A Linear Phase Modulator for Large Baseband Bandwidths

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(Manuscript received June 3, 1970)

A linear phase modulator with a stable carrier frequency would be a useful component in radio systems—especially in coherent phase-shift-keyed PCM systems with baud rates of the order of 100 megabauds per second.

The Armstrong modulator appears adequate for these applications; the circuit functions required for its realization are well understood and amenable to the techniques of integrated circuitry.

In this paper, an analysis of the signal and distortion properties of the Armstrong circuit and variations of it are presented and applied to three system applications: as a replacement modulator for existing low-index analog systems; for multilevel coherent phase-shift-keyed PCM systems; and for frequency-division frequency-modulation multiplex systems which are of interest in radio trunk systems.

I. INTRODUCTION

A linear phase modulator with a stable carrier frequency would be a useful component for the following three applications.

(*i*) As a replacement modulator for the reflex Klystron in an otherwise all solid-state repeater of the TL System.[1]

(*ii*) For frequency-division frequency-modulation multiplex systems with baseband bandwidths of the order of 100 MHz.[2]

(*iii*) For multi-level coherent phase-shift-keyed PCM systems with baud rates of the order of 100 megabauds per second.[3]

The modulator described in this paper appears adequate for these applications. It is based upon the original Armstrong circuit which is well suited to large baseband bandwidths and is reasonably linear for low modulation indexes.[4] An important feature of this method of modulation is that the carrier frequency can be stable with respect to ambient effects since it can be derived from a temperature-stabilized quartz crystal oscillator. The baseband bandwidths which may be achieved are those for which low-index double sideband amplitude modulators can be built.

An analysis of distortion is presented for the types of baseband signals used in the three applications discussed above, and a circuit is described in which the phase deviation can be increased to any desired value.

II. CIRCUIT DESCRIPTION

The Armstrong modulator is illustrated in Fig. 1. The baseband signal is modulated in a double-sideband suppressed-carrier amplitude modulator with a sufficiently low index of modulation to ensure suitable linearity. At the modulator output another carrier, 90° out of phase with the first, is added to the sidebands. The residual AM is removed by the limiter whose output is a low-index phase-modulated signal. The phase distortion can be made arbitrarily small by choice of the carrier to sideband power ratio at the limiter input; the result is a nearly linear, low-index phase-modulated signal.

Let the baseband signal be

$$e = v(t), \quad \text{with} \quad | v(t) | \leqq 1. \tag{1}$$

The output of the double-sideband suppressed-carrier amplitude modulator is

$$e_a = mv(t) \cos \omega_0 t \tag{2}$$

where $m \leqq 1$ is the index of modulation.

A quadrature carrier is added to e_a in approximately the correct phase to obtain

$$e_p = \sin (\omega_0 t + \epsilon) + mv(t) \cos \omega_0 t. \tag{3}$$

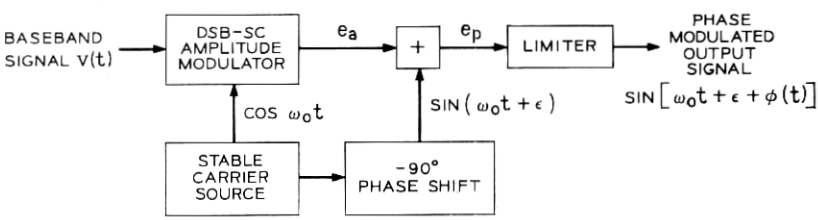

Fig. 1—Armstrong phase modulator.

$$e_p = \sqrt{1 + 2mv(t)\sin\epsilon + m^2v^2(t)} \cdot \sin\left[\omega_0 t + \epsilon + \tan^{-1}\frac{mv(t)\cos\epsilon}{1 + mv(t)\sin\epsilon}\right], \tag{4}$$

where ϵ is small and represents any error in carrier phase.

If this signal is passed through a perfect limiter the envelope becomes constant, leaving an angle modulated signal whose phase modulation is

$$\varphi(t) = \tan^{-1}\frac{mv(t)\cos\epsilon}{1 + mv(t)\sin\epsilon}. \tag{5}$$

When the nonlinear distortion is small, the controlling distortions will be second and third order so terms in the expansion of equation (5) beyond the third will be omitted and (5) becomes

$$\varphi(t) \approx mv(t)\cos\epsilon - m^2v(t)^2\sin\epsilon\cos\epsilon + m^3v(t)^3\sin^2\epsilon\cos\epsilon - \frac{m^3}{3}v(t)^3\cos^3\epsilon. \tag{6}$$

Ideally, $\epsilon = 0$ and the first term in equation (6) is the desired modulating signal; the second and third terms will be zero and the last term is the third-order distortion. When $\epsilon \neq 0$, second-order distortion occurs and the desired output signal amplitude is reduced by the factor $\cos\epsilon$.

It can be seen from equation (6) that the distortion can be made as small as desired by the proper choice of m, which is proportional to the phase deviation. In order to determine suitable values of m, $v(t)$ must be specified; we shall consider three signals of interest, corresponding to the three applications listed in Section I.

2.1 *Case I*

The signal $v(t)$ is gaussian noise uniformly distributed in a bandwidth extending from $O - W$ Hz.

For nonlinearities of the type described in equation (6) the desired results can be computed by well-known methods.[5]

$$\frac{S_0(f)}{S_2(f)} = \frac{1}{2m^2\sigma^2\sin^2\epsilon\left(1 - \frac{|f|}{2W}\right)}, \qquad 0 \leqq |f| \leqq W, \tag{7}$$

$$\frac{S_0(f)}{S_3(f)} = \frac{2}{m^4\sigma^4\cos^4\epsilon\left[1 - \frac{1}{3}\left(\frac{f}{W}\right)^2\right]}, \qquad 0 \leqq |f| \leqq W, \tag{8}$$

where,

$S_0(f) = m^2 \cos^2 \epsilon(\sigma^2/2W)$, with $-W \leqq f \leqq W$, is the spectral density of the phase of the output signal,

$S_2(f)$, $S_3(f)$ are the spectral densities of the second- and third-order distortion terms, respectively.

σ^2 is the mean square value of $v(t)$, that is, the power in $v(t)$, and $m\sigma$ is the rms phase deviation.

2.2 *Case II*

$$v(t) = \sum_{n=1}^{N} Q \cos (np + q_n)t. \tag{9}$$

The baseband signal, $v(t)$, is a frequency-division frequency-modulated multiplex signal. Each term in equation (9) is an FM carrier with its own frequency modulation q_n. Bennett has derived the number and types of modulation products produced by the last three terms of equation (6) for $v(t)$ as in equation (9).[6] The second-order term of largest amplitude has the form

$$e_2 = m^2Q^2 \sin \epsilon \cos \epsilon \cos [(m \pm n)p + (q_m \pm q_n)]t. \tag{10}$$

Similarly, the controlling third-order product has the form

$$e_3 = \frac{m^3}{2} Q^3 \cos^3 \epsilon \cos [(l \pm m \pm n)p + (q_l \pm q_m \pm q_n)]t. \tag{11}$$

The total power in the signal of equation (9) is

$$\sigma^2 = N \frac{Q^2}{2} \tag{12}$$

where N is the number of terms in equation (9). From equation (6) the output phase modulation for an individual channel is

$$e_1 = mQ \cos \epsilon \cos (np + q_n)t. \tag{13}$$

The ratios of signal-to-distortion power for single modulation products are,

$$\frac{|e_1|^2}{|e_2|^2} = \left[\frac{1}{mQ \sin \epsilon}\right]^2, \tag{14}$$

$$\frac{|e_1|^2}{|e_3|^2} = \left[\frac{2}{m^2Q^2 \cos^2 \epsilon}\right]^2. \tag{15}$$

In order to determine the total signal-to-distortion power ratios it is necessary to compute the number of products falling in the kth channel, $1 \leqq k \leqq N$. Assuming power addition for these products the total signal to distortion ratios become

$$\frac{S}{D_2} = \frac{1}{2m^2\sigma^2 \sin^2 \epsilon} \left(\frac{N}{N_2}\right) \tag{16}$$

$$\frac{S}{D_3} = \frac{2}{m^4\sigma^4 \cos^4 \epsilon} \left(\frac{N^2}{2N_3}\right) \tag{17}$$

where

N is the total number of channels, i.e., the number of terms in equation (9),

N_2 is the equivalent number of $m \pm n$ type products and includes other second-order products weighted in accordance with their contribution to the distortion power. It is a function of k and N, and

N_3 is the equivalent number of $l \pm m \pm n$ type products and includes other third-order products weighted in accordance with their contribution to the distortion power. It is a function of k and N.

Expressions (16) and (17) for the signal consisting of N sine waves are much like expressions (7) and (8) for the case of the noise-like signal. It has been shown by Bennett that the sum of randomly phased sine waves of equation (9) behave like noise as N increases without bound and if the power and bandwidth are finite.[7] It is of interest to see in the present context how fast expressions (16) and (17) approach (7) and (8) as N increases; this is shown in Figs. 2 and 3. It is evident from the figures that the signal-to-distortion ratios are not a strong function of the number of channels, the ratios changing a maximum of 2 dB while the number of channels goes from 10 to infinity.

For a more detailed look at the behavior of the distortion products, the number of the various types of products falling in the kth channel for the 500-channel case are shown in Figs. 4 and 5.

2.3 *Case III*

In this case the baseband signal is a sequence of pulses which phase modulate a carrier in the format of a phase-shift-keyed system. A 4-level polar baseband signal is written

$$v(t) = V_0 \sum_{n=-\infty}^{\infty} k_n p(t - nT), \tag{18}$$

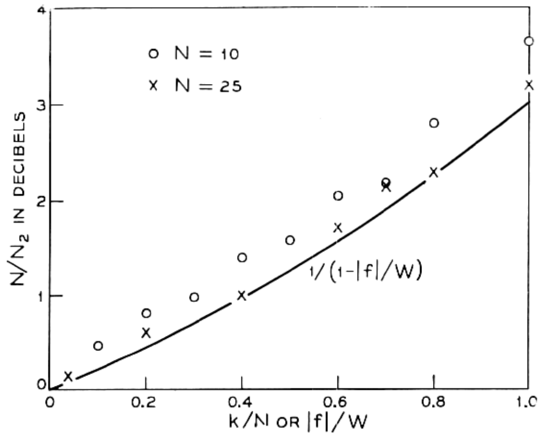

Fig. 2—The effect of the number of channels on the ratio of signal-to-second-order distortion.

where,

$p(t)$ is the pulse shape,
T is the time interval between adjacent pulses, and

$$k_n = \pm 1, \pm 3.$$

In a 4-level PSK system, a maximum peak deviation of $\pm 3\pi/4$ radians

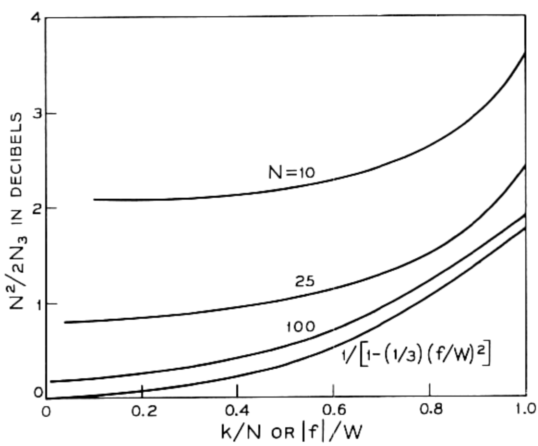

Fig. 3—The effect of the number of channels on the ratio of signal-to-third-order distortion.

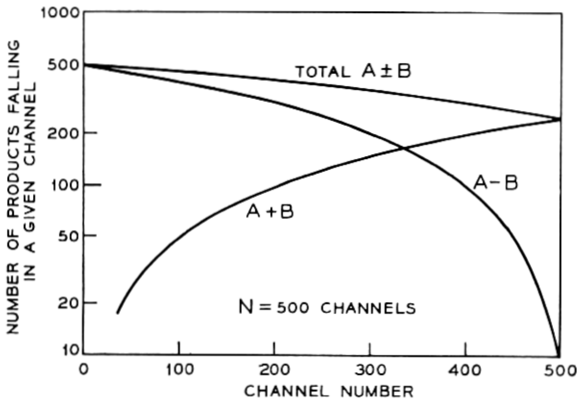

Fig. 4—Number of second-order distortion products.

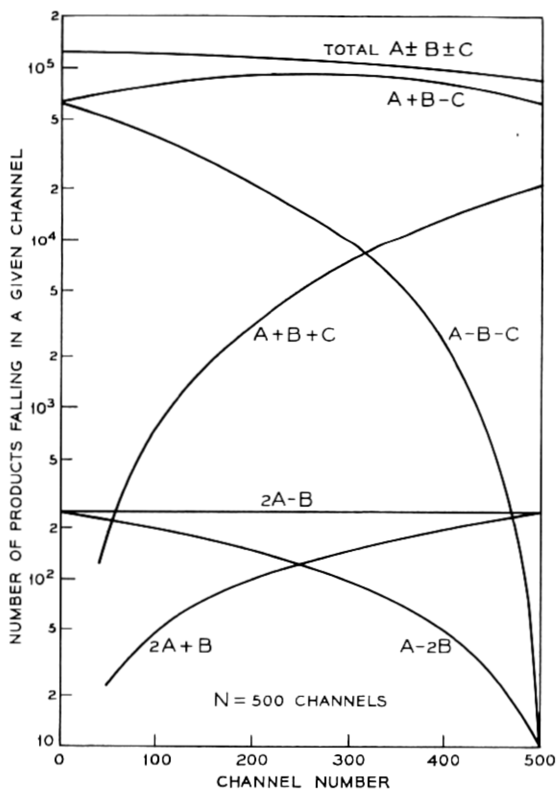

Fig. 5—Number of third-order distortion products.

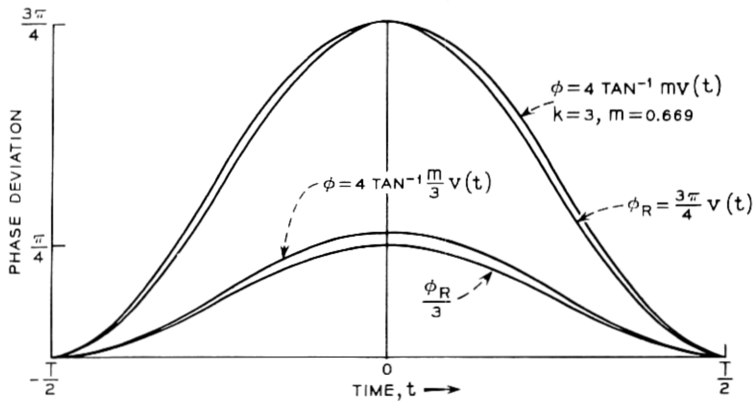

Fig. 6—Modulator input and output pulses.

is required. Deviations of this magnitude may be obtained by multiplying the output of the modulator in a resistive multiplier circuit.

As an example, suppose the modulator output is multiplied by four. The peak deviation required in the modulator is then $3\pi/16$ radians. Raised cosine input pulses, $v(t)$, and the corresponding phase deviations in the output of the modulator are shown in Fig. 6 for this case. The output pulses were computed from equation (5) for $\epsilon = 0$. The value of m was chosen to result in a peak deviation of $3\pi/16$ radians for the pulse corresponding to $k_n = 3$. For this example, $m = \tan 3\pi/16$, and

$$v(t) = \frac{k_n}{6}\left[1 + \cos\frac{2\pi t}{T}\right], \qquad -\frac{T}{2} \leqq t \leqq \frac{T}{2}.$$

In Fig. 6, the phase deviation, φ, is shown for pulses having $k_n = +1$, $+3$. Some pulse compression is present in the larger pulse and the parameter m has been chosen for the correct peak deviation. For the smaller pulse the peak deviation is seen to be too large by about five degrees. If uncorrected, this error would cause the system performance to be degraded a few tenths of a dB.[8] The peak deviation can be corrected by a gain adjustment in the circuits in which the smaller pulses are generated.[3]

III. MODIFIED ARMSTRONG MODULATORS

There may be applications in which it is desirable that the output carrier frequency equal the frequency of the source carrier. The circuit of Fig. 7 will accomplish this purpose while minimizing the degradation due to tones generated in the final mixer. The carrier frequencies of any high-order products of the two input signals which fall into the output band will be exactly at the carrier frequency of the output signal and result in minimum degradation.

If the times $(N - 1)$ frequency multiplier is replaced by a times M multiplier the flexibility in the choice of output carrier frequency is increased while the feature described above is retained. In either case the frequency multipliers should be resistive rather than reactive.

Finally, in the balanced modulator illustrated in Fig. 8 the phase deviation is doubled for a specified ratio of signal-to-third-order distortion.

IV. CONCLUDING REMARKS

The Armstrong modulator has three attractive features.

(*i*) The carrier frequency can be derived from a frequency stabilized oscillator. For example, a single source can be used in both modulators used to derive two cross-polarized channels for a short hop radio system or a satellite radio system. The identical carrier frequencies serve to minimize the effect of co-channel interference due to cross-polarization coupling.

(*ii*) The functions required to realize the modulator—limiting, mixing, and multiplication—are amenable to circuit integration.

(*iii*) The modulator is suitable for very large baseband bandwidths, particularly high-speed pulse sequences for PSK-PCM systems.

A short hop radio system has been described recently which has about the same communication capacity for either large index analog phase modulation or digital PSK-PCM.[9] In a system designed for either type of operation, it is convenient to do the digital processing at the inter-

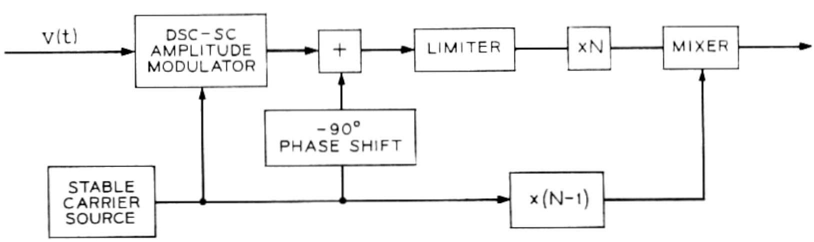

Fig. 7—A modulator with output frequency equal to frequency of stable carrier source.

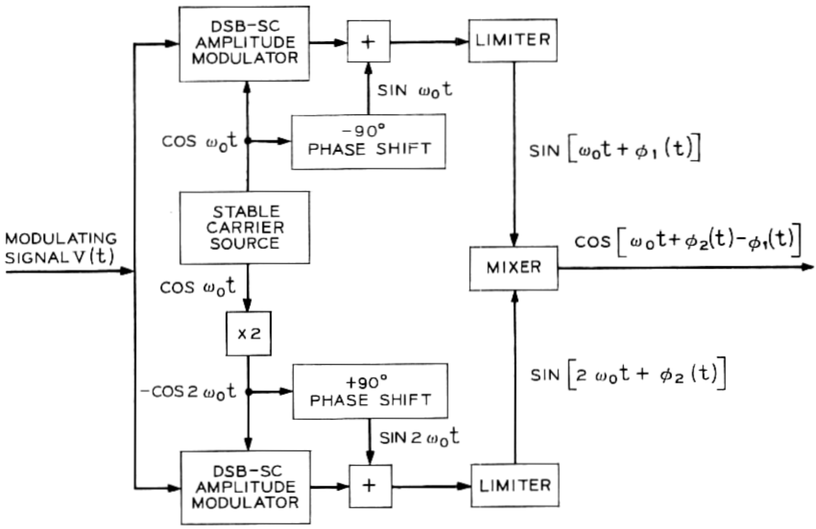

Fig. 8—Balanced Armstrong modulator.

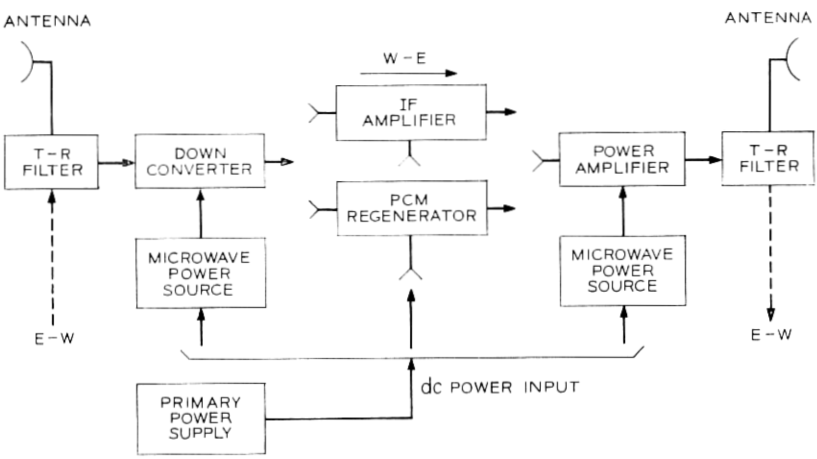

Fig. 9—Repeater of configuration for analog phase modulation or digital CPSK-PCM modulation.

mediate frequency; if PSK-PCM is to be used, the IF amplifier can be replaced by a digital regenerative repeater and no other changes need be made (See Fig. 9).

A digital regenerative repeater has been described which is appropriate for this application; it requires a phase modulator with requirements which are met by the configuration of Figure 7: that incidental AM be small, that the frequency be stable, that the linearity be adequate for multi-level operation, and that the power consumption be small.[3]

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