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Direct Distance Dialing: Call Completion and Customer Retrial Behavior

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Most customers placing a direct-distance-dialing (DDD) call in the United States are able to complete the call on the first attempt. However, it is reasonable to expect that the probability of an initial completion will be less than 1. When an initial attempt fails to complete, a customer may decide to abandon his desired telephone connection or to make one or more retrials. In general, a sequence of one or more attempts may be initiated by a customer in an effort to establish the desired connection. A study of DDD call completion and retrials is important to provide an overall characterization of network performance and customer behavior in setting up customers' desired telephone connections. A survey adopting a two-stage stratified sampling plan was undertaken to obtain DDD retrial statistics. Data associated with sampled DDD calls that were originated from one of 890 switching entities in the Bell System network were collected for a period of one week. The basic DDD retrial results reported here are initial attempt disposition probabilities, retrial probabilities, number of additional attempts, ultimate success probabilities, and distribution functions for retrial intervals following different types of incomplete initial attempts. Results of subclass analyses of retrial statistics by originating and terminating classes of service (residence and business) are also presented. Results obtained in this study are useful in many network planning applications. An application of significant importance is provision of a tool to evaluate the revenue and cost impact of call completion improvement programs. A technique to analyze the revenue and cost impact is outlined in the paper.

I. INTRODUCTION

Each day Bell System customers initiate about 780 million telephone call attempts to be carried from one part of the telephone network to the other. A very sophisticated network has been engineered to carry both local and toll calls. Though only 7 percent of those calls are toll calls, the number of toll calls is still very large indeed. For instance, on an average business day in 1977 about 52 million toll call attempts were placed by customers. Of those attempts, 36 million toll attempts (roughly 70 percent) led to successful telephone connections, with the other 16 million attempts resulting in failures because of the called customer being busy or not in, circuit blockage, and so on. A customer does not always succeed in establishing a desired connection on the initial attempt. Some customers will abandon their effort to establish desired connections after encountering incomplete attempts, others will make additional attempts. Therefore, a desired connection may lead to a sequence of more than one attempt, ultimately resulting in a connection or abandonment by the calling customer. Whether each attempt will result in a successful completion depends not only on network-dependent variables such as engineered blocking level, but also on customer-controlled variables such as telephone usage and on interactions between the two groups of variables.

The main purpose of this study is to obtain results needed to characterize both network performance and customer behavior not only on initial attempts but also on possible subsequent attempts in setting up a desired telephone connection. Retrial results are also useful in many practical traffic engineering and network management applications. As noted previously, roughly 70 percent of DDD call attempts are completed. An important question is how customers react after encountering the other 30 percent incomplete call attempts. Equally important is the question: How often do customers eventually establish their desired connections? It is shown in this paper that answers to those and other questions can provide a useful tool in evaluating the revenue and cost impact of efforts to improve network call completions.

Each individual call attempt may be characterized by a complex sequence of events involving actions and reactions by both customers and the network. A detailed characterization of network performance and customer behavior during the course of setting up a DDD call attempt was given recently by Duffy and Mercer.[1] In that study, no distinction was made on the attempt level, i.e., the order in a sequence of attempts associated with the same desired connection. The results therefore represent averages over all attempt levels. Also in that study, very limited information was given about customer reattempts following an unsuccessful attempt. Due to a restriction inherent in their data sources, only information of up to three fast retrials occurring within 60 seconds of a previous observed incomplete attempt was obtained. Thus, it was not possible to determine if any attempt had preceded the first observed attempt or might follow the last observed attempt.

The only substantial amount of published retrial data in the Bell System is what Wilkinson and Radnik[2] obtained in 1966 in a special study of Automatic Message Accounting (AMA) tapes associated with toll calls originated at Norfolk, Virginia, and Levitttown, Pennsylvania. In that study, data were obtained on whether and how many retrials would follow an initial unsuccessful attempt. However, the reason for failure of DDD attempts was not identified, so the description of the retrial phenomena was incomplete. Without identifying the cause of failure leading to subsequent retrials, it could not be determined under what circumstances those retrial data might be used properly.

It is clear that a sufficient description will be obtained if one monitors complete sequences of attempts from initial attempts to their ultimate successes or abandonments. Two significant factors should be considered in this type of approach. One is the amount of effort needed in the design of proper instrumentation that can monitor many customer lines simultaneously and gather detailed call set-up information of every call originated from the lines being monitored. The other factor is that, even with a proper instrumentation, a lengthy survey would be required to obtain a scientific sample of data that would characterize the DDD network. These two factors represented major stumbling blocks; therefore, an alternative approach was sought.

To carry out the retrial study, a new retrial model is introduced in which it is only necessary to determine the following: the disposition of one attempt in a sequence of attempts, which number it is in the sequence, how many attempts are made in the sequence, and the ultimate success or failure of the attempt sequence. The parameters of the model can be evaluated by combining service-observing data with AMA billing records from the same originating entities. In fact, for a DDD attempt observed through the normal course of Dial Line Service Observing (DLSO), we can classify the disposition of an attempt. From the called number recorded on the service observing card, the originating NPA/NNX, and the time the call was made, each observed DDD DLSO attempt can be matched with a corresponding AMA record. When a match is obtained, a search on appropriate AMA tapes will identify all previous or subsequent attempts. Once complete sequences of attempts are determined in this way, the attempt level—that is, initial attempt, first retrial, etc.—for each observed DLSO attempt in the corresponding sequence of attempts can be determined.

The outline of this paper is as follows. Data collection and processing procedure are explained in Section II. Section III presents the basic initial attempt retrial results. Dependence of the retrial characteristics upon both the originating and terminating classes of service is examined in Section IV. A class of service is used to designate a type of telephone service provided to a group of customers. In this paper, two broad classes of service are considered, residence and business. Potential network revenue gains resulting from reducing ineffective attempts are discussed in Section 5.1. Network revenue and cost impact of an increase in the network completion ratio are analyzed in Section 5.2. A special example of the impact of call waiting service is discussed in Section 5.3. The results presented in the paper are summarized in Section VI.

II. DATA COLLECTION AND DATA PROCESSING

At the outset of the sampling plan, it was recognized that existing DLSO bureaus should be considered as primary units and not be divided further into smaller units to minimize possible administrative difficulties. After comparing various possible sampling plans, it was discovered that a primary stratification based upon the number of average annual outgoing toll messages (AOTM) per switching entity in a DLSO bureau would provide a substantial gain in precision for the network call completion ratio (number of DDD messages/number of DDD attempts). A two-stage random sampling plan with stratification was therefore adopted. Under this sampling plan, all DLSO bureaus were divided into two strata. Within each stratum, the primary units, the DLSO bureaus, were selected with replacement and with probabilities proportional to the sizes of the bureaus, as measured by the total number of AOTM from the observed switching entities in each DLSO bureau. In the first stage sample, five DLSO bureaus were selected from the first stratum and thirty bureaus from the second stratum. A smaller number of bureaus was selected in the first stratum because of a smaller variance for the network call completion ratio among the bureaus within that stratum by design of the stratification. A total of 35 DLSO bureaus in 17 Bell System operating companies were selected. The second stage of sampling consisted of the selection of the actual observations of DDD call attempts from each switching entity. Each observed DLSO attempt was selected with equal probability. However, the traffic carried by one switching entity generally differed from that by others, therefore each DLSO observation from a switching entity had to be assigned a traffic weight proportional to the annual outgoing toll traffic from that entity for use in a proper estimation procedure. Estimates given in this paper are accompanied by 90-percent confidence intervals if the sample size is sufficiently large that the distribution of the ratio estimates used may be assumed to be approximately normal. Confidence intervals are not given when the sample size is too small for the normality assumption.

For the 890 switching entities in the 35 DLSO bureaus, the DLSO data, the corresponding AMA records, and information about the class of service for the calling and called customers were collected for a one-week period in 1976.

As explained in Section I, the collected DLSO data and AMA data were to be combined to match each DLSO observation with an AMA record. A total of 12,658 DLSO observations were successfully matched. In this paper, the time period for attempts to be considered as part of the same sequence are restricted to the same calendar day of the DLSO observation. (Effects due to an extension of the retrial period to include the next calendar day have been found to be small.)

III. BASIC RETRIAL RESULTS

Out of 12,658 matched observations, 10,672 observations were found to be initial attempts. The results presented in this paper are based upon those attempts and the subsequent reattempts, if any.

Each of those 10,672 matched first attempts defines a desired connection. For each desired connection, a sequence of attempts may be initiated by the calling customer. The number of attempts, completion probability, and retrial probability are listed in Table I for each attempt level in those 10,672 sequences of attempts. It is seen that the completion probability decreases steadily, while the retrial probability increases steadily as the attempt level increases. The steady decrease of the completion probability is a filtering effect. That is, as one observes higher attempt levels, the sequences still "active" are those directed to customers difficult to reach (or completion would have occurred earlier) by customers who are more determined to get through (or they would have abandoned earlier), so completion probability decreases and retrial probability increases.

The probability of ultimate success in completing a desired connection is found to be 0.885 ± 0.009. This means 88.5 percent of the desired connections are eventually established. Of the 88.5 percent successful sequences, 75.5 percent of them succeed on the initial attempt, while

Table I—Completion and retrial probabilities by attempt levels

Attempt Level	Number of Attempts	Completion Probability	Retrial Probability
1	10672	0.755 ± 0.015	0.665 ± 0.042
2	1749*	0.510 ± 0.051	0.743 ± 0.067
3	636*	0.415 ± 0.061	0.793 ± 0.081
4	295*	0.377 ± 0.052	0.871 ± 0.069
≥5	386*	0.119	0.957

Average retrial probability = 0.719 ± 0.020
Average attempts/initial attempt = 1.29 ± 0.04
Ultimate success probability = 0.885 ± 0.009

* The number of attempts shown here is an unweighted count of the number of remaining active sequences in the original 10,672 attempt sequences. To derive estimates such as the ratio of numbers of attempts on subsequent attempt levels, each attempt must be weighted by an appropriate traffic weight, as explained in the text.

the other 13 percent succeed after an average of 1.79 additional attempts. The call attempt time (excluding dialing time) is 18.2 seconds for a completed attempt, and 32.1 seconds for an incomplete attempt. The average call attempt time per successful sequence is (18.2 + 0.13/0.885 × 1.79 × 32.1) = 26.6 seconds. The other 11.5 percent of the sequences are abandoned after an average of an additional 0.52 attempts. The average call attempt time per incomplete sequence is 1.52 × 32.1 = 48.8 seconds. Those incomplete sequences contribute a substantial share of network load, or (0.115 × 48.8)/(0.115 × 48.8 + 0.885 × 26.6) = 19.3 percent of all nonconversation time in the DDD network. They also represent a very significant loss of potential revenue. (The revenue aspect is discussed further in Section V.) Overall, there are 1.29 call attempts per initial attempt, and 1.45 call attempts per message.

So far, the retrial results mentioned do not include the cause of failure. Information given on DLSO observations allows a detailed classification of the call disposition of each observation. For the present analysis, call dispositions are classified by the following five categories: complete (COMP), did not answer (DA), busy (BY), equipment blockage and failure (EB&F), and everything else (Other). The EB&F category encompasses all ineffective attempts due to network-caused problems such as no circuit/reorder, no ring, and miscellaneous equipment irregularities. The Other category is a mixture of several different categories of calls classified as one of the following dispositions: did not wait, correctly intercepted, no such number, customer-dialed wrong number, and no response due to customer omitting an access code.

Let q stand for one of the five dispositions listed above. The basic quantities determined from the study upon which all subsequent applications are based will be represented by the following:

p_{1q}, the probability that the initial attempt disposition is q.

r_q, the probability of a reattempt after an initial attempt with disposition q.

L_q, the average number of additional attempts after an initial attempt with disposition q including sequences with no additional attempts.

S_q, the probability that a connection is ultimately established given that the initial attempt disposition is q.

The retrial quantities along with overall disposition probabilities p_q are summarized in Table II.

The following comments about the results are in order:

(*i*) The initial completion probability of 0.755 is substantially higher than the overall completion probability of 0.69. For the incomplete attempts, the change in disposition probabilities

Table II—Basic retrial results

Initial Attempt Disposition q	Complete	Did Not Answer	Busy	EB&F	Other
p_{1q} (Initial attempt disposition probability)	0.755 ± 0.014	0.129 ± 0.10	0.059 ± 0.007	0.019 ± 0.004	0.037 ± 0.008
r_q (Retrial probability)	—	0.61 ± 0.06	0.72 ± 0.06	0.86 ± 0.07	0.66 ± 0.06
L_q (Number of additional attempts)	0	1.02 ± 0.08	1.62 ± 0.30	1.54 ± 0.42	0.95 ± 0.12
S_q (Ultimate success probability)	1.0	0.44 ± 0.04	0.66 ± 0.07	0.66 ± 0.11	0.54 ±0.10
p_q (Overall disposition probability)	0.69 ± 0.011	0.160 ± 0.011	0.094 ± 0.008	0.022 ± 0.003	0.034 ± 0.007

goes in the opposite direction. For example, 19 percent of initial attempts result in BY or DA, while 25.4 percent of all attempts end up in these two dispositions.

(*ii*) In terms of retrial parameters, r_q, L_q, and S_q, customer retrial behavior is quite different after encountering different initial dispositions. For example, the retrial probability ranges from 0.61 for calls encountering DA conditions to 0.86 for calls blocked due to network-caused problems.

(*iii*) A surprisingly large 39 percent of the customers did not make any reattempts at all after encountering initial DAs. As a result, only 44 percent of sequences initially encountering a DA were eventually completed. Since 12.9 percent of the initial attempts are DAs, this represents a 7.2-percent loss of all desired connections.

(*iv*) Customers tend to retry more often when encountering network-related problems, though the chance of an ultimate success is no better than that after BY.

A very important retrial characteristic is how fast a customer makes a reattempt. Retrial rate following EB&F is a critical factor influencing how fast a network congestion may build up. An immediate retrial following a BY condition is likely to lead to another BY. In the remainder of this section, retrial time distribution is described in terms of interarrival time between an initial attempt with a given incomplete disposition q and the subsequent reattempt.

The mean and the median retrial times following initial BY are 18.2 and 3.7 minutes, respectively; following DA, 67.3 and 36.2 minutes, following EB&F, 23.9 and 1.1 minutes. It is evident that in all three cases the median retrial time is much smaller than the corresponding mean retrial time. In terms of median retrial time, one can also see three different time scales for retrials. For instance, the reattempt time following an initial EB&F is relatively short (1.1 minutes), that following an initial BY is somewhat longer (3.7 minutes), and that following an initial DA is much longer (36.2 minutes).

If the interarrival time between an initial incomplete attempt and the second attempt of the same desired sequence behaves like an exponential distribution, then the ratio of the median retrial time to the mean retrial time can easily be shown to be ln 2 (=0.69). From the values listed above, none of the three cases satisfies this criterion. In fact, the three retrial time distribution functions following initial BY, DA, and EB&F shown in Fig. 1 have been fitted to three functional forms: (*i*) single exponential: $(1-e^{-\lambda t})$, (*ii*) two exponentials: $p(1-e^{-\lambda t})+(1-p)(1-e^{-\mu t})$, and (*iii*) log-normal:

$$\frac{1}{\sqrt{2\pi}\,\sigma}\int_0^t \frac{1}{x}\exp\left\{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right\}dx.$$

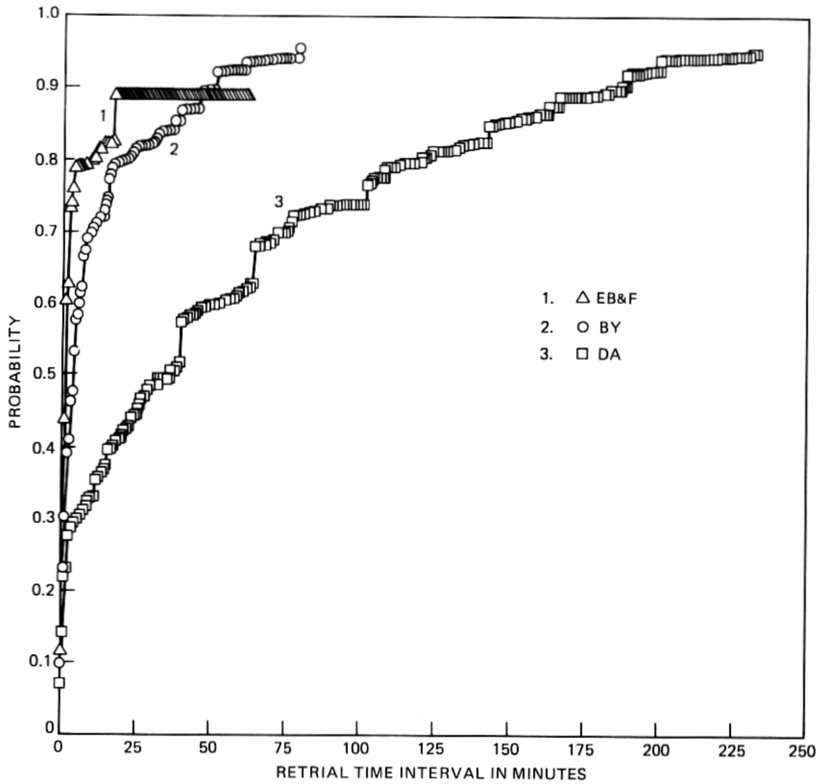

Fig. 1—Retrial time distribution functions.

The results of least-square fits are summarized in Table III. It can be seen that the two-exponential form gives a better fit in all three cases. In the case of BY, the log-normal form also gives a reasonably small mean-square deviation, considering the fact that it has only two parameters instead of three for the two-exponential form.

IV. INFLUENCE OF CLASS OF SERVICE UPON RETRIAL CHARACTERISTICS

The process of setting up a DDD call involves a complex interaction among a calling customer, a called customer, and the telephone network. The network, as complicated as it is, is outstanding in terms of its performance to successfully carry call attempts. On the average, only 2.2 percent of all DDD attempts fail to complete due to network-caused problems. For the most part, whether a desired connection will be completed or not is determined by the characteristics of the calling and called customers. The usage and capacity of a called customer's telephone equipment and the degree of readiness to answer an incoming call determine whether the incoming call will be properly answered, remain unanswered, or result in a busy condition. From the

Table III—Results of least-square fits of retrial distribution functions following initial BY, DA, and EB&F

Initial Attempt Disposition	Function Fitted	Mean Square Deviation
BY	Single exponential	1.0×10^{-2}
	Log-normal	2.6×10^{-4}
	Two exponentials	1.5×10^{-4}
DA	Single exponential	5.5×10^{-3}
	Log-normal	1.9×10^{-3}
	Two exponentials	2.4×10^{-5}
EB&F	Single exponential	1.5×10^{-2}
	Log-normal	1.2×10^{-3}
	Two exponentials	4.9×10^{-4}

viewpoint of the calling customer, his or her selection of time to place the calls and his or her behavior during the call setup and any subsequent reattempts certainly influence the outcome of these calls.

Customers in two important classes of service—residence and business—tend to show differences in the above characteristics. These customer characteristics may very well lead to substantial differences in the basic retrial characteristics. In particular, one may reasonably expect that a customer's class of service will influence retrial characteristics associated with customer-related failures such as BYs and DAs that are beyond the control of the network. In this section, the retrial parameters associated with calls originated from residence or business customers to residence or business customers will be examined.

Estimates of the percentages of initial attempt dispositions of DDD calls from residence to residence (R → R), residence to business (R → B), business to residence (B → R), and business to business (B → B) are summarized in Table IV. It can be seen from this table that within the confidence intervals obtained in this study, the percentages of initial attempts encountering BY, EB&F, and "Other" are quite similar in all four combinations of classes of service. The percentage of calls resulting in initial completions or initial DAs varies drastically from case to case. The initial completion percentages are 66.1 (R → R), 80.2 (R → B), 62.7 (B → R), and 84.6 (B → B). The corresponding initial DA percentages are 23.1, 6.4, 26.3, and 4.3. Clearly, the initial completion percentage is higher for calls terminating at business lines than those at residence lines. The main difference is due to a higher percentage of DAs for calls directed to residence customers. The sums of initial completion and DA percentages are 89.2, 86.6, 89, and 88.9. In all four combinations of classes of service, the initial DA rate and the initial completion rate practically complement each other.

Also listed in Table IV are the ultimate success probability, the number of attempts per initial attempt, and the overall completion ratio for calls between originating residence or business class of service and terminating residence or business class of service. Calls terminating at residence lines, whether they are originated by residence or

Table IV—Initial attempt dispositions by originating and terminating classes of service

Initial Attempt Disposition	Percentage			
	From Residence		From Business	
	To Residence	To Business	To Residence	To Business
Complete	66.1 ± 3.3	80.2 ± 3.5	62.7 ± 7.0	84.6 ± 3.0
Busy	5.6 ± 1.4	7.6 ± 2.2	6.0 ± 2.3	5.7 ± 1.6
Did not answer	23.1 ± 2.9	6.4 ± 1.8	26.3 ± 6.9	4.3 ± 1.3
EB&F	2.2	1.4	1.6	2.5
Other	3.0	4.4	3.4	3.0
S	0.84 ± 0.02	0.92 ± 0.02	0.79 ± 0.06	0.94 ± 0.02
L	1.46 ± 0.14	1.29 ± 0.10	1.39 ± 0.19	1.18 ± 0.05
C	56.3 ± 3.1	73.2 ± 4.4	55.9 ± 4.2	80.0 ± 3.0

S = Ultimate success probability.
L = Number of attempts per initial attempt.
C = Overall completion ratio (number of messages/number of attempts).

business customers, have poor initial and overall completion ratios. Almost 44 percent of all calls terminating at residence lines are incomplete attempts. The percentage of desired connections that are eventually established is 84 for R → R calls, and 79 for B → R calls. This means that 16 percent of desired connections from residence to residence and 21 percent of desired connections from business to residence are never completed.

In contrast, calls terminating at business lines have higher initial and overall completion ratios and better ultimate success probabilities. Though these parameters differ somewhat for residence-originated calls and business-originated calls, the differences are much smaller compared with the similar parameters for calls terminating at residence lines. Overall, calls from business to business have the best chance of completing, while those from business to residence have the worst.

Details of retrial characteristics for the four combinations of classes of service are given in Table V. The initial retrial probability (r_q), the number of additional attempts (L_q), and the ultimate success probability (S_q) have quite different values, depending upon the initial attempt disposition. However, for a given initial disposition, there are no drastic differences in most of those retrial characteristics among calls placed between different combinations of the originating and terminating classes of service. When a business customer encounters an initial DA, the probability of making one or more reattempts is 0.72 if he is calling another business customer. However if he is calling a residence customer, he will retry only with a probability of 0.46. Another interesting point is that residence customers tend to make a slightly higher number of reattempts for calls intended to residence customers when the initial attempt results in DA.

The relative importance of BYs and DAs in determining whether calls

Table V—Retrial characteristics by originating and terminating classes of service

Retrial Charac-teristic	From Residence: To Residence	From Residence: To Business	From Business: To Residence	From Business: To Business
r_{BY}	0.71 ± 0.11	0.74 ± 0.09	0.75 ± 0.02	0.74 ± 0.14
L_{BY}	1.50 ± 0.39	2.14 ± 0.89	2.18 ± 1.25	1.63 ± 0.49
S_{BY}	0.65 ± 0.10	0.67 ± 0.09	0.65 ± 0.16	0.70 ± 0.14
r_{DA}	0.58 ± 0.07	0.47 ± 0.12	0.46 ± 0.14	0.72 ± 0.10
L_{DA}	1.19 ± 0.19	0.94 ± 0.26	0.79 ± 0.30	0.86 ± 0.13
S_{DA}	0.44 ± 0.06	0.35 ± 0.11	0.39 ± 0.16	0.51 ± 0.13
$r_{EB\&F}$	0.80	0.97	0.86	0.79
$L_{EB\&F}$	2.58	1.39	1.71	1.05
$S_{EB\&F}$	0.72	0.84	0.67	0.72
r_{Other}	0.80	0.87	0.51	0.66
L_{Other}	1.44	1.10	0.64	0.94
S_{Other}	0.70	0.79	0.27	0.43
Sample size	1562	1252	750	2826

r_q = Retrial probability given that the initial attempt disposition is q.
L_q = Average number of additional attempts given that the initial attempt disposition is q.
S_q = The ultimate success probability given that the initial attempt disposition is q.

will be completed is demonstrated in Table VI which lists the percentage of sequences that are abandoned eventually by customers after encountering different initial incomplete attempts. For calls terminating at residence lines, whether they are initiated by residence or business customers, DAs are the main obstacles in preventing completions of customers' desired connections. For calls terminating at business lines, whether originated by residence or business customers, DAs and BYs are almost equally responsible for failures in completing desired connections.

The most important implication of the above discussions is that, as far as the initial attempt completion probability and the ultimate success probability are concerned, the terminating class of service is the dominant factor. The originating class of service has only a secondary effect on the results.

V. APPLICATIONS

Retrial results obtained in this study are very important in providing an overview of network performance and customer behavior in setting up a desired telephone connection. They are also useful in many practical applications. In traffic engineering, one has to take retrials into account to properly relate a carried load to the offered load for a given blocking objective. Especially in situations where repeated retrials may cause congestion in a local office or the network, information about retrial characteristics is particularly needed. Similarly, knowledge of customer behavior in reacting to network-caused failure should

Table VI—Percentages of sequences that are unsuccessful by originating and terminating classes of service

Initial Attempt Disposition	Percentages of Sequences That Are Unsuccessful			
	From Residence		From Business	
	To Residence	To Business	To Residence	To Business
Busy	2.0	2.5	2.1	1.7
Did not answer	12.9	4.2	16.0	2.1
EB&F	0.6	0.2	0.5	0.7
Other	0.9	0.9	2.5	1.7
Total	16.4 ± 2.0	7.8 ± 2.0	21.1 ± 6.0	6.0 ± 2.0

be helpful in designing proper network management techniques to minimize the impact of a temporary network problem.

Another application of significant importance is provision of a tool to evaluate the revenue and cost impact of call completion improvement programs. Several improvement programs have been adopted in the Bell System operating companies to increase the network call completion ratio, which is defined as the number of DDD messages divided by that of DDD attempts. An initial analysis on the revenue and cost impact of improving call completion on all DDD calls disregarding the originating and terminating classes of service is presented in this section.

As noted in Section III, 11.5 percent of all desired connections are never established. It will be shown that they represent a significant loss of revenue. The amount of revenue loss varies from disposition to disposition.

There are ways to recover some of the large revenue loss associated with unsuccessful sequences of attempts. For instance, network-related problems can be dealt with through a faster identification and correction of network problems. Customer-related problems (like BY or DA) may be attacked by various marketing strategies like selling customers new products or new services, or simply educating customers more effectively. There undoubtedly will be an associated cost for each program. It is not the purpose of the present paper to examine revenue and cost effects for every possible strategy to find out the most cost-effective way to improve the network completion ratio. Rather, the following analyses are intended to demonstrate how the retrial results reported here can be used to address two important aspects of the problem, namely, network revenue and network cost of carrying DDD messages and attempts. Questions like service charges and program costs are outside the scope of the current investigation.

Revenue loss associated with each category of incomplete dispositions is evaluated in the first subsection. In the other two subsections, two network improvement programs are discussed. These examples help to demonstrate the usage of the previously presented retrial quantities in other situations. One example discussed is the so-called worth of completion question. It deals with the relationship of network revenue gain to an increase in the network completion ratio. In the other example, revenue effects of the call waiting service are discussed. The call waiting service provides a subscriber with the option of answering an incoming call even when he is already engaged in a conversation.

5.1 Network revenue loss

Annual revenue loss for each category of ineffective initial attempts can be computed as follows. The number of annual initial attempts with disposition q is $N \times (N_1/N) \times p_{1q}$, where N is the number of annual DDD attempts and N_1 the number of annual initial DDD attempts. The ratio N_1/N is determined from the data in Table I to be 0.775. Out of these sequences of attempts, $N_1 \times p_{1q} \times (1 - S_q)$ desired connections are never established. Annual revenue loss for the category is $N_1 \times p_{1q} \times (1 - S_q) \times$ (revenue/message). The total annual DDD revenue may be written as $N_1 \sum_q p_{1_q} S_q$ (revenue/message). The ratio of annual revenue loss relative to this total revenue is 8.1 percent associated with initial DAS, 2.2 percent for BYS, and 0.7 percent for EB&FS. The predominant revenue loss can be attributed to DAS and BYS. This loss, which results from customer-controlled failures, will be recovered, not through direct improvements of the physical network, but only through marketing efforts to reduce the incidence of DAS and BYS. It may be achieved by selling customers more lines or new vertical services where available.

5.2 Worth of completion

To estimate the network revenue gain for a 1-percent increase in the network completion ratio, it will be assumed that the increase is made possible through the introduction of a call completion improvement program. The program will result in an increase in completion on the initial attempt for call sequences that would otherwise have resulted in an initial disposition q. Let $\tilde{N}_{1q}$ be the number of *initial* incomplete attempts with disposition q that will be converted into complete calls on the first attempts.* As a result of introducing the network improvement program, those $\tilde{N}_{1q}$ sequences will be affected.

* For dispositions other than DA and BY, the improvement program does not necessarily convert all $\tilde{N}_{1q}$ initial attempts into completions, but into a mixture of completions, DAS and BYS. The following equations are for the DA and BY dispositions. The modifications required for the other dispositions can easily be derived and are not shown here.

Before the conversion due to the program, $\tilde{N}_{1q}S_q$ connections were established, and an additional $\tilde{N}_{1q}L_q$ attempts were generated. After the conversion, all $\tilde{N}_{1q}$ sequences are completed without any additional attempts.

The total net additional gain consists of the net gain from the additional $\tilde{N}_{1q}$ $(1 - S_q)$ messages and the cost saving from $\tilde{N}_{1q}$ $(L_q + 1 - S_q)$ incomplete attempts. The net gain per message is defined as (revenue/message − cost/message). Therefore, the net worth (additional revenue − additional cost) may be written as

$$\begin{aligned}\text{Net worth} &= \tilde{N}_{1q}(1 - S_q)\ (\text{net gain/message}) \\ &\quad + \tilde{N}_{1q}(L_q + 1 - S_q)\ (\text{cost/noncompletion}) \\ &= \tilde{N}_{1q}(1 - S_q)\ (\text{revenue/message}) \\ &\quad - \tilde{N}_{1q}(1 - S_q)\ (\text{cost/message}) \\ &\quad + \tilde{N}_{1q}(L_q + 1 - S_q)\ (\text{cost/noncompletion}).\end{aligned}$$

If N is the total number of DDD attempts before the conversion and c is the completion ratio, then $\tilde{N}_{1q}$ is related to Δc, a change in the completion ratio, by the following equation:

$$\Delta c = \frac{Nc + \tilde{N}_{1q}(1 - S_q)}{N - \tilde{N}_{1q}L_q} - c = \frac{\tilde{N}_{1q}(1 - S_q + cL_q)}{N - \tilde{N}_{1q}L_q}$$

or

$$\tilde{N}_{1q} = \frac{N\Delta c}{1 - S_q + (c + \Delta c)L_q}.$$

The net worth can now be rewritten as

$$\begin{aligned}\text{Net worth} &= \frac{N\Delta c}{1 - S_q + (c + \Delta c)L_q} \\ &\quad \times [(1 - S_q)\ (\text{revenue/message}) \\ &\quad - (1 - S_q)\ (\text{cost/message}) \\ &\quad + (L_q + 1 - S_q)\ (\text{cost/noncompletion})].\end{aligned}$$

Thus, for a given change in the completion ratio, Δc, one can compute both $\tilde{N}_{1q}$ and the net worth, where $\tilde{N}_{1q}$ is the number of initial attempts with disposition q that need to be changed to achieve the new completion ratio. The number of ineffective attempts with disposition q disregarding the attempt level, N_q, that will be eliminated in the process can also be estimated. $\tilde{N}_{1q}/N_q$ may be approximated by the ratio of the number of initial DLSO observations with disposition q to that of the overall DLSO observations with a disposition q at any attempt level, i.e., $N_1 p_{1q}/Np_q$. An estimate of N_q is of practical interest, since in general the result of an improvement program is measured in terms of its effect on N_q, not $\tilde{N}_{1q}$, and it is often desirable to know the effect on revenues from such an improvement program.

Given appropriate revenue and cost figures per message and cost in setting up an incomplete attempt, the cost and revenue effects for a change in the completion ratio of DDD calls may be readily evaluated.

5.3 Call waiting service

The call waiting service is one of the vertical services available to customers served by ESS machines. It allows a subscriber to answer an incoming call from a second calling party when he is already engaged in a conversation. Normally, the second calling party would receive a busy signal. With the call waiting service, the subscriber can respond to the new incoming call, and thus change a would-be BY attempt into a complete call. If he decides to ignore the new incoming call, the attempt would appear as a DA instead of a BY to the second calling party. In the latter case, the second calling customer would presumably hold on much longer than if he encountered a BY signal. This choice of response available to the subscriber introduces into the problem an additional parameter β, the fraction of time that the subscriber will respond to the call-waiting signal. Let α be the marketing penetration factor for the call waiting service. One can show in a similar analysis, as given in the last two sections,*

$$\tilde{p}_{1,\text{BY}} = (1-\alpha)p_{1,\text{BY}}$$

$$\tilde{p}_{1,\text{COMP}} = p_{1,\text{COMP}} + \beta\alpha p_{1,\text{BY}}$$

$$\tilde{p}_{1,\text{DA}} = p_{1,\text{DA}} + (1-\beta)\alpha p_{1,\text{BY}}$$

$$\tilde{c} = \frac{1 + (\alpha p_{1,\text{BY}}/p_s)[\beta(1-S_{\text{DA}}) - (S_{\text{BY}} - S_{\text{DA}})]}{1 + (\alpha c p_{1,\text{BY}}/p_s)((1-\beta)L_{\text{DA}} - L_{\text{BY}})}\, c$$

Incremental messages $= N_1 \alpha p_{1,\text{BY}}[\beta(1-S_{\text{DA}}) - (S_{\text{BY}} - S_{\text{DA}})]$

Incremental ineffective attempts $= N_1 \alpha p_{1,\text{BY}}[L_{\text{BY}} + \beta(1-S_{\text{BY}}) - (1-\beta)L_{\text{DA}}]$

and

$$\text{Net worth} = N_1\alpha p_{1,\text{BY}}\{[\beta(1-S_{\text{DA}}) - (S_{\text{BY}} - S_{\text{DA}})] \times (\text{net gain/message}) + [L_{\text{BY}} + \beta(1-S_{\text{BY}}) - (1-\beta)L_{\text{DA}}]\,(\text{cost/noncompletion})\}.$$

* Here it is assumed that all incoming calls to call-waiting subscribers will not encounter BY conditions. In practice, a small percentage of incoming calls to call waiting subscribers will still result in BYs under certain conditions. The present analysis may be generalized to include the effect of those remaining BYs associated with calls to call waiting subscribers.

The net worth is directly proportional to the market penetration factor α. It is a linear function of the response factor β. For a small β, the net worth may bcome negative. The reason is that, if subscribers of the call waiting service do not respond to call-waiting signals, the calling customers may perceive DA conditions rather than would-be BY conditions and will react accordingly. Since the ultimate success probability after DA is lower than that after BY, the trade-off may not be favorable. Fortunately, if one uses reasonable estimates for various parameters, the net worth will be positive if β is greater than 0.25. Initial results from a recent special study indicates that β is approximately 0.63.

VI. SUMMARY

The important quantities needed to characterize retrial behavior are the initial disposition probability p_{1q}, the retrial probability r_q, the number of additional attempts L_q, and the ultimate success probability S_q, where q represents any of the various possible dispositions. These quantities, which are summarized in Table II, vary substantially for different initial dispositions. Depending upon the intial attempt incomplete disposition, whether it be a BY, DA, or EB&F, the time interval which a customer waits before initiating another attempt is quite different.

Besides the traditional traffic engineering application of these retrial parameters, several applications are included here to demonstrate the usefulness of these retrial statistics. In addition, the technique used in this paper has also been applied to several other problems in the network planning area.

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