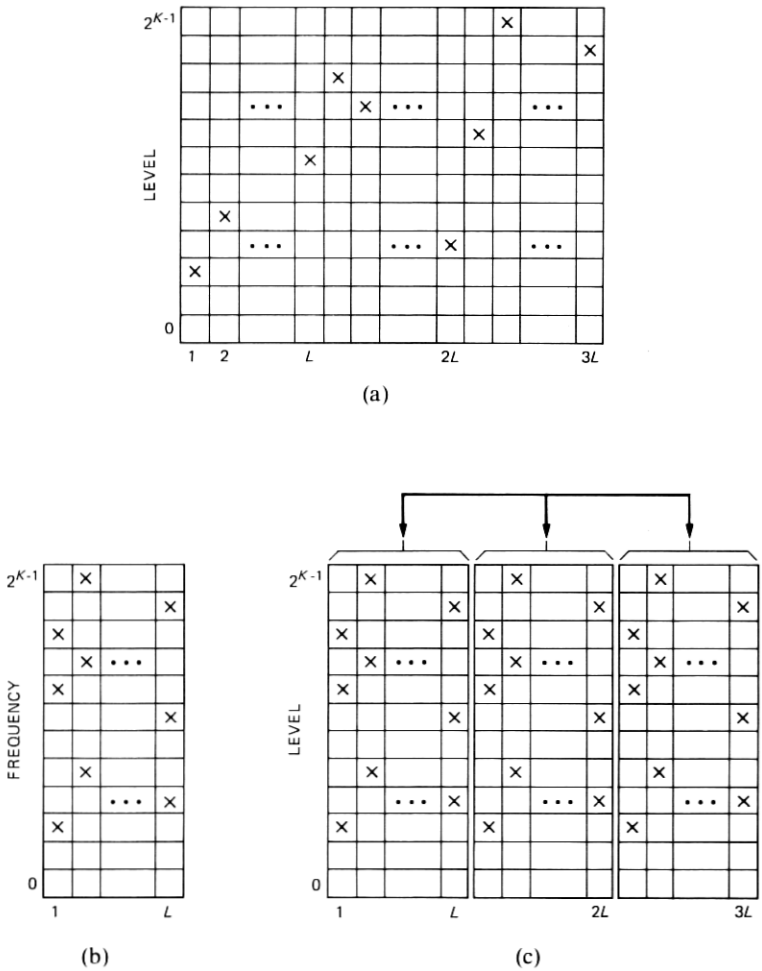

Fig. 4—Transmission of a sequence of length $3L$ over L time slots (chips). Three frequencies are transmitted on each chip. (a) Generated sequence. (b) Transmitted matrix. (c) Received matrix (expanded).

In analyzing the performance of the noiseless case we can follow the same approach as in the conventional scheme. Let M be the number of users and consider the decoded matrix of user m. The probability of having an entry at a particular frequency time slot because of interference from the other $M - 1$ users is

$$p_n = 1 - \left(1 - \frac{n}{2^K}\right)^{M-1}. \tag{8}$$

In addition, at each of the nL columns, we will have, with probability 1, self-interference entries at $(n - 1)$ frequency slots. Let us assume that the self-interference entries in different columns will appear at different rows. [If $(n - 1)nL \ll 2^K - 1$, this can be achieved with a proper address assignment.] Thus, we will have $(n - 1)nL$ rows containing one self-interference entry.

The probability of having a complete row (interference row) at one of those $(n - 1)nL$ rows is p_n^{nL-1}, while for the remaining rows it is p_n^{nL}. Using the union bound, the word-error probability can be upper-bounded by

$$P_W < (n - 1)nLp_n^{nL-1} + [2^K - 1 - (n - 1)nL]p_n^{nL}$$
$$P_W < (2^K - 1)p_n^{nL}(1 + S_n), \tag{9}$$

and the bit-error probability by[1]

$$P_b < 2^{K-2}p_n^{nL}(1 + S_n), \tag{10}$$

where

$$S_n = \frac{(n - 1)nL(1 - p_n)}{(2^K - 1)p_n}. \tag{11}$$

We can combine the two schemes described in Sections III and IV by dividing the total frequency band into j subbands and transmitting n frequencies per subband per chip. In the extreme case we will have a single chip (of duration T) and $L2^K$ frequencies divided into L subbands. Each user transmits n frequencies per subband for a total of nL frequencies transmitted simultaneously, with the performance given by (10).

V. SYSTEM PERFORMANCE

For a given bandwidth to user's rate ratio r, several (K, L) combi-

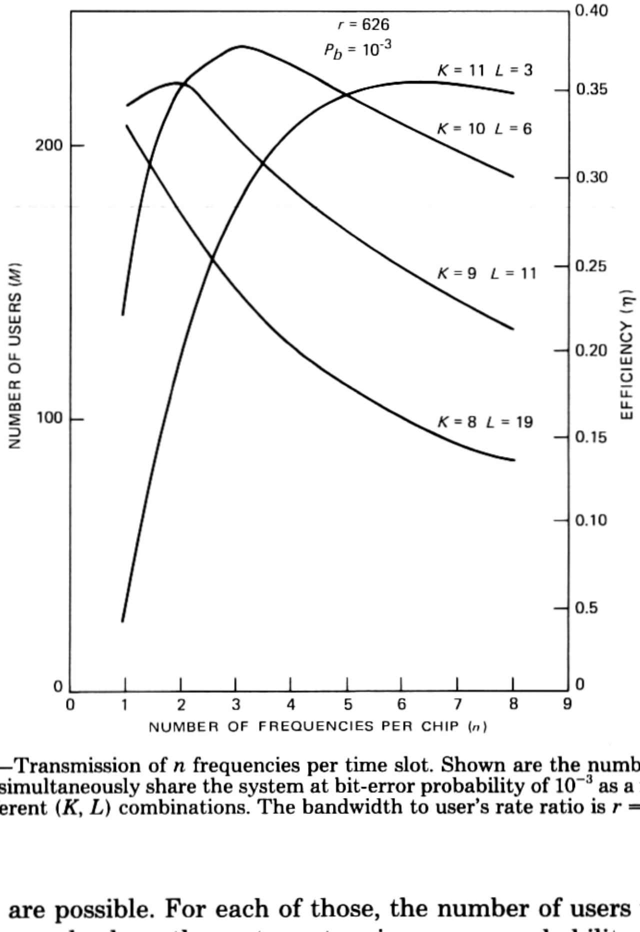

Fig. 5—Transmission of n frequencies per time slot. Shown are the number of users that can simultaneously share the system at bit-error probability of 10^{-3} as a function of n for different (K, L) combinations. The bandwidth to user's rate ratio is $r = 626$.

nations are possible. For each of those, the number of users that can simultaneously share the system at a given error probability depends on the number of frequencies n transmitted per chip.

Figure 5 depicts the number of users as a function of n for $r = 626$, bit-error probability of 10^{-3}, and several (K, L) combinations. Although for $n = 1$ only $K = 8$ or $K = 9$ are reasonable choices, we can now have good performance with $K = 8$, 9, 10, or 11. The optimum is at $K = 10$. The maximum number of users is increased by 10 percent from 216 ($K = 9$, $L = 11$, $n = 1$) to 237 ($K = 10$, $L = 6$, $n = 3$).

The bit-error probability as a function of the number of users for $r = 626$ and several (K, L, n) combinations [$n = 1$ and the optimum n for each (K, L)] is shown in Fig. 6.

Figure 7 depicts the efficiencies of a system with n frequencies per time slot (upper curve) and a conventional system ($n = 1$, lower curve) as a function of r. The best (K, L, n) and (K, L), respectively, were chosen for each r. Also shown are the system parameters K, L, n, and

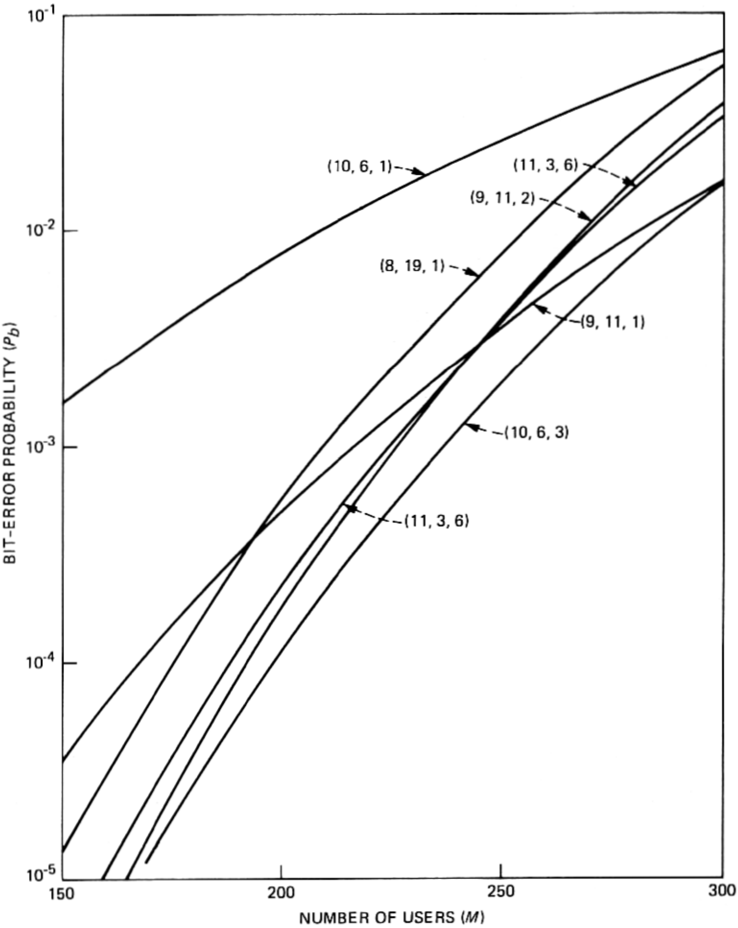

Fig. 6—Transmission of n frequencies per time slot. The bit-error probability as a function of the number of users for various (K, L, n) combinations is shown. The bandwidth to user's rate ratio is $r = 626$.

nL. We can see that for most r there is a 10- to 20-percent improvement in efficiency over the conventional system.

VI. CONCLUSIONS

A modified frequency-hopped multilevel FSK system for mobile radiotelephony that enables users to transmit multitone (instead of single tone) sequences has been presented. This scheme adds more flexibility to the system design by allowing a trade-off between the number of system frequencies and the number of time slots without reducing the efficiency of the system. This flexibility can be helpful in

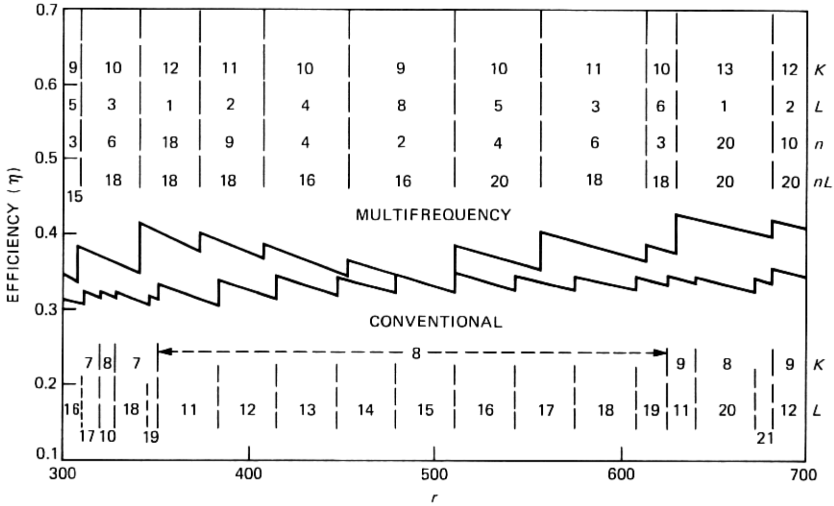

Fig. 7—The efficiencies of a spread spectrum system with n frequencies per time slot, and conventional spread spectrum system ($n = 1$) as a function of r for bit-error probability of 10^{-3}. The best (K, L, n) combination was taken at each r.

making the system less vulnerable to multipath delay spread. It also provides implementation advantages by admitting new combinations of hardware configuration and speed.

REFERENCES

1. A. J. Viterbi, "A Processing Satellite Transponder for Multiple Access by Low-Rate Mobile Users," Digital Satellite Commun. Conf., Montreal, October 23–25, 1978.
2. D. J. Goodman, P. S. Henry, and V. K. Prabhu, "Frequency-Hopped Multilevel FSK for Mobile Radio," B.S.T.J., *59*, No. 7 (September 1980), pp. 1257–75.