A Satellite System for Avoiding Serial Sun-Transit Outages and Eclipses

By C. W. LUNDGREN

(Manuscript received March 20, 1970)

The motions of satellites phased in particular, slightly inclined orbits are timed so that different satellites are north and south of the equator when sun-caused outages occur in geostationary equatorial systems.

I. INTRODUCTION

Communication satellite systems experience predictable service interruptions involving the sun. A sun-transit outage occurs when the pointing angles from a receiving earth terminal to a satellite and to the sun so nearly coincide that the additional noise power presented by the sun renders transmission unusable.[1] When a satellite passes through the earth's shadow, its solar primary power is interrupted and its sunlight-dependent heat balance is upset.

A geostationary system serving a common coverage region may include several satellites spaced less than 10° (175 mrad) in the synchronous equatorial orbit. Figure 1 illustrates the timing of sun transits and eclipses occurring in rapid series for three geostationary satellites during one day at the spring equinox, observed from an earth terminal located on the equator at longitude 0°W. One sun transit near noon and one eclipse 12 hours later are observed for each satellite served by this terminal. Eclipses of closely spaced satellites may occur at the same time, and sun transits of different satellites may also occur simultaneously within a large coverage region.

Daily sun transits of all geostationary satellites serving an earth terminal occur during one week in the spring and again in the fall. Service interruptions can last five minutes or more per satellite. Affected outage regions are large and move so rapidly that terrestrial restoration is unattractive.

Conversely, a minimum of one working and one spare geostationary satellite are required for restoration independent of terrestrial facilities.

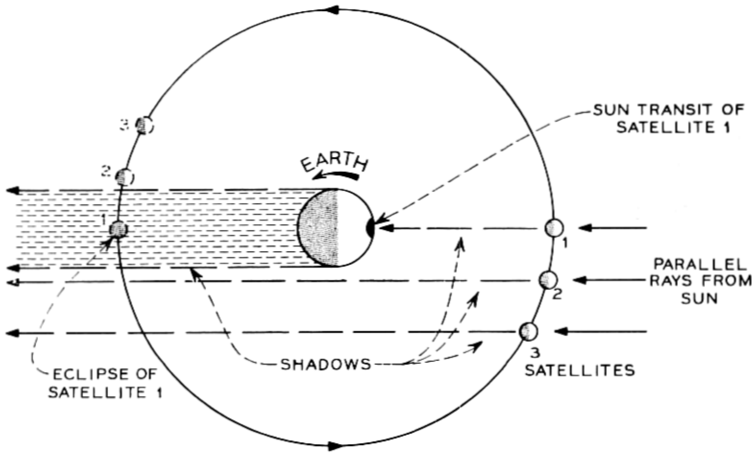

Fig. 1—Sun transits for earth terminal on equator, and eclipses of geostationary satellites at equinox.

Such redundancy is also required for adequate protection against satellite failure, since satellite replenishment intervals are prohibitively large.

A fully redundant geostationary system incorporates duplicate transmissions to working and spare satellites and duplicate reception from these satellites continuously at all earth terminals. Partially redundant systems depend upon redirection of earth antenna beams to spare satellites.*

Rapid, highly coordinated switching between geostationary satellites is required at all earth terminals to restore serial sun-transit outages. Numerous residual transmission "hits" result from such switching. Also, the orbit spacing must be sufficiently large to prevent simultaneous mutual outages of the different satellites at different locations within the coverage region to avoid additional switching complexity. A spacing as large as 8° (140 mrad) is necessary to prevent mutual sun-transit outages within the contiguous United States.†

Alternatively, serial sun transits are avoided by phasing the satellites in particular, slightly inclined orbits with motions timed so that one satellite is north of the equator and the other is south during both the spring and fall outage events. Only one switch of reception between

* If the earth terminals are equipped with duplicate antennas, transmitters, and receivers, the capacity of both satellites can be utilized except during outage periods.
† The 48 continental states, excluding Alaska and Hawaii.

the separated satellites is required per sun-transit season. The exact timing (hour) is unimportant and may be different for the convenience of each earth terminal. Except for these two switches, all earth terminals throughout the coverage region are afforded uninterrupted reception throughout the year. Mutual sun transits within the same coverage region are also avoided by this satellite diversity, and the large orbit spacing discussed above for geostationary satellites is unnecessary.‡

II. SUN TRANSITS AND ECLIPSES

Sun transits and eclipses of geostationary satellites occur during the spring and fall seasons. The exact dates of the former depend primarily upon the latitude of the receiving earth terminal.

2.1 *Sun Transits of Geostationary Satellites*

The geometry and duration associated with a sun transit are controlled by (*i*) the off-axis gain of a properly pointed earth antenna, (*ii*) the receiving system noise temperature, (*iii*) the solar noise power profile, and (*iv*) the minimum acceptable signal-to-noise ratio.

In Fig. 2 the sun's rays are assumed to be parallel; refraction cor-

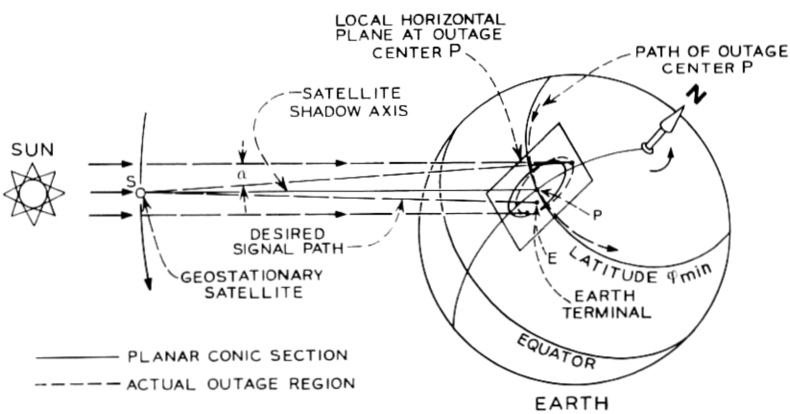

Fig. 2—Planar approximation of geography affected by a sun-transit outage.

rections are neglected, assuming a sufficiently large angle of incidence to the atmosphere for the desired ray **SE**. The affected outage region is defined approximately by the locus of all points on the illuminated earth's surface for which earth antennas aimed at satellite S also point

‡ See Sections 3.3 and A.5.

a prescribed minimum angular distance $\alpha°$ away from the sun's center.

An estimate of the geography involved is provided by the elliptical intersection of a cone of angular radius $\alpha°$, symmetrical about satellite-shadow axis **SP** with its apex at S, and the horizontal plane at P. It is elongated north-south in the figure.

The sun is assumed to be a uniform disk source of thermal noise about 0.5° in diameter.* Shapes and magnitudes of the solar noise power profile vary strongly with time and radio frequency. Edge brightening at the lower microwave frequencies approaches a factor of two, and comparable variations of total flux with time are common.[2,3]

A minimum solar noise temperature for the mean quiet sun (total flux averaged over the disk) is about 25,000 K for a single polarization, inferred from measurements at a wavelength of 10.3 cm.[2-4] This is approximately the minimum temperature presented to a sun-pointed ideal antenna at 4 GHz whose beamwidth is less than 0.5°.

Convolution of an appropriate solar noise profile with a known earth antenna gain pattern provides an estimate of increased noise versus angular displacement of the sun center from the main beam axis. Estimates for the minimum displacement permitting acceptable reception at 4 GHz range from about 0.6° (10 mrad) for very large earth antennas (30 m) to greater than 1° (18 mrad) for small antennas (8m).[1,5] Corresponding minor axes of outage regions range from 800 to 1300 km. Major axes occurring along satellite-earth longitudes are equal to the minor axes at the equator and approach 1.5 times the latter at high latitudes.

Because of synchronism between earth rotation and satellite revolution, each outage region appears to move. One at 41° north latitude traverses the contiguous United States from west to east in approximately one-half hour at noon of the time zone at the satellite's longitude (see Appendix A).

Figure 3 illustrates the path of an outage region. Each path is tangent to the latitude intercept of the center of the satellite's shadow at apparent noon at the satellite's longitude. For all other longitudes in the Northern Hemisphere, the path lies slightly to the north of this latitude.

Hence, in very late February or early March, short daily outages affect earth terminals situated near the United States–Canadian border. Two to three days later these terminals experience maximum outages lasting five minutes or more, depending upon transmission parameters and permissible signal-to-noise ratios. Outages at these terminals end

* The optical disk has a diameter of about 29 minutes of arc, in geocentered angular measure.

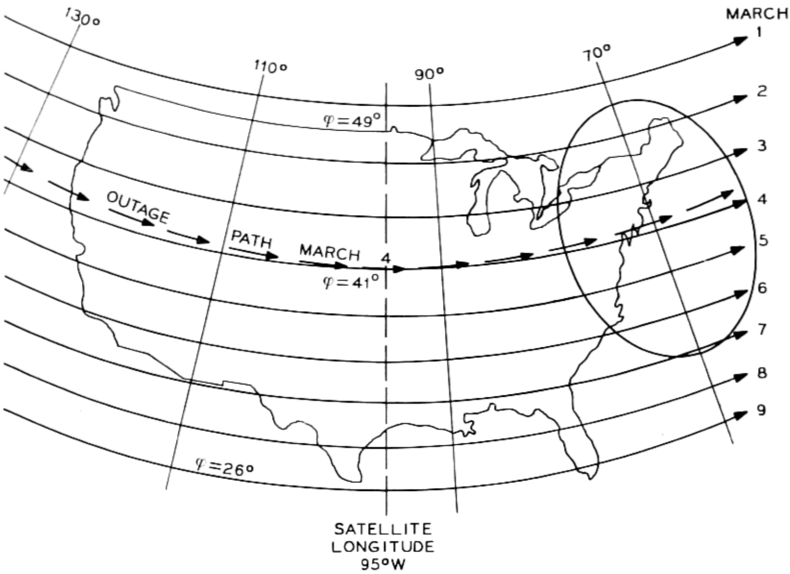

Fig. 3—Approximate paths of sun-transit outages for geostationary satellite.

after an additional two to three days, the outage paths progressing southward at a rate of about 3° latitude per day. All outages affecting United States earth terminals above north latitude 26° cease prior to mid-March.

Conversely, in the fall the daily outage paths progress from south to north, affecting southern United States terminals about October 1 and ending in the north about mid-October.

In Fig. 3, based on parameters adopted in Appendix A, a given earth terminal is affected about six days, twice yearly, while the contiguous United States experiences outages throughout a 14-day period, again twice yearly. If a multiple-feed antenna or a rapid-slewing antenna is employed to switch reception at an earth terminal from an affected satellite to another 6.8° (120 mrad) westward in the geostationary orbit, transmission from the latter satellite is interrupted only 30 minutes later.

2.2 *Eclipses of Geostationary Satellites*

Eclipses of geostationary satellites can be expected for a total of about 90 evenings per year in the spring and fall. Concurrent eclipses occur for geostationary satellites spaced less than 17.6° (310 mrad).

Eclipses occur near apparent midnight of the time zone at each satellite's longitude, beginning in late February or early March and ending by mid-April. Fall events begin about September 1 and end about mid-October. Eclipses lasting about 70 minutes occur on the dates of the spring and fall equinoxes; those lasting longer than one hour occur about 50 days per satellite per year.

Communication satellites are provided with batteries to prevent circuit outages and to maintain antenna pointing, attitude control, station keeping, telemetry, and command capabilities during eclipses. However, concomitant voltage and temperature fluctuations, loss of the solar reference for antenna pointing, and related ground command activities may contribute to an increased likelihood of satelite failure or a reduction in transmission capacity during eclipses.

III. DIVERSITY SYNCHRONOUS SATELLITE SYSTEM

A minimum arrangement of two slightly inclined, circular synchronous orbits with deliberate phasing of one working and one spare satellite in their respective orbits is suggested for providing space diversity during outage periods. The specific orbit parameters and satellite phasing are chosen so that they may remain unchanged throughout the year. Thus satellite station-keeping fuel expenditures are comparable to geostationary values. The parameters are also chosen so that only one noncritical handover of reception between satellites is required per sun-transit season.

3.1 *Basic Satellite Phasing in Specific Inclined Orbits*

Figure 4 illustrates the relationship between a "figure 8" pattern traced out by a synchronous satellite and the magnitude of its orbit inclination. Recent descriptions of such patterns are given by Rowe and Penzias,[6] treating the efficient use of orbit longitude.

Figure 5 illustrates the satellite phasing and timing of motions required for a two-satellite diversity system. The time reference selected for describing these motions is initial time t_0 mean solar hours, marking the advent of 12 o'clock noon (apparent, or sun time) on the date of the vernal equinox at average θ of mean longitudes θ_1 and θ_2 degrees west for satellites S_1 and S_2, respectively ($\theta = \langle \theta_1 + \theta_2 \rangle_{av}$). For satellites sharing radio frequency bands, a minimum orbit spacing between interfering satellites is generally specified consistent with resolving powers of the earth antennas. Accordingly, a minimum satellite spacing x degrees is assumed between mean longitudes θ_1 and θ_2.

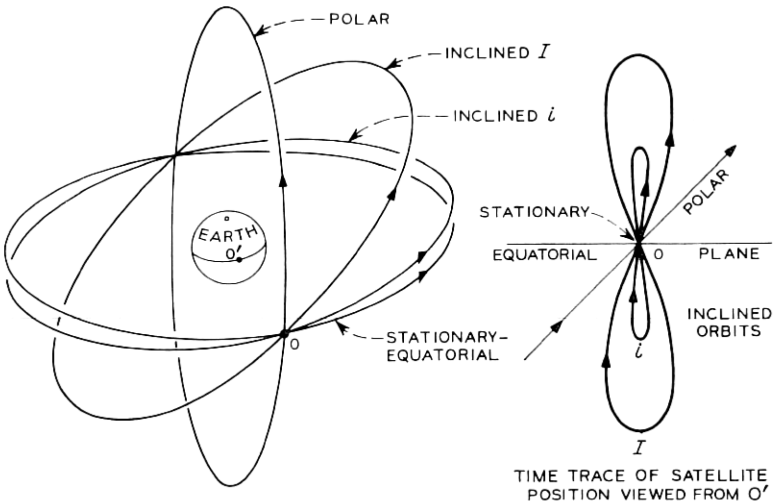

Fig. 4—Earth synchronous orbits and figure 8 patterns.

In terms of θ, chosen for service to a particular geographical region, the mean orbit longitudes shown in Fig. 5 are

$$\theta_1 = (\theta - x/2), \qquad \theta_2 = (\theta + x/2) \quad \text{degrees west.} \tag{1}$$

Dimensions of 8 patterns allowing adequate diversity between properly phased satellites are determined in Appendix B. Peak satellite displacements from the equatorial plane (geostationary orbit in Fig. 5) coincide in the spring and fall with sun transits of each satellite's mean longitude meridian.

For example, in Fig. 5(a) satellite S_1 is northernmost in its 8 pattern prior to apparent noon at average longitude θ. To an observer located at earth longitude θ_1, this coincides with alignment of the sun behind the 8 pattern for satellite S_1.

At apparent noon at longitude θ, satellite S_1 in Fig. 5(b) moves very slowly toward the geostationary orbit, while S_2 is approaching the southernmost point in its 8 pattern. The sun is located midway between the 8 patterns.

Shortly after apparent noon at longitude θ, the sun aligns behind the 8 pattern for satellite S_2, as observed from earth longitude θ_2. At this time, satellite S_2 reaches its peak excursion, while satellite S_1 moves more rapidly towards the geostationary orbit [Fig. 5(c)]. Tick marks

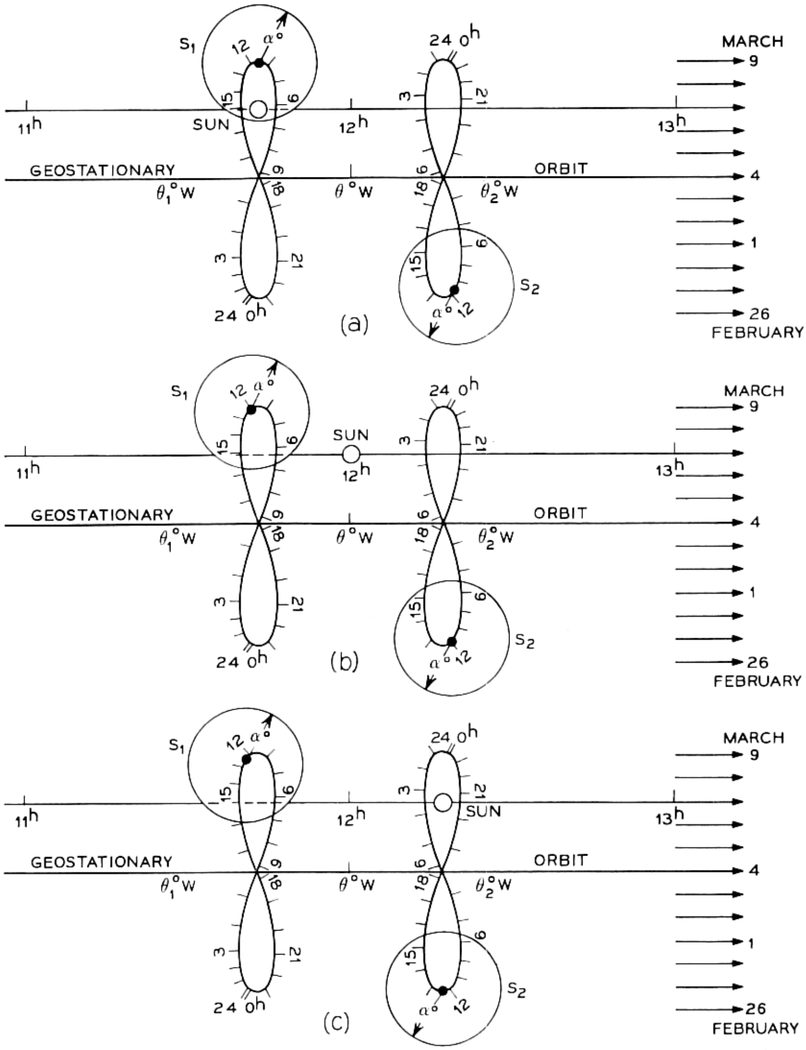

Fig. 5—Phased satellite motions.

on the 8 patterns are labeled according to each satellite's location at times referred to longitude θ.

Paths of the sun on consecutive days during the spring sun-transit season are also indicated. Note that these daily paths progress from south to north in accordance with a decreasing southern declination of the sun's rays at this time of year (cf., Fig. 3).

Circles of radius $\alpha°$ centered at each satellite define the minimum pointing angle to the sun for earth antennas directed at the satellites. Hence, reception from satellite S_1 is interrupted when the sun is within the circle for S_1. Tick marks give positions of the sun along its path, again at times referred to longitude θ.

In Fig. 5, uninterrupted reception from satellite S_1 is assumed throughout the late fall and winter, until March 7. At any convenient time between March 1 and March 7, an earth terminal observing these motions redirects its reception from satellite S_1 to satellite S_2. This allows uninterrupted reception from S_2 until the fall sun-transit season, during which this noncritical procedure is reversed.

Note that the 8 patterns in Fig. 5 are larger than required by a single earth terminal. The dimensions determined in Appendix B are sufficient to prevent serial sun transits throughout the entire latitude range of the coverage region, so that only one outage region from either satellite may traverse any part of the coverage region on any day. This simplifies switching between satellites in restoration schemes involving large numbers of working satellites and a minimum of one spare satellite.* However, for the basic scheme involving duplicate transmission via equal numbers of working and spare satellites, the dimensions of the 8 patterns may be reduced until the outage circles ($\alpha°$) are almost tangent to the geostationary orbit. Redirection of the earth antenna appropriate for Fig. 5 is required on or about March 4 for such reduced 8 patterns.

Note also that the satellites spend most of the time near the extremes of the 8 patterns, providing near-maximum diversity separation for several hours near noon. This tolerance to timing errors is particularly useful since the apparent alignments of the sun in Fig. 5 and the timing of transit events are somewhat different for observers at different locations within the coverage region. Allowances are made in Appendix B in the computation of required diversity separation for both latitude and longitude ranges of the coverage region, assuming that uninterrupted reception from the unaffected satellite is required continuously at all earth terminals throughout the coverage region.

The diversity performance is made nearly independent of arbitrary satellite spacing x by phasing each satellite so that its maximum latitude excursion occurs at sun transit of its mean longitude meridian.

Tick marks in Fig. 6 illustrate a daily progression of satellite positions at apparent noon at longitude θ throughout the year. This regular shift is observed in the *ideal* case at the earth terminals because such

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* Discussed in Section 3.3.

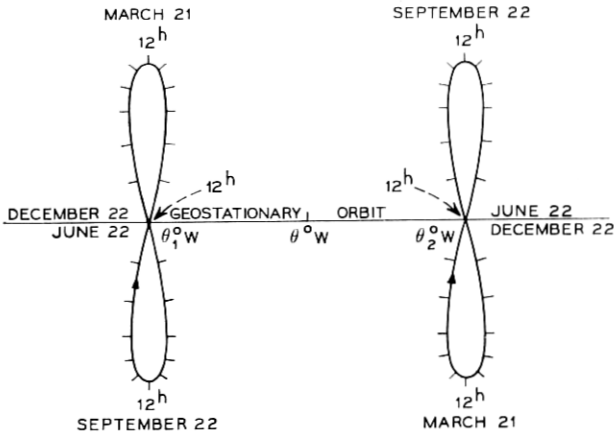

Fig. 6—Shift of daily satellite positions.

inclined synchronous orbits tend to maintain fixed orientations in space as the earth revolves around the sun (about 1° per day), illustrated in Fig. 7, by virtue of conservation of orbit angular momenta $\mathbf{m}_1$ and $\mathbf{m}_2$. Orbit perturbations, or departures from the above ideal motions, are approximately the same as those for geostationary orbits and are corrected by firing small station-keeping rocket motors at intervals throughout the lifetime of the satellites.

Specification of orbit stabilization with respect to the fixed stars is necessary to obtain properly timed satellite diversity automatically throughout the year; the precision required for diversity is needed only during outage seasons.

Hence, the daily period of satellite motions in their figure 8 patterns is less than 24 hours of civil time (mean solar hours). The actual sidereal period is $23^h\ 56^m\ 04^s.09054$ in mean solar time measure.

The daily shift of positions is utilized, by the deliberate orbit orientations and satellite phasing in the orbits, so that the apparent positions of satellites S_1 and S_2 are reversed automatically in time for diversity reception again during the fall outage season (see Fig. 7). Positions are also reversed daily, providing diversity for satellite eclipses near midnight, assuming sufficiently large orbit inclinations.

Conversely, lesser accumulated shifts must also be considered in computing the minimum diversity separation for sun transits for coverage regions located far from the equator, since sun transits occur either before, or after actual equinoxes (see Figs. 5 and 6, and Appen-

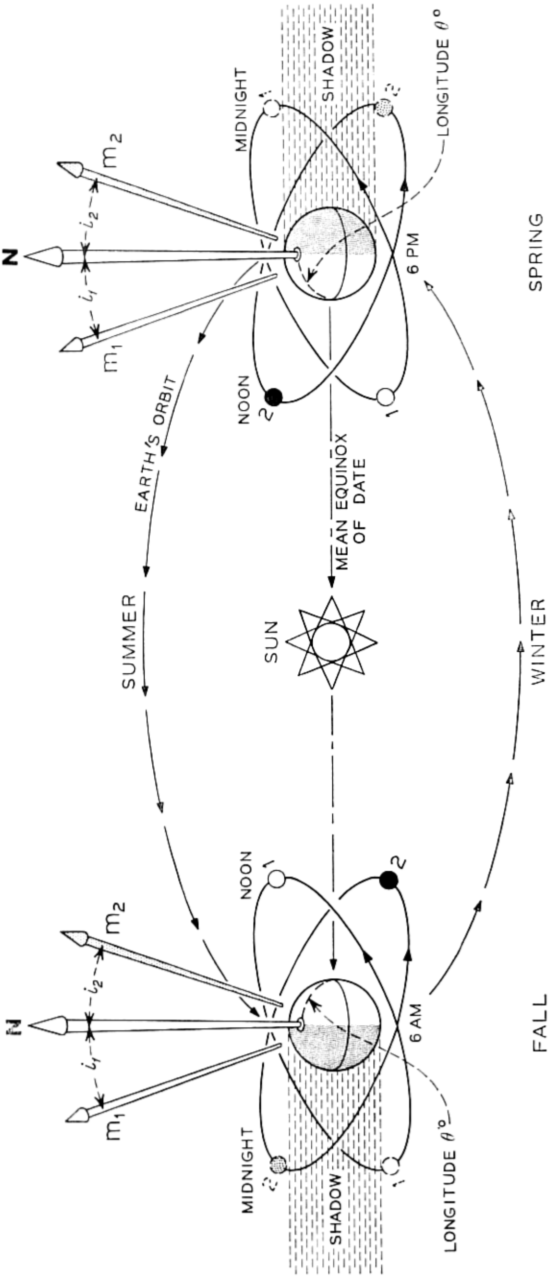

Fig. 7—Satellite space diversity with respect to the sun and seasons.

dix B). Sun transits are observed in the Northern Hemisphere prior to the vernal equinox and again after the autumnal equinox. Offsets of approximately two weeks from the symmetrical case are representative for the contiguous United States. Of course, dates of satellite eclipses are independent of earth latitude and the ideal symmetry is applicable.

3.2 *Orbit Parameters*

Satellite motions and initial conditions are illustrated in Figs. 7 and 8 for two diversity satellites.

3.2.1 *Inclination of Orbits*

The planes of orbits for satellites S_1 and S_2 are tilted slightly with respect to the earth's equatorial plane by inclination angles i_1 and i_2.

For the idealized case of equal inclinations, the minimum required magnitudes range from about 2 degrees for avoiding serial sun transits to about 9 degrees for avoiding serial and concurrent eclipses (see Appendix B).

3.2.2 *Alignment of Inclined Orbit Planes*

Positioning of the figure 8s is accomplished by aligning the orbit planes in slightly offset opposition as shown in Fig. 8. Two plane intersections with the earth's equator result, each forming acute angles $(90 - x/2)$ degrees symmetrically with the mean equinox axis (intersection of planes of the equator and of the earth's orbit around the sun; direction from earth towards the sun at the vernal equinox).

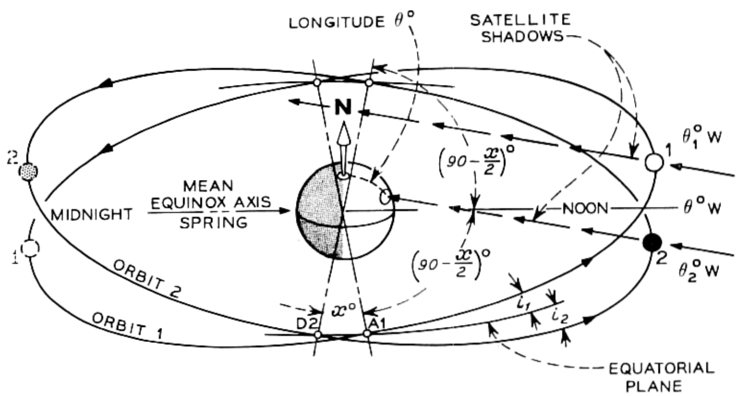

Fig. 8—Synchronous orbits phased for sun diversity.

3.2.3 *Phasing of Satellite S_1*

The time of the ascending node in orbit 1 for satellite S_1, for spacing x degrees is

$$t_1 = t_0 - [(6 + x/30)] \text{ mean solar hours,} \tag{2}$$

so that at time $t = t_0 - x/30$ hours satellite S_1 necessarily assumes its maximum north latitude (upper limit of excursion for left-hand figure 8 pattern in Fig. 5). From Fig. 8, note that the semi-major axis of the 8 in geocentered angular measure is equivalent numerically to orbit inclination i_1.

3.2.4 *Phasing of Satellite S_2*

The descending node in orbit 2 for satellite S_2 is specified by

$$t_2 = t_0 - [(6 - x/30)] \text{ mean solar hours,} \tag{3}$$

for which satellite S_2 assumes its maximum south latitude at time $t = t_0 + x/30$ hours.

3.2.5 *Satellite Motions Related to the Sun and Seasons*

By synchronizing satellite motions and timing with respect to the earth's revolution about the sun as shown in Fig. 7, the required space diversity is obtained during both spring and fall outage seasons.

Satellite motions and timing are specified above in terms of initial conditions at the vernal equinox. Of course, actual satellite launching is not restricted to any season, provided that satellite motions coincide with those for the specified system at the times when sun-caused outages occur in geostationary equatorial systems.

3.3 *Phased Multisatellite Systems*

Two satellites are required for the basic diversity system. The diversity satellites may be placed as desired in orbit longitude consistent with an assumed minimum orbit spacing x.

An obvious system growth is to add uniformly spaced, alternately phased working and spare satellites along the orbit (Fig. 5). Note that one of a diversity pair of spare satellites can restore all working satellites if fast switching may be employed daily at the affected earth terminals. Reception is transferred in sequence between transitted active satellites and the unaffected spare.* The orbit spacing between *second*-adjacent satellites (same phasing) should be sufficient to prevent mutual sun transits of the latter satellites within the coverage region.

* The affected spare is available as an additional working satellite.

For this case, only half of the orbit spacing required by geostationary satellites is required by the diversity satellites (about 4° for avoiding mutual outages within the contiguous United States).

Conversely, for satellites which may be closely spaced ($x \doteq 1°$), efficient use of the orbit may result from judicious incorporation, in a manner consistent with the satellite phasing and timing described above, of orbit loading techniques suggested by Rowe and Penzias.[6] Deliberate relative phases in adjacent 8 patterns may prevent major multiple sun transits of all satellites near the same latitude and minimize daily switching to different unaffected satellites (Section 3.1).

For large orbit inclinations,* (*i*) tracking of satellite and earth antennas is required, (*ii*) reduction in latitude of the coverage region results, (*iii*) transmission at low angles of arrival is more susceptible to atmospheric degradation, and (*iv*) the interference exposure between radio relay and satellite services is increased.

3.4 *Antenna Requirements for Earth Terminals and Satellites*

Only slight geometric departures from the geostationary case are required to obtain diversity for avoiding sun-transit outages; somewhat larger departures are required for avoiding eclipses. Hence, satellite radio transmission parameters appropriate for corresponding geostationary designs are essentially retained.

Earth antennas need follow only slow and very small periodic satellite motions. These motions are accomodated reliably by conventional 24-hour cyclic cam drives (sidereal time measure). Costs and maintenance for such antenna drives are virtually insignificant when compared with those for full automatic tracking. Cyclic drives are appropriate for a large deployment of small earth antennas requiring moderate beam-pointing precision, while costs for full automatic tracking are less significant for a smaller number of large antennas requiring precise beam pointing.

A minimum earth antenna steering requirement accommodating orbit inclinations up to 10° (175 mrad) and satellite longitude drifts from assigned orbit stations of ±10°, for satellite elevations of 5° or more, is reported by the Communications Satellite Corporation for quasi-stationary satellites.[7] Such earth terminals are compatible with the diversity satellites, since in the ideal application the smaller desired orbit inclinations are also maintained continuously.

The spin axis of a satellite is maintained perpendicular to its orbit plane, in the simplest wheel-mode attitude stabilization. Satellite

* For $x = 1°$, $i \doteq 10.7°$ and for $x = 5°$, $i \doteq 24°$, from equation (7) of Ref. 6.

antenna pointing referred to this axis benefits from partial compensation of pointing errors otherwise accompanying departures from the equatorial plane in the inclined orbits.*

IV. CONCLUSIONS

Space diversity is provided automatically at times of sun transits and eclipses by a convenient modification of a geostationary system in which the satellites appear to move in figure 8 patterns. Alternate satellites are oppositely phased, so that when one satellite is north the other is south. Orbit orientations and timing of satellite motions are arranged so that near the spring and fall equinoxes, when geostationary satellites transit the sun, the diversity satellites are at extreme north and south positions, allowing uninterrupted reception from at least one satellite.

The contiguous United States is cleared of serial sun-transit outages if orbit inclinations of about two degrees are employed. Concurrent satellite eclipses are also reduced in frequency and duration, and are avoided by increasing the orbit inclinations to about nine degrees.

Neglecting perturbations common to synchronous orbits including the geostationary orbit, the satellite deployment is steady state. Satellite launching requirements, mean station-keeping precision, and lifetimes are comparable to the geostationary case.

Diversity is provided automatically during both spring and fall outage seasons, requiring two noncritical switches between satellites per year.

Relatively minor modifications of earth terminals and satellites designed for geostationary service are required.

The diversity satellites are positioned as desired in orbit longitude without degrading system performance significantly, consistent with minimum orbit spacings to control interference from neighboring satellites.

Transmission via the unaffected satellite of a diversity pair can be switched in sequence daily to restore all transitted active satellites of a larger system.

One-half the minimum orbit spacing required by geostationary systems to prevent mutual outages of neighboring satellites within large coverage areas is required by the diversity system, since only alternate satellites experience outages on a given day.

Sun-transit outages in satellite circuits can be restored without involving terrestrial facilities.

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* For $i = 2°$, a peak uncompensated pointing error of 0.3° is representative.

V. ACKNOWLEDGMENTS

The writer wishes to express appreciation to H. W. Evans and to many colleagues who encouraged this presentation, to D. Jarett for many helpful suggestions, and to L. C. Thomas for suggesting a time-correction accounting for earth rotation during outages.

APPENDIX A

Simplified Geometry and Numerical Examples for Geostationary Satellites

A.1 *Minimum-Latitude Circle Tangent to Outage Path*

At the satellite longitude, conjunction of the sun and satellite occurs at apparent noon and the satellite's shadow intercepts a minimum latitude, shown in Fig. 9. On successive days in the spring, the shadow path becomes tangent to a smaller north latitude at the satellite's longitude, and lies slightly to the north of this latitude for all other longitudes.

Figure 9 illustrates the sun's rays on March 4, 1970. From an almanac, the apparent declination of the sun for 0 hours ephemeris time (E.T.) is $-6^\circ\ 40'\ 54''.5$ and on March 5, is $-6^\circ\ 17'\ 49''.0$.[8]

Ephemeris transit of the sun on March 4 is given as $12^h\ 11^m\ 50^s.39$ and the reduction ΔT from universal time (U.T.) to E.T. for the year 1970.5 is approximately 40^s. The ephemeris time corresponding to solar transit at west longitude λ° is

$$\text{E.T.} \doteq \text{E.T. (TRANSIT)} + [1.002738]\,\frac{\lambda}{360}\,(24^h)\ \text{hours}, \qquad \lambda < 180^\circ, \tag{4}$$

where the coefficient in brackets is the approximate ratio of the mean solar day to the mean sidereal day. Allowing for a 6-hour time difference from the Greenwich Meridian to the Central Time Zone,

$$\text{C.S.T.} \doteq \text{E.T.} - \Delta T - 6^h\ \text{hours}. \tag{5}$$

Assume a transit of geostationary satellite stationed at $\lambda = 95^\circ$W:

$$\text{C.S.T.} \doteq 12^h\ 11^m.84 + 6^h 20^m.93 - 0^h\ 0^m.67 - 6^h\ 0^m. \tag{6}$$

From equation (4), the ephemeris time of this event is $18^h\ 32^m.77$ on March 4. Interpolating between 0^h on March 4 and 0^h on March 5, the sun's apparent declination is

$$\begin{aligned} D &\doteq -6.682^\circ + \frac{18.55}{24.00}\,(0.385^\circ), \\ &\doteq -6.38^\circ. \end{aligned} \tag{7}$$

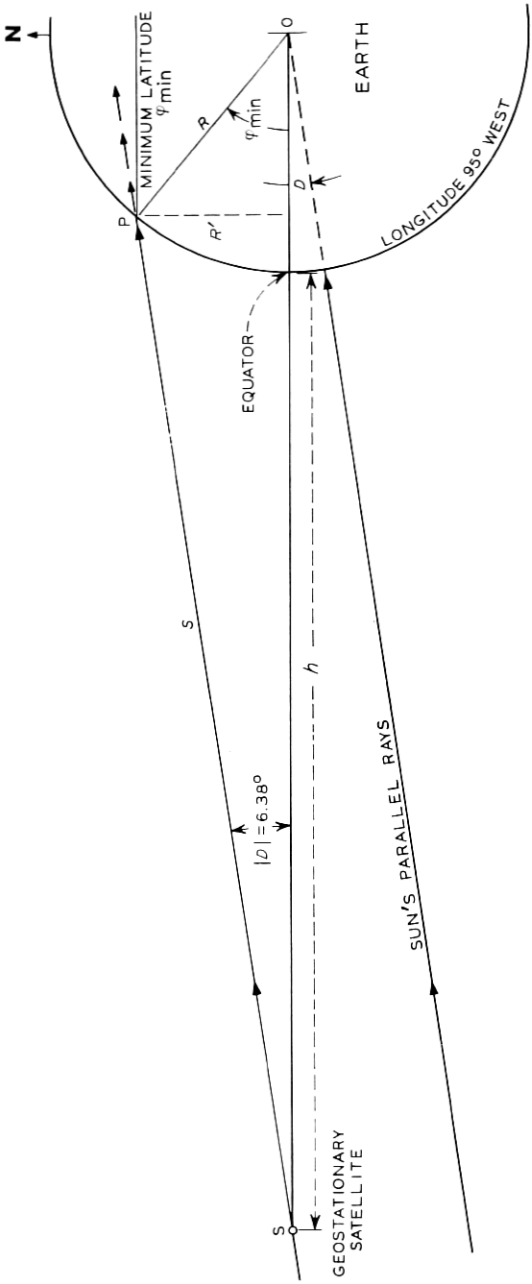

Fig. 9—Determination of minimum outage path latitude φ_{min}.

Note from Fig. 9,

$$\sin \varphi_{\min} = -\frac{S}{R} \sin D, \tag{8}$$

$$S = \frac{h + R(1 - \cos \varphi_{\min})}{\cos D} \text{ km}, \tag{9}$$

where $\varphi_{\min}$ is the north latitude of the satellite's shadow at the time of sun transit.

Then

$$\sin \varphi_{\min} = -\frac{\sin D\left[(1 - \cos \varphi_{\min}) + \frac{h}{R}\right]}{\cos D}, \tag{10}$$

from which it is determined that $\varphi_{\min} \doteq 41.0°$ north latitude, assuming geostationary orbit altitude $h = 35{,}900$ km and mean spherical earth radius $R = 6373$ km.

A.2 *Estimate of Speed with Which Outage Centers Traverse U.S.A.*

Figure 10 shows the contiguous United States represented by a longitude span of 60° centered at the satellite longitude and located at north latitude $\varphi_{\min}(41°)$. Consider a projection of the extreme longitude meridians (i.e., ±30° referred to the satellite longitude) parallel to an assumed shadow axis between the span center at B′ and the satellite at B, such that orbit arc intercept $\widehat{AC}$ is specified.

The geocentric orbit radius is

$$AO = CO = R + h \doteq 42{,}270 \text{ km}. \tag{11}$$

Then the radius of latitude circle $\varphi_{\min}$ is

$$R'' = R \cos \varphi_{\min} \doteq 4810 \text{ km}. \tag{12}$$

The approximate distance measured along latitude circle $\varphi_{\min}$ for this model of the United states is

$$|\widehat{A'C'}|_{60°} = \frac{2\pi 60}{360} R'' \doteq 5040 \text{ km}. \tag{13}$$

Recognizing equilateral triangle A′OC′, the orbit chord is

$$AC = A'C' = R'' \doteq 4810 \text{ km}. \tag{14}$$

The solution of an oblique triangle with sides a, b, c and opposite angles A, B, C is

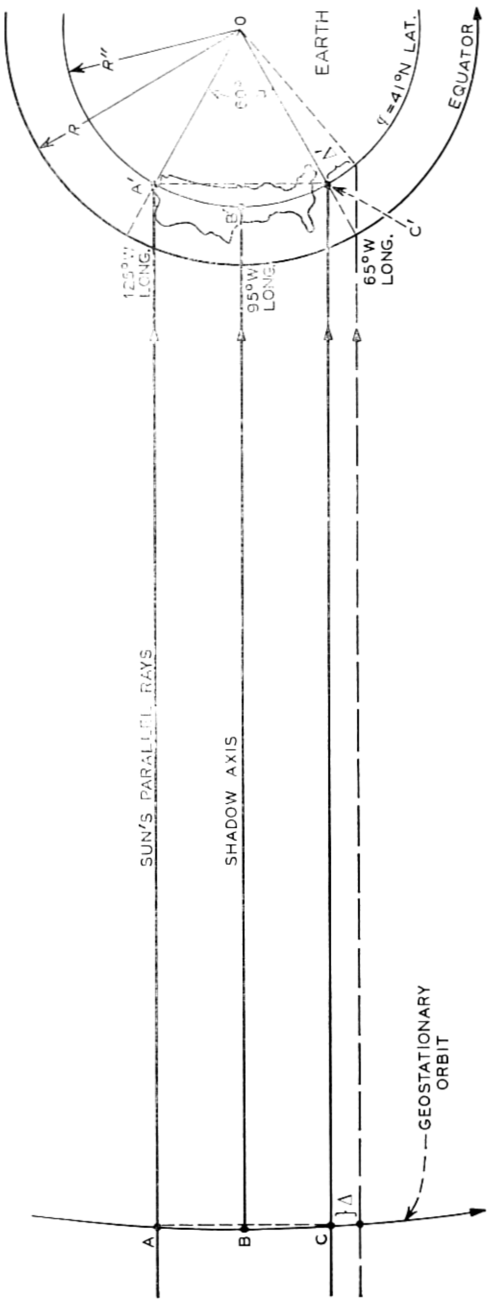

Fig. 10—Geometry describing motion of an outage region across a model of the contiguous United States.

$$\cos A = \frac{-a^2 + b^2 + c^2}{2bc}, \tag{15}$$

so that the orbit arc may be found from

$$\cos (A\text{-}O\text{-}C) = \frac{-(AC)^2 + 2(R + h)^2}{2(R + h)^2}. \tag{16}$$

Then the desired geocentered angle representing the orbit intercept of all parallel sun's rays simultaneously illuminating this model is

$$\widehat{AC} = \cos^{-1}(A\text{-}O\text{-}C) \doteq 6.5 \text{ degrees.} \tag{17}$$

The time required for a satellite's shadow to traverse a stationary representation of the United States is numerically equivalent to the time for the fractional revolution of a satellite from position A to C:

$$t_{sta} \doteq 6.5° \times \frac{(24 \times 60)^{m}}{360°} = 26.0 \text{ min.} \tag{18}$$

However, the actual elapsed time t_t is greater by virtue of earth rotation during this interval. The effective longitude span of the United States is very approximately

$$\widehat{A'C'} + \widehat{\Delta'} \doteq 60° + 26^{m}.0 \times \frac{15°}{60^{m}} \doteq 66.5°. \tag{19}$$

Accounting for a correspondingly enlarged orbit intercept,

$$t_t \doteq t_{sta} \times \frac{66.5°}{60°} \doteq 28.8 \text{ min,} \tag{20}$$

so that an outage region traverses the United States from west to east in approximately one-half hour. The exact interval depends primarily upon φ_{min}.

A.3 *Estimation of Size of Outage Region—Example*

A conic figure of revolution about axis **SP** in Fig. 2 defining the affected outage region subtends total angle 2α measured at the satellite. To enable example calculations without specific reference to antenna pattern data, a worst-case minimum angular separation $\alpha = 1°$ between a satellite and the sun center is adopted.*

*The value $\alpha = 1°$ is assumed for a hypothetical 4-GHz satellite system incorporating 55 percent efficient, 30-ft diameter parabolic reflector earth antennas, a receiving system noise temperature of 200 K, and a 3-dB allowable increase in received thermal noise power.

The horizontal plane at the location of the satellite's shadow P at time of transit is shown in edge view in Fig. 11. Slant range $S = SP$ is found from equation (9) to be about 37,470 km. The conic section defined by this plane and the outage cone is elliptical; point P specifies its motions.

The east-west semi-minor axis r is equivalent to the radius of the right circular intersection of the cone and a plane through P normal to satellite-shadow axis **SP**:

$$\begin{aligned} r &= S \sin \alpha \\ &\doteq 655 \text{ km.} \end{aligned} \tag{21}$$

The north-south semi-major axis r' in Fig. 11 is found from a projection of the above circular intersection upon the local horizontal plane at P:

$$\begin{aligned} r' &= \frac{r}{\cos (\varphi_{\min} - D)} \\ &\doteq 970 \text{ km.} \end{aligned} \tag{22}$$

A.4 *Estimate of Outage Duration*

The maximum duration of an outage occurring at an earth terminal located on latitude $\varphi_{\min}$ is approximately that fraction of time t_t [equation (20)] for the satellite's shadow to travel the 1310-km width of the

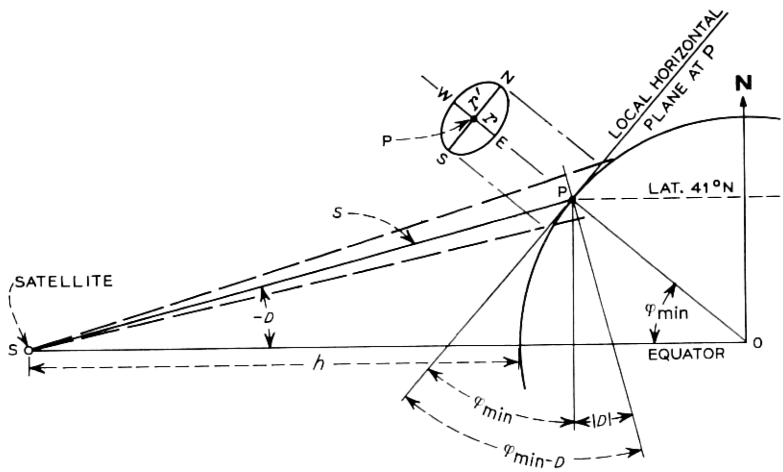

Fig. 11—Determination of outage region at P.

above outage pattern. Allowing for earth rotation, and noting that the 60° longitude span $\widehat{A'C'}$ at latitude φ_{min} corresponds to chord $A'C'$:

$$t_t \times \frac{2r}{|\widehat{A'C'}|} < t_d < t_t \times \frac{2r}{A'C'} ; \tag{23}$$
$$7.5 \text{ minutes} < t_d < 7.8 \text{ minutes}.$$

For the satellite stationed at longitude 95° west, the path of its shadow on March 4, 1970, approaches latitude 41° north near Omaha at 12:32 p.m. C.S.T. Taking dimensions of the outage region into account, the West Coast should just begin to experience outages north of Eureka, California, at about 10:16 a.m. Pacific Standard Time, and the last outages, near Boston, should cease about 1:50 p.m. Eastern Standard Time.

A.5 *Geostationary Satellite Spacing and Serial Outages*

Several identical satellites are assumed deployed along the geostationary orbit. Earth terminals are assumed capable of receiving signals from at least one pair of adjacent satellites, either simultaneously or one at a time. The orbit spacing between satellites is assumed to be uniform, but adjustable to alter the timing of serial sun transits. Numerical assumptions made in previous sections are retained for illustration; earth terminals are assumed to be located along the outage path (worst case).

A.5.1 *Case 1–Minimum of 30 Minutes Between Switches at an Earth Terminal*

If each satellite is assumed to possess spare circuit capacity adequate for the restoration of one transitted satellite, it is of interest to estimate the orbit spacing between satellites required for a prescribed outage-free interval between switches at an affected earth terminal. The interval between onsets of serial outages at a given earth terminal for satellites spaced 6.5° in orbit, allowing for earth rotation is about 28.8 minutes (Section A.2). Then, an approximate minimum satellite spacing for a 30-minute clear interval is $(30^{m}/28^{m}.8) \times 6.5° \doteq 6.8°$.

A.5.2 *Case 2–Minimum of 30 Minutes Between Adjacent Outages*

If multiple satellites are deployed without spare capacity and an earth terminal receives simultaneously from adjacent satellites, but does not switch between them, a 30-minute required clear time between outages of the adjacent satellites leads to a greater estimated satellite spacing. The elapsed time for the center of a first (easterly) outage region to depart an affected earth terminal and travel eastward until reception is regained (a distance equal to the semi-minor dimension; Section A.4) is approximately $7^{m}.6/2 \doteq 3.8$ minutes. The elapsed time for the center of a second outage region to approach the same earth terminal is also 3.8 minutes, measured from onset of the second outage. The sum of elapsed times and the required 30-minute clear interval is 37.6 minutes. The minimum satellite spacing, scaled from the 28.8-minute interval between arrivals of shadows at the terminal for satellites spaced 6.5° (Section A.2) is approximately $(37^{m}.6/28^{m}.8) \times 6.5° \doteq 8.5°$.

A.5.3 *Case 3–Minimum of 30 Minutes Free of United States Outages*

An estimate of the satellite spacing required for a 30-minute clear interval between outages of earth terminals throughout the contiguous United States for the case without switching is desired. A time equivalent of the satellite spacing for a 30-minute clear interval between adjacent outages at a single earth terminal is about 37.6 minutes (Section A.5.2). A satellite spacing of 6.5° is necessary for simultaneous sun transit of a first satellite at the extreme eastern terminal and a second satellite at the extreme western terminal; a time equivalent of this spacing is approximately 28.8 minutes (Section A.2). The sum of these intervals, 66.4 minutes, accounts for transits of all terminals within the assumed 60° longitude span at 41° north latitude. The approximate minimum satellite spacing for a 30-minute clear interval throughout the United States is $(66^{m}.4/28^{m}.8) \times 6.5° \doteq 15.0°$.

A.5.4 *Case 4–Minimum of 30 Minutes Free of Outages Throughout One Time Zone*

The time equivalent of spacing for a 30-minute clear interval at a single terminal without switching is 37.6 minutes. The time equivalent of spacing for simultaneous sun transits of adjacent satellites at eastern and western terminals bounding a 15° time zone is approximately $(15°/60°) \times 28^{m}.8 \doteq 7.2$ minutes. The required interval is about 44.8 minutes, accounting for outage dimensions and all terminals within one time zone. The resulting minimum satellite spacing is approximately $(44^{m}.8/28^{m}.8) \times 6.5° \doteq 10.1°$.

APPENDIX B

Estimation of Minimum Required Space Diversity

B.1 *Minimum Orbit Inclinations for a Prescribed Coverage Region*

Figure 12 relates the latitude extremes of a desired coverage region to limits of the sun's apparent declination angle for which sun transits

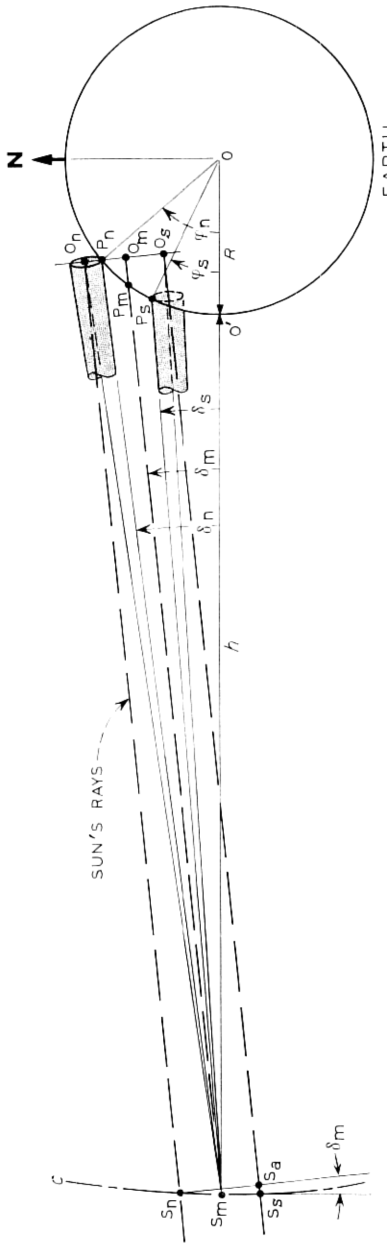

Fig. 12—Estimation of minimum required space diversity.

of diversity satellites can affect transmissions. First, two conic figures of revolution as described in Section A.3 having angular radius α define the northernmost and southernmost outage regions for geostationary satellite S_m. Minimum orbit inclinations for diversity satellites S_n and S_s are estimated by geometric construction. Parallel sun rays are assumed and atmospheric refraction is neglected for all but extreme latitudes in the presence of fairly large angles of incidence.[9]

One approximation implicit in the figure is that the satellites occupy the same mean orbit longitude. This enables a highly simplified geometrical analysis and the uncertainty introduced is shown later to be insignificant.

B.2 *Determination of Minimum Orbit Inclinations*

For geostationary satellite S_m in Fig. 12, apparent declination angle limits δ_n and δ_s are calculated for which the satellite shadows intercept north geographic latitude limits φ_n and φ_s of an assumed coverage region in the Northern Hemisphere between points P_n and P_s respectively. Slant range segment P_nS_m is determined from the solution of oblique triangle P_nOS_m:

$$P_nS_m = [R^2 + (R+h)^2 - 2R(R+h)\cos\varphi_n]^{\frac{1}{2}}, \tag{24}$$

and

$$P_sS_m = [R^2 + (R+h)^2 - 2R(R+h)\cos\varphi_s]^{\frac{1}{2}} \text{ km}. \tag{25}$$

The declination angles corresponding to northern and southern boundaries of the coverage region are

$$\delta_n = \cos^{-1}\left[\frac{-R^2 + (P_nS_m)^2 + (R+h)^2}{2(P_nS_m)(R+h)}\right] \text{degrees } S, \tag{26}$$

and

$$\delta_s = \cos^{-1}\left[\frac{-R^2 + (P_sS_m)^2 + (R+h)^2}{2(P_sS_m)(R+h)}\right] \text{degrees } S, \tag{27}$$

where the units designation S denotes angular displacement south from the celestial equator.

The angle measuring bisector $\mathbf{P}_m\mathbf{S}_m$ is denoted by δ_m, where

$$\delta_m = \langle \delta_n + \delta_s \rangle_{av} \quad \text{degrees S}. \tag{28}$$

Synchronous satellites S_n and S_s are shown in Fig. 12 located on great circle C of a geocentered sphere of radius $(R + h)$ whose plane contains the mean geopolar axis and an assumed common satellite meridian circle. The satellites are also assumed to be symmetrically opposite and equidistant from the equatorial plane. The required distance between parallel sun's rays through satellites S_n and S_s having mean apparent declination δ_m is determined by constructing segment $\mathbf{O_nO_s}$ perpendicular to $\mathbf{S_mP_m}$ through P_n. The base of isosceles triangle $O_nS_mO_s$ represents the required ray separation. Making the approximation

$$O_nS_m \equiv O_sS_m \doteq P_nS_m, \tag{29}$$

and denoting the angle O_n-S_m-O_s by γ,

$$\gamma = \delta_n - \delta_s + 2\alpha \quad \text{degrees.} \tag{30}$$

From the solution of an isosceles triangle,

$$O_nO_s \doteq [2(P_nS_m)^2(1 - \cos\gamma)]^{\frac{1}{2}} \text{ km.} \tag{31}$$

Constructing segment $\mathbf{S_nS_a}$ perpendicular to $\mathbf{S_mP_m}$ through S_n, its length is

$$S_nS_a \equiv O_nO_s \text{ km.} \tag{32}$$

The length of chord S_nS_s between the satellites on circle C is

$$S_nS_s = S_nS_a/\cos\delta_m \text{ km.} \tag{33}$$

The total geocentered arc $\widehat{S_nS_s}$ on circle C corresponding to chord S_nS_s is found from the solution of isosceles triangle S_nOS_s (not illustrated). Note that

$$OS_n \equiv OS_s \equiv OS_m = (R + h) \text{ km.} \tag{34}$$

Then

$$\cos(\widehat{S_nS_s}) = \frac{-(S_nS_s)^2 + 2(OS_m)^2}{2(OS_m)^2}. \tag{35}$$

Note from Fig. 12 that equal orbit inclinations i_n and i_s are determined by the minimum geocentered angular displacements of synchronous satellites S_n and S_s from the equatorial plane, necessary for avoiding simultaneous sun-transit outages between latitudes φ_n and φ_s. Hence,

$$i_n = i_s = (\widehat{S_nS_s})/2 \quad \text{degrees.} \tag{36}$$

While the simplified geometry of Fig. 12 results from an assumption that the satellites' mean longitudes are identical, recall from Section 3.1

that the maximum satellite excursions are made to occur at the instant of zenith transit viewed by an observer at each satellite's longitude. Thus, for earth terminals situated along the longitude meridian of—and receiving from—satellite S_1, the minimum required orbit inclination i_1 is identical to i_n. Very slightly increased inclinations are necessary to accommodate receiving earth terminals far from this longitude.

B.3 *Correction for Longitude Span and Latitude Location of Coverage Region*

The maximum time difference Δt_1 between sun transit of a geostationary satellite centered over the United States and observed along its longitude, and sun transit of the same satellite observed at a longitude displaced by $\pm 30°$, for a minimum latitude of 26°N is about ∓ 0.3 hour, allowing for earth rotation (Fig. 3; Section A.2). The magnitude of accumulated time shift Δt_2 (civil time versus sidereal time) relating positions of satellites at 0^h to 0^h at the vernal equinox, arising from location of the affected coverage region north of the equator, is about 1 hour (Fig. 6). An approximate worst-case adjustment of orbit inclinations providing the required displacement of diversity satellites from the equator at times when sun transits would otherwise be observed is

$$i' \doteq \frac{i_n}{\cos [(|\Delta t_1| + |\Delta t_2|)(360°/24^h)]} \text{ degrees.} \tag{37}$$

B.4 *Illustrative Calculation*

It is assumed that latitude limits φ_n and φ_s for the United States coverage region to be cleared of outages are 49°N and 26°N, respectively. A spherical earth model is assumed with radius $R = 6373$ km. The height of the geostationary orbit h is assumed to be 35,900 km. A conic sun-transit outage figure is assumed (Fig. 12), having a radius in angular measure of $\alpha = 1°$.

Numerical results are obtained using all preceding relationships:

From equations (24) and (25),	$P_nS_m = 38{,}394$ km,
	$P_sS_m = 36{,}652$ km.
From equations (26) and (27),	$\delta_n = 7.200°$,
	$\delta_s = 4.375°$.
From equation (28),	$\delta_m = 5.788°$.
From equation (30),	$\gamma = 4.825°$.
From equation (31),	$O_nO_s = 3{,}232$ km.
From equation (33),	$S_nS_s = 3{,}249$ km.

From equation (34), $OS_m = 42{,}273$ km.

From equation (35), $\cos(\widehat{S_n S_s}) = 0.997047.$

From equation (36), $i_n = i_s = 2.201°.$

From equation (37), $i' \doteq 2.337°.$

For larger earth terminals, assuming $\alpha = 0.7°$ for 25-m antennas, the corresponding worst-case minimum equal orbit inclinations providing the specified diversity is 2.045 degrees.

B.5 *Minimum Orbit Inclinations for Avoiding Serial Eclipses*

The earth's shadow is assumed to be a circular cylinder with a diameter equal to the mean diameter of the earth. This amounts to neglecting atmospheric refraction and the distinction between the umbra and penumbra shadow regions. For satellites with batteries, the net radiation energy lost per eclipse corresponds to a time integration of the actual solar-array power output. This is nearly the energy loss which would result if the solar source were completely obstructed while the satellite traversed the assumed cylindrical shadow.

An approximate relationship between declination D and the orbit arc eclipsed is illustrated by Figs. 13 and 14. The length of geostationary orbit radius OS is $(R + h)$ km, so that

$$O'S = (R + h) \mid \sin D \mid \text{ km}. \tag{38}$$

Angle ψ in Fig. 14 is thus determined:

$$\psi = \sin^{-1}\left(\frac{O'S}{R}\right) \text{ degrees}. \tag{39}$$

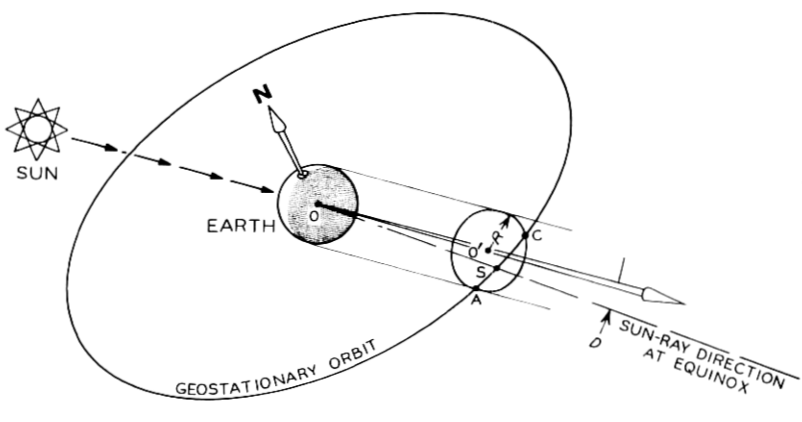

Fig. 13—Simplified geometry describing satellite eclipses.

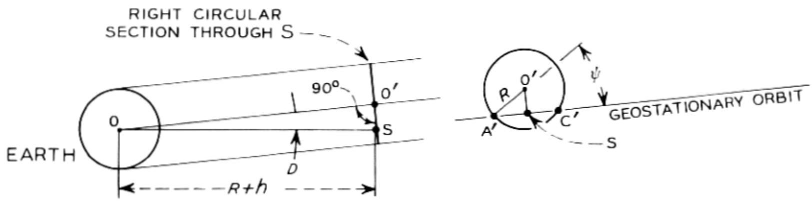

Fig. 14—Projection of points A, C upon right section of earth's shadow through S.

The length of the chord intercept common to both the orbit and the cylindrical earth shadow is determined by a normal projection of the orbit upon a right circular section of the shadow through S. From equations (38) and (39),

$$AC = A'C' \doteq 2R \cos\left\{\sin^{-1}\left[\frac{(R+h)\,|\sin D\,|}{R}\right]\right\}\text{km.} \tag{40}$$

If the fraction in brackets in equation (40) is smaller than unity for a given declination D, an eclipse of the orbiting satellite is indicated. For zero values of apparent declination, the chord AC is simply twice the mean earth radius R.

The corresponding orbit arc $\widehat{AC}$ is next calculated from the solution of oblique triangles:

$$\widehat{AC} = \cos^{-1}\left\{\frac{-(AC)^2 + 2R^2}{2R^2}\right\}\text{degrees.} \tag{41}$$

Hence, the minimum space diversity in geocentered angular measure necessary for avoiding serial satellite eclipses is identified numerically with the maximum orbit arc intercept, occurring for $D = 0°$. From equation (41), the maximum resulting geocentered angle, corresponding to one earth diameter, is approximately 17.6°. Then each minimum orbit inclination $i_1 = i_2$ necessary for avoiding serial eclipses in the manner of Section 3.2 is approximately $17.6°/2 = 8.8°$.

Finally, it is of interest to estimate the time required for the satellite to traverse shadow arc $\widehat{AC}$. The interval Δt_e is numerically equivalent to the resulting arc fraction times the orbit period, corrected for the earth's revolution about the sun:

$$\Delta t_e \doteq [1.002738](24 \times 60)^{\text{m}} \times \frac{\widehat{AC}}{360}\text{minutes.} \tag{42}$$

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