# Loop Plant Modeling:

# The Facility Analysis Plan: New Methodology for Improving Loop Plant Operations

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The Facility Analysis Plan is a new methods system developed to reduce facility provisioning and maintenance costs in the loop plant. It addresses comprehensively the set of operational costs incurred by all work forces. The Plan consists of three components: (i) an information processing system, (ii) an engineering applications system, and (iii) a control system. The information processing system comprises a set of reporting procedures for data which portray the operation of the loop network within geographical regions called allocation areas. defined so that each represents a virtually independent segment of the existing network. The engineering applications system provides methods for using data from the information processing system to identify those allocation areas in which high operating costs are incurred, to determine the cause of the high costs, and to select and evaluate economical means of reducing these costs. The control system uses data from the other two component systems to ensure the validity of the economic evaluations and to verify that predicted cost reductions are actually achieved.

#### I. INTRODUCTION

The Facility Analysis Plan is a new methods system designed to improve loop plant operations. The objectives of loop plant operations are to provide telephone service to the customer on demand at the lowest possible cost and to maintain that service without interruption until the customer requests its termination. Meeting this objective requires the combined efforts of many work forces, each of which performs a distinct set of functions.

Prior to the development of the Facility Analysis Plan, each work force had separate plans to monitor and improve its effectiveness in providing service. While these plans involved collecting and processing large quantities of data concerning loop network operations, they were in many cases only partially effective because of three basic weaknesses. First, the data were not organized for easy use in identifying and correcting specific problems. Second, the plans usually focused on the operation of a single work force even though there are substantial interdependencies among the various forces. Third, the plans generally lacked the detailed and comprehensive procedures needed to make them an integral part of the normal work process.

The Facility Analysis Plan was developed in response to these identified weaknesses. It is designed to reduce facility provisioning and maintenance costs by addressing comprehensively the set of operational problems encountered by all work forces associated with the loop plant. The Plan has three components: (i) an information processing system, (ii) an engineering applications system, and (iii) a control system. These systems use data gathered by monitoring specific loop plant work operations and their associated costs. Section II describes the cost measures used in the Plan. Subsequent sections describe each component system in more detail. We have tried in our descriptions to avoid a surfeit of telephone company terminology which would obscure the essential concepts and underlying models.

#### II. COST MEASURES

Our objective is to minimize the aggregate cost of providing and maintaining loop facilities.

The aggregate cost is the total cost of providing the facilities needed to satisfy demands for service. This cost has four components—basic operating cost, basic scheduled cost, marginal operating cost, and marginal scheduled cost.

Basic operating cost is that incurred to satisfy a specific customer's immediate request for service. In the case where service to an address is being established initially, this component includes all, and only, those costs associated with the work operations required, given that a spare facility is available for use at the terminal nearest the customer's address. In the case where service was discontinued and is being reestablished, this component includes all, and only, those costs associated with the work operations required, given that the idle facility previously serving the address has remained connected from the central office to the customer's premises.

Basic scheduled cost is that incurred to make additions to or rearrange those parts of the cable network where current capacity can no longer satisfy forecasted service requirements. Both the basic operating and basic scheduled costs are assumed to be fixed components of the aggregate cost throughout this paper.

Marginal operating cost (MOC) is that incurred in addition to the basic operating cost to satisfy a service request when the ideal facility conditions governing the basic operating cost are not met. This cost results from the additional work operations required to make a spare pair available to the customer's terminal and/or to reconnect the facility from the central office to the customer's premises. MOC is also incurred if the facility fails after service has been established, since work operations are then required to restore the service.

Marginal scheduled cost (MSC) is that incurred to make additions, rearrangements or other modifications to the network other than those associated with basic scheduled cost. Examples are the cost of replacing deteriorating facilities to provide more reliable service to existing customers, the cost of advancing cable relief, the cost of converting existing plant to the Serving Area Concept (see N. G. Long, this issue), or the cost of initiating new administrative procedures such as the connect through plan or the conformance testing program.

MOC is strongly influenced by MSC. Often, by incurring an MSC it will be possible to decrease the MOC.

In order to minimize the aggregate cost, then, it is necessary to identify those parts of the network where incurring a particular choice of MSC will reduce the MOC by an amount which more than compensates for the MSC. This is accomplished by use of the engineering applications system, described in Section IV, which operates on measures of the MOC provided by the information processing system, detailed in Section III. In this section we describe those work operations which generate the MOC and define a cost factor for each work operation.

# 2.1 MOC work operations

There are two types of work operations which generate the MOC: service provisioning and service restoration. We give only a few examples, since a full listing of each type is not essential to the remainder of the paper.

## 2.1.1 MOC work operations associated with service provisioning

Consider the following example. Service is requested at a given address. The terminal which is designated to serve the address has no spare pairs. However, at a second terminal there is an idle connect-through pair connected to a vacant residence (see H. T. Freedman,<sup>2</sup> this issue). Access to that pair is also possible at the terminal where service is desired. Service can be provided by the additional work operation of breaking the connection at the second terminal, thereby creating a spare pair at the desired terminal. An MOC is incurred because of the time spent by

the assignment force in determining how to provide the spare pair at the desired terminal and by the installation force in breaking the connection at the second terminal according to instructions provided by assignment. This operation is called a break connect-through (BCT) pair. Other examples of this type of work operation are the line-and-station transfer (LST) and the wired out-of-limits (WOL) described by Koontz,<sup>3</sup> this issue.

# 2.1.2 MOC work operations associated with service restoration

Assume that service to a customer is interrupted because of a faulty connection at a terminal. Service can be restored by the work operation of repairing the faulty connection. An MOC is incurred because of the time spent by the groups within the repair force to process the customer's report of service interruption, to determine the type and approximate location of the fault, and to physically make the repair. This example is called a found cable trouble. An alternative method of restoring service is to connect the customer to a different pair, provided that there is a spare pair at the customer's serving terminal. This second example is one of several types of operations that are called assignment changes.

## 2.2 Cost factors for MOC work operations

The cost factor for each work operation that results in an MOC is defined as the average cost to all force groups of an occurrence of the work operation. The cost factor  $K_i$  for work operation i is defined as

$$K_i = \sum_{j=1}^J t_{ij} l_j$$

where J = the number of different work forces involved in loop operations

 $t_{ij}$  = the average time spent by the jth work force on the ith work operation

 $l_j = \cos t$  of labor per unit of time for the jth work force These cost factors will be used in subsequent sections to compute MOCs associated with portions of the loop network.

#### III. INFORMATION PROCESSING SYSTEM

Having discussed the cost measures and rationale to be used in minimizing the aggregate cost of providing and maintaining loop facilities, we next consider the information processing system. This system processes data on the occurrences of MOC work operations and produces outputs used by the engineering applications system and the control system. Outputs include an ordering of allocation areas by their normalized yearly MOCs and a history of monthly levels of work operations for each allocation area. Together these outputs are used to identify those

allocation areas where the aggregate cost can be reduced. The latter output is also used to verify that the cost is, in fact, reduced. The information processing system also allows organization of the data so that patterns of work operations become apparent. These patterns are used to determine the type and extent of the MSC to be applied within the allocation area.

#### 3.1 Motivation for the allocation area

Individual work operations are essentially random occurrences. It was important, therefore, in structuring the information processing system to consider the size of the entity and time interval for which the MOC should be reported. As discussed in Ref. 4, the variability of any measure decreases with increases in the size of the area and the interval of time used. The statistical need for a large area and a long time interval must be balanced against the desire to quickly identify small portions of the network which exhibit a high MOC.

A suitable compromise is to measure the MOC for a period of one year in elemental geographic units called allocation areas (see N. G. Long, 1 this issue). Using twelve months' data is intuitively appealing, since seasonal variations will be effectively removed, but the interval is not so long as to mask actual changes occurring in the areas. Allocation areas must be large enough to give a statistically significant measure of cost for one year but not so large that actual concentrations of high cost are masked. Areas of 500 to 2000 assigned pairs are considered suitable. Allocation areas are fed by groups of 50 feeder pairs (called complements). The term "allocation" is used because the area is also the basic geographic unit to which feeder pairs are allocated (see B. L. Marsh,<sup>5</sup> this issue). Allocation areas are also defined so as to minimize the number of feeder pairs terminating in more than one area. This ensures that data collected by complement (see Section 3.2) are associated with the proper allocation area and that an MSC applied in one allocation area will not affect any others.

## 3.2 Data organization

The data to be collected are the number of occurrences of MOC work operations. For each complement, the number of monthly occurrences of each type of service provisioning work operation and those service restoration work operations known as assignment changes is recorded. These "initial data records" are retained for later use in the engineering applications system. They are also summarized by allocation area each month, by applying a transformation which maps each complement to a particular allocation area.

For each of the remaining service restoration work operations (those known as found cable troubles), the allocation area and the address of the trouble within the allocation area are recorded. These "initial data records" are retained for later use in the engineering applications system. They are also summarized by allocation area each month.

## 3.3 Allocation area data reporting

The data on monthly occurrences of each type of MOC work operation are presented in a historical report for each allocation area. The report for a given allocation area allows comparisons of monthly levels of work operations over as long as a two-year period in order to detect trends.

Semiannually, a report is generated listing the allocation areas in order of decreasing normalized yearly MOC. The normalized yearly MOC for an allocation area is obtained by dividing the yearly MOC (the total over the past year) by the size of the allocation area as measured by assigned pairs. The normalized yearly MOC is referred to in Ref. 4 as the cost penalty per assigned pair (CPPAP). The term "cost penalty" is used there in the sense that the MOC is a penalty over the basic operating cost. This semiannual report also shows for each allocation area the yearly MOC and the effective cable fill (number of assigned and defective pairs divided by the number of available pairs in complements feeding the allocation area).

The use of the historical report and the ordering report is described in Section 4.1. The further use of the "initial data records" is described in Section 4.2.

#### IV. ENGINEERING APPLICATIONS SYSTEM

The data processed and output by the information processing system are used primarily by members of the engineering work force to make decisions to incur an MSC so as to reduce the aggregate cost of the loop network. There are three basic processes involved. The first is to identify those allocation areas with cost reduction potential—usually those with a high MOC. The second is to determine the physical conditions causing the high MOC and the type of MSC that will significantly reduce it. The third is to predict the magnitude of the expected cost reduction and decide on a course of action.

The models governing these processes describe relationships that hold in general, but that are not always sufficient to describe specific situations. Their successful use requires engineers who are familiar with the loop environment. For this reason we shall generally refrain from citing specific applications or procedural techniques, but shall instead discuss, in abstract terms, basic relationships defined by the models.

## 4.1 Data screening process

The initial application of the data obtained from the information processing system is to identify allocation areas in which there is good potential for reducing the aggregate cost. As was described in Section II, the means to achieve such a cost reduction is by incurring an MSC that will be smaller than the resulting decrease in the MOC in the allocation area.

Recall that the information processing system only provides measurements of the MOC in each allocation area. The magnitude of the MSC required to reduce the MOC is not known a priori. This is a function of a large set of variables and network characteristics and can only be determined from a thorough study by an engineer familiar with the allocation area. The time involved precludes studying every allocation area in this manner. For this reason, a process has been developed to screen the data so as to identify those allocation areas with the greatest expected cost reduction potential.

This screening process involves comparing the allocation areas on the basis of three measures. Generally, the higher the level of a given measure, and the larger the number of measures at a high level, the greater the expected cost reduction potential in an allocation area. The three measures are:

- (i) The normalized yearly MOC. This is the most substantive of the three measures. As described in Section 3.3, the information processing system provides an ordering of the allocation areas according to this measure. Further refinement of this ordering can be achieved by examining, on the historical report for each allocation area, the month-to-month trends of the occurrences of MOC work operations. Thus, two areas which are in statistically close proximity (see D. M. Dunn and J. M. Landwehr,<sup>4</sup> this issue) in the ordering can be differentiated by comparing their trends. An area with an increasing trend has greater expected cost reduction potential than one with a flat trend. Furthermore, an area with a distinctly decreasing trend has little or no expected potential because such a trend pattern usually indicates that MSCs have already been applied to tap a previous cost reduction potential.
- (ii) The effective cable fill. High values of this measure indicate that the number of cable pairs available to meet future service requests is limited. This condition usually leads directly to an increase in marginal operating costs (see W. L. G. Koontz,<sup>3</sup> this issue). An ordering of the allocation areas based on this measure can be refined by considering the growth rate of the areas. If two areas have similar fill levels, the one with the higher growth rate has the greater expected cost reduction potential.
- (iii) The yearly MOC. Allocation areas with very high yearly MOC (relative to other allocation areas in the district), regardless of the nor-

malized value, may also have a high cost reduction potential. This is because the conditions causing the high costs may be concentrated in such a way as to enable substantial reductions in these costs with a modest MSC investment.

The validity of these measures as independent indicators of the relative expected cost reduction potential in allocation areas has been demonstrated empirically, and they have been applied successfully in the data screening process. However, we have not yet derived a specific quantitative relationship among these three measures, nor have we identified such desirable characteristics as "threshold" levels, i.e., absolute values of the measures above or below which allocation areas could be classified as possessing or not possessing cost reduction potential. More information from field locations using the Facility Analysis Plan is needed before work in this area can proceed.

## 4.2 Data interpretation process

An allocation area that has been identified as possessing a high expected cost reduction potential is examined in greater detail to determine the type of MSC required to reduce the MOC.

## 4.2.1 Establishing cause from effect

We have identified twelve specific network conditions that may cause the work operations which are the source of the MOC. Examples of these are insufficient spare pairs, imbalances in the network, rapid customer movement in and out of the allocation area, old and deteriorating cables and terminals, inadequately maintained records and unexpected growth of customer demand. Each of these network conditions creates the need for a certain type (or types) of work operation to provide or restore service. For this reason the network conditions in a given allocation area can usually be identified by noting which type(s) of work operation occurred during the previous year.

To facilitate identification, a matrix has been developed to illustrate the cause and effect relationship between the network conditions and work operations (see Fig. 1). The various work operations are grouped into 11 categories on the left. Each category comprises from one to seven different work operations. Those operations in a given category are considered indistinguishable in that any one or more of them may be caused by a particular category of network condition. The 12 network conditions are grouped into eight categories along the top of the matrix. Each of these categories comprises from one to four different network conditions. Any one of the network conditions in a given category may cause a particular category of work operation.

An "x" in the matrix indicates that the category of network condition

		CATEGORY OF NETWORK CONDITION							
		Α	В	C	D	E	F	G	н
CATEGORY OF WORK OPERATION	1	х							
	2	х	×						
	3		x						
	4		x	x					
	5			х	х	x			
	6			x		x			
	7				х	x			
	8					x			
	9						х		
	10							x	
	11								X

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Fig. 1—Matrix to identify cause and effect relationship between network conditions and work operations.

listed in that column can be expected to cause occurrences of the category of work operation listed in the associated row. Thus network condition B (high inward/outward subscriber movement) often causes work operations 2 (facility modifications such as BCT, LST, and WOL), 3 (reterminating service connections at established customer locations), and 4 (repair of faulty connections in terminals). By using the matrix in reverse, the observed set of work operations identifies the category of network condition likely to be present in the allocation area of interest. For example:

- (i) If work operations in categories 5 and 7 are observed, a network condition in category D is probably present.
- (ii) If, however, work operations in categories 5, 6, 7, and 8 are observed, a network condition in category E is most likely to be present and one in category D may be present as well.
- (iii) Finally, if work operations in categories 5, 7, and 9 are observed, a network condition in both categories D and F are probably present.

While the large majority of work operations appear in the patterns shown in Fig. 1, exceptions do occur occasionally. In such cases the engineer examining the allocation area must draw on a personal knowledge of the conditions in the area to determine the cause of the observed operations.

Given the general category of network condition present in an allocation area, it is usually a simple matter to examine other outputs from the information processing system (e.g., effective cable fills, defective pair rates) to further delineate the specific network condition.

## 4.2.2 Determining impact level

A network condition may impact at one of three levels. It may affect (i) all parts of the allocation area, (ii) all parts of one distribution area (see N. G. Long, this issue) within the allocation area, or (iii) only certain distribution cable areas within a distribution area. A distribution cable area is a small geographic region served by a single complement or unique set of complements. The cable comprising this (set of) complement(s) is usually referred to as a cable "leg." For a given network condition, the appropriate type of MSC is normally different for each impact level.

The impact level can be determined from information on the "initial data records" compiled by the information processing system (see Section 3.2). For certain work operations (those dealing with service provisioning as well as assignment changes), this record shows the number of occurrences within each complement. For the remaining operations (all types of found cable troubles), the record shows the street address of each occurrence. The data are organized so that information pertaining to complements or addresses within a given allocation area can easily be extracted.

To use these data, the complements serving an allocation area must be partitioned into groups and subgroups such that a group contains those complements which serve a particular distribution area and a subgroup, a particular distribution cable area. This allows work operations recorded by complement to be mapped geographically and therefore to be combined with those operations recorded by address.

The resulting data patterns can then be observed. If the work operations are distributed rather uniformly across the groups, the impact level is the allocation area. If they are concentrated within one group but distributed uniformly across the subgroups, the impact level is the distribution area. If they are concentrated within specific subgroups, the impact level is the distribution cable area.

Where more than one network condition is present, the process must be applied separately to the work operations caused by each condition. The result may be that the conditions are independent (at different impact levels or in different groups or subgroups within the allocation area) or dependent (same impact level and same group association). For those that are independent, separate MSCs should be applied. For those that are dependent, a type of MSC must be found that will reduce the combined set of MOCs.

# 4.2.3 Identifying network enhancements

The last step of the data interpretation process is to find the most effective means of reducing the aggregate operating costs in a given allocation area. An MSC is applied by making one or more planned enhancements to the network which are designed to correct the condition(s)

identified as causing the observed Moc. Seventeen specific types of network enhancements have been defined. These include clearing defective pairs, relieving the terminals or cables, adopting connect-through administrative procedures, instituting preventive maintenance techniques, and converting part or all of an allocation area to the serving area concept (see N. G. Long, this issue, for description). Each of these enhancements is effective for a specific set of network conditions at a particular level. The objective is to identify the least costly enhancement that will correct the network conditions.

For this purpose a three-tiered matrix has been developed (see Fig. 2). Each tier corresponds to one of the three impact levels. Within each tier the rows correspond to the 12 network conditions and the columns correspond to the 17 network enhancements. The enhancements are arranged from left to right in order of increasing complexity and expected cost. An "x" in the matrix indicates that the enhancement listed in that column will correct the network condition listed in the associated row at the impact level of the corresponding tier. Thus enhancement  $E_5$  will correct network conditions  $C_1$  and  $C_6$  at impact level  $L_3$ , condition  $C_6$  only at impact level  $L_2$ , and is not applicable at impact level  $L_1$ .

The matrix is used by locating the row and tier associated with the network condition and its impact level, and then moving across the columns from left to right to the first column that contains an "x." The enhancement identified with this column will generally result in the greatest reduction in the aggregate cost of the allocation area. For example, condition  $C_4$  at impact level  $L_2$  should be corrected by enhancement  $E_6$ .

If a set of dependent network conditions is present, an enhancement must be found that will correct all the conditions. This can be done by locating the corresponding set of rows on the proper tier and moving along these rows to the first column that contains an "x" in each. For example, network conditions  $C_1$ ,  $C_2$ , and  $C_6$  at impact level  $L_3$  should be corrected by enhancement  $E_8$ . If no single enhancement can be found that will correct all the network conditions, the combination of enhancements requiring the smallest expected scheduled cost should be chosen. For example, for network conditions  $C_7$  and  $C_8$  at impact level  $L_1$ , enhancements  $E_{14}$  and  $E_{15}$  should provide the most cost effective solution.

There are also cases where, even if there is a single enhancement that will correct all conditions, a combination of enhancements may be more economical. Where such a possibility exists, both choices should be analyzed according to the procedures in Section 4.3 to determine which is more economical.

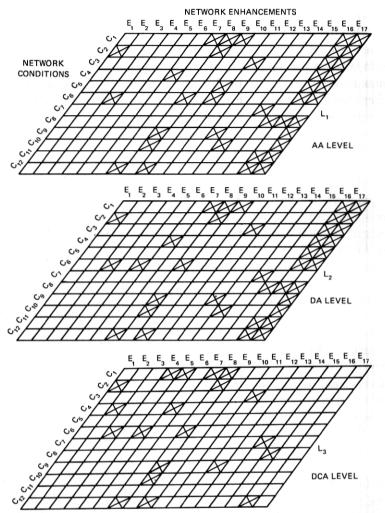


Fig. 2—Matrix to identify relationship between network enhancements and network conditions.

#### 4.3 Economic analysis process

The network enhancement(s) derived from the data interpretation process are those with the greatest possibility of reducing the aggregate costs in an allocation area. Whether such a reduction can in fact be achieved, however, can only be determined by numerically comparing the MSC to the expected reduction in the MOC. If the former is smaller than the latter, the aggregate cost can be expected to decrease, and the enhancements should be applied. If not, the aggregate cost may already be at its minimum level, or another, less extensive enhancement may prove cost effective.

There are two economic analysis techniques used to make the cost comparison. One has been developed expressly for cases in which the network enhancement involves conversion to the serving area concept (SAC). This enhancement is often the most desirable of the available choices because it results in permanent and stable distribution plant which virtually eliminates the need for future enhancements (see N. G. Long,¹ this issue, and also J. O. Bergholm and P. P. Koliss⁶). It also requires the largest commitment of MSC. For these reasons a complete analysis is required before this enhancement is applied. The other analysis technique is used for the non-SAC enhancements and, at present, makes use of a more rudimentary model. The non-SAC technique is described first since the concepts employed are fundamental and also apply to the more complex model of the serving area concept.

The two techniques are conceptually alike in that both are designed to estimate accurately the incremental change, A, in the aggregate cost of an allocation area resulting from an MSC of size S that reduces the MOC by amount O. Both techniques make use of empirical models for estimating the decrease in work operations that result when network enhancements are made. Work on theoretical models of these effects is described by W. L. G. Koontz<sup>3</sup> and H. T. Freedman<sup>2</sup> in this issue.

In the discussions to follow, all three costs will be expressed in present worth dollars, so that

$$A = S - O \tag{1}$$

In order to compute the value of A, it will be necessary to develop expressions for S and O in terms of parameters whose values are readily obtained. Acceptable parameters include those whose values are:

- (i) Set at the corporate level for the purposes of economic studies, such as the cost of money and the costs of the various work operations.
- (ii) Measured or forecast for individual allocation areas, such as the number of occurrences of particular work operations and the growth rate of assigned pairs.
- (iii) Estimated for the particular allocation area under study, such as the costs of the proposed enhancement and the expected reduction in work operations resulting from the enhancement.

# 4.3.1 Analyzing non-SAC enhancements

In this section we describe a general model which is applicable to the analysis of any non-SAC enhancement. We also present two simplified versions of the model that can be applied to the most common of the enhancements.

To determine the value S of the MSC, we first define the term  $\sigma_*$  to be the levelized equivalent annual cost (LEAC) of an expenditure, \*, which

contains a capital and associated expense element,  $c_*$  and  $x_*$ , and which is applied according to the repeated plant assumption (see J. Freidenfelds, Appendix, this issue). All expenditures used in the derivation of S are of this type. Thus

$$\sigma_* = \alpha_* c_* + \gamma_* x_* \tag{2}$$

where  $\alpha_*$  is the annual charge factor that applies to the type of plant placed by the capital expenditure, and  $\gamma_*$  is defined so that the present worth of a constant annuity of  $\gamma_*x_*$  dollars over the life of the plant placed equals the present worth of  $x_*$ . The present worth of annual charges (PWAC) of expenditure \* is then the present worth of the series of  $\sigma_*$  dollars applied from the time the expenditure is incurred,  $\tau_*$ , until the end of the study period, T (see J. Freidenfelds, Appendix, this issue). Thus

PWAC (expenditure \*) = 
$$\int_{\tau_*}^{T} \sigma_* e^{-rt} dt$$
$$= \sigma_* (e^{-r\tau_*} - e^{-rT})/r$$
(3)

where r is the force of interest  $[r = \ln(1 + \cos t)]$ .

In the most general case, the total cost of a network enhancement includes certain basic scheduled costs—typically for cable relief—that would normally be expended at time  $\tau_b$  in the future. The PWAC of these future costs is therefore deducted from the PWAC of the total cost of the enhancement in order to determine the value of the MSC.

Let  $\sigma_e$  be the LEAC of the total enhancement cost, which is incurred at time  $\tau_e$  (= 0), and let  $\sigma_b$  be the LEAC of the future basic scheduled cost, incurred at time  $\tau_b$ . Then the value of the MSC is

S = PWAC (enhancement cost) - PWAC (future basic scheduled cost) which can be expressed, from eq. (3), as

$$S = [\sigma_e (1 - e^{-rT}) - \sigma_b (e^{-r\tau_b} - e^{-rT})]/r \tag{4}$$

To determine the value of the MOC reduction, O in eq. (1), we first compute the expected reduction,  $\omega$ , in the annual MOC. For work operation i, let  $w_i$  be the number of annual occurrences, and let  $\rho_i$  be the fraction of these which the MSC is expected to eliminate.  $w_i$  is taken from the information processing system and  $\rho_i$  is estimated based on the empirical knowledge of the engineer designing the enhancement. More sophisticated estimates of  $\rho_i$  may result from the theoretical work described by H. T. Freedman<sup>2</sup> and W. L. G. Koontz<sup>3</sup> in this issue. Then  $\rho_i w_i$  is the expected reduction in the annual number of work operation i. Furthermore, if  $K_i$  is the cost of work operation i (see Section 2.2), then define  $k_i$  to be

$$k_i = \begin{cases} K_i, & \text{if the cost of operation } i \text{ is an expense} \\ \text{PWAC}(K_i), & \text{if the cost of operation } i \text{ is capitalized} \end{cases}$$
 (5)

where

$$PWAC(K_i) = \int_0^{L_k} \alpha_k K_i e^{-rt} dt = \alpha_k K_i (1 - e^{-rL_k})/r$$
 (6)

and where  $\alpha_k$  is the annual charge rate and  $L_k$  is the lifetime of the capitalized cost. PWAC( $K_i$ ) is computed only over the life of the investment because the repeated plant assumption does not apply in this case. These costs are derived from work operations associated with providing service to a customer, which represents a commitment of capital only to the point in time at which the customer requests that the service be disconnected. The average service life,  $L_k$  years, is generally less than the length of the study.

Then  $\rho_i w_i k_i$  is the expected reduction in the annual cost of operation i, and the expected reduction in the total annual MOC of the allocation area is

$$\omega = \sum\limits_{i} \rho_{i} w_{i} k_{i}$$

In many cases, incurring the basic scheduled cost would probably have achieved a portion of the annual cost reduction,  $\omega$ , starting at time  $\tau_b$ . Let  $\omega'$  represent the part of  $\omega$  attributable to the basic scheduled cost. The result of incurring the MSC then is to reduce annual costs by amount  $\omega$  until time  $\tau_b$  and by amount  $\omega - \omega'$  from time  $\tau_b$  to T.

The total reduction in the MOC is therefore

$$O = \int_{0}^{\tau_{b}} \omega e^{-rt} dt + \int_{\tau_{b}}^{T} (\omega - \omega') e^{-rt} dt$$

$$= [\omega (1 - e^{-rT}) - \omega' (e^{-r\tau_{b}} - e^{-rT})]/r$$
(7)

From eqs. (1), (4), and (7), we have the following general expression for A, the change in the aggregate cost of the allocation area:

$$A = [(\sigma_e - \omega)(1 - e^{-rT}) - (\sigma_b - \omega')(e^{-r\tau_b} - e^{-rT})]/r$$
 (8)

Equation (8) reduces to a simpler form for the two most common MSC applications (other than conversion to SAC, discussed in the next section). The first is where the MSC represents simply the advancement of future basic costs. In this case  $\sigma_b = \sigma_e$  and  $\omega' = \omega$ . Equation (8) reduces to

$$A = (\sigma_e - \omega)(1 - e^{-r\tau_b}) \tag{9}$$

The second application is where there is no basic scheduled cost component in the cost of the enhancement. In this case  $\sigma_b = 0$  and  $\omega' = 0$ . Equation (8) reduces to

$$A = (\sigma_e - \omega)(1 - e^{-rT}) \tag{10}$$

If the value of A is negative, the aggregate cost will be reduced, and

the network enhancement is justified. If A is positive, however, the enhancement should not be made. In either case if other reasonable enhancements exist, they should also be investigated and the best one chosen if it pays.

## 4.3.2 Analyzing conversion to SAC

This model and the analysis techniques built around it apply exclusively to the case where the network enhancement involves converting parts of an allocation area to the serving area concept (SAC). Refer to the article by N. G. Long<sup>1</sup> in this issue for a definition of SAC.

The simplest form of conversion to SAC is called "stabilization." This involves placing a serving area interface between the feeder and distribution networks, breaking all multipling (see N. G. Long, this issue) within the distribution network behind the interface, and providing enough distribution pairs to permanently connect one pair to each existing living unit and supply sufficient additional pairs to satisfy service demands for at least two years. In order to satisfy these criteria, it is usually necessary to add cables to the distribution backbone (the main cable paths extending out from the interface) and to rearrange the connections between the "leg" cables (the small cables containing the terminals serving the customers' premises) and the backbone cable (see Fig. 3). The network design changes affected by stabilization eliminate virtually all future work operations associated with providing service (see Section 2.1.1) as well as some operations associated with service restoration (see Section 2.1.2).

A more complex form of conversion to SAC involves replacing certain of the existing leg cables and terminals in the area in addition to doing the stabilization work described above. This additional work is simply called "replacement." The effect of replacement is to eliminate most of the remaining work operations associated with service restoration.

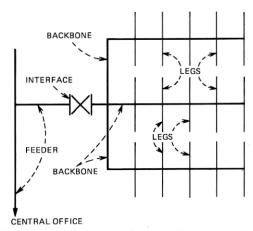


Fig. 3—Example of feeder and distribution (backbone and leg) cables.

The choice of the conversion design—either pure stabilization or some combination of stabilization and replacement—depends upon the type of work operations generating the MOC in the area and the location of the network conditions causing the operations. If a design combining stabilization and replacement is chosen, both parts of the enhancement must be shown to decrease the aggregate cost of the allocation area. If stabilization pays, but replacement does not, conversion to SAC is justified, but only if the replacement work is modified or eliminated. If replacement pays, but stabilization does not, then conversion to SAC is not justified and some other enhancement—such as merely replacing selected troublesome cables—should be examined.

Let  $A_S$  and  $A_R$  be the incremental changes in the aggregate cost of the allocation area that result from incurring MSCs  $S_S$ , the stabilization cost, and  $S_R$ , the replacement cost, respectively. Also denote by  $O_S$  and  $O_R$  the reductions in MOCs resulting from stabilization and replacement, respectively. As before, all costs will be expressed in present worth dollars, so

$$A_S = S_S - O_S, \quad A_R = S_R - O_R$$
 (11)

**4.3.2.1** Costs of stabilization and replacement. In this section we derive expressions for  $S_S$  and  $S_R$ . We begin with the following definition of the stabilization cost:

$$S_S \equiv I + B - E - D \tag{12}$$

where

- (i) I is the PWAC of the interface cost. This is the cost of placing the serving area interface between the feeder and distribution parts of the cable network.
- (ii) B is the present worth of the cost of advancing backbone cable relief. The relief is needed in order to enable the elimination of multipling conditions and provide enough pairs to satisfy the SAC distribution design criteria (see description of stabilization in Section 4.3.2). We only include the advancement cost in this case because the relief would ordinarily have been done at some future date.
- (iii) E is the present worth of the cost of future cable pair transfers that are eliminated by the stabilization work. Since this cost would otherwise be part of the future basic scheduled costs of the allocation area, it is credited to the stabilization work.
- (iv) D is the present worth of the deferred cost of feeder relief resulting from the stabilization work. Because SAC provides improved efficiencies in the utilization of feeder cables and also makes available certain previously unusable feeder pairs, future relief of the feeder network is deferred for a period of time. The value of deferring this basic scheduled cost is therefore credited to the stabilization work.

The interface cost includes both capital and expense components. Let  $\sigma_I$  be the LEAC of this cost as defined in eq. (2). Then we have

$$I = PWAC \text{ (interface cost)}$$
  
=  $\sigma_I (1 - e^{-rT})/r$  (13)

The relief of backbone cables is actually a basic scheduled cost that would have been required at time  $\tau_B$  in the future. For this reason only the advancement, B, of the backbone cable costs is included in the cost of stabilization. Letting  $\sigma_B$  be the LEAC of the backbone cost, B is given by

$$B = \text{PWAC}$$
 (backbone cost at time 0)  
 $- \text{PWAC}$  (backbone cost at time  $\tau_B$ )  
 $= \sigma_B (1 - e^{-r\tau_B})/r$  (14)

The value of  $\tau_B$  may be calculated from parameters of the allocation area.  $\tau_B$  is the time at which the assigned pair fill (assigned pairs/available pairs) of the backbone cables reaches the level at which relief is normally provided. This level is called the nominal fill-at-relief and is denoted f'. The current fill of the backbone cables is denoted  $f_B$ . If, for example, demand for additional pairs in the allocation area grows exponentially at the rate g,

$$f' = f_B e^{g\tau_B} \tag{15}$$

Solving eq. (15) for  $\tau_B$  gives

$$\tau_B = \ln(f'/f_B)/g \tag{16}$$

A cable pair transfer—sometimes called a cable throw—is the process of rearranging the physical cable pair connections within the network. This is done frequently in areas with multiple plant design. Specifically in such areas, as cable relief is provided, distribution pairs are transferred in order to maintain a balanced multipling arrangement between the distribution and feeder pairs. Assuming such a balanced arrangement exists initially, this means that the average number of pairs transferred each year must equal the number of distribution pairs grown during the year. Of course distribution pairs are actually added as relief is needed, and transfers are made in groups, many at the time of relief. We assume, however, that the cost of transfers can adequately be estimated using a continuous rate of transfers. Therefore, assuming again that the allocation area grows exponentially at rate g, the number of distribution pairs grown—and the number of pairs transferred—in year t is

$$de^{gt} - de^{g(t-1)} = d(1 - e^{-g})e^{gt}$$

where d is the number of distribution pairs at time zero. If  $x_E$  is the cost of a cable pair transfer—an expense—then the present worth of the cost of all transfers made during the study is

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$$E = \int_0^T x_E d(1 - e^{-g}) e^{gt} e^{-rt} dt$$

$$= x_E d(1 - e^{-g}) (1 - e^{-(r-g)T}) / (r - g)$$
(17)

Since the need for these transfers is eliminated by conversion to the SAC design, the stabilization cost is reduced by amount E.

The deferral of feeder relief results from two effects of stabilization. First, under SAC design the feeder cables can be worked to a higher assigned pair fill before requiring relief than under multiple plant design. This is because SAC design provides greater access to the feeder network. The first line to a customer's premises remains assigned as either a dedicated or CT pair (see H. T. Freedman, this issue) after service is discontinued, and requests for second lines or service at new premises can be satisfied by any spare feeder pair in the interface. Thus if  $\delta$  is the fractional increase in the fill at relief afforded by SAC and a is the total number of available pairs in the area to be converted to SAC, then  $\delta a$ additional feeder pairs are available for use before relief is required. Second, when the interface is placed between the feeder and distribution networks, feeder pairs that were previously unusable, because of defects in the distribution pairs to which they were connected, now become available for use. Let b be the number of pairs recovered in this way. The total effect of placing the interface, therefore, is to increase by  $\delta a + b$ the number of feeder pairs available and thus to postpone the time when each successive future basic scheduled cost for feeder relief must be incurred. The general form of the expression for D is

$$D = PWAC(future feeder relief costs)(1 - e^{-r \cdot deferral interval})$$

However, calculating values for feeder relief costs and the deferral interval are beyond the scope of the Facility Analysis Plan, because they are functions of parameters such as the spare pair levels, cable gauge and structure requirements, and growth rates of each section of the feeder route. For this reason a standard cost factor,  $x_D$  (based on average feeder route conditions), is used to approximate the value of feeder deferral. Specifically,  $x_D$  represents the present worth value per unit length of one feeder pair gained through stabilization. The expression for D is therefore

$$D = (\delta a + b)lx_D \tag{18}$$

where l is the length of the pairs (the distance from the central office to the location of the interface). Since D represents a reduction in basic scheduled costs afforded by conversion to SAC, the stabilization cost is reduced by amount D.

This completes the stabilization cost terms [eqs. (13), (14), (17), and (18)]. We turn next to the replacement costs.

The cable replacement cost,  $S_R$ , is associated with replacing those

"leg" cables in the allocation area which experience recurring work operations for service restoral purposes.

This cost would normally be expended at some point  $\tau_R$  in the future when relief of the leg cables is needed. If we let  $f_R$  be the assigned pair fill of the cables to be replaced, then

$$\tau_R = \ln(f'/f_R)/g \tag{19}$$

where f' is the nominal fill-at-relief and g is the exponential growth rate [see eqs. (15) and (16)].

The replacement cost,  $S_R$ , then is taken to be the cost of advancing this relief  $\tau_R$  years:

$$S_R = \sigma_B (1 - e^{-r\tau_R})/r$$
 (20)

where  $\sigma_B$  is the LEAC of the relief cost.

4.3.2.2. Reductions in marginal operating costs. In this section we derive expressions for  $O_S$  and  $O_R$ , the reductions in the MOC resulting from stabilization and replacement respectively. Recall that the MOC accrues from the occurrence of the work operations discussed in Section 2.1. Since these operations are affected in several distinctly different ways by conversion to SAC, we shall classify them into four groups for the purposes of this discussion.

Group 1 contains operations such as BCTs, LSTs and WOLs (see Section 2.1.1), which are caused by facility shortages and network imbalances. Since these conditions are corrected by stabilization, reductions in Group 1 operations are credited solely to the stabilization part of the conversion. The specific reduction factors applied to these operations are constants derived from studies of numerous conversion jobs in several operating telephone companies.

Group 2 contains operations caused primarily by activity in terminals and cable splices resulting from customer movement. Included in this group are cable troubles found in terminals and splices (see Section 2.1.2) as well as assignment changes made when a defective pair is encountered while installing service. The network activity causing these operations is substantially reduced, but not eliminated, by stabilization. For example, the activity due to reinstalling service at an existing customer's premises is largely or totally (depending on the choice of SAC design) eliminated. On the other hand the activity due to installing service to a new customer's premises is not reduced at all. The reduction factors, based on stabilization, that are applied to the operations in this group are variables. Their values are calculated based on the levels of network activity, the mix of subscriber demand (new vs. reinstallations) and the choice of SAC design.

Those Group 2 operations which are not directly affected by stabilization are eliminated, however, wherever the leg cables and terminals

are replaced in the serving area. The reduction factors based on replacement are therefore the unit complements of the stabilization factors.

Group 3 contains the operations associated with types of cable troubles other than those in Group 2. These are caused primarily by old and deteriorating cables (usually those with lead sheaths). Since this condition is not related to the network design or activity, these operations are reduced only where the cables are replaced. The reduction factors for this group are constants which reflect the vast improvement in the integrity of modern plastic sheathing materials as compared to lead.

Group 4 contains one type of operation—the assignment change made to restore service to a customer (described in Section 2.1.2). This operation is caused by the same conditions that cause both Group 2 and Group 3 operations. The reduction in this operation is therefore expressed as a weighted average—based on the relative numbers of Group 2 and Group 3 operations—of the reductions in the other two groups.

If unchecked by conversion, the operations in Groups 1 and 2 are assumed to increase over time at the same rate as the growth in assigned pairs. This assumption is made because these operations are related to the size and movement of the subscriber population. The operations in Group 3 on the other hand are assumed to remain constant since they are not affected by these factors. The Group 4 operation will be treated in two parts, one growing and the other remaining constant because of the composite nature of its causes.

Expressions for the marginal operating cost reductions for the operations in each of the four groups are developed below. For the purposes of this discussion, we define the following parameters:

- $k_i \equiv PWAC$  cost of work operation i as defined in eq. (5)
- $w_i \equiv$  number of annual occurrences of work operation i in the area to be converted (from the information processing system)
- $w_i^* \equiv$  number of annual occurrences of work operation i in the parts of the converted area to be replaced (from the information processing system)
- $\rho_i \equiv \text{fractional reduction in work operation } i \text{ resulting from stabilization (fixed or computed as described above)}$
- $\rho_i^* \equiv \text{fractional reduction in work operation } i \text{ resulting from replacement (fixed or computed as described above)}$
- $\omega_{Sj} \equiv$  expected annual reduction in the MOC for Group j work operations resulting from stabilization
- $\omega_{sj}^* \equiv \text{expected annual reduction in the MOC for Group } j \text{ work}$  operations resulting from replacement at the normal relief time and thus credited to stabilization
- $\omega_{Rj} \equiv \text{expected annual reduction in the MOC for Group } j \text{ work}$

operations resulting from replacement at the time of conversion

 $O_{sj}$  = present worth of total reduction in the MOC for Group j work operations resulting from stabilization

 $O_{Rj}$  = present worth of total reduction in the MOC for Group j work operations resulting from replacement

Reductions in Group 1 operations are assumed to hold for the length of the study, T. While some reductions might coincide with regular reliefs (in which case our assumption may overestimate SAC related reductions), these would only be temporary if the basic design of the network is not changed. On the other hand, if the network is not currently in need of relief, operations can be expected to increase as relief nears (in which case our assumption may underestimate SAC related reductions). Improvements in this assumption may be possible in the future as a result of theoretical models currently under investigation (see W. L. G. Koontz,  $^3$  this issue). Based on the present assumption,

$$\omega_{S1} = \sum_{Gp1} k_i \rho_i w_i$$

as in Section 4.3.1, and since the exponential growth rate, g, applies in this case,

$$O_{S1} = \int_0^T \omega_{S1} e^{gt} e^{-rt} dt$$
  
=  $\omega_{S1} (1 - e^{-(r-g)T}) / (r - g)$  (20)

Since reductions in these operations are all due to stabilization,

$$O_{R1} = 0 \tag{21}$$

Reductions in Group 2 operations result from both stabilization and replacement. The stabilization component can be viewed as consisting of two parts. The first applies to the entire study period:

$$\omega_{S2} = \sum_{Gp2} k_i \rho_i w_i$$

The second part is a further reduction which begins at time  $\tau_R$  when leg cables would have normally been replaced:

$$\omega_{S2}^* = \sum_{Gp2} k_i \rho_i^* w_i$$

The MOC reduction due to stabilization then is

$$O_{S2} = \int_0^T \omega_{S2} e^{gt} e^{-rt} dt + \int_{\tau_R}^T \omega_{\bar{S}2}^* e^{gt} e^{-rt} dt$$

$$= \omega_{S2} (1 - e^{-(r-g)T}) / (r - g) + \omega_{S2}^* (e^{-(r-g)\tau_R} - e^{-(r-g)T}) / (r - g)$$
(22)

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For those leg cables which are replaced now (advanced from  $\tau_R$ ), a reduction of

$$\omega_{R2} = \sum_{G\rho 2} k_i \rho_i^* w_i^*$$

is obtained from now until year  $\tau_R$ . This is the MOC reduction due to replacement:

$$O_{R2} = \int_0^{\tau_R} \omega_{R2} e^{gt} e^{-rt} dt$$

$$= \omega_{R2} (1 - e^{-(r-g)\tau_R}) / (r-g)$$
(23)

Reductions in Group 3 operations result solely from replacement. In this case

$$\omega_{R3} = \sum_{Gp3} k_i \rho_i^* w_i^*$$

As for Group 2, these replacement reductions are only obtained from now until year  $\tau_R$ . No stabilization reductions exist here because these effects are independent of the network design. The above cost reductions would be realized, beginning in year  $\tau_R$ , even without SAC design. The MOCs for Group 3 (remember that growth does not apply here) are therefore

$$O_{R3} = \int_0^{\tau_R} \omega_{R3} e^{-rt} dt$$

$$= \omega_{R3} (1 - e^{-r\tau_R})/r$$
(24)

and

$$O_{S3} = 0 \tag{25}$$

Reductions in the Group 4 operation are expressed as weighted averages of the reductions in the Group 2 and 3 operations, and the costs are prorated accordingly. Thus we have four components of the annual reduction in the MOC for the Group 4 operation. These are  $\omega_{S4}(2)$ ,  $\omega_{S4}(2)$ ,  $\omega_{R4}(2)$  and  $\omega_{R4}(3)$ , and they correspond respectively to  $\omega_{S2}$ ,  $\omega_{S2}^*$ ,  $\omega_{R2}$  (from Group 2) and  $\omega_{R3}$  (from Group 3). The MOC reductions for Group 4 are therefore

$$O_{S4} = \int_{0}^{T} \omega_{\underline{S4}}(2)e^{gt}e^{-rt}dt + \int_{\tau_{R}}^{T} \omega_{\underline{S4}}^{*}(2)e^{gt}e^{-rt}dt$$

$$= \omega_{S4}(2)(1 - e^{-(r-g)T})/(r - g) + \omega_{\underline{S4}}^{*}(2)(e^{-(r-g)\tau_{R}} - e^{-(r-g)T})/(r - g)$$
(26)

and

$$O_{R4} = \int_0^{\tau_R} \omega_{R4}(2) e^{gt} e^{-rt} dt + \int_0^{\tau_R} \omega_{R4}(3) e^{-rt} dt$$

$$= \omega_{R4}(2)(1 - e^{-(r-g)\tau_R})/(r-g) + \omega_{R4}(3)(1 - e^{-r\tau_R})/r$$
 (27)

The total marginal operating cost reductions due to stabilization and replacement are

$$O_S = \sum_{j=1}^4 O_{Sj}$$
 and  $O_R = \sum_{j=1}^4 O_{Rj}$  (28)

where the  $O_{Sj}$  and  $O_{Rj}$  are given by eqs. (20) through (27).

4.3.2.3 Changes in aggregate costs. Expressions for the changes in the aggregate cost due to conversion,  $A_S$  and  $A_R$ , can be derived by substituting into eq. (11) the expressions derived for  $S_S$ ,  $S_R$ ,  $O_S$ , and  $O_R$ . If the value of  $A_S$  is positive, the aggregate cost due to stabilization will not be reduced, and the area should not be converted to SAC. An alternative, less costly network enhancement should be sought to correct the most serious of the network conditions.

If  $A_S$  is negative, stabilization can be expected to reduce the aggregate cost of the allocation area. If  $A_R$  is also negative, then the conversion work may be undertaken as designed. However, if  $A_R$  is not negative, then more selective (or no) replacement work should be investigated until a negative or zero value of  $A_R$  is achieved. The resulting design may be adopted for the conversion in this case.

#### V. CONTROL SYSTEM

#### 5.1 Feedback

The control system is essential to achieving a reduction in the aggregate cost. Data from the information processing system are used in the control system as feedback to verify that the reduction in aggregate cost, predicted by the engineering applications system, is actually achieved.

In considering whether a particular enhancement is economical, the engineering applications system utilizes reduction factors for each type of MOC work operation. These reduction factors are now used to set an objective level for occurrences in the allocation area of each type of work operation. The objective level is computed by multiplying the reduction factor by the semiannual level of occurrences which existed in the portion of the allocation area to be affected by the MSC in the period prior to incurring the MSC and then adding the result to the semiannual level of occurrences in the portion of the allocation area that will not be affected by incurring the MSC.

By using data on the historical report for the six-month period after the work associated with the MSC has been completed, the actual levels of occurrences of each MOC work operation are compared with the objective. If the objectives are met and continue to be met, the aggregate cost has been reduced. If not, the techniques of the engineering applications systems may need to be reapplied to determine what is preventing the cost reduction. Work along the feeder, for instance, may be causing service interruptions that increase the number of work operations. Another possible explanation is that the level of occurrences of MOC have increased in the portion of the allocation area not treated. It may now be desirable in that portion of the allocation area to incur an additional MSC that will reduce the aggregate cost.

Field experience has shown that objectives are usually met. When they are not, the cause can usually be determined and successful corrective action taken.

## 5.2 Management procedures

Other elements of the control system are management procedures. One is that use of the engineering applications systems be required for approval of all MSC items. Another is a forum in which managers of the various forces involved in loop provisioning and maintenance regularly discuss the status of high cost allocation areas and agree on the MSCs which will reduce the aggregate cost. In some cases the areas treated are different than if the costs to only one work force are considered. Such cases demonstrate the importance of considering costs to all forces when deciding where to incur MSCs.

#### VI. APPLICATION

The Facility Analysis Plan is being applied successfully in several telephone companies. In most it has been introduced with manual data manipulation procedures. In one company, the plan has been introduced using prototype computerized versions of both the information processing system and the economic analysis segment of the engineering applications system. The benefits of computerization have been lower cost, improved accuracy, and the ability to produce report formats that are not practical to produce manually. One such report makes it possible for the company to allocate money for MSCs to its operating divisions by depicting the distribution of the normalized yearly MOC in the allocation areas of each division.

Experience from these applications is being used to further refine the models and processes in the plan, and to expand its application to other companies. In all cases, application of the plan has reduced the aggregate cost of providing and maintaining loop facilities by much more than the relatively small cost of applying the plan, while at the same time improving service to the customer.

#### VII. ACKNOWLEDGMENT

Many people contributed to the development of the Facility Analysis Plan. Some have authored other papers in this issue. The authors are grateful to all, but wish in particular to acknowledge the constant inspiration of Joseph O. Bergholm, without whom the plan would never have been developed.

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