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Almost-Coherent Detection of Phase-Shift-Keyed Signals Using an Injection-Locked Oscillator

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We analyze a proposed scheme of detection of phase-shift-keyed signals using an injection-locked oscillator the bandwidth of which is much less than the modulation rate. The output of the oscillator is a carrier with essentially all of its modulation removed. We analyze the effect of noise and signal modulation on the phase of this reference tone and compute its effect on the probability of detection error. If a suitable encoder and decoder are used for the transmitted signal, this technique can provide nearly ideal coherent demodulation.

I. INTRODUCTION

The two generally recognized methods of detection of phase-shift-keyed (PSK) signals are coherent detection and differential detection. Coherent detection has been shown to be optimum in the presence of Gaussian noise,[1] but, due to the difficulty of storing an absolute phase reference at the receiver, it is seldom used in practice.

In a recent paper,[2] B. Glance showed that an injection-locked oscillator, the locking bandwidth of which is much less than the modulation rate can, under certain conditions, be used to derive a phase reference from the input signal itself. This is actually a form of a quadrature reference system, where the phase of one quadrature remains essentially unkeyed, and is used to provide the reference tone.[3,4] In this paper, we examine the effectiveness of this scheme for a two-phase PSK system where the modulation is a random digital signal and additive Gaussian noise is present. We derive an expression for the probability distribution of the reference phase, and from this calculate the average probability of a detection error. We find that if the modulation rate is much greater than the bandwidth of the oscillator, and if a suitable encoder and decoder are used, the method very nearly approaches the ideal performance of coherent detection.

II. LOCKING EQUATION ANALYSIS—ZERO ORDER SOLUTION

A portion of the received signal is used as the input to the injection-locked oscillator. This signal may be represented as

$$x(t) = \sqrt{2}\, A \cos [\omega t + \theta(t)] + n(t), \tag{1}$$

where A is the signal power, ω is the carrier frequency, and $\theta(t)$ is the phase modulation. The received signal is assumed to be contaminated by additive white Gaussian noise, $n(t)$, with double-sided spectral density $N_0/2$. If ω is sufficiently close to the natural oscillator frequency, ω_0, locking will occur and the output of the oscillator will be

$$y(t) = \sqrt{2}\, B \cos [\omega t + \theta(t) - \phi(t)], \tag{2}$$

where B can be assumed to be constant.[5] $\phi(t)$ is the phase difference between the input and output signals of the oscillator. In the case of interest, the total phase modulation of the oscillator output, $\eta(t) = \theta(t) - \phi(t)$, is small, and $y(t)$ is used as the phase reference in the coherent detection of the remaining portion of the received signal.

In the absence of noise, the phases of the input and output signals of the oscillator are related by the well-known locking equation[6]

$$\frac{d\phi(t)}{dt} + \Delta \sin \phi(t) = \omega - \omega_0 + \frac{d\theta(t)}{dt}, \tag{3}$$

where 2Δ is the locking bandwidth of the oscillator. This equation takes the same form as that for a first order phase-locked loop.[7] The effect of the noise at the input has been analyzed by Viterbi.[8] The effect is to add an additional term to eq. (3),

$$\frac{d\phi(t)}{dt} + \Delta \sin \phi(t) = \omega - \omega_0 + \frac{d\theta(t)}{dt} - \frac{\Delta}{A} n'(t), \tag{4}$$

where $n'(t)$ has the same statistics as $n(t)$. We rewrite eq. (4) in terms of $\eta(t)$.

$$\frac{d\eta(t)}{dt} + \Delta \sin [\eta(t) - \theta(t)] = \omega_0 - \omega + \frac{\Delta}{A} n'(t). \tag{5}$$

For the case of zero input phase modulation, i.e., $\theta(t) = 0$, eq. (5) becomes

$$\frac{d\eta(t)}{dt} + \Delta \sin \eta(t) = \omega_0 - \omega + \frac{\Delta}{A} n'(t). \tag{6}$$

Using Fokker-Plank techniques, Viterbi derived from this equation

$p(\eta)$, the steady-state probability density of η. For $\omega = \omega_0$ the solution is

$$p(\eta) = \frac{e^{\alpha^2 \cos \eta}}{2\pi I_0(\alpha^2)}, \tag{7}$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind, and $\alpha^2 = 4A^2/N_0\Delta$ is the signal-to-noise ratio in the bandwidth of the oscillator. For $\alpha \gg 1$, this distribution for η small is nearly Gaussian with mean zero and standard deviation $1/\alpha$. For $\omega \neq \omega_0$ the distribution becomes centered about the point $\beta = \sin^{-1} (\omega_0 - \omega)/\Delta$. In the case, $\alpha \gg 1$ and $|(\omega_0 - \omega)/\Delta| \ll 1$, $\beta \approx (\omega_0 - \omega)/\Delta$ and for η small, the distribution is very nearly equal to

$$p(\eta) \approx \frac{e^{\alpha^2 \cos (\eta - \beta)}}{2\pi I_0(\alpha^2)}. \tag{8}$$

We now consider the case $\theta(t) \neq 0$. In a binary PSK signal, $\theta(t)$ is a waveform of the form

$$\theta(t) = \sum_n a_n p(t - nT), \tag{9}$$

where $a_n = \pm 1$, and $p(t)$ is assumed to be a pulse which is nonzero only over the range $0 < t < T$. For the moment we assume zero noise. The resulting equation is

$$\frac{d\eta(t)}{dt} + \Delta \sin [\eta(t) - \theta(t)] = \omega_0 - \omega \tag{10}$$

which we rewrite as

$$\frac{d\eta(t)}{dt} + \Delta \sin \eta(t) \cos \theta(t) - \Delta \cos \eta(t) \sin \theta(t) = \omega_0 - \omega. \tag{11}$$

Our technique for the solution of this case will suggest a method of handling the stochastic problem when the noise term is reintroduced.

The exact solution to eq. (11) is difficult or impossible to obtain for general $\theta(t)$. Let us therefore take advantage of the fact that $T \ll 1/\Delta$ to derive a differential equation, the solution of which approximates that of eq. (11).

We assume that there exists an interval of length τ with the property that $T \ll \tau \ll 1/\Delta$, and we take the average of eq. (11) over τ. Letting $\langle \ \rangle$ denote this averaging operation, i.e.,

$$\langle \eta(t) \rangle = \frac{1}{\tau} \int_{t-(\tau/2)}^{t+(\tau/2)} \eta(u) \, du \tag{12}$$

there results

$$\frac{d\langle\eta(t)\rangle}{dt} + \Delta\langle\sin\eta(t)\cos\theta(t)\rangle - \Delta\langle\cos\eta(t)\sin\theta(t)\rangle = \omega_0 - \omega. \tag{13}$$

Choosing $\tau \ll 1/\Delta$ insures that $\eta(t)$ is very nearly constant over the interval of averaging, for, from eq. (10), we have $|d\eta(t)/dt| \leqq \Delta + |\omega_0 - \omega| \leqq 2\Delta$. Consequently, $\eta(t) \approx \langle\eta(t)\rangle$ and the quantities $\sin\eta(t)$ and $\cos\eta(t)$ may be taken outside the averaging operator.

$$\frac{d\langle\eta(t)\rangle}{dt} + \Delta\sin\langle\eta(t)\rangle\langle\cos\theta(t)\rangle - \Delta\cos\langle\eta(t)\rangle\langle\sin\theta(t)\rangle \approx \omega_0 - \omega. \tag{14}$$

On the other hand, the choice $\tau \gg T$ insures that there will be many data pulses over the interval of averaging. Thus the quantity $\langle\cos\theta(t)\rangle$ is very nearly constant and is equal to

$$C \equiv \langle\cos\theta(t)\rangle = \frac{1}{T}\int_0^T \cos p(t)\,dt. \tag{15}$$

(Table I lists the value of this quantity for three important pulse shapes.)

Since we usually have no control over the transmitted message, the quantity $\langle\sin\theta(t)\rangle$ will not, in general, be time-independent. However, a scheme has been suggested by C. L. Ruthroff and W. F. Bodtmann[9] in which, through the use of a simple encoder and decoder, this quantity can be made very nearly equal to zero.

TABLE I—VALUES OF C AND σ FOR THREE PULSE SHAPES

$p(t)$	$C = \frac{1}{T}\int_0^T \cos p(t)\,dt$	$\sigma = \frac{1}{T}\int_0^T \sin p(t)\,dt$
raised cosine $\frac{\theta_0}{2}\left[1 - \cos\frac{2\pi t}{T}\right]$	$J_0\left(\frac{\theta_0}{2}\right)\cos\frac{\theta_0}{2}$	$J_0\left(\frac{\theta_0}{2}\right)\sin\frac{\theta_0}{2}$
positive sine $\theta_0\sin\frac{\pi}{T}t$	$J_0(\theta_0)$	$\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{J_{2k+1}(\theta_0)}{2k+1}$
rectangular $\theta_0,\ 0 \leqq t \leqq T$	$\cos\theta_0$	$\sin\theta_0$

The encoder is a device which stores a block of N bits of the message, and counts the net difference of $+1$'s and -1's contained in the block. It also keeps a separate count of the net difference in $+1$'s and -1's which have been sent over the entire past history of the message. The entire block is transmitted either with normal polarity, or with reversed polarity, in such a way as to cause the accumulated count to come as close as possible to zero. Preceding each block is a single "code" bit which specifies the polarity of that block. (The code bits are included in the counts.) The decoder decodes the message in an obvious manner.

We assume that $\theta(t)$ is the output signal of such an encoder. Assuming $\tau \gg NT$, we have

$$\langle \sin \theta(t) \rangle \approx 0, \tag{16}$$

so that our approximating differential equation becomes

$$\frac{d\eta_0(t)}{dt} + \Delta C \sin \eta_0(t) = \omega_0 - \omega. \tag{17}$$

We call the solution to this equation the "zero-order approximation to $\eta(t)$".

We note that this equation has the identical form as eq. (10) for the case of zero modulation. Thus the "zero-order effect" of the modulation is to reduce the effective value of the locking bandwidth by a factor C.

In Figs. 1 and 2, we demonstrate the above results. In both figures,

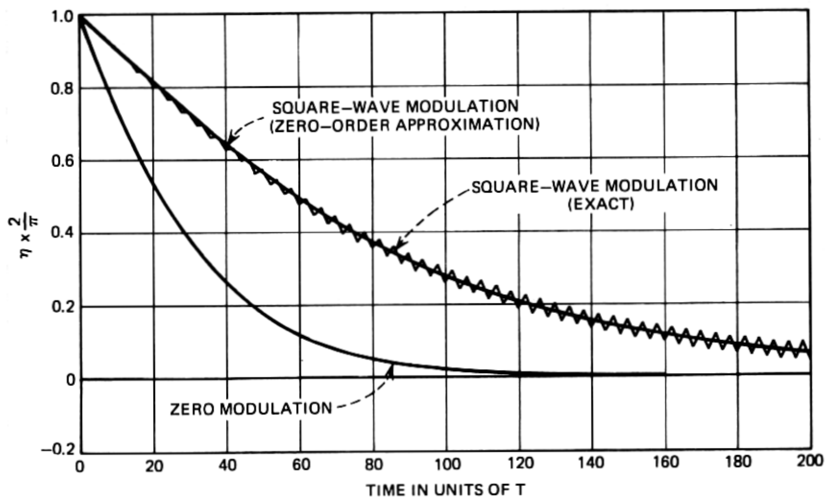

Fig. 1—$\eta(t)$ with square-wave modulation.

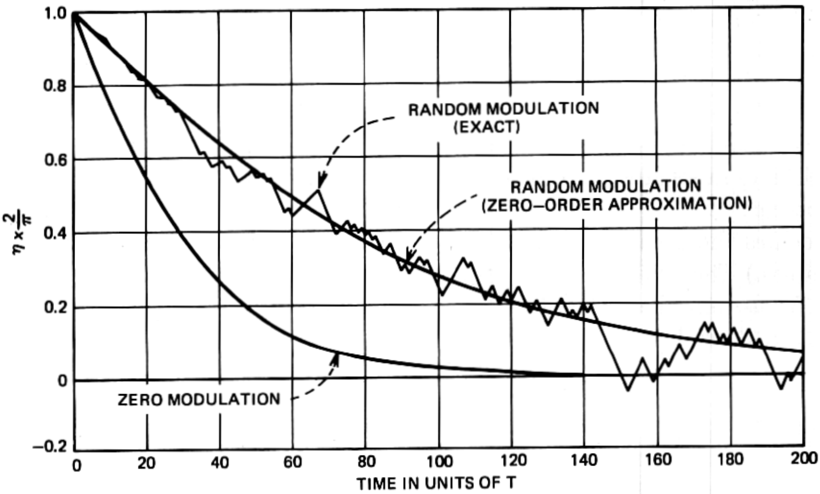

Fig. 2—$\eta(t)$ with random modulation.

$\Delta = \pi/80T$. The lower curve of Fig. 1 shows the behavior of $\eta(t)$ versus time for the case of zero modulation, i.e., $\theta(t) = 0$. The saw-tooth wave shows the behavior of $\eta(t)$, that is, the exact solution to eq. (10), for the case where $\theta(t)$ is a square wave with amplitude $3\pi/8$ and period $4T$. The smooth curve drawn over the saw-tooth wave shows the zero-order approximation obtained from eq. (17). As can be seen, the main effect of the modulation has been an increase in the decay time of the resulting curve. This is a result of the decrease in the effective locking bandwidth caused by the modulation, as predicted above.

In addition to this, the true curve has a "wiggle" which seems to increase in size as the curve approaches zero, and which reaches a maximum magnitude of about $0.02 \times \pi/2$.

Figure 2 shows the true curve and the zero-order approximation where the phase modulation is a random binary signal with Prob $(+1) =$ Prob $(-1) = \frac{1}{2}$ which has then been passed through an encoder with $N = 5$; and a rectangular pulse shape of amplitude $3\pi/8$ is used. We note the curve follows the same overall path as in Fig. 1. The "wiggles" are now random in character; we note that their amplitude again appears to grow as the curve approaches zero.

III. FIRST ORDER SOLUTION

To better understand the behavior of these "wiggles" we now derive a correction term, $\eta_1(t)$, which, when added to $\eta_0(t)$, improves the accuracy of our solution. We call $\eta_0(t) + \eta_1(t)$ the "first-order approximation to $\eta(t)$."

We define $\eta'(t) = \eta(t) - \eta_0(t)$. Subtracting eq. (17) from eq. (10), there results

$$\frac{d\eta'(t)}{dt} + \Delta \sin \eta'(t) \cos [\eta_0(t) - \theta(t)] = \Delta C \sin \eta_0(t) - \Delta \cos \eta'(t) \sin [\eta_0(t) - \theta(t)]. \tag{18}$$

If the assumptions we have made above are valid, the error in the zero-order approximation will be small, $|\eta'(t)| \ll 1$, and we may linearize eq. (18).

$$\frac{d\eta'(t)}{dt} + \Delta\eta'(t) \cos [\eta_0(t) - \theta(t)] = \Delta C \sin \eta_0(t) - \Delta \sin [\eta_0(t) - \theta(t)]. \tag{19}$$

Since $|\eta'(t)| \ll 1$, the second term on the left-hand side may be neglected, and we have approximately

$$\eta'(t) \approx \int_0^t \{\Delta C \sin \eta_0(t) - \Delta \sin [\eta_0(t) - \theta(t)]\} \, dt, \tag{20}$$

where we assume the initial conditions are accounted for in $\eta_0(t)$. $\eta_0(t)$ varies slowly compared to $\theta(t)$, so we treat it as a constant in the integral. Our correction term is

$$\eta_1(t) = \Delta \sin \eta_0(t) \cdot \int_0^t [C - \cos \theta(t)] \, dt + \Delta \cos \eta_0(t) \int_0^t \sin \theta(t) \, dt. \tag{21}$$

Equation (21) explains the behavior of the "wiggles" in Figs. 1 and 2. If rectangular pulses are used, $\cos \theta(t) = C$, and

$$\eta_1(t) = \Delta \cos \eta_0(t) \int_0^t \sin \theta(t) \, dt. \tag{22}$$

For a square wave of period $4T$ and amplitude $3\pi/8$, $\eta_1(t)$ is a saw-tooth wave with amplitude $\Delta T \cos \eta_0(t) \sin 3\pi/8 = 0.924 \, \Delta T \cos \eta_0(t)$. The amplitude of the saw-tooth wave is seen to vary as the cosine of $\eta_0(t)$, and reach a maximum amplitude of $0.924 \, \Delta T$, which is $0.023 \times \pi/2$ for Fig. 1. This behavior agrees with our earlier observations. The accuracy of our approximation in this case is quite good. A plot of $\eta_0(t) + \eta_1(t)$ superimposed on the figure can not be distinguished by eye from the true curve $\eta(t)$.

We remark that the first term of eq. (21), which is zero for a rectangular pulse shape, may in general be ignored relative to the second term. The first term equals zero at the beginning and end of each pulse, and never achieves magnitude greater than $\Delta T \sin \eta_0(t)$. The second term, however, increases monotonically during a positive pulse and decreases monotonically during a negative pulse. The magnitude of this term can reach a maximum of $\frac{3}{2} N \Delta T \sigma \cos \eta_0(t)$, where

$$\sigma = \frac{1}{T} \int_0^T \sin p(t)\, dt. \tag{23}$$

(The factor $\frac{3}{2}$ arises because of the possibility of having a message block consisting of N +1's followed by a block consisting of $N/2$ +1's and $N/2$ −1's). For the case of interest, $\eta_0(t)$ will be near zero and N will be greater than about 10 or 20. This means that the second term of eq. (21) will predominate, and thus eq. (22) may be used for arbitrary pulse shapes. (Table I lists the value of σ for three important pulse shapes.)

IV. THE EFFECT OF NOISE

We now consider a system where both noise and modulation are present. We saw in eq. (17) that the zero-order effect of the modulation was simply to reduce the locking bandwidth by a factor C.

Accordingly, we take as our zero-order approximation the solution to the stochastic differential equation

$$\frac{d\eta_0(t)}{dt} + \Delta C \sin \eta_0(t) = \omega_0 - \omega + \frac{\Delta}{A} n'(t). \tag{24}$$

In this case, we are interested in the steady-state probability density for η_0, $p(\eta_0)$. By comparison with eqs. (6) and (8), the solution can be written down immediately.

$$p(\eta_0) \approx \frac{e^{\alpha_e^2 \cos (\eta_0 - \beta_e)}}{2\pi I_0(\alpha_e^2)}, \tag{25}$$

where α_e^2, the effective signal-to-noise ratio in the presence of modulation, is higher than the zero-modulation signal-to-noise ratio by a factor of $1/C$.

$$\alpha_e^2 = \frac{\alpha^2}{C} = \frac{4A^2}{N_0\, \Delta C}. \tag{26}$$

The average phase shift due to frequency offset is increased.

$$\beta_e = \sin^{-1} \frac{\omega_0 - \omega}{\Delta C} \approx \frac{\omega_0 - \omega}{\Delta C} = \frac{\beta}{C}. \tag{27}$$

To get a correction term in our solution, we proceed as before, by taking the difference between eqs. (24) and (5). We notice that the noise term cancels, and we again obtain eq. (18) and the approximate solution, eq. (22). For the case of large signal-to-noise ratio, α_e , η_0 will, with high probability, be in the vicinity of zero (assuming $\beta_e \approx 0$). Since η_0 effects η_1 only as the cosine, η_1 is essentially independent of η_0 in this case and is equal to

$$\eta_1(t) = \Delta \int_0^t \sin \theta(t)\, dt. \tag{28}$$

$\eta_1(t)$ depends on the particular digital signal being transmitted. However, the use of the encoder described earlier insures that

$$|\eta_1(t)| < \tfrac{3}{2} N\Delta T\sigma. \tag{29}$$

Thus if each of the components is small, the output phase is seen to consist of the sum of three essentially independent parts:

(*i*) A constant $(\omega_0 - \omega)/\Delta C$ resulting from the difference between the carrier frequency and the natural oscillator frequency.
(*ii*) A time varying part which depends upon the digital modulation, and which has a maximum magnitude of $\frac{3}{2} N\Delta T\sigma$.
(*iii*) A random part, due to noise, the distribution of which has a standard deviation of $1/\alpha_e = \sqrt{N_0 \Delta C}/2A$.

Thus we can write the probability distribution of the reference phase approximately as

$$p(\eta) = \frac{e^{\alpha_e^2 \cos(\eta - \psi)}}{2\pi I_0(\alpha_e^2)}, \tag{30}$$

where ψ, the nonrandom portion of the phase, has a magnitude less than or equal to $\frac{3}{2} N\Delta T\sigma + |(\omega_0 - \omega)/\Delta C|$.

V. CALCULATION OF ERROR PROBABILITY

If η has a known value, the probability of a decoding error, assuming equal likelihood detection, is

$$P_e = \tfrac{1}{2} \operatorname{erfc}(\rho \cos \eta), \tag{31}$$

where ρ^2 is the signal-to-noise ratio in the bandwidth of the signal.[10] If the receiving filter has a bandwidth $2W$, then

$$\rho^2 = \frac{A^2}{N_0 2W}, \tag{32}$$

where A is the rms signal amplitude, and $N_0/2$ is the double-sided spectral density of the noise. The average error probability $\bar{P}_e$, is obtained by averaging the quantity in eq. (31) over the possible values of η.

$$\bar{P}_e = \int_{-\pi}^{\pi} \frac{1}{2} \operatorname{erfc}(\rho \cos \eta) \frac{e^{\alpha_e^2 \cos(\eta - \psi)}}{2\pi I_0(\alpha_e^2)} d\eta. \tag{33}$$

This integral was performed using an expansion technique similar to that described in Ref. 10.

VI. DEMONSTRATION OF RESULTS

In order to reduce the noise as much as possible, the bandwidth of the receiving filter in a PSK system is usually set at the value which allows the signal to pass essentially undistorted. This bandwidth $2W$ is roughly given by

$$2W \approx \frac{1.6}{T}. \tag{34}$$

Thus the signal-to-noise ratio in the bandwidth of the loop, $\alpha_e^2 = (4A^2/N_0 \, \Delta C)$, is related to the signal-to-noise ratio of the received signal, $\rho^2 = (A^2/N_0 2W)$, as

$$\alpha_e^2 = \frac{6.4}{\Delta TC} \rho . \tag{35}$$

The effect of noise on the output phase is thus reduced by a factor of $\Delta TC/6.4$ from its input value. We have already seen that the effect of the phase modulation on the output phase was proportional to $\Delta T\sigma$. Thus the size of the quantity ΔT is important in determining the performance of the system: it should be kept as small as possible. The extent to which this can be done, however, is limited by the need to keep the difference between the carrier frequency and the natural oscillator frequency small relative to ΔC. This frequency difference may be reduced by the use of a negative feedback loop.[11] A reasonable value of ΔT presently obtainable in the laboratory for which these conditions can be satisfied is $\Delta T \approx 10^{-3}$.

Using the value $\Delta T \approx 10^{-3}$, and the relationship of eq. (35), the methods of the preceding section were employed to compute the average error probability. Under the assumption that the nonrandom portion of the output phase is 10 degrees, the results of this computation for a raised cosine pulse shape are plotted in Fig. 3. Also plotted in the figure are the curves representing true coherent detection, $\bar{P}_e = \frac{1}{2} \operatorname{erfc}(\rho)$, and

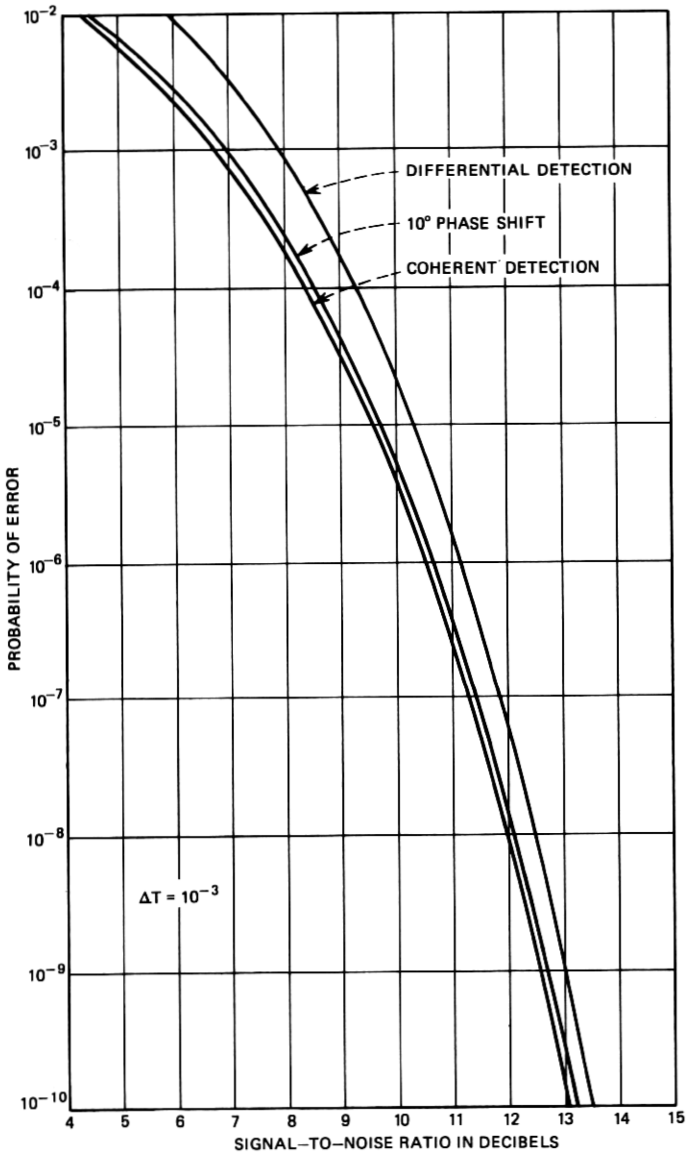

Fig. 3—Probability of error as a function of signal-to-noise ratio.

differential detection $\bar{P}_e = \frac{1}{2}e^{-\rho^2}$.[12] As can be seen, the curve comes quite close to the ideal of coherent detection. The curve corresponding to a nonrandom phase shift of 5 degrees was also computed but is not plotted in Fig. 3 because it comes so close to the coherent curve that the two cannot be distinguished on the scale of the drawing.

By taking the nonrandom part of the output phase to be zero, it is possible to determine the increase in error probability (over the coherent case) which is due to noise. We found the error probability to be virtually identical to coherent detection in this case. This indicates that the effect of the noise on the output phase shift is negligible.

For reasonably small values of $\Delta T (\leqq 10^{-3})$, and for the range of signal-to-noise ratios usually of interest (>5 db) the output phase shift resulting from noise may be safely ignored: the increase in error probability of the proposed system over coherent detection is almost entirely due to the effects of modulation and the offset in the carrier frequency. Consequently, under these conditions an approximate expression for the error probability, which is very nearly correct is $\bar{P}_e = \frac{1}{2} \operatorname{erfc} (\rho \cos \psi)$, where ψ is the phase shift resulting from the modulation and carrier offset. In Fig. 3, for example, the plotted curve is almost identical to $\frac{1}{2} \operatorname{erfc} (\rho \cos 10 \text{ degrees})$.

For $\Delta T = 10^{-3}$, a total of 289 consecutive raised-cosine pulses of maximum amplitude $\pi/2$ with the same polarity are needed to shift the output phase by 10 degrees. For $\Delta T = 10^{-2}$, this number is reduced to 29. For positive sine pulses of maximum amplitude $\pi/2$, the corresponding numbers are 241 and 24 respectively. We remark that increasing ΔT from 10^{-3} to 10^{-2} also increases the effect of the noise on the output phase, but that this effect remains negligible.

In order to demonstrate the effect of the noise on the output phase, we must consider an extremely high-noise example. Figure 4 plots the average error probability versus the signal-to-noise ratio over a range of from −7 to +7 db, for the case $\Delta T = 10^{-1}$. Nonrandom phase shifts of zero and 10 degrees were assumed respectively. As can be seen, the zero-phase shift curve almost coincides with the coherent curve for $\rho \geqq 4$ db, even for this large value of ΔT.

VII. CONCLUSIONS

We have analyzed the proposed system of PSK detection for the case of random modulation and additive Gaussian noise. If the modulation rate is much greater than the bandwidth of the oscillator ($\Delta T \ll 1$), and if a suitable encoder and decoder are used, we have shown that the

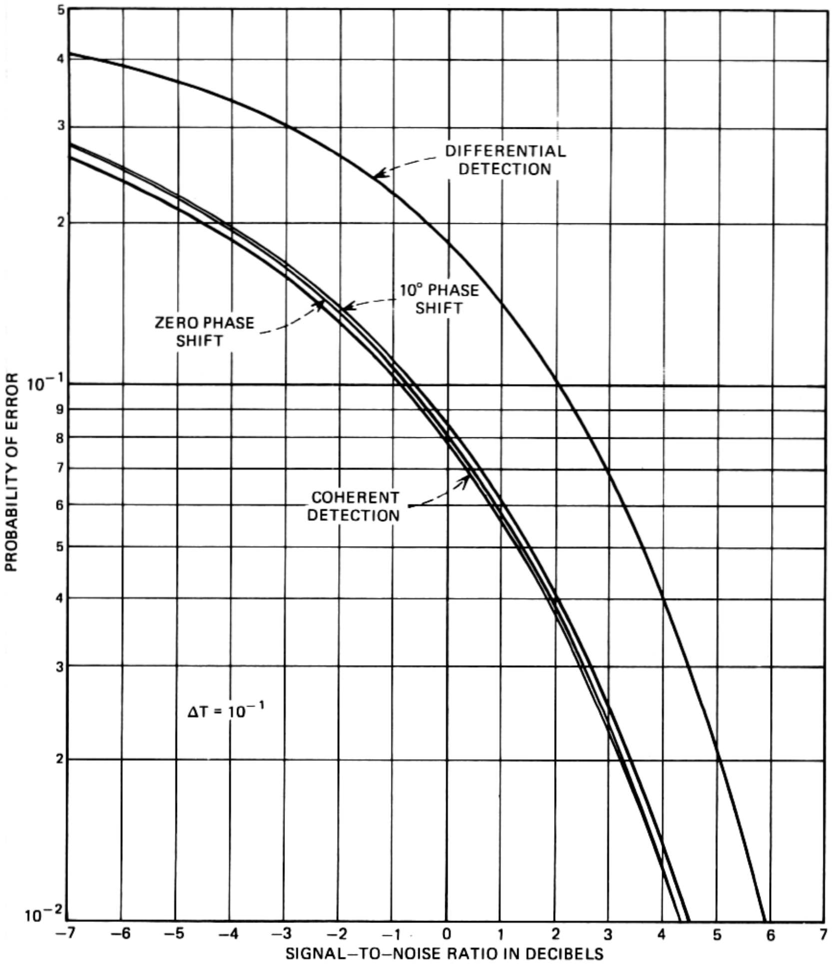

Fig. 4—Probability of error as a function of signal-to-noise ratio.

system will perform as almost a perfect coherent detector. For a binary system, this means a power savings of about $\frac{1}{2}$ db over the presently employed method of differential detection. This is not very great. However, with only a slightly more complex encoder and decoder, this same technique may be utilized for higher level systems. (The analysis requires only minor modifications.) For a 4-level system, for example, the power savings over differential detection is about 3 db which is significant.

Construction of an experimental encoder and decoder is presently being carried out in the laboratory, and tests of the system are planned to confirm the theoretical results.

VIII. ACKNOWLEDGMENTS

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