Organic Synchronization: Design of the Controls and Some Simulation Results

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Organic synchronization is a method for phase-locking the signals of an extensive digital communication network. Each clock in the network is made to depend on the phase drift of signals arriving and departing its station. This work demonstrates the practicality of such schemes. A four station simulation operating in real time with realistic parameters for a transcontinental network is used to evaluate various types of linear and nonlinear controls and to study effects of changing clock frequency and transmission delays. Considerable attention is given to the analysis of linear organic systems in order to pave the way for reasonable choices of design parameters and to make the results more easily understandable. The experiments show that the systems are very stable and easy to implement. No difficulty was experienced in starting the systems or in modifying their structures and they were immune to large scale breakdown caused by local faults.

I. INTRODUCTION

A model for certain mutually-synchronized systems of clocks and transmission links has been described by M. Karnaugh.[1] He calls systems that conform to this model "organic systems." The work on organic synchronization has been motivated chiefly by a desire to synchronize the sampling and switching operations in a geographically widespread pulse code modulation communication network.

Broad sufficient conditions for the stability of nonlinear organic systems have not yet been mathematically established. Nevertheless, there is reason to believe that systems having readily achievable clock stabilities and transmission delays of terrestrial magnitude can be well behaved. Because of this, and because performance under a variety of starting conditions, parameter choices, and perturbations is of interest, an analog simulator for organic synchronization has been constructed and put to use.

Details of the simulator hardware are described in the preceding article.[2] The present discussion will center on what has been learned from experimenting with the simulator. Preliminary attention is given to the analysis of linear organic systems; this aids the selection of reasonable control parameters for the design and study of some representative systems.

The experiments have supported the conjecture that systems of continental, or even global, dimensions will be stable and easy to implement. No great difficulty in starting the systems or in modifying their structures has been encountered.

II. SYSTEM ORGANIZATION

2.1 *Discussion*

The simulator contains four oscillators. Each of these corresponds to the local clock at a geographically distinct switching station. These stations may be interconnected through selected delay lines in any or all of the twelve possible directed paths. The sinusoids transmitted through these paths correspond in period to the data frames in a pulse code modulation communication system.

Each transmission path includes a large, fixed delay and a small, continuously variable, delay. The latter is used to simulate the slow variations in transmission delay which may occur in cables. In addition, there is another continuously variable delay line at the receiving terminal of each path. This simulates the buffer store which is needed to retime all data arriving at a switching station. The buffer store must synchronize the incoming data frames with the local switching actions. The latter are timed by the local clock.

For example, consider the two stations illustrated in Fig. 1 which shows a signal transmitted from station j to station i. The arriving signal is held in close phase agreement with the ith clock by the servoloop that automatically adjusts the buffer delay. Intuitively, we see that a constant frequency difference between the two clocks would drive the buffer at constant speed to one end, where synchronism would be lost. To prevent such failure, the clock frequencies are controlled by voltages derived from the position of the buffer, as shown. Gain factors a and b are placed in the frequency control paths.

A complete network is composed of many links resembling the one we have described. When the frequency of the clock at each station depends only upon the states of the buffers in transmission links arriving at that station, $b = 0$, and the controls will be called "one-sided."

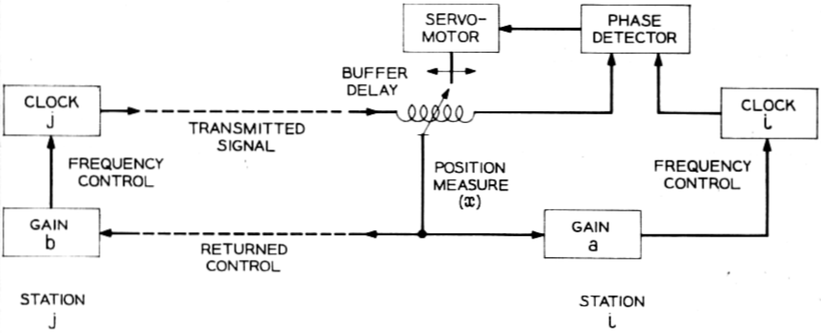

Fig. 1 — A transmission from station j to station i.

When the clocks are also controlled by the states of buffers in links leaving their stations, $b \neq 0$, the controls will be called "two-sided." Two-sided controls require the transmission of narrowband control signals between nearest neighbors in the system. In the simulator, separate baseband delays are used for this purpose.

The simulator incorporates filters, h, in the control paths to the clocks. They are used to explore the possibility of shaping the dynamic response of the network. Also included in the control paths is an amplitude limiter, ρ, which places a bound on the frequency deviations. Another nonlinearity, v, is placed in series with the buffer-position output, x. This causes the effective gain to vary with buffer position, so that the control influence of buffers near overflow may be made greater than those near their center position.

2.2 *Analysis*

Karnaugh has analyzed one-sided controls to determine the settling state after switching on from specified initial values. Now we shall examine changes in the settling conditions resulting from disturbances of transmission delays and clock frequencies, with two-sided controls. The model used by Karnaugh,[1] with a very slight change in notation, is illustrated in Fig. 2. Mathematically,

$$f_i(t) = F_i + \rho\left[h_i(t)*\left\{\sum_{j=1}^{N} a_{ij}v[x_{ij}(t)] - \sum_{j=1}^{N} b_{ji}v[x_{ji}(t - \bar{\tau}_{ij})]\right\}\right] \quad (1)$$

That is, the frequency of the ith clock equals its natural frequency F_i plus a control function of x, the buffer states at the i^{th} station and its nearest neighbors.

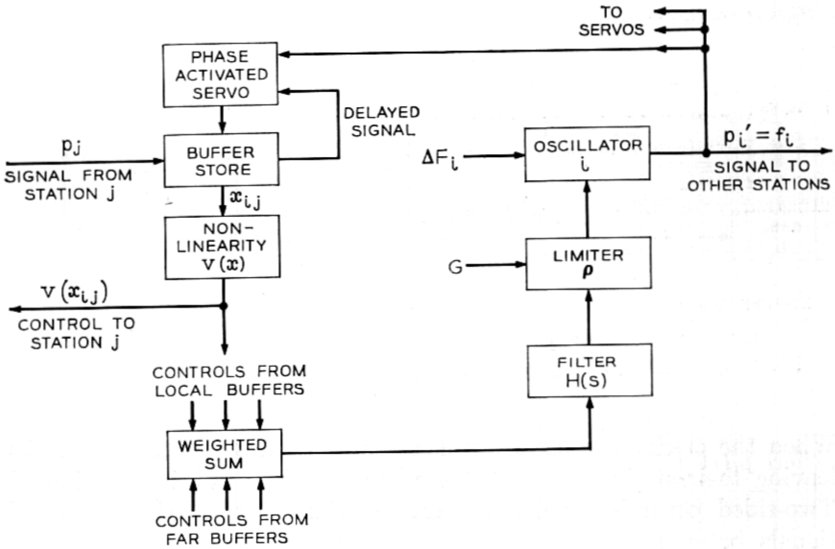

Fig. 2 — Station i and its control with respect to station j.

The function $\rho(\cdot)$, which represents a limiter, is defined by

$$\begin{aligned} \rho(x) &= x, && \text{if} \quad |x| \leqq G \\ &= G, && \text{if} \quad x > G \\ &= -G, && \text{if} \quad x < -G \end{aligned} \tag{2}$$

It is stipulated that the control filters, h, have unity gain for dc:

$$H_i(0) \equiv \int_0^\infty h_i(t)\, dt = 1, \qquad i = 1, 2, \cdots, N \tag{3}$$

where $h_i(t)$ is the impulse response and $H_i(0)$ is the dc response. Mathematical convolution is indicated by "*."

The gain coefficients a_{ij}, b_{ij} are nonnegative values. They are both equal to zero when there is no transmission link to station i *from* station j. When the link is present, $a_{ij} > 0$ and, if and only if the controls are two-sided, $b_{ij} > 0$.

The function $v(\cdot)$ is monotonic and of odd symmetry. It is also assumed that

$$v(\pm 1) = \pm 1. \tag{4}$$

Simple examples of such functions are

$$v_n(x) = x^{2n+1}, \qquad n = 0, 1, 2, \cdots \tag{5}$$

This is linear in the special case, $n = 0$.

The instantaneous state of the buffer delay in the link *to* center i *from* station j is denoted by $x_{ij}(t)$. Let the number of clock cycles stored in this buffer be $y_{ij}(t)$, and let its capacity be $2D_{ij}$. Then

$$x_{ij}(t) = [y_{ij}(t) - D_{ij}]D_{ij}^{-1} . \tag{6}$$

This represents the fractional deviation of the buffer delay from its half capacity. For synchronism to exist the buffers must lock the phase of the incoming signal to that of the local clock.

$$x_{ij}(t) = D_{ij}^{-1}[p_j(t - \tau_{ij}) - p_i(t)] + C_{ij} . \tag{7}$$

Where $p_i(t)$ is the phase of the ith clock, therefore

$$p_i'(t) = f_i(t). \tag{8}$$

The constant C_{ij} is determined by the initial switch-on conditions.[1]

τ_{ij} is the transmission delay in the link to station i from station j. $\bar{\tau}_{ij}$ is the delay in sending control signals to station i from station j. It might or might not be approximately true that the two delays are equal when both are defined, but this has been true of the simulations that have been done.

Now assume that the system has settled down to a common frequency f and that the phase differences remain finite. Thereafter

$$p_i(t) = ft + r_i \tag{9}$$

where r_i is independent of time. Also, because the buffers are quiescent

$$x_{ji}(t + \tau) = x_{ji}(t) \quad \text{for all} \quad \tau > 0$$

and

$$h_i(t)^*x = x$$

so equation (1) becomes time independent,

$$f = F_i + \rho \sum_{j=1}^{N} \{a_{ij}v(x_{ij}) - b_{ji}v(x_{ji})\}, \tag{10}$$

and (7) becomes

$$x_{ij} = D_{ij}^{-1}[r_j - r_i - f\tau_{ij}] + C_{ij} , \tag{11}$$

which is also time independent.

For the purpose of investigating the changes in settling frequency of a system, F_i and τ_{ij} will be considered variables, and such variations are possible in the simulator. It is also possible to perturb the simulator by connecting or disconnecting transmission links, or by adding or removing an entire station.

III. LINEAR SYSTEMS

3.1 *Analysis*

The linear subclass of systems is of very special importance. Study of these systems has added much to our knowledge of organic synchronization.

One-sided linear systems have been very extensively studied. V. E. Beneš, in unpublished work, was the first to establish a very interesting sufficient condition for stability of these systems, and to study their settling frequency. A. Gersho and B. J. Karafin[3] have recently simplified the derivation and proof of these results. M. B. Brilliant[4] has shown that the requirement for network connectedness can be weakened, if master-slave relations between subsystems are permitted. He also has determined dynamic responses for some networks.[5] Karnaugh has formulated a model that makes explicit the influence of the starting conditions in the formulas for settling frequency.[1]

Here, we shall extend the analysis to two-sided controls, considering the network to be switched on and in a quiescent state and then determining the change of quiescent conditions resulting from certain parameter changes. The result is a linear expression for the small changes in frequency that result from small changes in delay and in natural frequencies of the clocks.

The model defined in the previous section is linearized by removal of the limiter and by assuming $v(x)$ linear for the small changes. Total derivatives of equations (10) and (11) show how the settling state defined by the frequency f and the buffer states x is affected by changes. Thus for small changes, Δ, from a given quiescent state we have

$$\Delta f = \Delta F_i + \sum_{j=1}^{N} \{a_{ij}v'(x_{ij})\,\Delta x_{ij} - b_{ji}v'(x_{ji})\,\Delta x_{ji}\} \tag{12}$$

and

$$\Delta x_{ij} = D_{ij}^{-1}[\Delta r_j - \Delta r_i - \Delta(f\tau_{ij})]. \tag{13}$$

Now use

$$\alpha_{ij} = a_{ij} D_{ij}^{-1} v'(x_{ij})$$

and (14)

$$\beta_{ji} = b_{ji} D_{ji}^{-1} v'(x_{ji})$$

which are assumed constants for a given settling state. Also use

$$g_i = \sum_{j=1}^{N} (\alpha_{ij} + \beta_{ji}) \tag{15}$$

and

$$\tau_i = \sum_{j=1}^{N} (\alpha_{ij}\tau_{ij} - \beta_{ji}\tau_{ji}) \tag{16}$$

to get

$$\Delta f = \Delta F_i + \sum_{j=1}^{N} (\alpha_{ij} + \beta_{ji})\, \Delta r_j - g_i\, \Delta r_i - \Delta(f\tau_i). \tag{17}$$

Now measure phases with respect to the first station

$$q_i = p_i - p_1 = r_i - r_1\,, \qquad \Delta q_i = \Delta r_i - \Delta r_1\,. \tag{18}$$

Then

$$\Delta f + \sum_{j=2}^{N} \{g_i\, \delta_{ij} - (\alpha_{ij} + \beta_{ji})\}\, \Delta q_j = \Delta F_i - \Delta(f\tau_i) \tag{19}$$

where δ_{ij} is the Kronecker delta

$$\delta_{ii} = 1, \qquad \delta_{ij} = 0 \quad \text{for} \quad i \neq j.$$

The solution of equation (19) by determinants, d, can be carried out to yield, after some simplification and index permutation,

$$\Delta f = \frac{\sum_{i=1}^{N} d_i\, \Delta F_i}{\sum_{i=1}^{N} d_i} - \frac{\sum_{i,j=1}^{N} (d_i\, \alpha_{ij} - d_j\, \beta_{ij})\, \Delta(f\tau_{ij})}{\sum_{i=1}^{N} d_i}. \tag{20}$$

This equation is not solved for Δf explicitly because the term $\Delta(f\tau_{ij})$ depends on Δf, however this result is a convenient one for practical use. Let us investigate its meaning.

By definition, $f\tau_{ij}$ is the phase delay in the transmission link connecting center j to center i. Therefore equation (20) expresses the change of settling frequency as a weighted sum of the changes in clock natural frequencies, ΔF, less a weighted sum of the change in phase delays during transmission, $\Delta(f\tau_{ij})$.

The usefulness of the result derives from the fact that in most practical networks the phase delay variations are determined by changes of delay alone, variations resulting from changes of frequency, $\tau\Delta f$, are usually negligible, that is

$$\Delta(f\tau_{ij}) \cong f\,\Delta\tau_{ij}$$

and this is true for the simulations.

Returning to the general result, for small changes we can write

$$\Delta(f\tau_{ij}) = f\,\Delta\tau_{ij} + \tau_{ij}\,\Delta f. \tag{21}$$

Then equation (20) becomes

$$\Delta f = \frac{\sum_{i=1}^{N} d_i(\Delta F_i - f\,\Delta\tau_i)}{\sum_{i=1}^{N} d_i(1 + \tau_i)}. \tag{22}$$

This is similar to the result given by Beneš. It demonstrates the surprising property that the sensitivity of the network to changes is small when the delays are large. However, the effect has only theoretical interest because in practice $\tau_i \ll 1$.

Each weighting coefficient, d_i, is the cofactor of the element in the ith row, first column, of the matrix $[\Gamma - \lambda]$, defined by

$$\lambda_{ij} = \alpha_{ij} + \beta_{ji}$$

$$\Gamma_{ij} = g_i\,\delta_{ij}.$$

It can be shown, following Brilliant,[4] that $d_i > 0$ when

$$\left[\sum_{n=1}^{N} \lambda^n\right]_{ji} > 0 \quad \text{for all} \quad j \neq i$$

and $d_i = 0$ otherwise. The criterion for positive d_i is, heuristically speaking, that a chain of transmission links of λ shall run from station i to every other station. If the ith station provides a master clock for the system, $d_i > 0$ and $d_j = 0$, $j \neq i$. The strongly connected systems of Beneš, on the other hand, have $d_i > 0$ for all i. The system cannot be synchronized unless at least one of these coefficients is positive.

The solution, (22), of the linear equations depends upon their being nonsingular. This is equivalent to the nonvanishing of the denominator. This condition will be seen to hold in systems of practical interest.

3.2 *The Expression for Settling Frequency*

To give direction to the simulation experiments, we summarize the salient properties of equation (20) in a set of rules that can be readily confirmed experimentally without need for solving determinants. They are also important because they describe conditions that might be used in a real network.

Rule 1

In the case of two-sided controls that are "proportional" so that $K_j\alpha_{ji} = K_i\beta_{ji}$ for all $i \neq j$, where $(K_1K_2 \cdots K_N)$ is a set of N positive numbers, then it can be shown that

$$\frac{K_1}{d_1} = \frac{K_2}{d_2} = \frac{K_N}{d_N}$$

and that

$$\Delta f = \frac{\sum_{i=1}^{N} K_i \, \Delta F_i}{\sum_{i=1}^{N} K_i}. \tag{23}$$

The settled frequency is a weighted average of the oscillator center frequencies and is independent of delay.

Rule 2

In the particular case of "balanced" two-sided control defined by $\alpha_{ij} = \beta_{ij}$ for all $i \neq j$,

$$\Delta f = \frac{1}{N} \sum_{i=1}^{N} \Delta F_i \, . \tag{24}$$

The settled frequency of a balanced two-sided organic system is always the unweighted average of the clock center frequencies.

Rule 3

In the case of a reciprocal control defined by $K_i(\alpha_{ij} + \beta_{ji}) = K_j(\alpha_{ji} + \beta_{ij})$ for all $i \neq j$, it can be shown that

$$\frac{K_1}{d_1} = \frac{K_2}{d_2} = \frac{K_N}{d_N}$$

and that

$$\Delta f = \frac{\sum_{i=1}^{N} K_i \, \Delta F_i - \sum_{i,j=1}^{N} \{K_i\alpha_{ij} - K_j\beta_{ij}\} \, \Delta(f\tau_{ij})}{\sum_{i=1}^{N} K_i}. \tag{25}$$

Having reciprocal control requires that the controls also be linear, $v(x) = x$. One sided controls, $\beta = 0$, can be reciprocal if $K_i\alpha_{ij} = K_j\alpha_{ji}$.

Rule 4

In the case of proportional control, as in rule 1, if a transmission link is symmetrical, $\tau_{ij} = \tau_{ji}$ then, in response to a change $\Delta\tau_{ij} = \Delta\tau_{ji}$

$$\Delta x_{ij} = D_{ij}^{-1} \, \Delta\tau_{ij} f \quad \text{and} \quad \Delta x_{ji} = D_{ji}^{-1} \, \Delta\tau_{ji} f$$

and there is no change in the position of any other buffer in the network.

Rule 5

The net phase delay around a closed loop in a synchronized network is a constant integer.

To prove this we notice that the net phase delay in link ij is

$$\phi_{ij} = D_{ij}(1 + x_{ij}) + \tau_{ij} f. \tag{26}$$

Therefore

$$\Delta\phi_{ij} = D_{ij} \, \Delta x_{ij} + \Delta(f\tau_{ij})$$

using (13)

$$\Delta\phi_{ij} = \Delta r_j - \Delta r_i \, . \tag{27}$$

Then around a closed loop in the network the net phase delay will be

$$\Delta\phi_{ij} + \Delta\phi_{jk} + \cdots , \Delta\phi_{hi} = \Delta r_j - \Delta r_i + \Delta r_k - \Delta r_j \, , \cdots , + \Delta r_i$$
$$= 0.$$

The definition of synchronization requires that the net phase delay be an integer. Notice that changes in the phase delays must be given a direction. That is, changes in delays introduced into signals flowing in one direction around the loop have the opposite sign from changes in delays introduced in signals flowing in the other direction.

IV. DYNAMIC RESPONSE AND STABILITY

In the foregoing analysis, stability was tacitly assumed and the actual form of the dynamic response to the parameter changes was ignored. Here we shall explore conditions for stability of two-sided controls for a linear network having no limiter. The clock frequencies will be considered variables but for convenience we shall assume transmission delays are fixed.

The transient response and stability of linear organic systems may be studied by means of the Laplace transforms of equations (1) and (7). Combining these equations and putting

$$r_1(t) = p_i(t) - p_i(0) \tag{28}$$

we obtain the result

$$sR_i(s) = H_i(s) \sum_{j=1}^{N} (\hat{\alpha}_{ij} + \hat{\beta}_{ji})R_j(s) - H_i(s)R_i(s) \sum_{j=1}^{N} (\alpha_{ij} + \bar{\beta}_{ji}) + V_i(s) \tag{29}$$

where capitals denote Laplace transforms, and

$$\hat{\alpha}_{ij} = \alpha_{ij}e^{-s\tau_{ij}} \tag{30}$$

$$\hat{\beta}_{ji} = \beta_{ji}e^{-s\bar{\tau}_{ij}} \tag{31}$$

$$\bar{\beta}_{ji} = \beta_{ji}e^{-s(\tau_{ji}+\bar{\tau}_{ij})} \tag{32}$$

$$V_i(s) = \frac{F_i(s)}{s} + C_i(s) \tag{33}$$

and $C_i(s)$ represents initial conditions of the network. Following Gersho and Karafin,[3] we notice that equation (29) may be put in the form

$$R_i(s) = B_i(s) \sum_{j=1}^{N} \frac{\hat{\alpha}_{ij} + \hat{\beta}_{ji}}{g_i} R_j(s) + \frac{B_i(s)}{g_iH_i(s)} V_i(s), \tag{34}$$

where

$$B_i(s) \equiv \frac{g_iH_i(s)}{s + H_i(s) \sum_{j=1}^{N} (\alpha_{ij} + \bar{\beta}_{ji})}. \tag{35}$$

Let M be the matrix whose (ij)th element is

$$M_{ij} \equiv B_i(s) \frac{\hat{\alpha}_{ij} + \hat{\beta}_{ij}}{g_i}, \tag{36}$$

and let Q be the diagonal matrix with

$$Q_{ii} = \frac{B_i(s)}{g_i H_i(s)}. \tag{37}$$

The system is now seen to obey the vector equation

$$R(s) = [I - M]^{-1} Q V(s). \tag{38}$$

When $V(s)$ is specified, we can compute the solution $R(s)$ from this relation.

Arguments which differ from those of Gersho and Karafin only in minor detail can now be used to establish a sufficient condition for the stability of connected systems. The proof will not be repeated here. The resulting condition is that

$$| B_i(s) | < 1 \quad \text{for} \quad s = \omega\sqrt{-1} \neq 0 \qquad i = 1, 2, \cdots, N, \tag{39}$$

where ω is real.

$B_i(s)$ is independent of the system delays when the controls are one-sided. This is not the case for two-sided controls, as one may see from equations (32) and (35). However, the sufficient condition may be checked for any particular network and is easily satisfied in practice.

M. B. Brilliant has simplified (39) for a simple but revealing case:

$$H(s) = 1, \qquad g_i = g, \qquad \tau_{ij} + \bar{\tau}_{ji} = \tau \quad \text{for all} \quad i \neq j.$$

The sufficient condition for stability is then

$$g\tau < 0.5. \tag{40}$$

Thus, the largest product of the delay and the gain can, if it is less than 0.5, guarantee stability of the unfiltered two-sided linear network.

V. GAIN AND BUFFER DELAY

It will be shown later that, for one-sided controls, limiting of the frequencies can cause synchronization failures when the transmission delays are changing. Therefore, we are motivated to use small enough values of gains, g_i, to avoid limiting.

When there are no filters, or only single-pole filters, equations (1), (2), (4), and (6) show that limiting will not occur if

$$\sum_{j=1}^{N} (a_{ij} + b_{ji}) < G. \tag{41}$$

Notice that G would be made equal to the maximum tolerable frequency deviation. For simplicity, suppose that

$$g_i = g \quad \text{for} \quad i = 1, 2, \cdots, N$$

and

$$D_{ij} = D \quad \text{for all} \quad (i, j).$$

Then the inequality, (41), becomes

$$gD < G. \tag{42}$$

Now consider the buffer size. The buffers deflect to compensate for delay changes and to correct oscillator drift. Let the greatest transmission delay variation from midrange be denoted by $\hat{\Delta}\tau$ and let the maximum error magnitude of the oscillator center frequency be denoted by $\hat{\Delta}F$ so that

$$\hat{\Delta}F = \max_i \mid \Delta F_i \mid.$$

Then we require

$$D > F\,\hat{\Delta}\tau + \frac{\hat{\Delta}F}{g} \tag{43}$$

and we wish to keep the buffer sizes small by making g large. Then most of the buffer delay capacity is used to compensate for transmission delay variations

$$F\,\hat{\Delta}\tau > \frac{\hat{\Delta}F}{g}. \tag{44}$$

Inequalities (44), (43), and (42) lead to

$$\frac{\hat{\Delta}F}{F\,\hat{\Delta}\tau} < g < \frac{G}{D} < \frac{G}{F\,\Delta\tau + \dfrac{\hat{\Delta}F}{g}}. \tag{45}$$

Values of D that satisfy the last two inequalities in (45) will exist, provided that

$$g < \frac{G}{F\,\hat{\Delta}\tau + \dfrac{\hat{\Delta}F}{g}}$$

whence,

$$\frac{\hat{\Delta}F}{F\,\hat{\Delta}\tau} < g < \frac{G - \hat{\Delta}F}{F\,\hat{\Delta}\tau}. \tag{46}$$

Inequalities (46) are satisfied over a positive interval of values of g whenever

$$G > 2\,\hat{\Delta}F. \tag{47}$$

Finally, D must be chosen in the interval,

$$F\,\hat{\Delta}\tau + \frac{\hat{\Delta}F}{g} < D < \frac{G}{g}. \tag{48}$$

For some specific examples, let us make the following choices:

$$G = 4\,\hat{\Delta}F$$

$$g = \frac{2\,\hat{\Delta}F}{F\,\hat{\Delta}\tau}$$

$$D = \tfrac{7}{4}F\,\hat{\Delta}\tau.$$

Let us further assume the largest single transmission delay to be 2×10^{-2} seconds, which is of transcontinental magnitude, and let $\hat{\Delta}\tau = 2 \times 10^{-5}$ seconds. The latter is almost surely an overestimate for underground coaxial cable, but is reasonable if about 10 per cent of the cable is above ground.

Two values of F will be used, corresponding roughly to voice and video sampling rates. For each of these, two values of $\hat{\Delta}F$ will be used, corresponding roughly to the accuracies of simple crystal oscillators and atomic oscillators. Table I shows the resulting parameters.

The greatest value of g encountered in Table I is 3×10^{-2}. Let us examine the consequences of this with respect to the stability condition, equation (40). The product τg is less than 12×10^{-4} for all cases. Therefore in satisfying the stability condition $\tau g < 0.5$ we have a factor of 400 to spare.

From (35), we see that $B_i(s)$ is the transfer function for the ith clock with respect to equal perturbations of phase in all arriving

TABLE I—Parameters for Voice and Video Sampling

F	$\hat{\Delta}F$	$\hat{\Delta}\tau$	G	g	D
(Hz)	(Hz)	(seconds)	(seconds^{-1})		(cycles)
10^4	3×10^{-3}	2×10^{-5}	12×10^{-3}	3×10^{-2}	3.5×10^{-1}
10^4	10^{-5}	2×10^{-5}	4×10^{-5}	10^{-4}	3.5×10^{-1}
6×10^6	18×10^{-1}	2×10^{-5}	7.2×10^0	3×10^{-2}	2.1×10^2
6×10^6	6×10^{-3}	2×10^{-5}	2.4×10^{-2}	10^{-4}	2.1×10^2

signals. In the absence of delays,

$$B_i(s) = \frac{g_i H_i(s)}{s + g_i H_i(s)}.$$

Therefore, if $H_i(s)$ is either constant or is monotone decreasing with little phase shift in

$$s = \omega\sqrt{-1}, \qquad 0 \leqq \omega \leqq g_i ,$$

the response radian bandwidth is no more than g_i. When $H_i \equiv 1$, the response has the simple time constant, g_i^{-1}.

This enables us to interpret τg as the ratio of the delay between stations to their time constants of response. When this ratio is very small, the transmission delays have negligible effect on the dynamic response of the network. This is another strong reason for using relatively low gains.

Another useful approximation that applies to most practical networks concerns the phase delay during transmission $f\tau_{ij}$. The change of phase delay is

$$\Delta(f\tau_{ij}) = f\,\Delta\tau_{ij} + \tau_{ij}\,\Delta f. \tag{21}$$

We have seen that delays change about 0.1 per cent* while clock frequencies change about 10^{-7}, thus the second term in the expression is negligible in most cases and this is true for the simulations. Equation (20) then becomes

$$\Delta f \cong \frac{\sum_{i=1}^{N} d_i(\Delta F_i - f\,\Delta\tau_i)}{\sum_{i=1}^{N} d_i}. \tag{49}$$

VI. A TWO STATION NETWORK

6.1 *Response of the System to Small Change*

Many revealing properties of the organic synchronizing scheme can be illustrated with two stations connected in a loop as in Fig. 3. Such a system is characterized by the following equations

$$\Delta f \cong \frac{g_i\,\Delta F_i + g_i\,\Delta F_j}{g_i + g_j} - \frac{(\alpha_{ij}\alpha_{ji} - \beta_{ij}\beta_{ji})f\,\Delta(\tau_{ij} + \tau_{ji})}{g_i + g_j} \tag{50}$$

* For short delays the fractional change can be much larger.

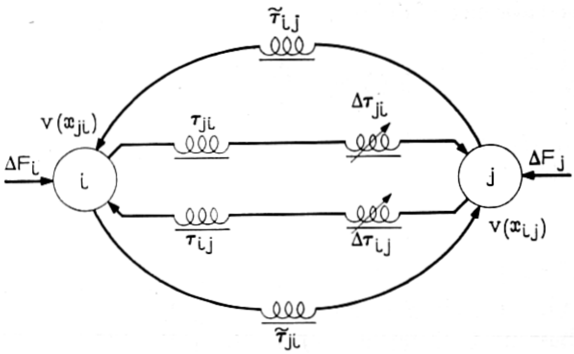

Fig. 3 — Two coupled stations, i and j.

$$\Delta x_{ij} \cong \frac{D_{ij}^{-1}}{g_i + g_j} [\Delta[F_j - F_i] + (\alpha_{ji} + \beta_{ji})f\ \Delta(\tau_{ij} + \tau_{ji})]$$

$$\Delta x_{ji} \cong \frac{D_{ji}^{-1}}{g_i + g_j} [\Delta[F_i - F_j] + (\alpha_{ij} + \beta_{ij})f\ \Delta(\tau_{ij} + \tau_{ji})]. \tag{51}$$

Equation (50) was derived from (49), and equations (51) follow from (13), (18), (19), and (49).

Our simulation of this network has a nominal 1 megahertz center frequency with ±1 hertz control range. The gains $(g_i + g_j)$ are in the range 0.1 to 0.001 $\sec^{-1}$. Transmission delay around the loop can be preset in the range 0 to 0.1 second and be varied continuously by ±100 microseconds in each link.

In all setups the settling states agree well with prediction. They confirm that fixed delay has negligible effect ($\ll$ one percent) on buffer and frequency deflections. Therefore the approximation in deriving (49) from (20) is justified.

For illustration, Table II gives some typical results. The data have been normalized to represent unit amplitude disturbances. The incremental delay changes, $\Delta\tau$ are expressed as a fraction z of the associated buffer capacity such that

$$z_{ij} = \frac{\Delta\tau_{ij}F}{D_{ij}}. \tag{52}$$

In the simulations the gain delay product τg is less than 10^{-2}. This should guarantee stability with reasonable valued filters, and indicates that fixed delays are too small to have appreciable effect on transients. Observations of responses after switch-on and of subse-

TABLE II—Settling States of Two Stations

Control Type	Gain Setting				Disturbance		Result		
	α_{ij}	β_{ij}	α_{ji}	β_{ji}	ΔF_i	Δz_{ji}	Δx_{ij}	Δx_{ji}	Δf
Balanced	1	1	1	1	1	0	−1/4	+1/4	1/2
	1	1	1	1	0	−1	1/2	1/2	0
	2	2	2	2	1	0	−1/8	+1/8	1/2
	2	2	2	2	0	−1	1/2	1/2	0
	2	2	1	1	1	0	−1/6	−1/6	1/2
	2	2	1	1	0	−1	1/3	2/3	0
Proportioned	2	1	1	2	1	0	−1/6	1/6	1/3
	2	1	1	2	0	−1	1/2	1/2	0
	1	2	2	1	1	0	−1/6	1/6	2/3
	1	2	2	1	0	−1	1/2	1/2	0
One Sided	1	0	1	0	0	−1	1/2	1/2	1/2
	2	0	2	0	1	0	−1/4	1/4	1/2
	2	0	2	0	0	−1	1/2	1/2	1
	2	0	1	0	0	−1	1/3	2/3	2/3

Unit gain = 10^{-2} sec^{-1}
Unit frequency = 1 Hz
Unit delay = 10^2 μsec
Nominal clock frequency = 1 MHz

quent disturbances confirm this conclusion. Indeed, delay had insignificant effect when various one- and two-pole low pass filters were included in the control loops. The filters investigated had cut-off frequencies from 0.01 to 50 hertz and q-factors up to 10.

When fixed delays are neglected, the response of the linear system to small change can be expressed as

$$D_{ij}X_{ij}(s) \cong \frac{F_i(s) - F_j(s) - [\alpha_{ji}(s) + \beta_{ji}(s)][\tau_{ij}(s) + \tau_{ji}(s)]F}{[s + g_i(s) + g_i(s)]} \tag{53}$$

$$f(s) \cong \alpha_{ij}(s)X_{ij}(s) - \beta_{ji}(s)X_{ji}(s) \tag{54}$$

where

$$\alpha_{ij}(s) = \alpha_{ij}H_i(s), \qquad \beta_{ij}(s) = \beta_{ij}H_j(s)$$

and

$$g_i(s) = g_iH_i(s).$$

When no filters are used these responses have a simple time constant $1/(g_i + g_j)$. Fig. 4 gives some typical response curves for a system with no filters and using various gain values. Including delays up to 0.1 second had no noticeable effect on these curves.

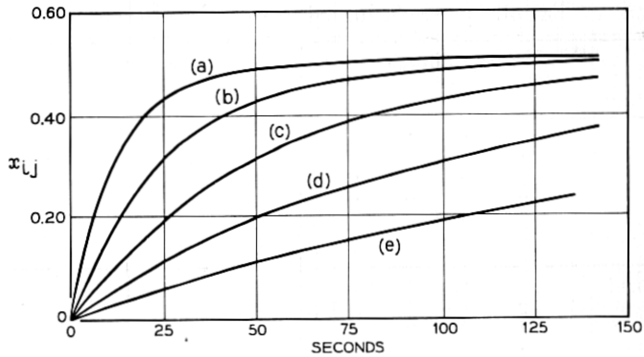

Fig. 4 — Response of buffer x_{ij} to a step decrease in delay $\Delta\tau_{ij}$. The controls are balanced, and the net gain is such, that $(g_i+g_j)^{-1}$ is 12 seconds for curve a, 24 for b, 48 for c, 108 for d, and 216 for e.

Fig. 5 shows responses with similar one-pole low-pass filters in both oscillator circuits. Notice that use of filter time constants near $1/g$ speeds the response. We do not anticipate that filters will play an important role in the operation of the networks. We have seen that they are not needed to stabilize the controls, nor will they be needed to speed responses because most changes will be thermally induced, and will therefore occur very slowly. Some filtering may be needed

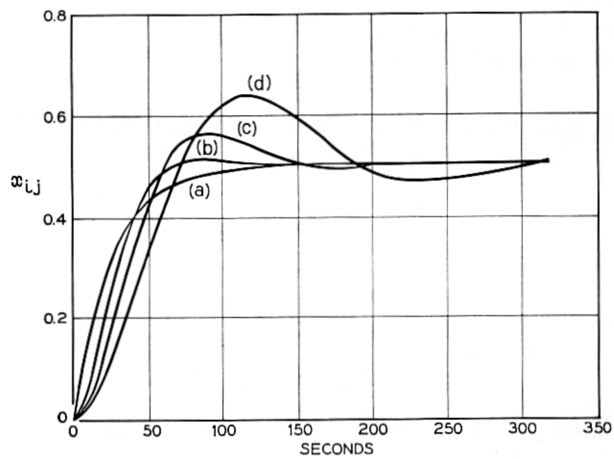

Fig. 5 — Response of buffer x_{ij} to a step decrease in τ_{ij} with various filters $1/(1+s\tau)$. The controls are balanced and the net gain $(g_i+g_i) = 1/24$ sec.$^{-1}$. The filter time constants τ are 0 seconds for curve a, 10 for b, 20 for c and 40 for d.

to smooth the output of phase detectors and to reduce noise. For this, time constants not greater than $1/g$ should be adequate.

6.2 *When Control Signals Limit*

A saturating limiter ρ is placed in the control path to the oscillators in order to place a bound on the frequency deviations. To demonstrate its effect, Fig. 6 shows a buffer response to a ramp change of delay using various gain values in a symmetrical one-sided control system with no filters

$$\alpha_{ij}(s) = \alpha_{ji}(s) = g \qquad \beta_{ij}(s) = \beta_{ji}(s) = 0.$$

At the start of the experiment the network is quiescent with the two clock natural-frequencies offset from one another and the two buffers defected by amounts that provide sufficient control voltage to align the running frequencies.

$$\Delta x_{ij} = \frac{\Delta F_{ij}}{g_i} D_{ij} \; .$$

This initial buffer-displacement decreases proportionately with increased gain. For this purpose a large gain is attractive. However, when the transmission delay varies, limiting occurs if the gain is large, as shown in curve c of Fig. 6, where there is loss of control and ultimately loss of synchronization. Clearly, saturation must be avoided, that is, equation (41) satisfied, when one-sided controls are used.

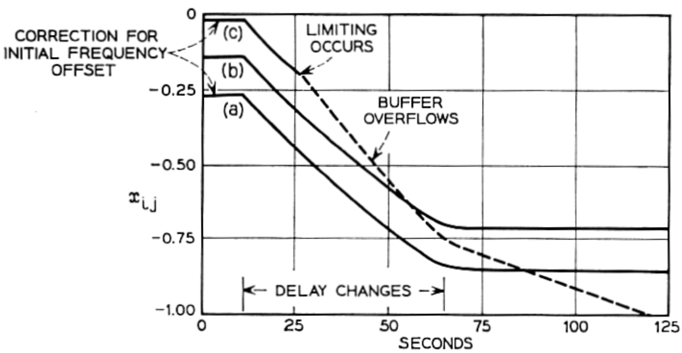

Fig. 6 — Response of x_{ij} to similar ramp delay increases $\Delta\tau_{ij}$ and $\Delta\tau_{ji}$ in a system having initial frequency discrepancy. The controls are one-sided and the gains are such that g_i^{-1} and g_j^{-1} are both equal to 50 seconds for curve a, 25 for b, and 5 for c.

With two-sided "proportioned" control, changes of delay do not cause change of settling frequency. See rule 1. Any frequency changes that might occur during the dynamic response will usually be small because large parameter changes will occur slowly in a practical network. Thus with "proportioned" control, delay changes will not cause limiting, the signal in the limiters being determined only by the differences of oscillator center frequencies.

Larger gains can be used with two-sided control systems than with one-sided. However, in practice equation (41) should always be obeyed so that systems continue to operate reliably if faults interrupt the distant control paths. The reasons for avoiding limiting are equally valid for more complex networks, and this is not a hard restriction on design. It has already been seen in Table I how limiting can be avoided while using gains that satisfy the main requirements of a practical network.

6.3 *Nonlinear Control*

Use of nonlinearity in the control loop has been proposed[1] as a means for exaggerating the correcting influence of buffer stores that are near overflow, at the expense of those nearer center. For this purpose the nonlinear circuit $v(x)$ is included in Fig. 2. It makes the incremental gains a function of buffer position, as illustrated by equation (14). We shall examine separately the nonlinear response to incremental delay and incremental frequency change as given by equations (50) and (51).

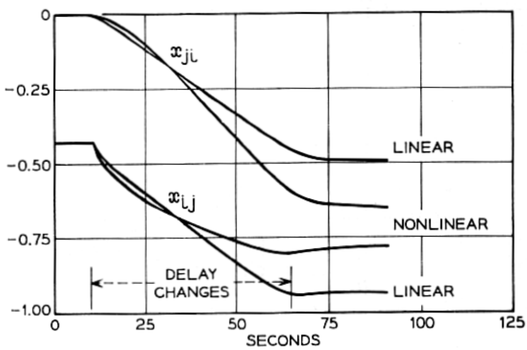

Fig. 7 — The response of both buffers to a ramp increase in delay $\Delta\tau_{ij}$ with linear and nonlinear controls. Controls are balanced with net $g = 0.04$ second^{-1} and $\mu = 1.2$.

When x_{ij} is nearer zero than x_{ji} the nonlinearity can be selected to make $|v'(x_{ij})|$ very small compared with $|v'(x_{ji})|$. Then $(\alpha_{ij} + \beta_{ij}) \ll (\alpha_{ji} + \beta_{ji})$. Therefore x_{ij} will approximately track the delay change and x_{ji} will have little change.

To illustrate this, Fig. 7 shows the response of both buffers to a steadily increasing delay commencing with $x_{ij} = 0$ and $x_{ji} \cong 0.5$. The nonlinearity successfully makes the buffer with the most reserve compensate for most delay change. In this and subsequent experiments

$$v(x) = \frac{x(\mu - 1)}{(\mu - x^2)}, \qquad \mu > 1. \tag{55}$$

A useful method for demonstrating the response of nonlinear control is a locus on a graph of x_{ji} plotted against x_{ij}. Fig. 8 shows such loci for various starting conditions, and Fig. 9 is an enlargement of a section of Fig. 8 repeated for different values of nonlinearity. These results show that the nonlinearity can prevent overflow in some circumstances.

Next, consider the response to frequency change with constant delays. Equation (51) shows that when the deflections are expressed in cyles

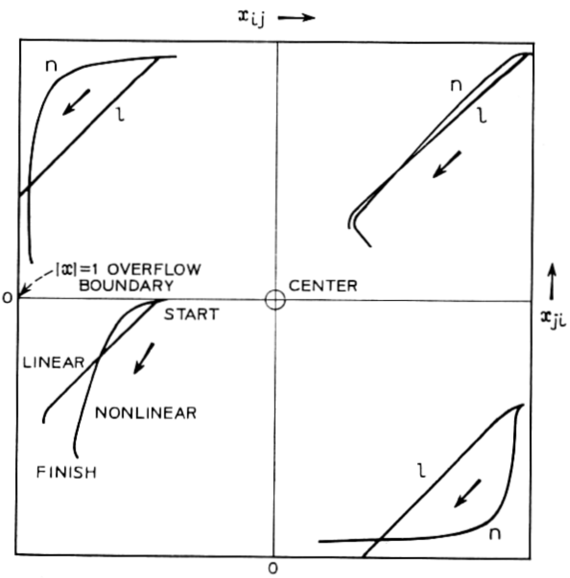

Fig. 8 — Loci of x_{ji} against x_{ij} for a slow ramp change of delay $\Delta\tau_{ij}$ in a two-station network; l: balanced linear controls $g_i = g_j = 0.04$ second^{-1}; n: balanced nonlinear controls $\mu = 1.2$ $g_i = g_j = 0.04$ second^{-1}.

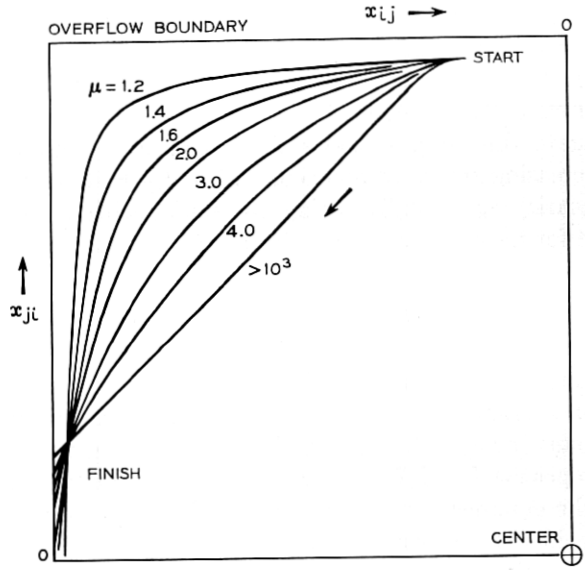

Fig. 9 — An enlargement of the upper left quadrant of Fig. 8 for various values of nonlinearity.

they are the same at both stations. We need to minimize their value when a buffer is near overflow. Consider a worst case with $x_{ji} = 0$ and $x_{ij} \cong 1$. Then there is improvement over the linear control if the net gain $(g_i + g_j)$ exceeds the linear gain. That is, if

$$[v'(0) - 1](\alpha_{ji} + \beta_{ji}) + [v'(1) - 1](\alpha_{ij} + \beta_{ij}) > 0.$$

For example, when $(\alpha_{ji} + \beta_{ij}) \cong (\alpha_{ij} + \beta_{ji})$ there is improvement if $1 < \mu < 3$ using nonlinearities described by equation (48) or if $n > 2$ using nonlinearities described by equation (7). An illustration is given in Fig. 10.

The degree of nonlinearity used will be limited by the need to maintain some minimum gain for controls near center.

VII. A CHAIN OF FOUR STATIONS

The simulation is extended by adding two more stations to form the chain shown in Fig. 11. In designing controls for this network it seems sensible to make controls associated with each link reciprocal and symmetric, that is,

$$(\alpha_{ij} + \beta_{ji}) = (\alpha_{ji} + \beta_{ij}) \quad \text{for all} \quad i \neq j.$$

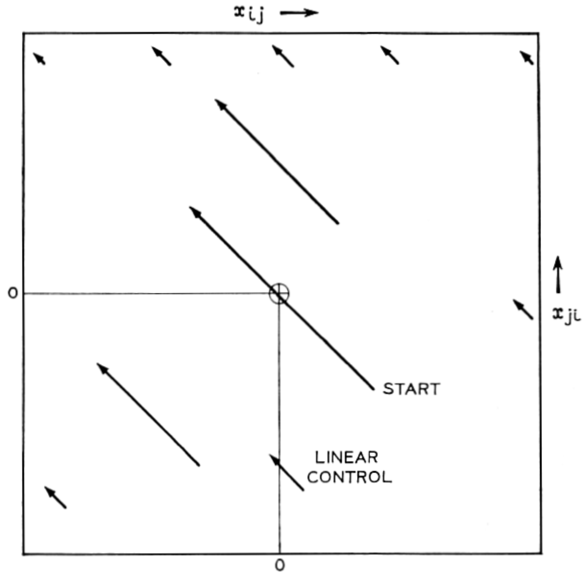

Fig. 10 — Loci of x_{ji} against x_{ij} for a step change in frequency ΔF_i in a two-station network with balanced nonlinear control $\mu = 1.2$.

Then with the buffers at center the settling frequency is equally sensitive to frequency drift in each oscillator. With such control the net gain, g, of the relay stations j and k exceeds that of the end stations i and l. The relay stations therefore have greater risk of limiting, but this risk is lessened by the larger possibility of averaging and canceling of disturbances.

Some typical settling states for this chain are given in Table III. The top of Fig. 12 shows how a sudden change of station i frequency causes transients through the system. The bottom shows the corresponding response with nonlinearities added. Notice the slowing down of the response and the increased deflections particularly at remote stations. These effects are a consequence of the low control gains when buffers are near center.

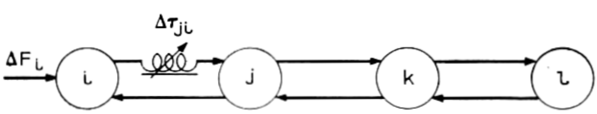

Fig. 11 — The four station chain.

TABLE III—Settling States of a Four Station Chain

Control Type	Gain Setting				Disturbance			Result						
	α_{ij} α_{lk}	β_{ji} β_{kl}	α_j α_k	β_j β_k	ΔF_i	ΔF_j	Δz_{ij}	Δx_{ij}	Δx_{ji}	Δx_{jk}	Δx_{kj}	Δx_{kl}	Δx_{lk}	Δf
Balanced	1	1	1	1	1	0	0	−3/8	3/8	−1/4	1/4	−1/8	1/8	1/4
	1	1	1	1	0	1	0	1/8	−1/8	−1/4	1/4	−1/8	1/8	1/4
	1	1	1	1	0	0	−1	1/2	1/2	0	0	0	0	0
Proportioned	1	1	1/2	1/2	1	0	0	−5/12	5/12	−1/4	1/4	−1/12	1/12	1/6
	1	1	1/2	1/2	0	1	0	1/6	−1/6	−1/2	1/2	−1/6	1/6	1/3
	1	1	1/2	1/2	0	0	−1	1/2	1/2	0	0	0	0	0
One-sided Reciprocal	2	0	2	0	1	0	0	−3/8	3/8	−1/4	1/4	−1/8	1/8	1/4
	2	0	2	0	0	1	0	1/8	−1/8	−1/4	1/4	−1/8	1/8	1/4
	2	0	2	0	0	0	−1	1/4	3/4	−1/2	1/2	−1/4	1/4	1/2

Code: $\alpha_j = \alpha_{jl} = \alpha_{jk}$ $\quad \beta_j = \beta_{ij} = \beta_{kj}$
$\alpha_k = \alpha_{kl} = \alpha_{kj}$ $\quad \beta_k = \beta_{lk} = \beta_{jk}$

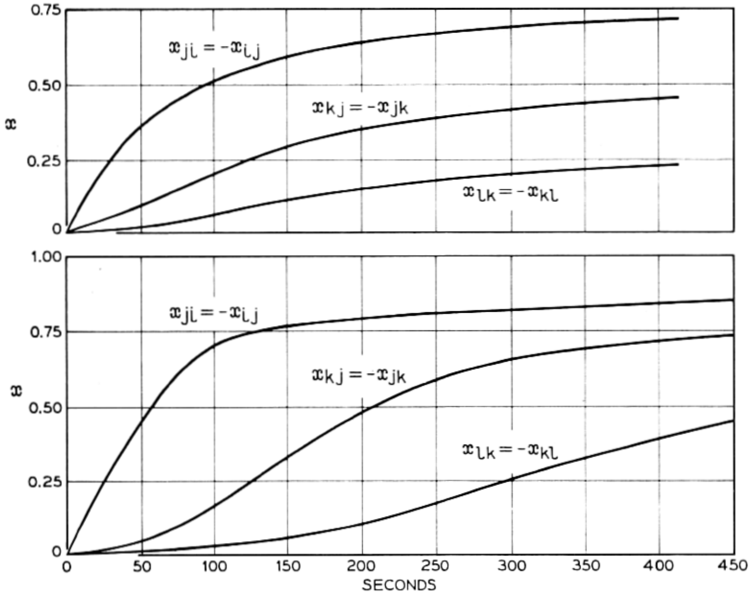

Fig. 12 — Buffer responses to a step change of frequency ΔF_i in a four station chain. Top: linear balanced controls $\alpha = \beta = 0.02$ second^{-1}. Bottom: nonlinear balanced controls $\alpha = \beta = 0.02$ second^{-1} $\mu = 1.2$.

However, we have said that frequency drift is small in practice, more significant are effects of delay change. These are demonstrated in Fig. 13 which shows loci of x_{ji} against x_{lk} for a delay change $\Delta\tau_{ji}$. For simplicity, buffer x_{ij} starts with the same content as x_{ji}. The content of x_{ji} and x_{lk} are apparent from the graph; all other buffers start at center. Curve a is the response with two-sided balanced control. For slow change, it is a vertical line whose shape is independent of position on the graph or of nonlinearity value. Curve b is the response with one-sided linear control. The others are for nonlinear one-sided controls.

VIII. VARIOUS CONNECTIONS OF FOUR STATIONS

Figs. 14, 15, and 16 show four stations connected as a loop, star, and complete network, respectively. The settling states of these three networks are illustrated in Tables IV, V, and VI. These results were virtually independent of fixed delay values and confirm the prediction of equation (49).

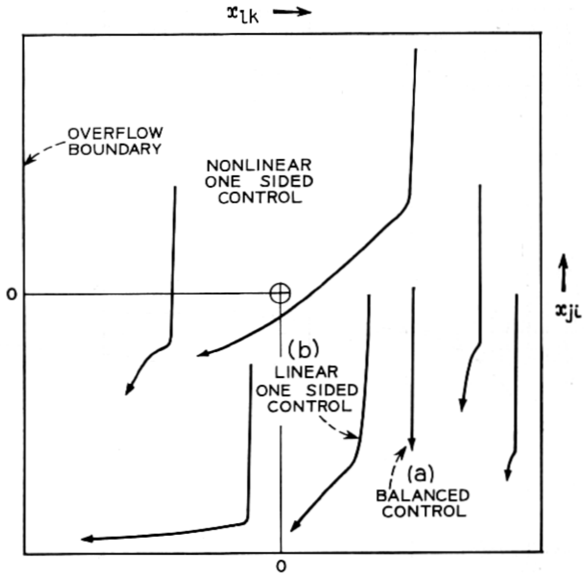

Fig. 13 — Loci of x_{ji} against x_{lk} for a slow ramp change of delay $\Delta\tau_{ji}$ in a four station chain.

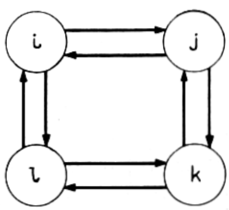

Fig. 14 — Ring network.

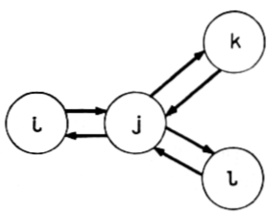

Fig. 15 — Star network.

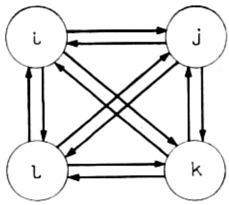

Fig. 16 — Complete network.

Fig. 17 gives some typical transient responses of the loop connection for both linear and nonlinear controls. Fig. 18 demonstrates the effect of nonlinearity on loci of x_{ji} to x_{kj} for various starting positions. A corresponding graph for the fully connected network is shown in Fig. 19.

In studying the responses of all these networks no noticeable effects were observed when delays up to 0.1 second were included in signal and control paths. The responses were always stable even with single pole filters in the control loop.

Obtaining proper statistical data on the response of the synchron-

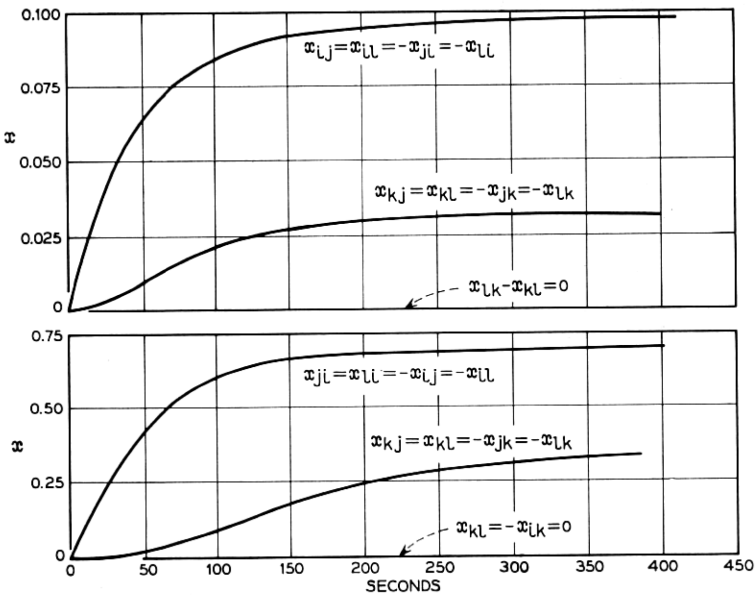

Fig. 17 — Buffer responses to a step change of frequency ΔF_i in a four station ring. Top: balanced linear controls $g = 0.012$ second^{-1}. Bottom: nonlinear controls $\mu = 1.2$ $g = 0.012$ second^{-1}.

TABLE IV — Settling States of a Four Station Ring

	Gains		Disturbance			Result					
Control Type	α	β	F_i	z_{ji}	z_{ij}	Δx_{ij}	Δx_{ji}	Δx_{kj} $-\Delta x_{jk}$	Δx_{lk} $-\Delta x_{kl}$	Δx_{il} $-\Delta x_{li}$	Δf
Balanced	1	1	1	0	0	−3/16	3/16	1/16	−1/16	−3/16	1/4
	1	1	0	−1	0	3/8	5/8	1/8	1/8	1/8	0
	1	1	0	−1	−1	1	1	0	0	0	0
One Sided Reciprocal	2	0	1	0	0	−3/16	3/16	1/16	−1/16	−3/16	1/4
	2	0	0	−1	0	3/8	5/8	3/8	1/8	−1/8	1/2
	2	0	0	−1	−1	1	1	1/2	0	−1/4	1/2

ized network to disturbances is an extensive project because of the large number of interacting parameters available and because of the slow response of the system to change. To short cut this work we have examined tendencies in the response in some extreme conditions.

The fully connected network was disturbed by driving the delays and the oscillators individually with triangular waves having unrelated frequencies in the range 0.01 to 0.001 Hz. The maximum deflection of the twelve buffers and their likelihood of overflow under various control settings were observed.

The following conclusions were made from these experiments.

(*i*) The chance of buffers overflowing is decreased by use of increased control gain, provided limiting is avoided. Increasing the gain past the limiting point increased the chance of overflow.

(*ii*) The chance of overflow increases when limiting levels are decreased.

(*iii*) The chance of either limiting or overflow occurring is less with balanced control than with one-sided control.

(*iv*) Use of low pass filters with long time constants increases the chance of overflow.

(*v*) Use of nonlinearities ($\mu \cong 1.2$) in a balanced control system reduces the chance of overflow.

(*vi*) Use of nonlinearity with single ended control increases the chance of overflow.

(*vii*) Use of nonlinearity increases the chance of overflow when disturbances change at rates comparable with the control bandwidth.

IX. RESPONSES IN FAILURE

If a single transmission link is severed or a signal is lost for any reason the servo in Fig. 1 normally drives its buffer to an extremity and

TABLE V — Settling States of a Four Station Star

Control Type	Gain Setting				Disturbance			Result				
	α_j	β_j	α_{ij} α_{kj} α_{lj}	β_{ji} β_{jk} β_{jl}	ΔF_i	ΔF_j	Δz_{ji}	Δx_{ij}	Δx_{ji}	Δx_{kj} $-\Delta x_{jk}$	Δx_{jl} $-\Delta x_{lj}$	Δf
Balanced	1	1	1	1	1	0	0	−3/8	3/8	1/8	1/8	1/4
	1	1	1	1	0	1	0	1/8	−1/8	1/8	1/8	1/4
	1	1	1	1	0	0	−1	1/2	1/2	0	0	0
Proportion	1/3	1/3	1	1	1	0	0	−5/12	5/12	1/12	1/12	1/6
	1/3	1/3	1	1	0	1	0	1/4	−1/4	1/4	1/4	1/2
	1/3	1/3	1	1	0	0	−1	1/2	1/2	0	0	0
One Sided	2	0	2	0	1	0	0	−3/8	3/8	1/8	1/8	1/4
	2	0	2	0	0	1	0	−1/8	1/8	1/8	1/8	1/4
	2	0	2	0	0	0	−1	1/4	3/4	−1/4	−1/4	1/2
	1	0	3	0	0	0	−1	1/6	5/6	−1/6	−1/6	1/2

Code:
$\alpha_j = \alpha_{ji} = \alpha_{jk} = \alpha_{jl}$
$\beta_j = \beta_{ij} = \beta_{kj} = \beta_{lk}$

TABLE VI — Settling State of Four Stations Fully Connected

Control Type	Gains		Disturbance			Result							
	α	β	ΔF_l	Δz_{ij}	Δz_{ji}	Δx_{ij}	Δx_{ji}	Δx_{ik} $-\Delta x_{ki}$	Δx_{il} $-\Delta x_{li}$	Δx_{jk} $-\Delta x_{kj}$	Δx_{jl} $-\Delta x_{lj}$	Δx_{lk} Δx_{kl}	Δf
Balanced	1	1	1	0	0	−1/8	1/8	−1/8	−1/8	0	0	0	1/4
	1	1	0	−1	0	1/4	3/4	1/8	1/8	−1/8	1/8	0	0
	1	1	0	−1	−1	1	1	0	0	0	0	0	0
One Sided Reciprocal	2	0	1	0	0	−1/8	1/8	−1/8	−1/8	0	0	0	1/4
	2	0	0	−1	0	1/4	3/4	0	0	−1/4	−1/4	0	1/2
	2	0	0	−1	−1	1	1	−1/4	−1/4	−1/4	−1/4	0	1

thus deflects the frequency of the whole network. In the simulator this is prevented by auxiliary equipment that takes the buffer to center in case of signal loss. Meanwhile, the remainder of the network can function correctly.

Another interesting fault occurs when a buffer is driven to overflow by the normal controls. In the simulator it rests at full deflection until the phase drift brings another frame within half a cycle of synchronism with the local clock, the system locks to that frame.

Little difficulty has been experienced in switching stations in and out of an active network. The switching transient can be reduced by adjusting the station frequency so that its buffers are near center.

X. CONCLUSION AND RECOMMENDATION

There seems to be no good reason for using nonreciprocal controls, moreover using linear-symmetric controls $(\alpha_{ij} + \beta_{ji}) = (\alpha_{ji} + \beta_{ij})$ makes the network equally sensitive to frequency drift in each oscillator (rule 3). Then if the oscillators have similar properties this minimizes the frequency displacement caused by oscillator drift.

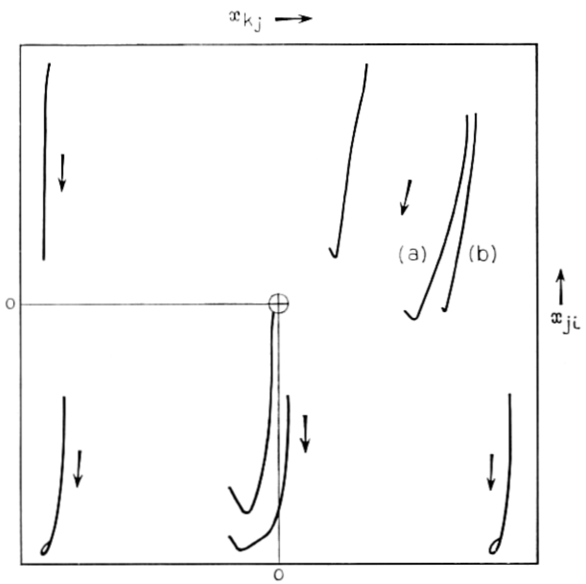

Fig. 18 — Responses of buffers to a ramp change in delay $\Delta\tau_{ji}$ in a four station ring: (a) linear balanced control; (b) linear one-sided control. All other curves for nonlinear balanced control $\mu = 1.2$ starting with $x_{jk} = x_{kj}$, $x_{ij} = x_{ji}$ and all other buffers at center.

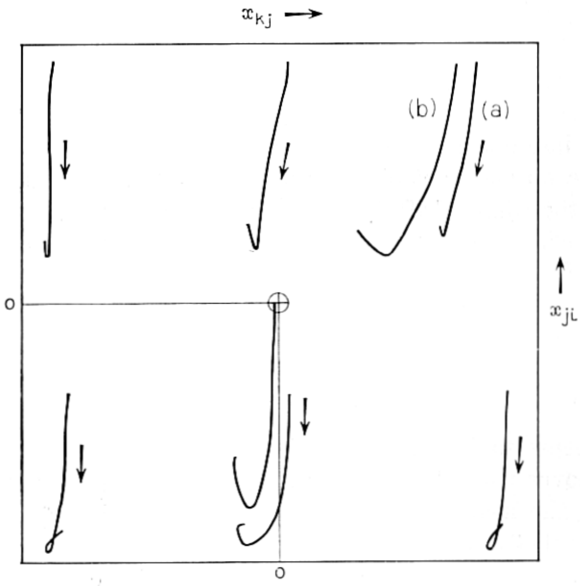

Fig. 19 — Responses of buffers to a ramp change of delay $\Delta\tau_{ji}$ for the complete network with the same controls as in Fig. 18.

There is additional advantage in also balancing controls ($\alpha_{ij} = \beta_{ij}$). Then slow delay change will not affect the system frequency and the chance of buffers overflowing is reduced. The disadvantage of using two-sided, and hence also of balanced controls, is the need for control paths between centers. These paths require little bandwidth (< 1 Hz) but must pass dc; they could be included in the framing and signaling codes of a PCM system.

A commercially useful network will almost certainly contain both sending and receiving links in each path used, and these links will be similar in opposite directions. For example, they will expand and contract together so that $\Delta\tau_{ij} = \Delta\tau_{ji}$. Rule 4 shows that if the controls in such a network are proportional then each delay change is corrected directly by the buffer in its link and no disturbance propagates through the network.

If the buffers in such a symmetric path start at corresponding positions they will approximately track one another, deviating only to correct small frequency drift and the misbalance of the links. There will be little use for nonlinear shaping to equalize their positions, besides, nonlinearities lower sensitivity when buffers are near center and increases the practical difficulty of keeping controls proportional.

Use of symmetric links and linear balanced controls provide for near optimum design of buffers. Each one corrects only the delay change of its own link and the small frequency drifts. They can be designed independently of the network.

XI. ACKNOWLEDGMENTS

The simulator used for the experiments described here was designed and built in cooperation with the authors of the preceding article that describes the hardware.[2] We are especially grateful to M. B. Brilliant for an extremely painstaking review and many valuable and poignant comments.

REFERENCES

1. Karnaugh, M., A Model for the Organic Synchronization of Communications Systems, B.S.T.J., *45,* December 1966, pp. 1705–1735.
2. Bosworth, R. H., Kammerer, F. W., Rowlinson, D. E., and Scattaglia, J. V., Design of a Simulator for Investigating Organic Synchronization Systems, B.S.T.J., this issue, pp. 209–226.
3. Gersho, A. and Karafin, B. J., Mutual Synchronization of Geographically Separated Oscillators, B.S.T.J., *45,* December 1966, pp. 1689–1704.
4. Brilliant, M. B., The Determination of Frequency in Systems of Mutually Synchronized Oscillators, B.S.T.J., *45,* December 1966, pp. 1737–1748.
5. Brilliant, M. B., Dynamic Response of Systems of Mutually Synchronized Oscillators, B.S.T.J., *46,* February 1967, pp. 319–356.