A Decomposition of a Transfer Function Minimizing Distortion and Inband Losses

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A rational transfer function to be realized by an RC-*active network is usually decomposed into functions of at most second degree. We present a method for achieving this—which maximizes the dynamic range of the whole network while minimizing inband losses. The method is based on the "bottleneck problem."*

I. INTRODUCTION

A given rational transfer-function $T(s)$ of a passive network, which is real for real s, is to be realized by an inductorless two-port. This is usually done by breaking down $T(s)$ into functions $T_i(s)$ of the first or second degree in s. All functions of the first and those of the second degree with poles on the negative real axis are realized by passive RC-networks with buffer amplifiers between the different stages. Those of second degree but with poles not on the negative real axis are realized by RC-active networks containing amplifiers. We deal at first with the second group of functions. The extension to the general case follows easily. The voltage swing at the input of the different stages with functions $T_i(s)$ is often tightly limited by the threshold above which overdriving of the amplifiers (that is, distortion) occurs. A further result in many cases is high inband loss of the overall filter which cannot be overcome by amplification because of both distortion and a too low signal/noise ratio.

Our task is to find a method of factoring $T(s)$ into the different functions $T_i(s)$ such that the allowable voltage swing at the input is as high as possible without creating distortion and the inband losses as low as possible. We confine ourselves first to transfer functions

$$T(s) = \frac{V_2}{V_1} = K \frac{s^n + a_{n-1}s^{n-1} + \cdots a_1 s + a_0}{s^m + b_{m-1}s^{m-1} + \cdots b_1 s + b_0} \quad \text{with} \quad n \leqq m \qquad (1)$$

which, as mentioned, have no poles on the negative real axis. Thus m is even. If we count the zeros of $T(s)$ including those at infinity, then $T(s)$ has also m zeros. Let the number of zeros including the origin and infinity on the real axis be r_z. Then r_z is even, since m and the number of zeros not on the real axis are even. We are mainly dealing with transfer-functions, which belong to the class of networks having a passband. In the case of two ports without a passband a slightly different approach will be necessary. The functions $T_i(s)$ of second order have the general form.

$$T_i(s) = K_i \frac{s^2 + \frac{\omega_z}{q_z} s + \omega_z^2}{s^2 + \frac{\omega_p}{q_p} s + \omega_p^2}. \tag{2}$$

After normalization by

$$\frac{s}{\omega_p} = p \tag{3a}$$

with

$$p = \sigma + j\Omega \tag{3b}$$

we get from equation (2)

$$T_i(\omega_p \cdot p) = \overline{T_i}(p) = K_i \frac{p^2 + \frac{c}{q_z} p + c^2}{p^2 + \frac{1}{q_p} p + 1} \quad \text{with} \quad c = \frac{\omega_z}{\omega_p}. \tag{3c}$$

To meet the aforementioned requirements as to distortion and inband losses, we have these possibilities:

(*i*) There are in general a large number of ways of finding the different functions $T_i(s)$, because there are many methods of choosing pairs of poles and zeros in forming $T_i(s)$. In Sections II and III we discuss the best choice for our task.

(*ii*) In RC-active two-ports there is some freedom in evaluating the constant K_i in equation (2).

(*iii*) The functions $T_i(s)$ and their realizations once found, there are many possibilities for the sequence in cascading the different stages. In Section IV, we discuss some guidelines for this point as well as for *ii*.

II. A CRITERION FOR THE GOODNESS OF AN ASSIGNMENT OF POLES AND ZEROS

We need a criterion which tells us when a chosen assignment meets the requirement of voltage swing and inband losses. For this reason we are looking for the shape of the function $| T_i(j\Omega) |^2$, which from equation (3c) has the normalized form,

$$\frac{| T_i(j\Omega) |^2}{K_i^2} = F_i(x) = \frac{(c^2 - x)^2 + \dfrac{c^2}{q_z^2} x}{(1 - x)^2 + \dfrac{x}{q_p^2}} \quad \text{with} \quad x = \Omega^2. \tag{4}$$

We are interested in the shape of the function in equation (4) for real values of Ω, that is, for real nonnegative values of x. The extrema of $F_i(x)$ occur, as can be easily calculated, at the values

$$x_{a,b} = \frac{c^4 - 1 \pm \left[(c^4-1)^2 - c^2 \left\{ 2(c^2-1) + \dfrac{1}{q_z^2} - \dfrac{c^2}{q_p^2} \right\} \left\{ 2(c^2-1) + \dfrac{1}{q_p^2} - \dfrac{c^2}{q_z^2} \right\} \right]^{\frac{1}{2}}}{2(c^2-1) + \dfrac{1}{q_p^2} - \dfrac{c^2}{q_z^2}} \tag{5a}$$

and

$$x = \infty \tag{5b}$$

with

$$F_i(\infty) = 1. \tag{5c}$$

If $x_{a,b}$ are real and nonnegative, extrema occur with the ordinate values $F_i(x_{a,b})$ from equations (4) and (5a) as follows:

$$F_i(x_{a,b}) = \frac{(c^2 - x_{a,b})^2 + \dfrac{c^2}{q_z^2} x_{a,b}}{(1 - x_{a,b})^2 + \dfrac{x_{a,b}}{q_p^2}}. \tag{6}$$

For the moment we need not know if $F_i(x_a)$ or $F_i(x_b)$ is a maximum or a minimum. It is sufficient to note that besides the extremum at $x = \infty$, at most two other extrema of $F_i(x)$ can occur.* Let the maximum be at x_m and the minimum at x_0. We note from equation (4):

$$F_i(0) = c^4 \tag{7}$$

with $F_i(0) \lesseqgtr F_i(\infty) = 1$ depending on c. An example for a function

* For Ω as abscissa, a further extremum can occur at $\Omega = 0$. For $c = 0$, one of the extrema in equation (5a) lies at $x = 0$.

$F_j(x)$ with $F_j(x_m) > c^4$ and $F_j(0) > F_j(\infty)$ is shown in Fig. 1a, while Fig. 1b represents a function $F_j(x)$ with $F_j(x_m) < 1$ and $F_j(0) < F_j(\infty)$.

The passband of the whole filter may be in $x \in [x_1, x_2]$ with $x_1 \geqq 0$ and $x_2 > x_1$, as shown in Fig. 1. Let us first assume that a peak $F_{\max}$ of $F_j(x)$ may occur at x_m with $x_m \in [x_1, x_2]$. Considering only frequencies in the passband, we are stating that overdrive of the amplifiers will first occur at the maximal value $F_{\max}$ of $F_j(x)$, when the spectrum of the input signal is assumed constant at least in $x \in [x_1, x_2]$. To prevent overdrive of the amplifiers $F_{\max}$ should exceed the "mean" values in the passband as little as possible. On the other hand, we have to regard the minimum value $F_{\min}$ of $F_j(x)$ in $x \in [x_1, x_2]$. $F_{\min}$ gives us the strongest attenuation of the signal, which we have to overcome by amplification. When this is not possible because of overdrive or a too low signal/noise ratio, then a low $F_{\min}$ yields high inband losses. For this reason, $F_{\min}$ should be as close to the "mean" values in the passband as possible. It would seem at first sight that both requirements can be met, namely that $F_{\max}$ and $F_{\min}$ be as close to the mean values in the passband as possible, if we look for a transfer function $T_j(s)$ such that $d_j = F_{\max} - F_{\min}$ be minimized. But this criterion does not always cover our requirements as can be seen in Fig. 2. Both functions $F_j(x)$ have the same value $d_j = F_{\max} - F_{\min}$. Their practical behavior however is very different. The two port with the transfer function of Fig. 2a almost entirely cuts off the frequencies in the passband in the neighborhood of x_1 and x_2 and we would need a very high gain to bring them up, which is not true in the case of Fig. 2b. To avoid this error, we redefine the d_j-value by the ratio

$$d_j = \frac{F_{\max}}{F_{\min}} \tag{8}$$

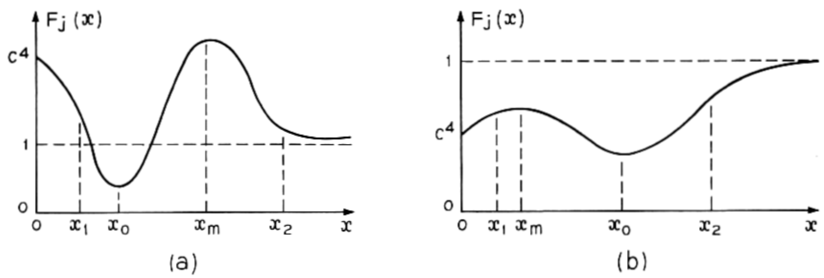

Fig. 1 — Functions $F_j(x)$ with (a) $F_j(x_m) > c^4$ and $F_j(0) > F_j(\infty)$; and (b) $F_j(x_m) < 1$ and $F_j(0) < F_j(\infty)$.

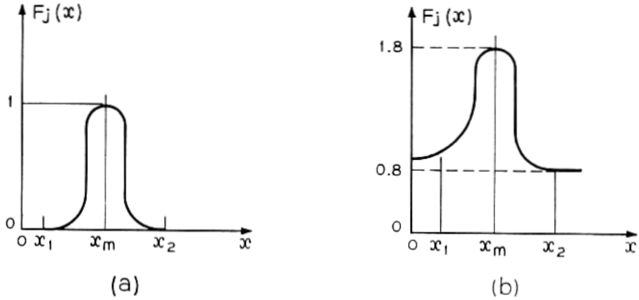

Fig. 2 — Two-port almost entirely cuts off the frequencies in passband in neighborhood of x_1 and x_2 in (a) but not in (b).

with $F_{\max}$ and $F_{\min}$ for $x \in [x_1, x_2]$ and require it to be as close to unity as possible. In order to compress the range of values we can let

$$d_j = \log \frac{F_{\max}}{F_{\min}} \tag{9}$$

where d_j is obviously a positive number. The values d_j in equation (9) should be as close as possible to zero, that is, as small as possible.

Until now, we have assumed $x_m \in [x_1, x_2]$. If x_m is outside the passband we have to argue in a slightly different manner. Now we could have two points of a peak value, one at $x = x_m$ and one at the boundary $x = \infty$. We look for that point among these two with the highest $F_{\max}$ value and we denote this point by x_m'. Let us assume that the amplitude of a signal from a neighboring channel occurring at x_m' is so high that the amplifiers are overdriven. This will change the operating points of the amplifiers resulting in an impaired transmission of signals in the passband. This can sometimes be avoided by inserting the stage under consideration at such a place in the cascade, that the amplitude of the input signal at x_m' is not too high. This, however restricts the freedom of choosing the cascade sequence. We therefore request that the maximum $F_{\max}$ at x_m' be as close to the "mean" values in the passband as possible, even if $x_m' \notin [x_1, x_2]$. Thus we look for $F_{\max}$ for $x \in [0, \infty]$. Minimum values of $F_j(x)$ for $x \notin [x_1, x_2]$ are of no importance since we don't have to amplify those values outside the passband. For these reasons the d_j-values in equations (8) and (9) are replaced by

$$d_j = \frac{F_{\max}}{F_{\min}} \tag{10}$$

or

$$d_j = \log \frac{F_{\max}}{F_{\min}} \tag{11}$$

with

$$F_{\max} \quad \text{for} \quad x \in [0, \infty] \tag{12a}$$

and

$$F_{\min} \quad \text{for} \quad x \in [x_1, x_2]. \tag{12b}$$

Thus each assignment of a pair of poles to a pair of zeros, that is, each function $T_j(s)$, is described by a number d_j, defined by one of the equations (10) or (11), which in the case of (11) should be as small as possible for all $j = 1, 2, \cdots, m/2$. In other words

$$\max \{d_j\}_{j=1,2,\cdots m/2} \rightarrow \min \tag{13}$$

for the d_j in equation (11).

M. Segal pointed out that this problem is an assignment problem of the bottleneck type.[1] O. Gross has given a solution which is convenient also for large numbers of poles and zeros.[2] This algorithm was adopted by S. Halfin to find an optimal pairing and an optimal nested solution.[3] He also presented a method of listing all equivalent solutions. A further solution suitable for smaller numbers of poles and zeros (for example, $\leqq 20$) has been described in Ref. 4, where also all equivalent solutions may be found. The next paragraph shows how the various types of transfer functions should be treated as to this assignment problem.

III. THE PAIRING OF POLES AND ZEROS

We have to check all possibilities of assigning a pair of poles to a pair of zeros. For simplicity, we first assume all zeros including the origin and infinity to lie off the real axis. We consider the case of zeros on the real axis later. Remember that all poles of $\bar{T}(p)$* lie in the interior of the left half plane of p but not on the negative real axis. If we assign a pole at $p = p_\gamma$ to a zero at $p = z_\mu$ as shown in Fig. 3, we have to assign, as is well known, the conjugate complex pole $p = p_\nu^*$ to the conjugate complex zero $p = z_\mu^*$, thus forming the second order function $T_j(p)$ in equation (3c). Therefore we need only regard in Fig. 3 the assignments of the poles $p_1, p_2 \cdots p_{m/2}$ to the zeros z_1,

* $\bar{T}(p)$ is the function $T(s)$ of equation (1) normalized by equation (3a).

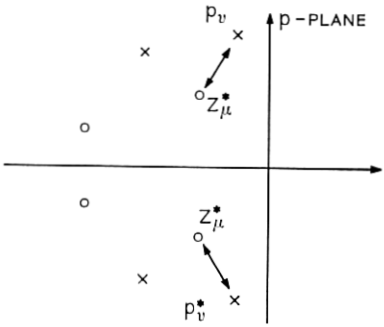

Fig. 3 — An example for a pole-zero assignment.

$z_2 \cdots z_{m/2}$ where all values have positive imaginary part. To each assignment (p_ν, z_μ), ν, $\mu = 1, 2 \cdots m/2$, belongs a number defined in equations (10) or (11), which will here be denoted by $d_{\nu,\mu}$. All possible assignments (p_ν, z_μ), with their associated $d_{\nu,\mu}$ are listed in Table I. Obviously their total number is $(m/2)^2$. The solution of the assignment problem[3,4] starts with Table I.

Now we have to regard the case of zeros lying on the real axis including the origin and infinity, while as before, all poles are assumed to be complex. The problem now is to assign a pair of zeros to each conjugate complex pair of poles. The zeros on the real axis with number r_z, where r_z is even, can be arranged pairwise in many ways. For example if we have the four distinct zeros 1, 2, 3, 4 in Fig. 4, where one of them may be at infinity, then we have these three possibilities to arrange them in pairs: (1, 2)(3, 4); (1, 3)(2, 4); (1, 4)(2, 3). In general, if we have r_z different zeros on the real axis with r_z even, then we have

$$a_0 = (r_z - 1)(r_z - 3)(r_z - 5) \cdots 5 \cdot 3 \cdot 1 \tag{14}$$

TABLE I

	z_1	z_2	-------	z_h
p_1	d_{11}	d_{12}	-------	d_{1h}
p_2	d_{21}	d_{22}	-------	d_{2h}
⋮	⋮	⋮		⋮
p_h	d_{h1}	d_{h2}	-------	d_{hh}

$h = \frac{m}{2}$

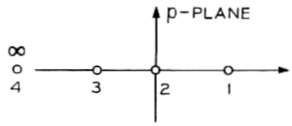

Fig. 4 — We have these possibilities to arrange zeros in pairs: (1, 2) (3, 4); (1, 3) (2, 4); (1, 4) (2, 3).

different possibilities of a pairwise arrangement. In the case of multiple zeros on the real axis we have less than a_0 possibilities of pairwise assignments. For the zero arrangement shown in Fig. 5, we have only the following two possibilities: (1, 2)(3, 4) and (1, 3)(2, 4).

Now we have to complete Table I by regarding also zeros on the real axis. We pick out one pairwise combination of the zeros, for example, the arrangement (1, 2)(3, 4) of Fig. 5, and add each pair of these zeros to the pairs of conjugate complex zeros in Table I, where we treat them like the other zeros. From that, one solution of the assignment problem will be found. However, we did not yet regard all possible assignments. We have to replace the pairwise combinations of the real zeros by another possible combination, for example, by (1, 3)(2, 4) in Fig. 5. This provides a second table like Table I from which a further solution can be found, and so on. The solution with the least maximum value of the $d_{\nu,\mu}$ is the solution to the whole problem.

Finally we have to deal with the case in which poles are also located on the real axis. We consider first the simplest and most important case of only one pole and r_z different zeros on the real axis. The pole on the real axis can be assigned to one of the zeros on the real axis. There are r_z ways of doing this.

The $r_z - 1$ zeros left can be pairwise arranged according to equation (14) in $a_1 = (r_z - 2)(r_z - 4) \cdots 5 \cdot 3 \cdot 1$ ways where each pair of these a_1 sets is handled like a conjugate complex pair of zeros. Thus we get $r_z a_1$ sets of zeros to be assigned to the poles, which means $r_z a_1$ different tables of the kind of Table I. The solution with the least maximum value of the $d_{\nu,\mu}$ is the solution of the whole problem. The case where the r_z zeros on the real axis are not different is handled in a

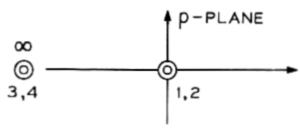

Fig. 5 — We have here two possibilities: (1, 2) (3, 4) and (1, 3) (2, 4).

similar way. The more general case where $r_p > 1$ poles, with r_p odd or even, are on the real axis is usually not so important and is therefore only described briefly.

If r_p is odd, we assign one pole to one zero on the real axis, which can be done in several ways. The zeros and poles left on the real axis are arranged in pairs, which once more can be done in several ways. Then we form for each pairwise arranged set of poles and zeros a table like Table I from which we get the solution. If r_p is even, we start with the pairwise arrangement of poles and zeros and proceed as above.

Incidentally more first order functions of the kind described could be formed with negative real poles and zeros. However this is undesirable because it would require more buffer amplifiers.

In some cases the assignment of one particular pole to one particular zero is prescribed by the realization procedure. Then we simply assign them and eliminate this pole-zero pair from consideration. On the other hand, if a particular assignment of one pole to one zero is forbidden, we provide it a high $d_{\nu,\mu}$-value.

IV. THE CHOICE OF THE FACTORS K_i AND OF THE SEQUENCE OF CASCADING

In most realization procedures the factor K_i in equation (2) can be chosen within certain limits by evaluating the gain of the amplifiers. We describe here one way to do that. The choice of K_i should be made in such a way that the "gain" in the passband of all stages is as close together as possible. This prevents one stage from having a much lower gain than the others which results in a lower signal/noise level in that stage. This is only a short hint because work on this point is continuing. Within the building block concept of G. S. Moschytz[5] there is enough freedom to choose the appropriate K_i. In the case of the low pass, bandpass and the high pass, this has been shown in Ref. 6.

Some guidelines for the sequence in cascading the different stages follow where we use observations by Moschytz. The first stage should be a low pass or a bandpass, thus keeping higher frequencies from the amplifiers and avoiding slew rate problems. Also the last stage should be a low or a bandpass for the purpose of suppressing noise created by the amplifiers themselves. Where a peak in the frequency response of a stage cannot be avoided, this stage should be preceded by stages delivering attenuation at the peak point. In special cases different considerations for cascading could be necessary.

If the assignment problem gives us several solutions, then the one best meeting these guidelines for cascading should be chosen. If the two port has no passband, then x_1 and x_2 should define the frequency range in which the network has to operate. The choice of the values K_i should also be made in such a way that there is no great difference in gain in the frequency range $x \in [x_1, x_2]$.

V. AN EXAMPLE

Given a transfer function $T(s)$ for a Single-Side-Band (SSB) filter with the following poles $p_1 \cdots p_5$ and the zeros $z_1 \cdots z_5$:

$$\begin{aligned}
z_1 &= \pm j0.32233523 \cdot 10^6, \\
z_2 &= \pm j0.36742346 \cdot 10^6, \\
z_3 &= -0.31480 \cdot 10^5 \pm j0.3132295 \cdot 10^6, \\
z_4 &= 0 \text{ (twice)}, \\
z_5 &= \infty \text{ (twice)}, \\
p_1 &= -0.276100 \cdot 10^5 \pm j0.2961048 \cdot 10^6, \\
p_2 &= -0.31480 \cdot 10^5 \pm j0.3132295 \cdot 10^6, \\
p_3 &= -0.8706 \cdot 10^4 \pm j0.314697 \cdot 10^6, \\
p_4 &= -0.93340 \cdot 10^5 \pm j0.18670202 \cdot 10^6, \\
p_5 &= -0.25280 \cdot 10^5 \pm j0.62888167 \cdot 10^5.
\end{aligned}$$

The zero z_3 and the pole p_2 are a phantom pair which have been introduced for the realization procedure.

The four zeros on the real axis z_4 and z_5 can be pairwise arranged in two ways: (z_4, z_4), (z_5, z_5) or (z_4, z_5), (z_4, z_5). Let the pairs of the first arrangement be denoted by $z_{4,1} = (z_4, z_4)$ and $z_{5,1} = (z_5, z_5)$ and the pairs of the second assignment by $z_{4,2} = (z_4, z_5)$ and $z_{5,2} = (z_4, z_5)$.

Now we calculate the d_j-values of equation (11) from which we obtain Table II, corresponding to Table I. Then we obtain with the help of Ref. 4, the following pairings*

$$(z_5, p_4); \quad (z_{41}, p_5); \quad (z_{51}, p_4); \quad (z_2, p_2); \quad (z_1, p_1) \cdot \tag{15}$$

In the realization procedures there is usually a constraint such that one particular assignment of a pole to a zero is prescribed. In the realization by building blocks,[5] pole p_3 has to be assigned to zero z_3. The rest of the assignments are free. In this case the solution is

$$(z_3, p_3); \quad (z_{41}, p_5); \quad (z_{51}, p_4); \quad (z_2, p_2); \quad (z_1, p_1). \tag{16}$$

* The pairings in this example have been calculated by a procedure described in Ref. 4, which is suitable for small numbers of poles and zeros.

TABLE II

	z_1	z_2	z_3	z_{41}	z_{51}	z_{42}	z_{52}
p_1	0.15	0.57	0.25	3.7	1.36	2.47	2.47
p_2	0.18	0.3	0	3.8	1.49	2.69	2.69
p_3	0.18	1.25	1.1	4.9	2.5	3.77	3.77
p_4	1.87	1.4	1.6	1.9	0.56	0.87	0.87
p_5	4.1	3.3	3.7	0.4	2.64	1.5	1.5

Solution (16) has the five transfer functions T_j , $j = 1, 2, \cdots 5$ listed and drawn with full lines in Figs. 6a through 6e. There the factors K_j , $j = 1, 2, 3, 4, 5$ in equation (3b) have been chosen in accordance with Section IV such that the whole filter has an attenuation of 0 dB at 30 kHz. The sequence in cascading the different stages is as described in Section IV, using the denotations for the different stages in Figs. 6a through 6e

$$T_4(s)\,T_1(s)\,T_2(s)\,T_3(s)\,T_5(s).$$

T_4 , T_1 and T_2 deliver the attenuation for the peak of T_3 . Since it is not possible to have a low pass as both the first and the last sections, we chose the low pass to be the first because in this case, noise coming in at the input terminals was stronger than noise created by the amplifiers. The magnitude of the transfer function of the whole filter can be seen in Fig. 7.

We wish to compare the solution described above with an earlier solution realizing the same transfer function in a different way. In the earlier version the phantom pair (z_3 , p_2) had the following location

$$z_3 = -0.12\cdot 10^6 \pm j0.28\cdot 10^6; \quad p_2 = -0.12\cdot 10^6 \pm j0.28\cdot 10^6.$$

In the new realization we shifted this phantom pair closer to the pole p_3 and thus were able to decrease the peak in the FEN* section as shown in Fig. 6c.

The earlier realization had the following assignment of poles and zeros; $(z_1p_2)(z_2p_1)(z_3p_3)(z_4p_4)(z_5p_5)$, which leads to the following transfer functions of Ref. 7. T_j' , $j = 1, 2. \cdots 5$, are listed in Figs. 6a through 6e. The magnitude in dB of these functions is plotted as dotted lines in Figs. 6a through 6e. The passband lies between 12 and 46 kHz. The functions T_1 , T_5 and especially T_4 of the new version have obviously less attenuation in the passband, while the functions T_2 and especially T_3 have a lower peak than in the earlier version. The whole filter has

* Frequency Emphasizing Network.

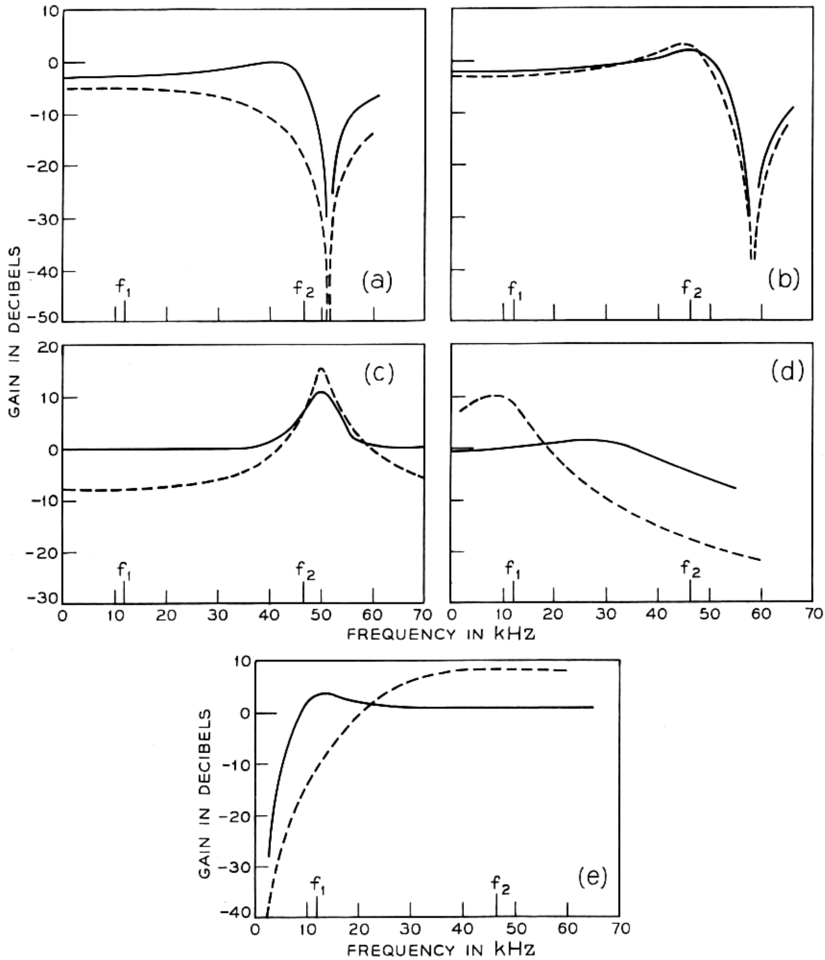

Fig. 6a — Transfer function T_j, $j = 1$.

$$T_1(s) = 0.85 \frac{s^2 + 0.1039 \cdot 10^{12}}{s^2 + (0.5522 \cdot 10^5)s + 0.8844 \cdot 10^{11}} \text{ (solid line);}$$

$$T_1'(s) = \frac{.575s^2 + 59.742 \cdot 10^9}{s^2 + (25.909 \cdot 10^4)s + 9.911 \cdot 10^{10}} \text{ (dotted line).}$$

Fig. 6b — Transfer function T_j, $j = 2$.

$$T_2(s) = 0.75 \frac{s^2 + 0.135 \cdot 10^{12}}{s^2 + (0.06296 \cdot 10^6)s + 9.911 \cdot 10^{10}} \text{ (solid line);}$$

$$T_2'(s) = .4586 \left[\frac{s^2 + .135 \cdot 10^{12}}{s^2 + (.5522 \cdot 10^5)s + .8844 \cdot 10^{11}} \right] \text{ (dotted line).}$$

the frequency response of Fig. 7, where the full line represents the new, the dotted line the earlier version. The new version has a minimum attenuation of 0 dB in the passband, instead of −17 dB in the earlier case. If needed, the new version is able to deliver some amplification in the passband. In the earlier version a maximum voltage swing of $0.3V_{pp}$ at the input was admissible because of overdriving, while in the new version the maximum voltage swing is limited by the amplifiers and not by the peaks of the transfer-functions. Using the op. amp. RCA 3015A, the maximum voltage swing in the new version is $1.8V_{pp}$, since the amplifiers alone have a voltage swing of $1.8V_{pp}$. A way to improve the dynamic range of the amplifiers by minimizing the current drain can be found in Ref. 8. This method can also be used in connection with the transfer-functions $T_j(s)$ found by the method presented in this paper.

VI. CONCLUSIONS

The given transfer function of a filter, which is to be realized by an RC-active two-port, is generally factored into second order functions. A method has been presented to achieve this so that the whole filter has minimum inband losses and maximum dynamic range in which no overdrive of the amplifiers (that is, no distortion) occurs. The problem led to an assignment problem of the bottleneck type. The efficiency of the method has been shown in the case of an SSB-filter, where the in-

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Fig. 6c — Transfer function T_j, $j = 3$.

$$T_3(s) = \frac{s^2 + 0.06296\cdot 10^6 s + 9.911\cdot 10^{10}}{s^2 + (1.7412\cdot 10^4)s + 9.911\cdot 10^{10}} \text{ (solid line);}$$

$$T_3'(s) = \frac{4015^2 + 10.412\cdot 10^4 s + 39.84\cdot 10^9}{s^2 + (1.7412\cdot 10^4)s + 9.911\cdot 10^{10}} \text{ (dotted line).}$$

Fig. 6d — Transfer function T_j, $j = 4$.

$$T_4(s) = 3.277\cdot 10^{10} \frac{1}{s^2 + (1.8668\cdot 10^5)s + 4.357\cdot 10^{10}} \text{ (solid line);}$$

$$T_4'(s) = \frac{15.287\cdot 10^7}{s^2 + (5.056\cdot 10^4)s + 4.594\cdot 10^9} \text{ (dotted line).}$$

Fig. 6e — Transfer function T_j, $j = 5$.

$$T_5(s) = 0.834 \frac{s^2}{s^2 + (5.056 - 10^4)s + 4.594\cdot 10^9} \text{ (solid line);}$$

$$T_5'(s) = \frac{2.106 s^2}{s^2 + (1.8668\cdot 10^5)s + 4.357\cdot 10^{10}} \text{ (dotted line).}$$

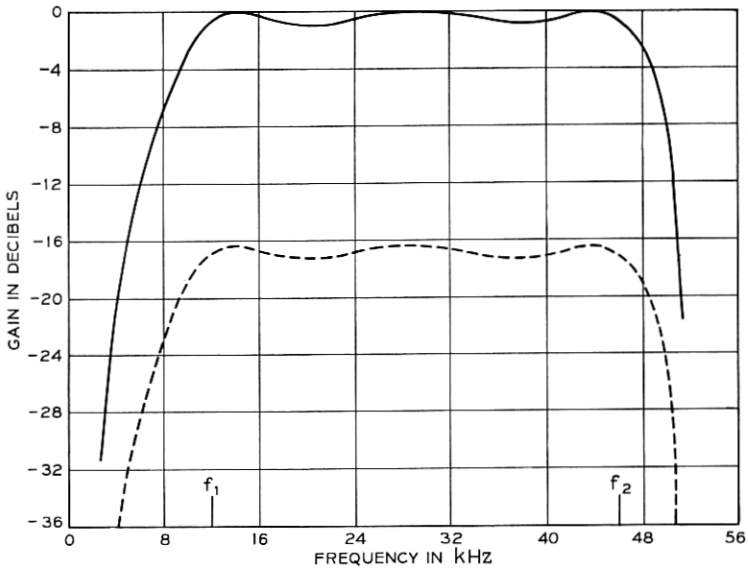

Fig. 7 — Gain of the bandpass. Earlier version with dotted line; new version with solid line.

band loss could be reduced to 0 dB instead of −17 dB in an earlier version and where the dynamic range of the input signal could be increased to $1.8V_{pp}$ instead of $0.3V_{pp}$ as before.

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