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The Equivalent Group Method for Estimating the Capacity of Partial-Access Service Systems Which Carry Overflow Traffic

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We present a technique, called the Equivalent Group method, for estimating the capacities of partial-access service systems which carry overflow traffic. The basic idea is to find a full-access group which has the same capacity as the partial-access system when the arrival process is Poisson. We consider these groups to be "equivalent" and use the capacity of the full-access group when it is offered overflow traffic to estimate the capacity of the partial-access group if it is offered the same overflow traffic.

The Equivalent Group method is used to estimate the capacity of Step-by-Step graded multiples which carry overflow traffic. The resulting procedures can account for day-to-day variations in the offered load, and the computations can be carried out by appropriate use of existing engineering tables.

I. INTRODUCTION

In telephone traffic engineering, it is frequently necessary to understand the behavior of partial-access service systems, i.e., systems in which arriving customers do not have access to all servers. The analysis of such systems is difficult even when the arrivals are adequately approximated by a Poisson process. However, when the arrivals are the overflows from some other service system, there are no simple methods for estimating capacity.

In this note, we present a new procedure, called the Equivalent Group method, for estimating the capacity of partial-access service systems which carry overflow traffic. The basic idea is to find a full-access group of servers which has the same capacity as the partial-access system when the arrival process is Poisson. We consider these groups to be "equivalent" and use the capacity of the full-access group when it is offered overflow traffic to estimate the capacity of the partial-access group if it is offered the same overflow traffic.

We then consider the Step-by-Step switching system, wherein the trunk groups that interconnect the selectors are sometimes arranged so as to form partial-access systems called graded multiples.[1,2] Although graded multiples have been studied extensively, almost all of the results have had to be based on the assumption that arrivals occur according to a Poisson process. When graded multiples are used as alternate routes, the arrivals are not adequately approximated by a Poisson process.[3] Consequently, the standard approximations (which assume Poisson arrivals) are inadequate. Here we apply the Equivalent Group method to estimate the capacities of such gradings.

Graded multiples which serve overflow traffic have been studied in two other instances. S. Neal[3] and A. Lotze[4] have obtained approximations but their results require extensive calculations. Moreover, their methods are not applicable when grading capacity is significantly affected by the properties of the system in which the grading is imbedded, eg., the Step-by-Step switching system.[1]

II. THE EQUIVALENT GROUP METHOD

A partial-access service system can be viewed as a service device; customers arrive and are either served or blocked.* Hence, to each offered load a for a partial-access group $\mathcal{G}$, there corresponds an "equivalent" full-access system of $s = s(a, \mathcal{G})$ servers† which will experience the same blocking at the load a. That is, if $B_{\mathcal{G}}(a)$ denotes the blocking probability for $\mathcal{G}$ when the offered load is a erlangs, then s is determined by the relation $E_{1,s}(a) = B_{\mathcal{G}}(a)$ where $E_{1,s}(a)$ is the first Erlang loss-function.

If an overflow process having mean a but variance $v > a$ were offered to $\mathcal{G}$, a blocking probability $B_{\mathcal{G}}(a, v) > E_{1,s}(a)$ would result. If the same overflow stream were offered to the equivalent full-access group of $s(a, \mathcal{G})$ servers, the blocking probability would be $B_s(a, v) > E_{1,s}(a)$. Since the blocking probabilities $B_s(a, v)$ have been tabulated,[5] it would be very useful if $B_s(a, v)$ could be used to estimate $B_{\mathcal{G}}(a, v)$.

* All service systems under consideration are in statistical equilibrium and obey a blocked-calls-cleared discipline; a customer arriving to find no server available is blocked, leaves the system, and does not return. Unless specified otherwise, arrivals constitute a Poisson process. Service times are independent and identically distributed according to a negative-exponential distribution.

† There is no apparent relation between $s(a, \mathcal{G})$ and the equivalent number of servers obtained by the extended Equivalent Random method.[3]

The phrase "Equivalent Group method" denotes the entire procedure outlined above. That is, determine $s = s(a, \mathcal{G})$ so that $E_{1,s}(a) = B_{\mathcal{G}}(a)$ and then use $B_s(a, v)$ to estimate $B_{\mathcal{G}}(a, v)$. Of course, we have not described exactly how $B_s(a, v)$ should be used. This point is covered below.

It is not our intention to determine the range of applicability of the Equivalent Group method. We desire only to point out the method and to show how it was used successfully to solve certain engineering problems in the Step-by-Step system.

III. APPLICATIONS

The Equivalent Group method evolved from a search for a simple procedure for engineering Step-by-Step graded multiples which serve overflow traffic. Initially we wanted to make an analytical comparison between $B_{\mathcal{G}}(a, v)$ and $B_s(a, v)$ to determine how $B_s(a, v)$ should be used to estimate $B_{\mathcal{G}}(a, v)$. However, we did not find a feasible analytical method. Having no alternative, we resorted to a numerical study using a simulation.

3.1 *Capacity Estimates for Step-by-Step Gradings*

We chose Step-by-Step gradings of 11, 19, 25, 37, and 45 trunks for experimentation (the gradings used in Ref. 1). The inherent load-balancing present in Step-by-Step systems was represented by the approximate model described in Ref. 1. The arrival processes having specified mean and variance were generated by using the swinging-gate approximation, as analyzed by A. Kuczura,[6] for an overflow process.

For each grading-selector configuration, we used a simulation to generate estimates of the grading load-loss relations for several values of the variance-to-mean ratio $z = v/a$ of the input (overflow) process. Figure 1 contains the results for the 25-trunk grading with 40 selectors. (We will come back to Fig. 1 later on.) The results of the other cases are of a similar nature. The results for $z = 1$ were obtained from Ref. 1.

The first step of the Equivalent Group method is to determine $s = s(a, \mathcal{G})$ so that $E_{1,s}(a) = B_{\mathcal{G}}(a)$ ($\mathcal{G}$ denotes the 25-trunk graded multiple). The results are displayed in Fig. 2. [Notice that $s(a, \mathcal{G})$ is not very sensitive to changes in the load a.] Next, we find $B_s(a, v)$ for each value of $z = v/a$ of interest. This can be done by using Wilkinson's tables[5] together with linear interpolation to obtain values of $B_s(a, v)$ for noninteger s. Figure 1 displays $B_s(a, v)$ as a first approximation to $B_{\mathcal{G}}(a, v)$. Again, the relations for the other gradings were of a similar nature.

For the Step-by-Step applications, we desire results for $1 \leqq z \leqq 3$. Comparing the curves in Fig. 1, we see that $B_s(a, v) > B_{\mathfrak{G}}(a, v) > B_{\mathfrak{G}}(a)$ for all a and $z > 1$. However, the differences between the load-loss relations obtained by simulation and those obtained from $B_s(a, v)$ appear to be simply related to the size of z. In fact, if we merely decrease z by $\Delta z = 0.2\ (z - 1)$ when carrying out the computations, the modified results are in excellent agreement with the simulation results (see Fig. 3). Moreover, except for the gradings having only two first-choice subgroups, the same adjustment was just as successful for the other cases. The Δz correction was not necessary for the gradings with only two first-choice subgroups.

3.2 *An Engineering Procedure for Step-by-Step Graded Multiples*

Any method used for the engineering of Step-by-Step systems must be able to incorporate into the system model day-to-day variations

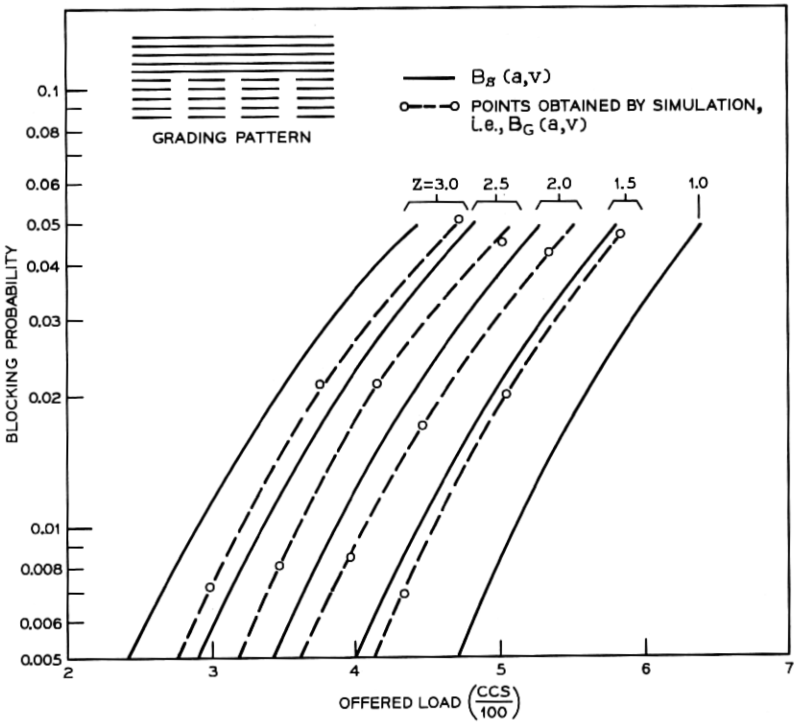

Fig. 1—Approximate load-loss relations for the 25-trunk graded multiple with 40 selectors.

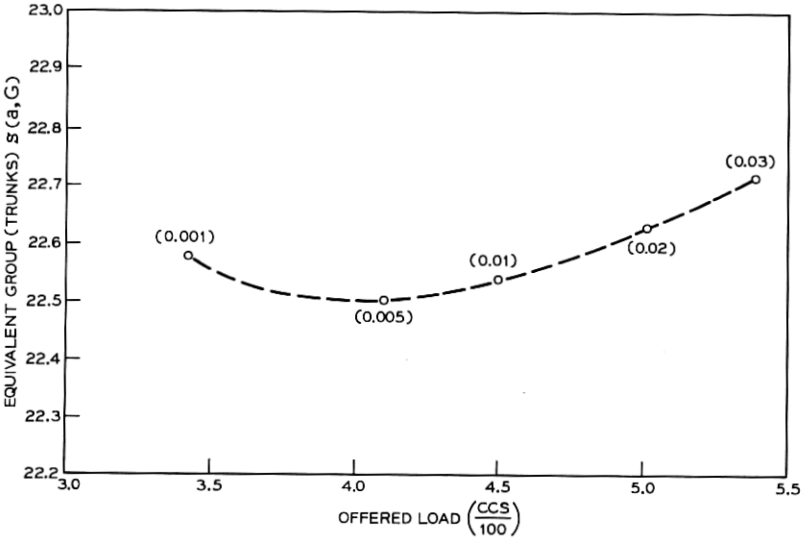

Fig. 2—Equivalent group vs offered load for the 25-trunk graded multiple with 40 selectors. Corresponding blocking enclosed in parentheses.

in the offered load.[7] We account for day-to-day variations by being consistent in the determination of $B_{\mathcal{G}}(a)$, $s(a, \mathcal{G})$, and $B_s(a, v)$. That is, if one is interested in results for day-to-day variations in the offered load, then the same level (low, medium, or high) of day-to-day variations must be assumed at each of the steps* outlined above. An example should clear up this point.

Suppose we are asked to decide which Step-by-Step graded multiple should be connected to 320 selectors in order to carry 13 erlangs (468 CCS) of overflow traffic having $z = 1.7$. We know that high day-to-day variations in the offered load are present and that the average blocking should not exceed 2 percent.

From the preceding, $B_s(a, v) = 0.02$, $a = 13$, and $z = 1.7$. For the first step, decrease z by $\Delta z = 0.2\ (z - 1) = 0.14$.† Using the $\bar{B}$ tables in Ref. 5 we see that 24.4 trunks will serve 13 erlangs under the specified conditions. In the same table, observe that 24.4 trunks will serve 14.5

* In the past, Step-by-Step selector-multiple tables have not explicitly accounted for day-to-day variations in the offered loads. However, revised tables that include the effects of day-to-day variations have been made and will soon be distributed[1].

† We are anticipating that one of the larger gradings will be required. Recall that the Δz correction is not required for the graded-multiples having only two first-choice subgroups.

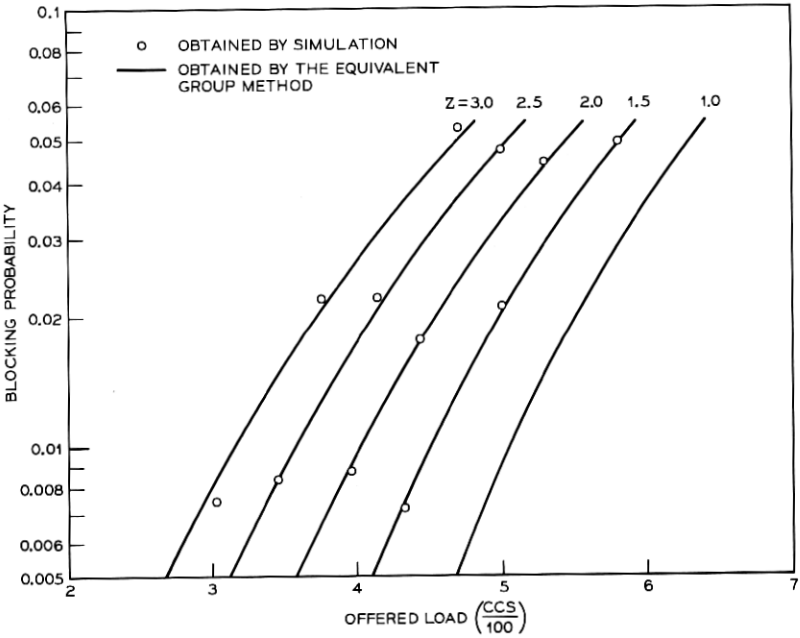

Fig. 3—Load service relations for the 25-trunk graded multiple with 40 selectors.

erlangs of Poisson traffic at $\bar{B}.02$.* Thus, the equivalent group contains 24.4 trunks and the equivalent load is 14.5 erlangs. Consequently, we must find a Step-by-Step grading which, when connected to 320 selectors, will serve an average of 14.5 erlangs of Poisson traffic at $\bar{B}.02$ when high day-to-day variations in the load are present.

Consulting the section for high day-to-day variation in the new Step-by-Step selector-multiple tables, we see that the closest entry is 29 trunks. Thus, the 29-trunk graded multiple connected to 320 selectors will serve an average of 13 erlangs of traffic with $z = 1.7$ at a slightly lower average blocking probability than $\bar{B}.02$, when high day-to-day variations in the offered load are present.

IV. CONCLUSIONS AND SUMMARY

The Equivalent Group method can be used to estimate the capacity of partial-access service systems which serve overflow traffic. In general,

* Since $s(a, \mathcal{G})$ changes very little as a function of a, for computational simplicity we are holding it constant during the computations for this example. The results are not sensitive to this approximation.

we can only say that the Equivalent Group method seems reasonable; an analytical study of the method seems very difficult. We were able to use the method to obtain good estimates of the capacities of Step-by-Step graded multiples for $1 \leqq z \leqq 3$. For the Step-by-Step application, the Equivalent Group method is easy to apply. Moreover, there is no reasonable alternative presently available.

In our applications of the Equivalent Group method, an empirical adjustment (i.e., the Δz correction) was required to maintain accuracy throughout the range of interest, $1 \leqq z \leqq 3$. Hence, one should be very cautious when attempting to use the Equivalent Group method for something other than the Step-by-Step applications discussed above.

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