THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 51, No. 7, September, 1972
Printed in U.S.A.

Methods for Designing Differential Quantizers Based on Subjective Evaluations of Edge Busyness

By J. C. CANDY and R. H. BOSWORTH

(Manuscript received February 18, 1972)

This is a study of the visibility of television noise and its dependence on the instantaneous rate of change of the video signal. Noise added to a picture tends to be least noticeable in regions where the brightness changes rapidly, but the relationship between visibility-of-noise and slope-of-the signal is dependent on the scene being displayed. Measurements are presented for four different scenes, and these data are used to design companding laws that minimize the visibility of noise from Differential Quantizers. The designs agree with those that have been satisfactory in practical applications.

I. INTRODUCTION

Experience with Differential Quantizers[1,2,3] teaches us that noise superimposed on brightness boundaries of a picture is less objectionable than similar noise on areas of uniform brightness. This work is an attempt to evaluate the phenomenon and make efficient use of it when designing quantizing scales for Differential Coders. We know that there is advantage in companding the quantization levels so that the largest steps are used for reconstructing rapidly changing signals and the smallest steps for slowly changing signals. In the past, the companding laws for differential quantizers have usually been obtained experimentally by trial and error, but these techniques are inadequate when there are a large number of levels. A more objective method is described here; it provides a basis for theoretical synthesis and evaluation of companding laws independent of the number of levels that are required.

The essence of the method is a subjective evaluation of the dependence of the visibility of noise on the rate of change of the signal. The measurement is used to determine how the visibility of the net quantization noise in a picture depends on the companding law. Laws that minimize this visibility are then obtained and they agree very well with those that have been obtained experimentally. Reference 4 describes a design of differential quantizers that is based on the probability of slopes occuring in pictures and on a weighting function for the visibility of noise.

II. EVALUATING EDGE BUSYNESS

2.1 *Simulating Edge Busyness in Pictures*

Noise added to a video signal at times when it changes rapidly has the appearance of "busyness" near brightness boundaries of the scene.[1,3] In order to measure its visibility, the noise must be introduced into the signal in a controlled fashion. A convenient method compares the rate of change of the signal amplitude with a threshold value, and adds calibrated noise at times when the threshold is exceeded.

Figure 1 illustrates the circuit used for the experiment. The input, a signal that simulates that of a *Picturephone*® system, passes through two parallel paths. In the upper path the signal is first differentiated, and then full-wave rectified to obtain a voltage that is proportional to the magnitude of the rate of change of the input. Whenever this voltage exceeds a threshold value, a trigger circuit fires and gate G conducts. A random signal is then fed through a high-pass filter and added, under control of switch S, to the video signal. In the picture, this noise resembles the edge busyness which is characteristic of differential coding. Its amplitude is controlled by calibrated attenuator A1.

The observer has the choice of two pictures by means of switch S: in position A, edge busyness is added to the scene; in position B, white noise is added to the entire scene. The observer varies the setting of attenuator B1 until, in his judgment, the quality of the picture is the same for the two positions of the switch. The experiment is repeated for various settings of the threshold and of the attenuator A1.

Attenuators A1 and B1 were calibrated to read the ratio of the rms amplitude of the noise to the peak signal. The rms value of the "edge-noise" was measured when the threshold was set at zero, in order that the noise be present at all times. This gives a measure of the noise in areas where it is displayed as opposed to a measure of the noise in the entire picture.

The video signal used in the experiment was derived from a picture scanned with 271 interlaced lines at 30 frames a second. The visible portion of the picture was about 13 cm high and 14 cm wide. The peak

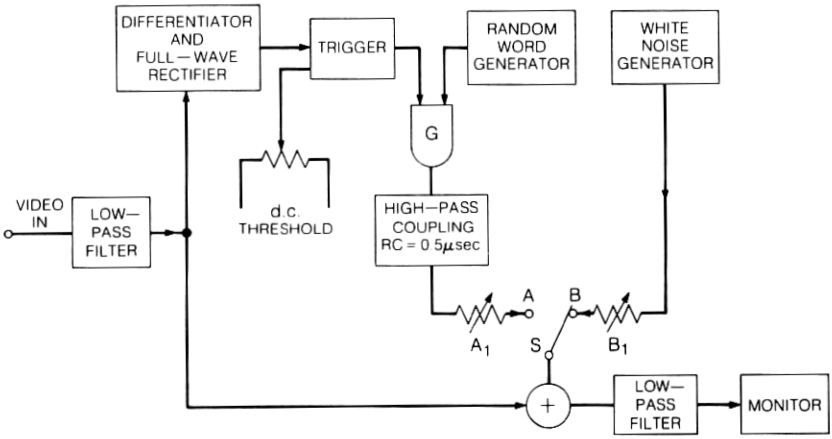


Fig. 1—The circuit used for simulating edge busyness and comparing it with white noise.

luminance was about 70 foot lamberts and the room illuminance about 60 foot candles. The white noise used in the experiments was approximately Gaussian with a flat spectrum from 100 Hz to 1 MHz. The edge noise was obtained from a generator of random binary-words, driven at 2 MHz. The low-pass filters placed at the input and output of the system had frequency characteristics approximating a fourth-order Butterworth filter. At 1 MHz, the gains of the input and output filters had fallen by 6 dB and 15 dB, respectively.

White noise with Gaussian amplitude distribution is used as a standard of comparison because techniques already exist[5] for describing the impairments introduced by uniform quantizing as an equivalent white noise. Moreover, it is a convenient and readily available reference for testing quantizers. A quasi-random binary signal is used as the source of edge noise because it is easily gated and may be synchronized to the scanning waveforms.

2.2 *Comparing Edge Busyness with White Noise*

The success of the experiments depends on the ability of observers to compare different impairments in a picture. These, and previous experiments[5,6] have demonstrated that experienced observers have little difficulty in making the measurements. Inexperienced observers either tire or go through a learning period during which their measurements are not always reproducible. The results reported here were obtained by experienced observers. Their measurements tend to be consistent and reproducible provided the picture quality is reasonably good;[7] that is, having signal-to-noise ratios in excess of 40 dB. Eight observers took part in this study, and the differences between their opinions were small. The difference could usually be expressed as a variation of less than ±1 dB in the sensitivity to noise. Most of the results presented are those obtained by one of the observers. His measurements have been compared with the other observers and are representative of this population. In future studies it may be useful to examine the differences between observers more carefully, but here we shall investigate the much larger differences that occur when scenes, and the type of noise is changed.

Three main experiments are described, their purpose is to find the following:

(*i*) The relationship between the amplitude of the equivalent white noise and the amplitude of the edge busyness for fixed threshold values. (It will be shown that their amplitudes are proportional to one another.)

(*ii*) The relationship between the amplitude of the equivalent white noise and the threshold setting. (It will be demonstrated that the relationship is strongly dependent on the scene.)

(*iii*) An expression for the net equivalent white noise when several unrelated distortions are introduced into the picture. (It will be shown that the power of the white noise that is equivalent to the combined distortions equals the sum of the powers of the white noises that are equivalent to the individual components of the distortions.)

III. EFFECT OF VARYING THE AMPLITUDE OF THE EDGE NOISE

The amplitude of the white noise that was considered to be equivalent to edge busyness was measured for a number of amplitudes of the busyness. Typical results obtained by eight observers are plotted in Fig. 2. The ordinate of this graph is divided into four regions which correspond to subjective classifications of the noise: Objectionable, Annoying, Tolerable, and Undetectable. We are mostly interested in the region where the noise is tolerable; here the results are represented quite well by a straight line having unit slope. This indicates that the amplitude of the equivalent white noise is proportional to the amplitude of the edge noise. The constant of proportionality depends on the observer, but the variation is only ±5 percent.

The data in Fig. 2 were obtained for a threshold value of 0.07; that is,

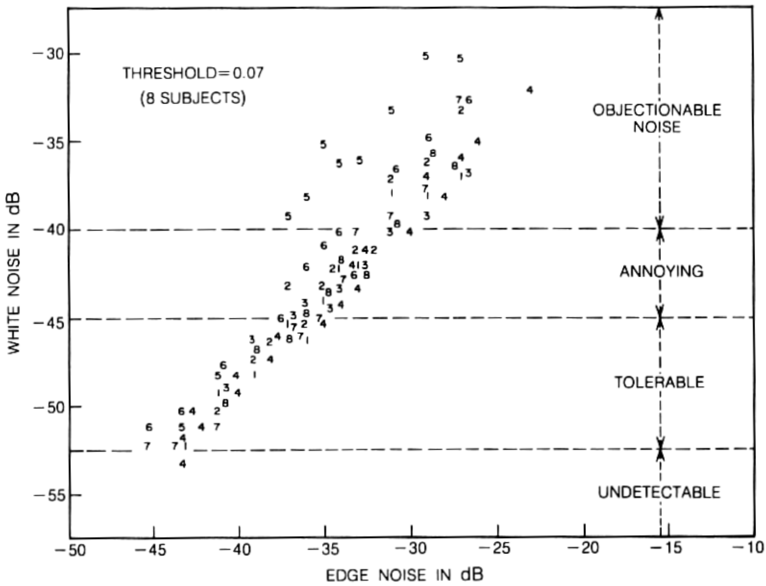


Fig. 2—The white noise that is equivalent to various amplitudes of edge noise for one threshold. There were eight observers.

the noise was added to the signal whenever it changed by more than 7 percent of peak amplitude in a Nyquist Interval (0.5 μs). Figure 3 shows similar graphs for different threshold values. Notice that for zero threshold, the white noise equals the edge noise; this is a consequence of the method used for calibrating the system. As the threshold increases, the amplitude of the equivalent noise decreases but for each threshold, the amplitudes of the two noises, as indicated by the attenuator settings, are proportional to one another.

We now define $F(x)$ as the relative power of the white noise for a threshold x. It is the power of the white noise, $V_w(x)$, that is equivalent to unit noise added to the picture at times when the video signal changes at a rate greater than x, ie.,

$$F(x) = \frac{V_w^2(x)}{V_E^2}. \tag{1}$$

IV. THE RELATIVE WHITE NOISE AS A FUNCTION OF THRESHOLD

The relative white noise is plotted against threshold values in Fig. 4b for the scene shown in Fig. 4a. Also shown in Fig. 4b is the probability

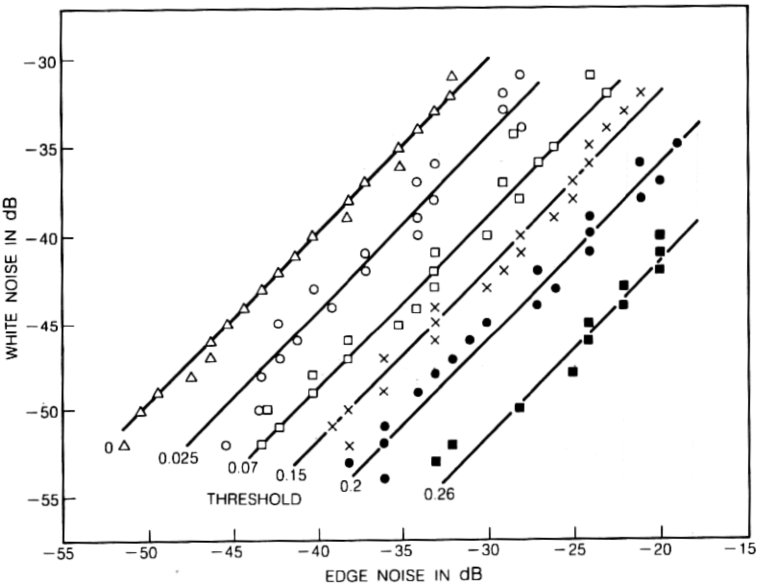
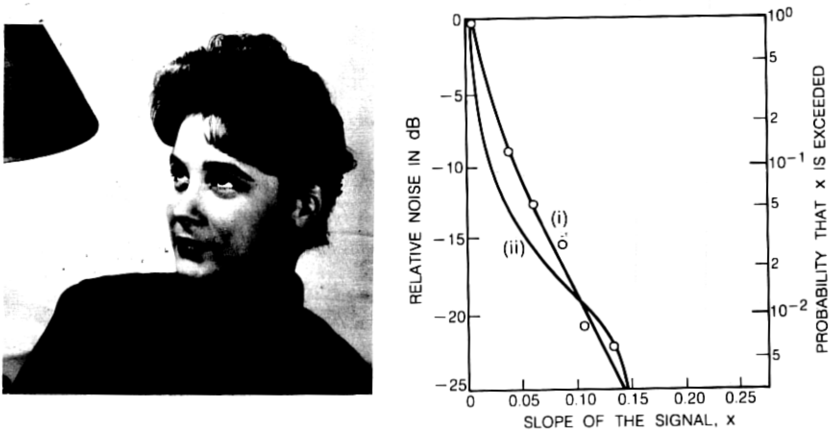


Fig. 3—The white noise that is equivalent to edge noise for six threshold values. There was one observer.



(a) (b)

Fig. 4a, b—Picture of a girl with a lamp: (*i*) The relative white noise plotted against threshold. (*ii*) The probability that the threshold is exceeded.

that the slope of the signal exceeds the threshold value. The relative noise for this scene may be represented approximately by the expression

$$F(x) = \exp(-x/\lambda) \tag{2}$$

where λ is a parameter. A similar exponential relationship is obtained for other scenes that contain a variety of detail. The parameter λ depends on the amount of detail in the scene: λ is small for flat scenes and larger for more highly detailed scenes.

The relationship (2) does not apply to scenes made up of abrupt changes separated by regions of uniform brightness such as a page of graphics, as illustrated in Fig. 5. At very small threshold values $F(x)$ falls rapidly with increasing threshold because it is determined largely by noise that is visible on the flat background, but when x exceeds 0.02, the noise is concentrated on the characters. Thereafter, further increases of threshold have less effect on $F(x)$. Other interesting scenes with their graphs are shown in Figs. 6 and 7. Notice that signals with slopes exceeding 0.25 of the maximum possible rarely occur.

V. COMBINING NOISES FROM SEVERAL SOURCES

In order to design and analyze systems, we need an expression for the visibility of noise that is added to the signal at times when its slope has a particular value. Our measurements describe the visibility

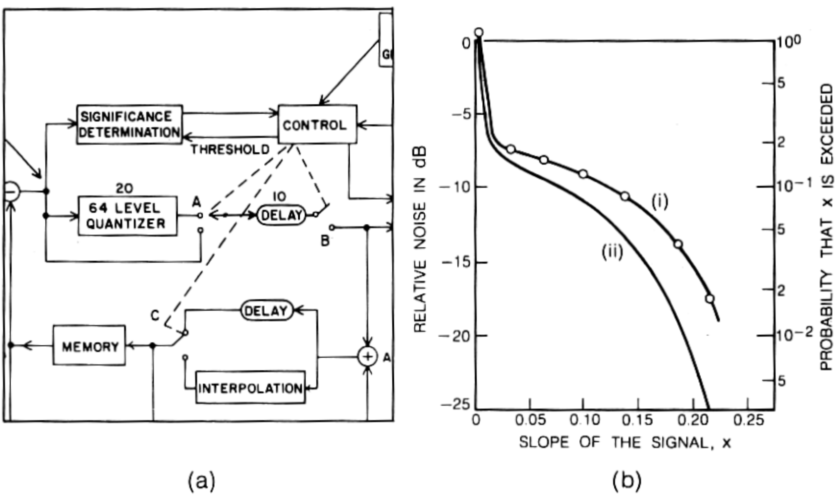


Fig. 5a, b—Picture of graphics: (*i*) The relative white noise plotted against threshold. (*ii*) The probability that the threshold is exceeded.

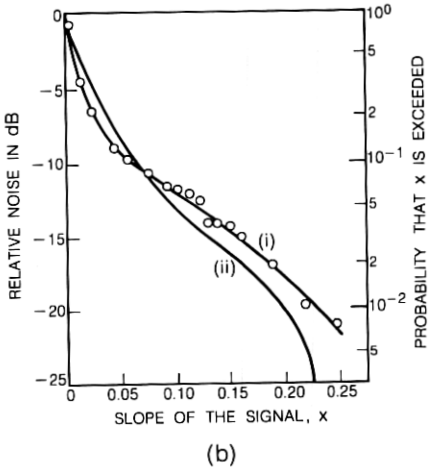
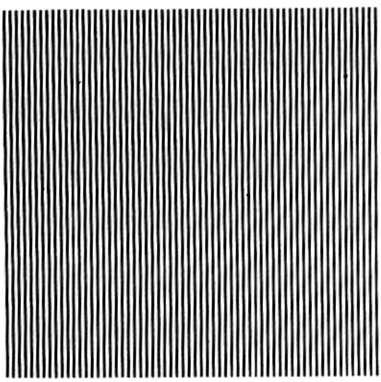


Fig. 6a, b—Picture of Karen: (*i*) The relative white noise plotted against threshold. (*ii*) The probability that the threshold is exceeded.



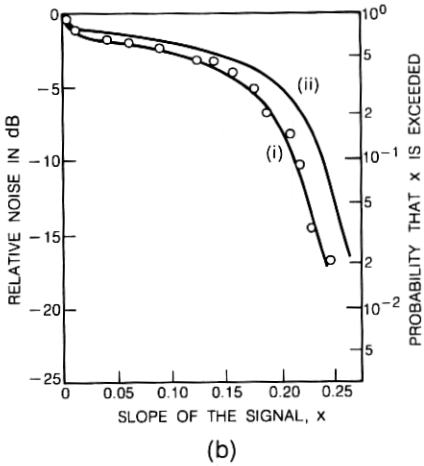


Fig. 7a, b—Picture of stripes: (*i*) The relative white noise plotted against threshold. (*ii*) The probability that the threshold is exceeded.

of noise added to the signal at times when the magnitude of its slope exceeds specified values: it is an accumulation of the required function. In order to obtain the required function we need to know how the visibility of a combination of noises depends on the visibility of its components.

There is strong evidence that, for a combination of distortions, the net equivalent white noise has power equal to the sum of the powers of white noise that are separately equivalent to the component distortions. This hypothesis has been checked by measuring the equivalent white noise for two uncorrelated distortions, separately and in combination. The results are shown in Fig. 8. The abscissa of this graph is the amplitude of the white noise that was equivalent to the least visible component; and the ordinate is the amplitude of the white noise that was equivalent to the combined distortions. Both values are expressed as a ratio of the white noise that was equivalent to the more visible component. The continuous curve is the result expected assuming addition of power. Evidently the hypothesis is a reasonable one, although there is a tendency for the more visible distortion to mask the less visible one to a greater degree than power addition predicts.

The distortions used for these measurements were:

(*i*) White noise added to the entire picture
(*ii*) White noise added to sections of the picture
(*iii*) Noise added when the slope of the signal exceeds a threshold value
(*iv*) Noise added when the slope of the signal lay in a certain interval.

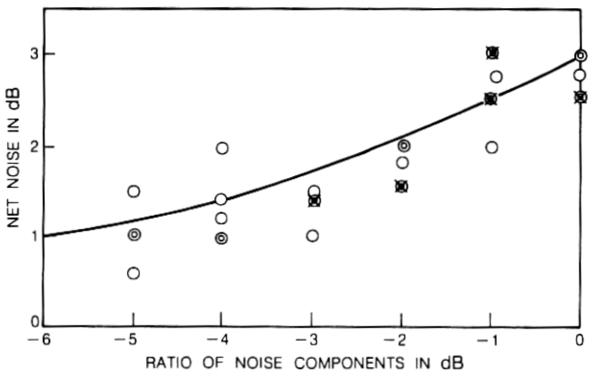


Fig. 8—The white noise that is equivalent to a combination of two distortions plotted against the least visible distortion. Both noise amplitudes are expressed as a ratio of the more visible distortion.

When two distortions of the same kind were introduced into the picture, the result always corresponded with points on the continuous curve in Fig. 8. Such results are not plotted on the graph.

VI. THE VISIBILITY OF NOISE AS A FUNCTION OF SIGNAL SLOPE

The expression $f(x)$ will be used to describe the visibility of unit noise power added to the signal at times when the magnitude of its slope is x. $f(x)$ is defined such that, when rms noise $\alpha(x)$ is added to the signal, the net equivalent white noise has power given by

$$N^2 = \int_0^1 \alpha^2(x) f(x)\, dx. \tag{3}$$

No attempt has been made to separate the visibility into two components dependent on the sign of x. If it is necessary to accommodate one scene, it will surely be necessary to also accommodate its mirror image; it therefore is unwise to make the noise dependent on the sign of x. The expression $f(x)$ can be determined from the subjective measurements of $F(x)$. $F(x)$ is the power of the white noise that is equivalent to unit noise added to the signal at times when the magnitude of the slope exceeds x. Therefore,

$$F(x) = \int_x^1 f(x)\, dx \tag{4}$$

or

$$f(x) = -\frac{dF(x)}{dx} = -F'(x). \tag{5}$$

Values of $f(x)$ are plotted in Fig. 9a for the scenes shown in Figs. 4 through 7. Notice that the visibility of noise tends to decrease as the slope of the signal increases. This decrease is caused by a combination of three effects: It is well known that the ability of the eye to resolve detail decreases near abrupt changes of brightness,[4,8,9] thus we expect the noise to be less visible for large values of x. Moreover, the fraction of the signal that changes rapidly is usually small, as Figs. 4 through 7 demonstrate; thus, for large values of x, noise is added only to a small fraction of the scene. These effects are counteracted to some extent by the fact that the eye tends to concentrate on regions of the scene that carry essential information[10], thus the brightness boundaries tend to be more significant than the overall probability of occurrence indicates. In general, there is not a simple relationship between the function, $f(x)$ and the probability density of slopes, $p(x)$.

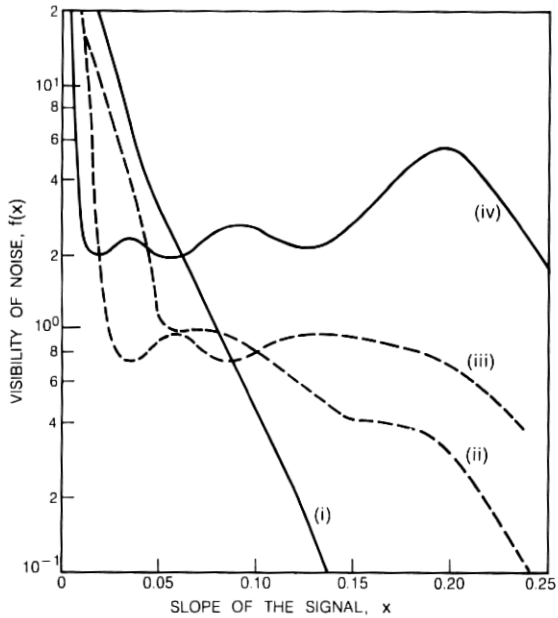


Fig. 9a—The relative visibility of noise $f(x)$ plotted against slope: (i) Girl with a lamp. (ii) Karen. (iii) Graphics. (iv) Stripes.

These functions are compared in Fig. 9b for two scenes; a page full of printed characters and a page containing only one line of characters.

Attempts to measure $f(x)$ directly instead of deriving its value from $F(x)$ have been unsatisfactory for practical reasons. It is much easier to detect the times when the slope of the signal exceeds a threshold value than it is to detect the times when it lies in a narrow range of values. Viewers find that it is easier to measure $F(x)$ than $f(x)$.

VII. OPTIMUM COMPANDING

Knowing the visibility function $f(x)$, it is possible to find distributions of the noise, $\alpha(x)$, that minimize the net equivalent noise given by eq. (3). Before discussing applications of eq. (3), we must emphasize that its derivation assumes that the noise in the picture can be expressed as a function, $\alpha(x)$, of the local slope of the input. This will be true only in special cases, because of storage properties of the integrator that are needed at the receiver. In general, integration will retain the average value of the noise, thus causing streaks to emanate from edges of the scene and persist into flat areas. The above discussion is valid only

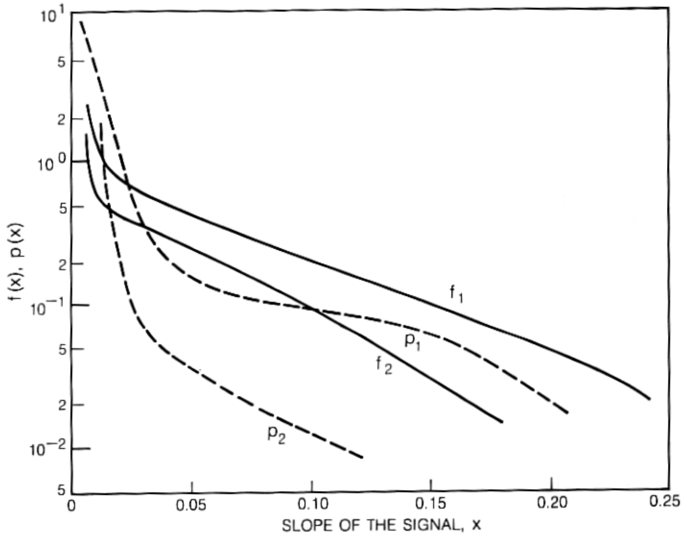


Fig. 9b—A comparison of the function $f(x)$ with the probability density of slopes $p(x)$, for two scenes.

when the average value of the noise added to each edge is small. This is true for Differential Quantization and it can be true for systems of the type illustrated in Fig. 10a, provided the channel noise has only high-frequency components.

The advantages of transmitting video signals as their first derivative are easily realized in digital systems in which the dominant source of noise is the quantization process. The average value of this noise can be kept small by placing a feedback loop around the quantizer, as in a conventional Differential Quantizer.[11,12]

In order to set up equations, let us use the model of a Differential Quantizer that is shown in Fig. 10b. In this circuit, the impairments introduced by quantization are represented as an added noise,[13] $n(t)$. To represent companding of the quantizer levels, the differential signal, x, is compressed[14] according to the function $g(\cdot)$ before noise is added, and expanded afterwards according to the function $h(\cdot)$, which is the inverse of $g(\cdot)$.

The signal emerging from the expandor is given by

$$y = h[g(x) + n], \tag{6}$$

and when the distortion is small, $n \ll g(x)$, it may be written as

$$y \cong h[g(x)] + nh'[g(x)], \tag{7}$$

Fig. 10a—Companding the derivative of a signal.

$h'(\cdot)$ and $g'(\cdot)$ are the first derivatives of $h(\cdot)$ and $g(\cdot)$. Because $g(\cdot)$ and $h(\cdot)$ are monotonic and inverses of one another

$$h[g(x)] = x \quad \text{and} \quad h'[g(x)]g'(x) = 1 \tag{8}$$

therefore,

$$y(t) = x(t) + n(t)/g'(x). \tag{9}$$

Now, let $u(t)$ and $v(t)$ be the input and output signals of the system, then

$$v(t + \tau) = y(t) + v(t) \tag{10}$$

and

$$x(t) = u(t) - v(t) \tag{11}$$

therefore

$$v(t + \tau) = u(t) + n(t)/g'(x) \tag{12}$$

and

$$x(t) = u(t) - u(t - \tau) + n(t)/g'(x). \tag{13}$$

Of particular interest are conditions where

$$x(t) \cong u(t) - u(t - \tau), \tag{14}$$

that is, where x approximates the slope of the input signal. Then the noise present in the output signal is a function of the slope of the input and the result in eq. (3) applies. For convenience, normalize the expansions by assuming $n(t)$ has unit rms value, then the output noise is

$$\alpha(x) = 1/g'(x). \tag{15}$$

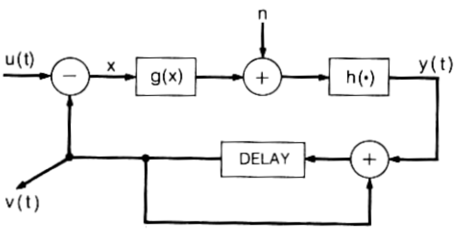


Fig. 10b—Model of a Differential Quantizer.

In practical systems, it is convenient to place constraints on the value of $g(\cdot)$. For example, to have fixed signal levels, let

$$g'(x) > 0, \quad g(0) = 0 \quad \text{and} \quad g(1) = 1. \tag{16}$$

We therefore require that

$$\alpha(x) > 0 \quad \text{and} \quad \int_0^1 \frac{1}{\alpha(x)}\, dx = 1. \tag{17}$$

With these properties, the noise given by eq. (3) can be minimized using variational calculus. (Similar functions are minimized in Refs. 14 and 15.) The visibility of the net noise is a minimum when

$$\alpha_0(x) = f^{-\frac{1}{3}}(x) \int_0^1 f^{\frac{1}{3}}(x)\, dx \tag{18}$$

ie,

$$\alpha_0(x) \propto [f(x)]^{-\frac{1}{3}}.$$

The optimum noise amplification factor $\alpha_0(x)$ is plotted in Fig. 11 for four scenes. Curve (i) shows that, for simple scenes, a compandor can be used to increase the noise continuously as the slope of the signal increases. For very busy scenes, curve (iv) shows that the edge noise can be made about four times larger than the flat-area noise.

VIII. DIFFERENTIAL QUANTIZATION

We have represented the quantization process as additive noise; this is an oversimplification, as is the approximation in eq. (14). However, the conclusions do apply to practical Differential Quantizers. Their action is illustrated in Fig. 12, which shows a typical response to an input rising at a constant rate. The output voltage is seen to step up at each sample time by an amount equal to a quantization level. If the signal change is not matched exactly by an available level, then an appropriate sequence of levels is used to approximate it. In general, the noise introduced by this process is determined by the spacing of the levels that are used; and a well-designed quantizer will use the levels closest to the local slope of the signal. We will now assume that the quantization noise, $\alpha_q(x)$, is proportional to the spacing of the quantization levels that are adjacent to the slope, x, of the signal. This function is plotted in Fig. 13 for an eight-level and a sixteen-level quantizer. These scales were selected by experimental trial and error; the eight-level quantizer[1,2,3] was optimized for pictures of faces and

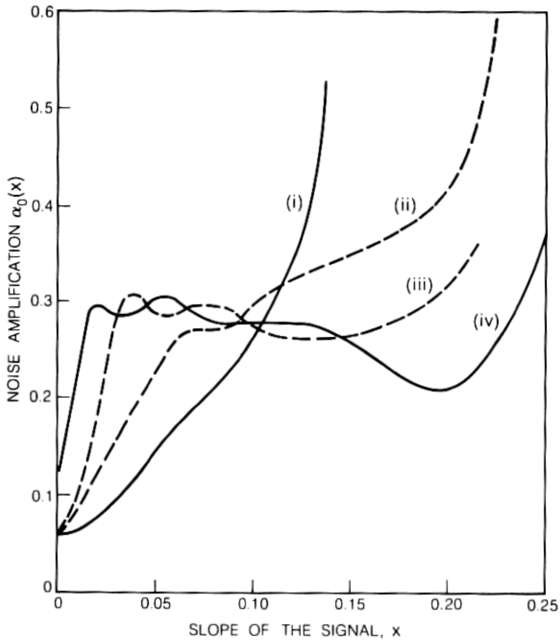


Fig. 11—The optimum noise amplification factor α (x): (i) Girl with a lamp. (ii) Karen. (iii) Graphics. (iv) Stripes.

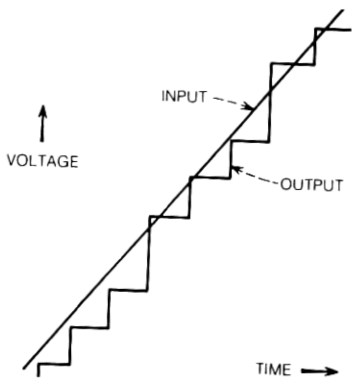


Fig. 12—The action of a Differential Quantizer.

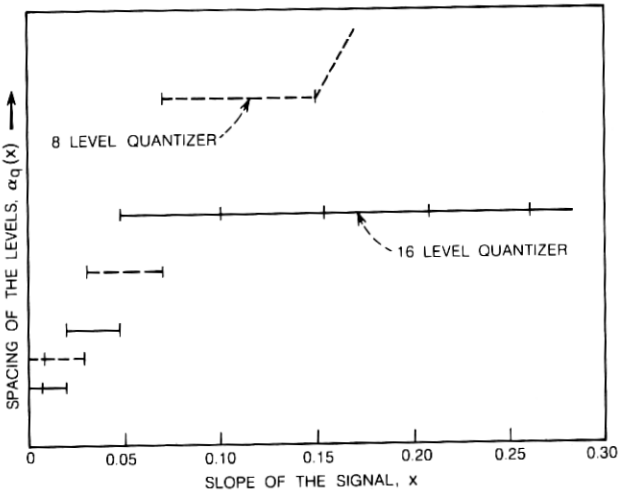


Fig. 13—The quantization intervals for an 8-level and a 16-level quantizer.

is inadequate for graphics. More emphasis was given to the reproduction of high-detail graphics in the design of the sixteen-level scale.[16] The shape of the graphs in Fig. 13 corresponds quite well with the graphs in Fig. 11; the eight-level quantizer approximates curve (i) and the sixteen-level quantizer approximates curve (iv).

The assumption that quantizing noise is proportional to the spacing of the quantization levels is true if $\alpha_q(x)$ is taken to be the average noise for values of x in each quantization interval. In reality, the variation of quantization noise with slope is very complex and has much fine structure. Similarly, any attempt to match the curves in Fig. 11 by line segments, such as in Fig. 13, must necessarily ignore the finer variations of $\alpha(x)$. This is not a disadvantage because the finer variations are very dependent on picture content and gain settings. Indeed, it is advisable to match the quantizer to an average of $\alpha(x)$ in order that a range of scenes can be accommodated. If the intent is to make the spacing of the quantization levels $\alpha_q(x)$ equal to the average value of $\alpha(x)$, there is an easy routine for designing quantizers. It is described in Appendix B.

IX. VISIBILITY OF THE NET QUANTIZATION NOISE

It is useful to have the ability to calculate an expression that describes the net impairment of the received picture; for this purpose we will use the amplitude of an equivalent white noise. The equivalent noise for uniform quantization is easily calculated if overloading is neglected. The error at the decision time ranges with uniform probability between plus and minus half a step interval. Therefore,

$$N_1 = \frac{d}{2\sqrt{3}}, \tag{19}$$

where d is the level spacing. The advantage of optimally companding the level of the Differential Quantizer is shown in Appendix A to be*

$$\left[\int_0^1 f^{\frac{1}{3}}(x)\, dx\right]^3. \tag{20}$$

The predicted advantage over uniform quantization is 17 dB for the picture shown in Fig. 6. It is customary to separate the advantage into two parts. One part is a consequence of the fact that quantization levels need not be provided for slopes greater than certain maximum values. For example, no slopes exceed 0.25 in this scene. The second advantage is a consequence of tapering the levels; this is predicted to be 5 dB for the picture in Fig. 6, about 5 dB improvement has been realized by companding an eight-level quantizer.[17] A similar improvement has also been obtained by weighting the steps of a delta modulator.[6]

A method for calculating the equivalent signal-to-noise ratio for particular quantizers is described in Appendix C. It predicts 50 dB signal-to-noise ratio for the sixteen-level quantizer and 42 dB for the eight-level quantizer when processing the picture in Fig. 6. Both results agree reasonably well with practical measurements.

X. THE VISIBILITY OF NOISE IN DIFFERENT PICTURES

White noise has been used as a standard for measuring impairments of a video signal and has proven to be very useful for evaluating distortions of a given scene. Care is needed, however, in using it to compare the visibility of distortions of different scenes because the visibility of the white noise itself depends on the picture content.[18] Indeed[9], it is very difficult to obtain numerical comparisons of the visibility of a given distortion in different pictures. Table I lists the main properties of the pictures used in this paper, together with an estimate of the visibility of white noise. For example, a given amplitude of white noise was judged to be 2.5 dB less annoying in the picture of Fig. 4 than it was in the graphics of Fig. 5.

* This assumes that the signal-to-noise ratio lies within the range for which our results apply, ie, 40–53 dB.

TABLE I—MAIN PROPERTIES OF PICTURES SHOWN IN FIGS. 4–7.

Scene	Peak/RMS	Visibility of Noise
Girl with lamp (Fig. 4)	11 dB	0
Graphics (Fig. 5)	11 dB	−2.5 dB
Karen (Fig. 6)	13 dB	−1.5 dB
Stripes (Fig. 7)	12 dB	−1.5 dB

The visibility of noise is also dependent on the motion in the scene. We have estimated that white noise is 3 dB less visible on the face of a person talking normally than it is when the face is stationary.

It has also been observed that, for the picture in Fig. 6, the edge busyness is about 2 dB less visible when the noise is synchronized to the scanning waveforms than when it is random. The advantage of synchronizing the noise is fully realized only in stationary scenes.

XI. DISCUSSION AND CONCLUSIONS

We have described a method for measuring and evaluating how the visibility of noise depends on the instantaneous rate of change of the signal. The visibility of quantization noise can be significantly reduced by making its amplitude depend on the slope of the signal. The optimum distribution is very dependent on picture content, therefore measurements have been presented for several different scenes. These scenes were chosen as being typical of important classes and their properties should be useful for characterizing system requirements.

The measurements have been used to design quantization scales for Differential Coders and to predict their performances. The results agree very well with scales that were obtained experimentally and with measured signal-to-noise ratios.

It would be useful to relate the visibility of noise, as measured for this work, to basic properties of scenes and to psychophysical properties of viewers. This has proven to be very difficult. For example, attempts to relate the visibility of noise to the probability density of slopes of the signal have been successful only for scenes of uniform texture, such as Fig. 7a.

The work reported here relates to the rate of change of brightness in the horizontal direction. Comparable results could be expected for the vertical direction.[19] It will be interesting to see similar measurements obtained for other types of pre-emphasis besides simple differentiation. The experiments should also be repeated with other inputs such as color and audio signals, and frame-to-frame changes.

XII. ACKNOWLEDGMENTS

We thank our colleagues at Bell Laboratories for inspiration and assistance, particularly R. C. Brainard, E. F. Brown, D. J. Connor, A. J. Goldstein, B. G. Haskell, J. O. Limb, and F. W. Mounts. We are especially grateful to M. R. Schroeder who suggested the measuring technique, and T. V. Crater for advice on the presentation of our results.

APPENDIX A

Net Noise in an Optimally Companded System

An optimally companded system is understood to be one in which the noise is distributed according to eq. (18), ie,

$$\alpha_0(x) = f^{-\frac{1}{3}}(x) \int_0^1 f^{\frac{1}{3}}(x)\, dx. \tag{21}$$

The net relative-noise power is obtained by substituting in eq. (3) to get

$$N_0^2 = \int_0^1 \alpha_0^2(x) f(x)\, dx = \left[\int_0^1 f^{\frac{1}{3}}(x)\, dx\right]^3 . \tag{22}$$

When there is no companding, $\alpha(x) = 1$ the noise is given by

$$N_1^2 = \int_0^1 f(x)\, dx. \tag{23}$$

Therefore, the advantage of companding can be expressed as

$$A = \left[\int_0^1 f^{\frac{1}{3}}(x)\, dx\right]^{-3} \tag{24}$$

because the measurements have been calibrated so that

$$\int_0^1 f(x)\, dx = F(1) = 1. \tag{25}$$

Most viewers, when judging pictures, are conscious of two components in the noise: flat area noise which corresponds approximately to areas of the picture where $x < 0.02$, and "edge-noise" which corresponds to $x > 0.02$. With optimum companding, the "edge-noise" usually predominates because

$$\int_0^{0.02} f^{\frac{1}{3}}(x)\, dx < \int_{0.02}^1 f^{\frac{1}{3}}(x)\, dx. \tag{26}$$

APPENDIX B

Determining Quantization Levels

The objective is to design a quantizer for which the relative spacing of the levels $\alpha_q(x)$ represents a given value of the function $\alpha_0(x)$ in eq. (10). An example is shown in Fig. 14. The function $\alpha_0(x)$ is plotted as a continuous curve and is extended to negative values of x by assuming it is an even function. The rectangles superimposed on this curve have the following properties: First, their height is proportional to the average value of $\alpha_0(x)$ during the interval defined by the width of the rectangle. Second, the inclination of the diagonal is the same for all the rectangles; this makes their width proportional to their height.

The quantization levels are set equal to the value of x that corresponds with the sides of the rectangles. Then the spacing of the levels is proportional, in turn, to the width of the rectangle, the height of the rectangle, and to the average value of $\alpha_0(x)$; thus satisfying our requirements. The number of levels in the quantizer is one more than the number of rectangles and this is determined by the inclination selected for the diagonals. Quantizers with given numbers of levels can be obtained by an iteration of the design method. The decision levels would be placed midway between the quantization levels.[20]

A "worst case" design is required for systems that must accommodate a variety of scenes. For example, for small values of x, the levels would be chosen to accommodate scenes with properties given by curve (i) of Fig. 11, and for large values of x, the levels would be chosen to accommodate scenes with properties (iv).

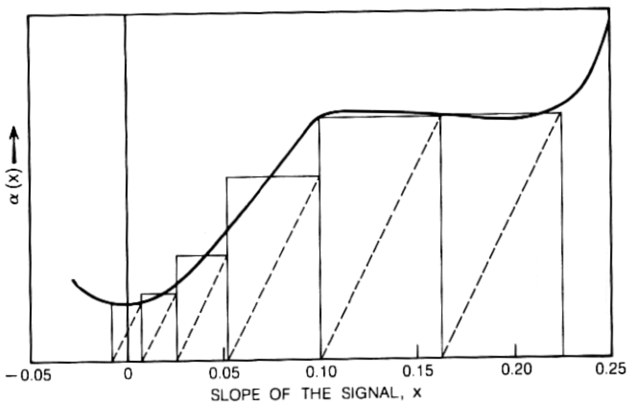


Fig. 14—A method for selecting quantization levels.

APPENDIX C

Determining the Visibility of Quantization Noise

Having determined the values of the quantization levels it is useful to be able to calculate the visibility of net quantization noise. The net quantization noise power will be given by

$$N_q^2 = \int_0^1 \alpha_q^2(x) f(x)\, dx \tag{27}$$

where $\alpha_q(x)$ is the rms quantization noise which is assumed to be proportional to the spacing of the levels. The constant of proportionality is given by eq. (11). If the nth quantization level is ln then

$$\alpha_q(x) = \frac{(l_n - l_{(n-1)})}{2\sqrt{3}}, \qquad l_{n-1} < x \leqq l_n . \tag{28}$$

The constraints (16) can be interpreted as requiring a fixed number of quantization levels, which are numbered in ascending order according to their magnitude. Therefore

$$N_q^2 = \frac{1}{12} \sum_n \left[(l_n - l_{n-1})^2 \int_{l_{n-1}}^{l_n} f(x)\, dx \right]. \tag{29}$$

Assuming that the companding is symmetrical, $l_n = -l_n$, then using eq. (4) and neglecting overloading we obtain

$$N_q^2 = \frac{1}{12} \left[4l_1^2(1 - F_{l_1}) + \sum_{n=2}^{N} (l_{n-1} - l_n)^2 [F(l_{n-1}) - F(l_n)] \right] \tag{30}$$

for an even number of levels $2N$, and

$$N_q^2 = \frac{1}{12} \left[\sum_{n=1}^{N} (l_{n-1} - l_n)^2 [F(l_{n-1}) - F(l_n)] \right] \tag{31}$$

for an odd number of levels $(2N + 1)$.

These expressions are easily evaluated when l_n and F are known. Values of the quantization levels l_n are found by using the approximate method described in Appendix B because we do not have an easy method for selecting values of l that minimize N_q^2. It has been found that levels, determined by the method described, give signal-to-noise ratios that are only 2 dB less than that predicted by eq. 14, that is, for optimum shaping of the noise using the continuous function $\alpha_0(x)$ instead of the discontinuous $\alpha_q(x)$.

Important cases are ones for which the quantization levels are sufficiently close that $f(x)$ is effectively constant during each interval. Then the method shown in Fig. 14 for obtaining levels gives optimum companding $\alpha_q = \alpha_0$. Equation (30) shows that every quantization interval then contributes the same amount to the visible noise.

REFERENCES

1. Millard, J. B., and Maunsell, H. I., "Digital Encoding of the Video Signal," B.S.T.J., *50*, No. 2 (February 1971), pp. 459–480.
2. Limb, J. O., and Mounts, F. W., "Digital Differential Quantizer for Television," B.S.T.J., *48*, No. 7 (September 1969), pp. 2583–2599.
3. Brown, Earl F., "A Sliding-Scale Direct—Feedback PCM Coder for Television," B.S.T.J., *48*, No. 5 (May–June 1969), pp. 1537–1553.
4. Limb, J. O., "Source-Receiver Encoding of Television Signals," Proc. IEEE, *55* (April 1967), pp. 364–379.
5. Brainard, R. C., "Subjective Evaluation of PCM Noise-Feedback Coder for Television," Proc. IEEE, No. 3 (March 1967), pp. 346–353.
6. Bosworth, R. H., and Candy, J. C., "A Companded One-Bit Coder for Television Transmission," B.S.T.J., *48*, No. 5 (May–June 1969), pp. 1459–1479.
7. Crater, T. V., "The PICTUREPHONE® System: Service Standards," B.S.T.J., *50*, No. 2 (February 1971), pp. 235–269.
8. Brown, E. F., and Kaminski, W., "Television: Visual Threshold to Errors Near Brightness Boundaries," Private Communication, July 1970.
9. Novak, S., and Sperling, G., "Visual Thresholds Near a Continuously Visible or Briefly Presented Light-Dark Boundary," Optica Acta, *10* (April 1963), pp. 87–91.
10. Yarbus, Alfred L., *Eye Movements and Vision*, New York: Plenum Press, 1967, Chapter VII, pp. 171–211.
11. Cutler, C. C., "Differential Quantization of Communication Signals," Patent No. 2, 605, 361 (June 1950).
12. Graham, R. E., "Predictive Quantizing of Television Signals," IRE Wescon Conv. Rec. 2, Part 4, 1958, pp. 147–157.
13. Brainard, R. C., and Candy, J. C., "Direct-Feedback Coders: Design and Performance with Television Signals," Proc. IEEE, *57*, No. 5 (May 1969), pp. 776–786.
14. Smith, Bernard, "Instantaneous Companding of Quantized Signals," B.S.T.J., *36*, No. 3 (May 1957), pp. 653–709.
15. Panter, P. F., and Dite, W., "Quantization Distortion in Pulse-Count Modulation with Nonuniform Spacing of Levels," Proc. IRE. *39* (January 1951), pp. 44–48.
16. Abbot, R. P., "DPCM Codec for Video Telephony Using 4-Bits Per Sample," IEEE, Trans. Commun. Tech. (December 1971).
17. Kaminski, W., Private Communication.
18. Hacking, K., "The Relative Visibility of Random Noise Over the Grey-Scale," Journal Brit., I.R.E. (April 1962).
19. Connor, D. J., Pease, R. F. W., and Scholes, W. G., "Television Coding Using Two-Dimensional Spacial Prediction," B.S.T.J., *50*, No. 3 (March 1971), pp. 1049–1061.
20. Max, Joel, "Quantizing for Minimum Distortion," IRE Trans., *IT-6* (March 1960), pp. 7–12.