Predictive Quantizing Systems (Differential Pulse Code Modulation) for the Transmission of Television Signals

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Differential pulse code modulation (DPCM) and predictive quantizing are two names for a technique used to encode analog signals into digital pulses suitable for transmission over binary channels. It is the purpose of this paper to determine what kind of performance can be expected from well-designed systems of this type when used to encode television signals. Systems using both previous sample and previous line feedback are considered.

A procedure is presented for the design of nonadaptive, time invariant systems which are near optimum in the sense that the resulting signal to unweighted quantizing noise ratios (S/N) are nearly maximum. Simple formulas are derived for these S/N ratios which apply to DPCM as well as standard PCM. Standard PCM is shown to be a special case of DPCM. These formulas are verified by digital computer simulation.

Any advantage of DPCM stems from removing the redundancy from the signal to be transmitted. Redundancy in a signal, however, affords a certain protection against noise introduced in the transmission medium. The penalty for removing this redundancy, through DPCM or other means, is that the transmitted signal becomes more fragile and requires a higher-quality transmission medium than would otherwise be required. This penalty is discussed in quantitative terms.

I. INTRODUCTION

In this paper, the terms predictive quantizing and differential pulse code modulation (DPCM) will be used interchangeably. They describe a special kind of predictive communications system. A predictive communications system is one in which the difference between the actual signal and an estimate of the signal, based on its past, is transmitted. Both the transmitter and the receiver make an estimate or prediction of the signal's value based on the previously transmitted signal. The transmitter subtracts this prediction from the true value of the signal and transmits this difference. The receiver adds this prediction to the received difference signal yielding the true signal. Highly redundant signals, such as television, are well suited for predictive transmission systems because of the accuracy possible in the prediction. If the signal is sampled, and if the difference signal is quantized and encoded into PCM, then the system is a predictive quantizing or DPCM system.

A block diagram of systems of this type is shown in Fig. 1. Although delta modulation which uses the feedback principle was introduced somewhat earlier,[1] DPCM systems are based primarily on an invention by Cutler.[2] In his original patent in 1952, Cutler used one or more integrators to perform the prediction function. His invention is based on transmitting the quantized difference between successive sample values rather than the sample values themselves. The invention is a special case of a predictive quantizing system and it turned out to be a special case admirably matched to the statistics of television signals.

In the early nineteen forties Weiner[3] developed the theory of optimum linear prediction. By 1952 Oliver, Kretzmer and Harrison at the Bell Telephone Laboratories, realized the importance of linear prediction in feedback communications systems and proposed that it be used to reduce the redundancy, and, therefore, lower the required power in highly periodic signals such as television. Oliver[4] explained how linear prediction could be used to reduce the bandwidth required to transmit redundant signals. Realizing that knowledge of the statistical properties of television signals was necessary in the design of linear prediction sys-

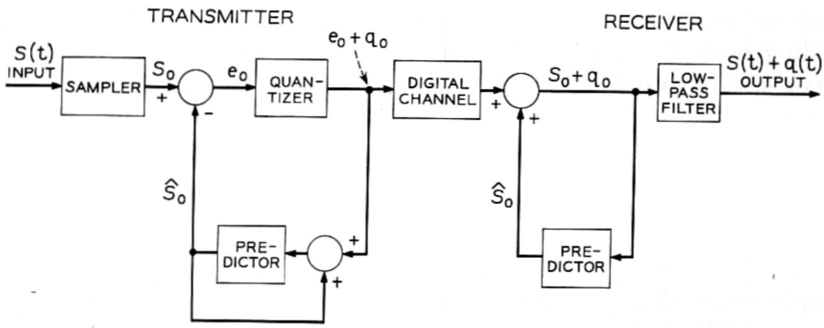

Fig. 1 — Block diagram of a DPCM system.

tems, Kretzmer[5] determined some statistics of typical television picture material. Harrison[6] actually built a signal processing system for television signals and illustrated how redundancy could be removed from these signals using linear prediction. Concurrently with this work at the Bell Telephone Laboratories, but published somewhat later, Elias[7] at MIT was developing this theory of predictive coding which explained the use of linear prediction in PCM systems.

Graham[8] recognized that the theory of prediction could be incorporated into the system described by Cutler. Since Graham's work in 1958, much effort has been expended to devise and build such a system for the transmission of television signals. Although a few experimental systems have been constructed, it is a discouraging fact that such systems have never proved to be very useful for high quality television transmission. Television signals are still being transmitted over transmission systems which do not take advantage of the signals' inherent redundancy.

It is the purpose of this paper to determine the advantages and disadvantages of well-designed DPCM systems. Such information is needed in order to establish whether or not DPCM systems are potentially useful for the transmission of television signals. To do this we present a procedure for the design of some DPCM systems which are near optimum for three television scenes and determine what kind of performance can be expected from such systems. The results obtained are verified by simulating some DPCM systems on an IBM 7094 digital computer and using, as an input, television signals derived from a flying spot scanner. Our study is restricted to nonadaptive systems using linear prediction in the feedback loop and a quantizer whose characteristics do not depend on the instantaneous value of the input signal. Both previous-sample and previous-line feedback in the prediction operation are considered in detail.

II. SUMMARY OF RESULTS AND CONCLUSIONS

Some of the more important results and conclusions about DPCM systems designed for the transmission of television signals are enumerated below. Throughout this paper the term S/N refers to the ratio of signal to quantizing noise.

(*i*) A simple formula for the S/N ratio is derived. If the sync pulses need not be transmitted, then standard PCM is shown to be a special case of DPCM and its S/N ratio is also given by this formula.

(*ii*) When the horizontal resolution is equal to the vertical resolution, line feedback, when used in addition to previous-sample feedback, can give no more than a 1.9-db additional improvement in S/N ratio. For FCC standard monochrome entertainment television the improvement due to line feedback will be considerably less than 1.9 db.

(*iii*) Differential PCM provides more of an advantage for high resolution television systems than for low resolution systems. For monochrome entertainment television, previous-sample feedback DPCM transmission systems can provide a signal-to-quantizing noise ratio approximately 15 db higher than standard PCM. This improvement may easily vary as much as 2 or 3 db depending on picture material. A 2.8-db improvement in S/N ratio can be realized in standard PCM systems if the sync pulses can be reconstructed by the decoder and need not be transmitted. The improvement of previous-sample feedback DPCM over sync-less PCM is, therefore, only about 12 db for monochrome entertainment television. The effect of line feedback has not been included in the above numbers.

(*iv*) Since 6 db of quantizing noise is equivalent to one bit per sample, the advantage in DPCM can also be expressed in terms of bit rate. For a constant signal-to-quantizing noise ratio, a DPCM system designed for entertainment television can provide a saving of about 18 megabits (2 bits per sample) over standard PCM. This assumes a sampling rate of 9 megacycles, which is twice the bandwidth, and that the noise added by bit errors in the transmission medium is negligible. These bit rate reductions are nearly independent of the signal-to-quantizing noise ratios required.

(*v*) A signal encoded into DPCM is more vulnerable to noise in the transmission medium (bit errors) than one encoded into PCM. It is characteristic of DPCM systems that, if they decrease the quantizing noise by k db over standard PCM, then the noise in the decoded signal caused by errors in the digital transmission channel is increased by k db. This penalty means that, if DPCM is used to reduce the quantizing noise by k db, then the error rate in the digital channel required for satisfactory transmission is reduced by a factor of $(1.26)^k$. This does not imply that DPCM offers no advantage. If the limiting degradation is quantizing noise, and this is generally true for digital systems, then decreasing this quantizing noise, even at the expense of increasing the noise introduced in the transmissoin medium, is desirable. Digital transmission lines designed for PCM encoding, however, may be unsatisfactory for DPCM encoding. This result applies to DPCM systems designed for any type of signal.

(*vi*) The power spectrum of the quantizing noise is approximately flat. The amplitude density function of the quantizing noise is found to be somewhat flatter than a Gaussian function.

(*vii*) For television input signals the amplitude density function of the quantizer input in a well-designed DPCM system is approximately Laplacian.

Television picture material which has meaning to a human observer has certain patterns which cause statistical redundancy in the resulting television signals. Differential PCM takes advantage of this statistical redundancy and the performance of DPCM systems varies with this redundancy. Conclusions (*iii*) and (*iv*) above are based on measured statistics of television signals derived from three scenes which have detail typical of television picture material.

III. PERFORMANCE CRITERION

The performance criterion used is the ordinary signal-to-quantizing noise ratio, S/N, present in the video part of the composite signal. Noise present in the sync pulses is seldom a limiting factor in television transmission. While it has often been argued that the S/N ratio is not an adequate performance criterion for television systems, a better alternative for analytical study has never been proposed. Furthermore, when used with discretion, the S/N ratio is a useful measure in determining the performance of television systems. It is especially useful in helping to decide which kinds of systems should be built and evaluated subjectively. The subjective test is the final arbiter in determining the usefulness of DPCM for the transmission of television signals.

Unless otherwise stated, the term noise used in this paper implies quantizing noise. We are concerned here with designing DPCM encoding and decoding systems which minimize the mean square difference between the decoded output signal and the analog input signal. This optimization is based on an analytical, i.e., objective, criterion, not a subjective one. Thus, the S/N ratios used are unweighted. All sampling is assumed to be at twice the bandwidth of the baseband input signal, and all the resulting quantizing noise is considered to be in-band. Systems have been proposed[9] which shape the power spectrum of the quantizing noise to make it less objectionable to the human observer. This approach, however, is complicated by the difficulty in determining the proper weighting function for noise which is not independent of the signal. In most DPCM systems the quantizing noise is highly correlated with the derivative of the signal.

IV. DESIGN PROCEDURE

The design procedure used herein is to first design the predictor ignoring the presence of the quantizer. Then the quantizer is designed to match the amplitude distribution of the signal coming from the subtractor. This procedure will result in a system which is very nearly optimum because when the number of quantizing levels is large, the inclusion of the quantizer in the circuit has very little effect on the amplitude distribution of the signal coming out of the subtractor. The predictor will be restricted to be a linear time invariant device and the theory of linear prediction will be used to optimize it. The quantizer will be designed in accordance with procedures first proposed by Panter and Dite.[10]

V. THE PREDICTOR

It is true that nonlinear prediction is superior, by the S/N ratio criterion, to linear prediction for television signals. It has never been determined, however, just how much the S/N ratio can be improved by using nonlinear prediction techniques. Graham[8] suggested one nonlinear predictor and simulated it on the computer. Fine[11] discusses the general case where both nonlinear prediction and quantization are allowed. In this paper, however, only linear prediction is used.

5.1 *Theory of Linear Prediction*

The following brief explanation of the procedure of linear prediction is based on the terse exposition of this subject given by Papoulis.[12]

Let a stationary signal $S(t)$ with mean 0 and rms value σ be sampled at the times $t_1, t_2, \cdots, t_n, \cdots$ and let the sample values be $S_1, S_2, \cdots, S_n, \cdots$, respectively.

A linear estimate of the next sample value S_0 based on the previous n sample values $S_1, S_2, \cdots, S_n$ is defined to be

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + \cdots + a_n S_n . \tag{1}$$

For simplicity, we assume here that the a's and S's are real numbers. A linear predictive encoder forms this estimate $\hat{S}_0$ and transmits the difference or error

$$e_0 = S_0 - \hat{S}_0 . \tag{2}$$

A block diagram of such a system is shown in Fig. 2. The D's represent delay elements.

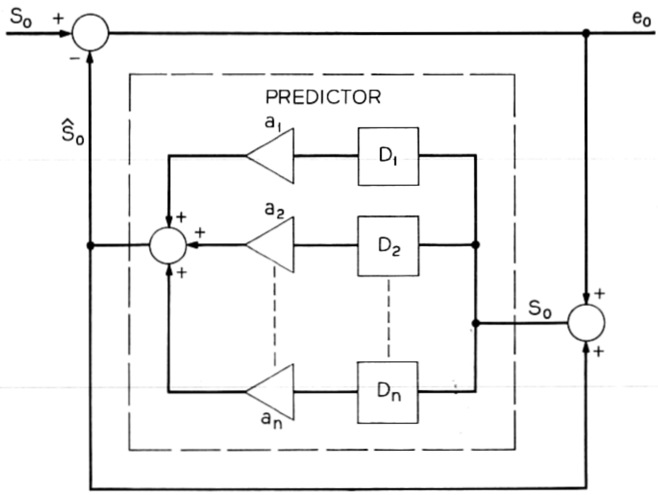

Fig. 2 — Block diagram of a linear predictive encoder.

We define the best estimate of S_0 to be that value of $\hat{S}_0$ for which the expected value of the squared error is minimum. To find the values of the a's which satisfy this condition we first take the partial derivatives of $E[(S_0 - \hat{S}_0)^2]$ with respect to each one of the a's. $E[x]$ denotes the expected value of x.

$$\frac{\delta E[(S_0 - \hat{S}_0)^2]}{\delta a_i} = \frac{\delta E[(S_0 - (a_1 S_1 + a_2 S_2 + \cdots + a_n S_n))^2]}{\delta a_i}$$

$$= -2E[(S_0 - (a_1 S_1 + a_2 S_2 + \cdots + a_n S_n))S_i]$$

$$i = 1, 2, \cdots, n.$$

To find an extremum, in this case a minimum, we set this equal to zero giving

$$\begin{aligned} E[(S_0 - (a_1 S_1 + a_2 S_2 + \cdots + a_n S_n))S_i] &= 0 \\ E[(S_0 - \hat{S}_0)S_i] &= 0 \qquad i = 1, 2, \cdots, n. \end{aligned} \tag{3}$$

If we represent the covariance of S_i and S_j by

$$R_{ij} = E[S_i S_j], \tag{4}$$

then from (3) we can rewrite the conditions for the best linear mean square estimate as

$$R_{0i} = a_1 R_{1i} + a_2 R_{2i} + \cdots + a_n R_{ni} \qquad i = 1, 2, \cdots, n. \tag{5}$$

Equation (5) defines a set of n simultaneous linear equations in the n unknowns a_i, $i = 1, 2, \cdots, n$, which can be found if the covariances R_{ij} are known. These covariances are found from the autocovariance $\psi(\tau)$ of the signal itself,

$$R_{ij} = \psi(t_i - t_j). \tag{6}$$

If $\hat{S}_0$ is the best linear mean square estimate of S_0, then the expected value of the square of the error signal e_0 is

$$\sigma_e^2 = E[(S_0 - \hat{S}_0)^2] = E[(S_0 - \hat{S}_0)S_0]$$

$$\sigma_e^2 = R_{00} - (a_1R_{01} + a_2R_{02} \cdots + a_nR_{0n}). \tag{7}$$

In (7), R_{00} is simply the variance σ^2 of the original sequence S_0, S_1, $\cdots = \{S_i\}$.

The sequence of transmitted error samples is $e_0, e_1, \cdots = \{e_i\}$ where

$$e_i = S_i - \hat{S}_i \qquad i = 0,1, \cdots, \tag{8}$$

and

$$\hat{S}_i = a_1S_{i+1} + a_2S_{i+2} + \cdots + a_nS_{i+n}.$$

The error sequence $\{e_i\}$ is less correlated and has smaller variance than the signal sequence $\{S_i\}$. The use of linear prediction has produced a sequence $\{e_i\}$ from which the sequence $\{S_i\}$ can be reconstructed. The variance σ_e^2 of the error sequence $\{e_i\}$ is less than the variance of the original sequence $\{S_i\}$ by the amount shown in the parenthesis in (7). If the number of samples n used in forming the estimate is unlimited, then the sequence of error samples can always be made completely uncorrelated. If the sequence of samples $S_0, S_1, \cdots = \{S_i\}$ is an rth order Markoff sequence, then only r samples need be used in forming the best estimate of S_0 and the resulting sequence of error samples will be uncorrelated.

As an example of particular relevance to television, consider the 1st order Markoff sequence formed by sampling a signal whose autocorrelation is the exponential function $e^{-\alpha t}$. In this case, even if all previous sample values are available, the estimate of S_0 which minimizes σ_e^2 is $\hat{S}_0 = (R_{01}/\sigma^2)S_1$ where S_1 is the most recent sample value available. It is easy to show that, in this case, the error sequence $\{e_i\}$ is completely uncorrelated, i.e.,

$$\begin{aligned} E[e_ie_j] &= 0 \qquad i \neq j \\ &= \sigma_e^2 \qquad i = j. \end{aligned}$$

The autocorrelation function of one line of a television signal is very similar to $e^{-\alpha t}$ so in this case we expect that basing our estimate only on the previous sample value will be almost as good as using many sample values on the same line. It will be shown, however, that, if we have access to sample values on the adjacent line and/or on the previous frame, we can improve our prediction.

5.2 *Application to Television Signals*

The samples $S_1, S_2, \cdots, S_n$ used in (1) to form the estimate $\hat{S}_0$ need not be the most recently transmitted ones and they need not be in any particular order. They are simply n sample values which have been transmitted in the past. Fig. 3 illustrates 7 sample values which can be used to form a reasonably good estimate of S_0. Such an estimate would be

$$\hat{S}_0 = a_1S_1 + a_2S_2 + a_3S_3 + a_4S_4 + a_5S_5 + a_6S_6 + a_7S_7, \quad (9)$$

where the a's are chosen to satisfy (5). Also shown in the figure, are covariances between these samples.

It will be shown that there is little advantage in using samples S_3 through S_7 for, once S_1 and S_2 are used in the prediction, the other five

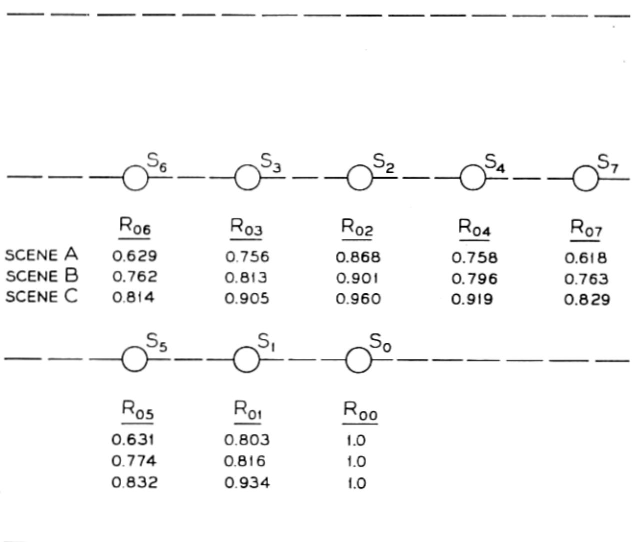

Fig. 3 — Television scan showing sample values near S_0. Covariances for the three scenes in Fig. 6 are also shown.

samples contain little additional information about S_0. Most DPCM systems built in the past use only the previous sample S_1 and form the estimate

$$\hat{S}_0 = a_1 S_1 .$$

In this simple case, it is clear from (5) that the constant a_1 should be R_{01}/σ^2, the covariance between adjacent sample points divided by the mean square value of the input sequence. DPCM systems of this type are called previous-sample feedback systems, and a block diagram of the predictor used in such a system is shown in Fig. 4.

A DPCM system which forms its estimate of S_0 by using the previous sample S_1 and the adjacent sample on the previous line S_2 will be called a line-and-sample feedback system. In this case,

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 . \tag{11}$$

A block diagram of the predictor for this system is shown in Fig. 5.

This concept can easily be extended to take advantage of frame-to-frame correlation. A frame-line-and-sample feedback system would form its estimate of the next sample value by

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + a_f S_f \tag{12}$$

where S_f is the sample value which is equivalent to S_0 but on the previous frame. Frame feedback systems are not considered in detail in this paper primarily because statistics of frame-to-frame correlations are not available.

5.3 *Statistics of Television Signals*

In order to determine some statistics of television signals and to use television signals as inputs to DPCM systems simulated on the IBM 7094 digital computer, some television signals were obtained from a slow-speed flying-spot scanner.[13] These signals were sampled and encoded into 11 bit PCM and placed on a magnetic tape suitable as an

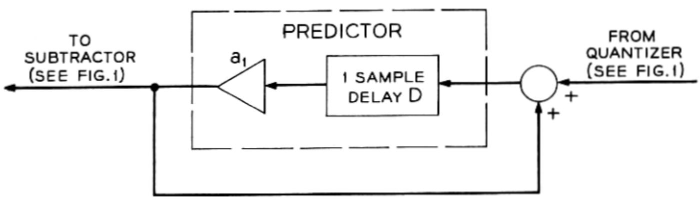

Fig. 4 — Previous-sample predictor.

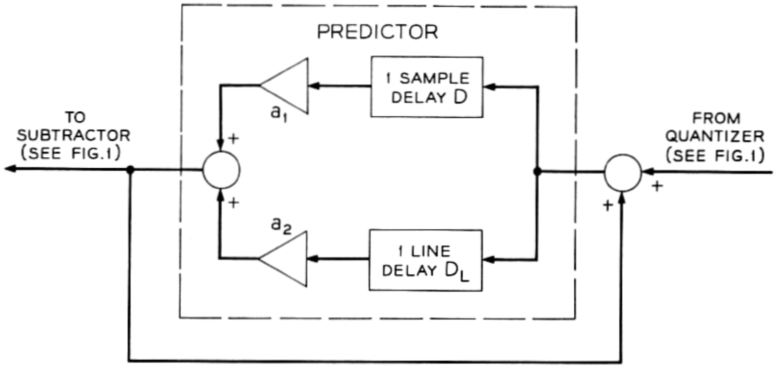

Fig. 5 — Previous line-and-sample predictor.

input to the computer. The signals were obtained by scanning the three square slides shown in Fig. 6 and represent only one frame of a television signal. In conformity with television practices, the video signal was a function of the 0.4 power of the brightness of the original scene. The standards used gave 100 lines and 100 samples per line for the visible part of the pictures and all samples taken were on a symmetric lattice or grid.

The signals on magnetic tape were composite signals, i.e., they consisted of the video signal and a train of sync pulses. Noise and distortions in the sync pulses do not govern the quality of a television signal as long as synchronization is maintained. Therefore, DPCM systems should be matched to the statistics of the video part alone. For this reason, the horizontal sync pulses were ignored and the autocovariance functions of the video part of the signals were obtained. For convenience, the signals were first normalized so that the rms value σ of the video was 1 and its mean value was 0. The autocovariance functions $\psi(\tau)$ are shown in Fig. 7. For small values of τ, these functions are very similar to exponential functions. Since we are dealing with sample values rather than with continuous signals, the autocovariance is actually a set of points at integer values of the lag τ and these points represent the values of R_{0i}, $i = 0, 1, \cdots$. Fig. 7 was constructed by finding these points and drawing lines between them. This is also true of Figs. 8 and 16. The peaks at $\tau = 100$ are due, of course, to the high correlation between adjacent lines of the television signals. Correlations between adjacent frames are not illustrated in the figure because the signals used represented only one frame of a television signal.

Fig. 3 illustrates some of the covariances between neighboring points

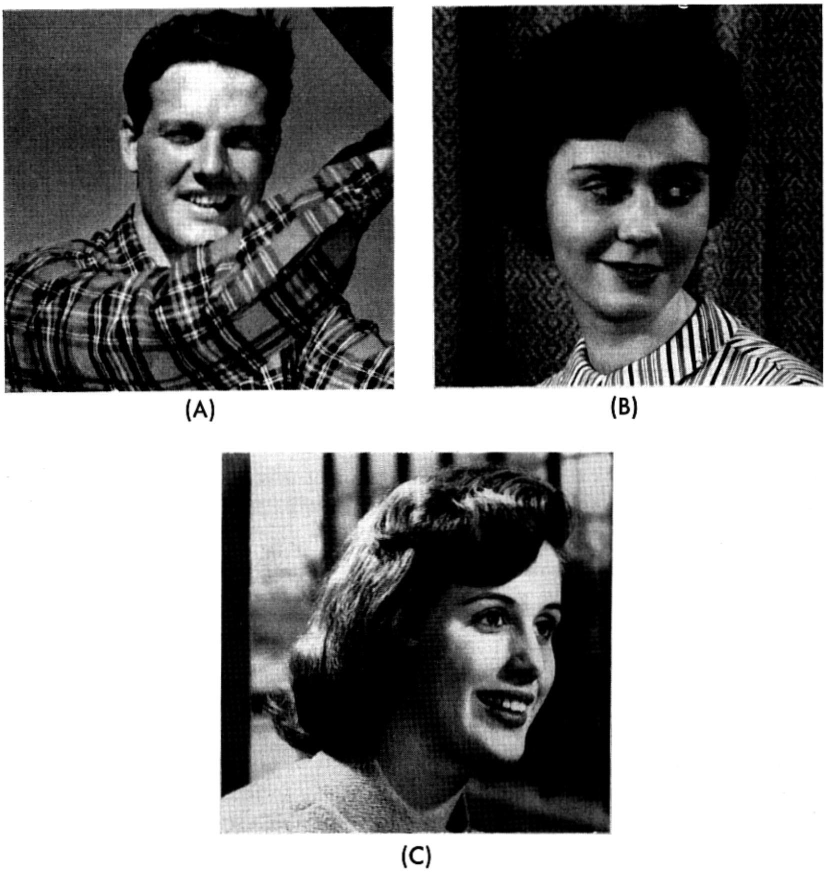

Fig. 6 — Pictures of three slides scanned to obtain television signals.

for the three scenes. For example, the covariance between points S_0 and S_4 in scene B is $R_{04} = 0.796$. The three pictures used had higher vertical than horizontal correlation.

A transmission system is useful only if it can satisfactorily transmit a vast ensemble of signals and its performance must be judged on the basis of its ability to transmit almost all members of this ensemble. The statistics we use here have been obtained from only three members of this ensemble and, since the members of this ensemble are derived from a nonergodic process, we cannot obtain the statistics of the ensemble by examination of these three members. Nevertheless, it is useful to determine the design and performance of DPCM systems when used to transmit these members which, in some sense at least, are representative of the whole ensemble.

The autocovariance functions in Fig. 7 are averages over the time for each of the three signals used. The autocovariance function of the random process from which these three signals could be derived could not be determined here. Franks,[14] however, has proposed a model for this random process in which the autocovariance function of the picture material is exponential in both the horizontal and vertical directions. Data obtained in this study, some of which is illustrated in Fig. 7, indicates that this is a good approximation for the three scenes used here.

5.4 *Linear Predictors*

Using the data in Fig. 3, we can solve (5) and (7) for the a_i and σ_e for several practical linear predictors. Table I illustrates the optimum values of the a's and the resulting mean square error signals for 8 different predictors. The relative positions of the sample values in this table are those of Fig. 3. For example, if the prediction of S_0 is based on the three sample values S_1, S_2, and S_4 (predictor number 6) then the linear

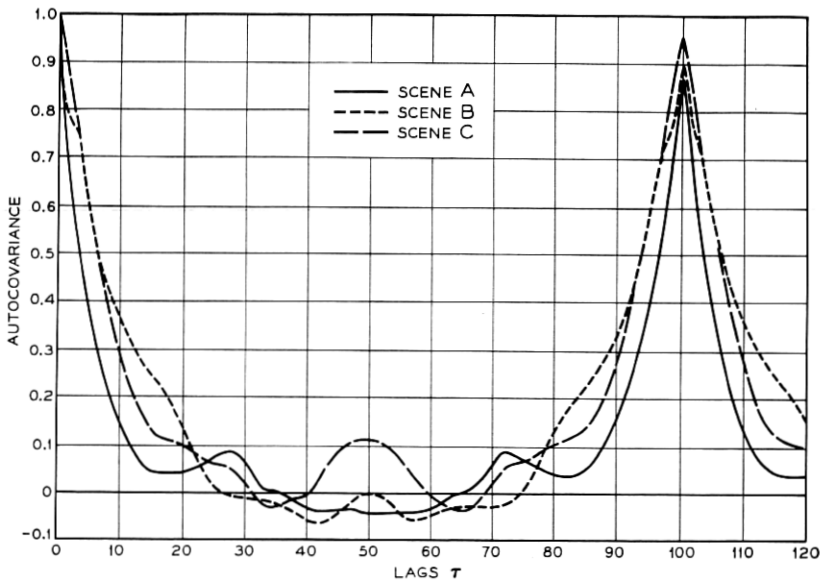

Fig. 7 — Autocovariances of the 100 line, 100 samples per line video signals obtained from the three scenes of Fig. 6. (The sync pulses were not included in computing these functions.)

predictor giving the smallest value of σ_e for scene C forms $\hat{S}_0$ by the equation

$$\hat{S}_0 = 0.383\ S_1 + 0.362\ S_2 + 0.263\ S_4 .$$

When no quantization is present, such a system results in an error signal whose rms value σ_e is 0.230. This is 12.8 db below the rms value of the signal itself whose rms value σ is 1. The transmitted error sequence will be much less correlated than the original signal sequence. We may think of this signal processing as the removal of redundancy, in this case, 12.8 db of redundancy.

Examination of Table I reveals that once samples S_1 and S_2 have been used in the prediction, there is little advantage in using any others. This means that samples S_1 and S_2 provide almost all of the information about sample S_0 which can be obtained from the previous samples. Samples S_3 and S_4 contain almost no additional information. For a system with line feedback (a system which can store and, therefore, has access to the previous line), there is little point in using any samples but S_1 and S_2. Similarly, for a system without line feedback there is little point in using any sample but S_1 to predict S_0. Furthermore, for the pictures tested, line feedback itself provides only about a 3-db improvement in the estimate of S_0. This is somewhat disappointing especially in view of the fact that the scenes tested had higher vertical than horizontal correlation. Exactly what can be obtained from frame feedback must await the availability of frame-to-frame covariance statistics. Although the above conclusions about line feedback are based on statistics obtained from some 100 line and 100 samples per line pictures it will be shown in section VIII that they apply to television systems in general.

This study suggests that the sequence of sample values derived from one frame of a television signal (or a facsimile signal) may be approximated well by a second-order Markoff sequence.* Furthermore, studies by Deriugin[15] indicate that the sequence derived from many frames of a typical television signal may be, approximately, a third-order Markoff sequence.* In this case, the state (value) of the next sample S_0 may be statistically dependent only on S_1, S_2 and S_F, where these are the sample values adjacent to S_0 on the same line, the previous line, and the previous frame, respectively. More work is required to determine just how well Markoff sequences can represent sample values of television signals.

* This might more properly be called a distant second (or third) order Markoff sequence because, although S_1 is the previous sample value, there are many intervening samples between S_1, S_2, and S_F.

5.5 *Computer Simulation of Predictors*

In order to determine how effectively redundancy could be removed from a television signal by using prediction, the predictors number 1, 3, and 8 shown in Table I were simulated on the computer for all three scenes. The actual rms value σ_e of the errors in the prediction agreed well with those shown in Table I. The autocovariances of the error signals were found and for scene C they are illustrated in Fig. 8. The autocovariance functions shown in Fig. 8 are also representative of what was found for scenes A and B.

Figs. 9 and 10 show the amplitude distribution of the error sequence

TABLE I — VALUES OF THE AMPLIFIER GAINS AND RMS PREDICTION ERROR FOR 8 PREDICTORS MATCHED TO THE 3 PICTURES OF FIG. 6

Predictor Number	Samples Used in Prediction (see Fig. 3)	Scene	Theoretical* rms Prediction Error		a_1	a_2	a_3	a_4	a_5
			σ_e	$-20 \log \sigma_e$					
1	S_1 (see Fig. 4)	A	0.597	4.5	0.803				
		B	0.578	4.8	0.816				
		C	0.358	8.9	0.934				
2	S_2	A	0.498	6.1		0.868			
		B	0.434	7.2		0.901			
		C	0.279	10.1		0.960			
3	S_1, S_2 (see Fig. 5)	A	0.444	7.0	0.341	0.610			
		B	0.402	7.9	0.270	0.686			
		C	0.247	12.1	0.333	0.654			
4	S_1, S_5	A	0.595	4.5	0.834				−0.039
		B	0.547	5.2	0.552				0.324
		C	0.339	9.4	1.229				−0.316
5	S_1, S_4	A	0.494	6.1	0.541			0.423	
		B	0.512	5.8	0.499			0.415	
		C	0.246	12.2	0.550			0.463	
6	S_1, S_2, S_4	A	0.443	7.1	0.337	0.481		0.163	
		B	0.398	8.0	0.238	0.629		0.101	
		C	0.230	12.8	0.383	0.362		0.263	
7	S_1, S_2, S_3	A	0.439	7.2	0.432	0.660	−0.149		
		B	0.401	7.9	0.227	0.670	0.062		
		C	0.224	13.0	0.606	0.793	−0.417		
8	S_1, S_2, S_3, S_4	A	0.429	7.3	0.419	0.533	−0.134	0.155	
		B	0.398	8.0	0.210	0.620	0.047	0.097	
		C	0.214	13.4	0.598	0.544	−0.346	0.203	

* The rms value σ of the input signal is 1. This table is concerned only with prediction error and does not consider the eff ects of quantization.

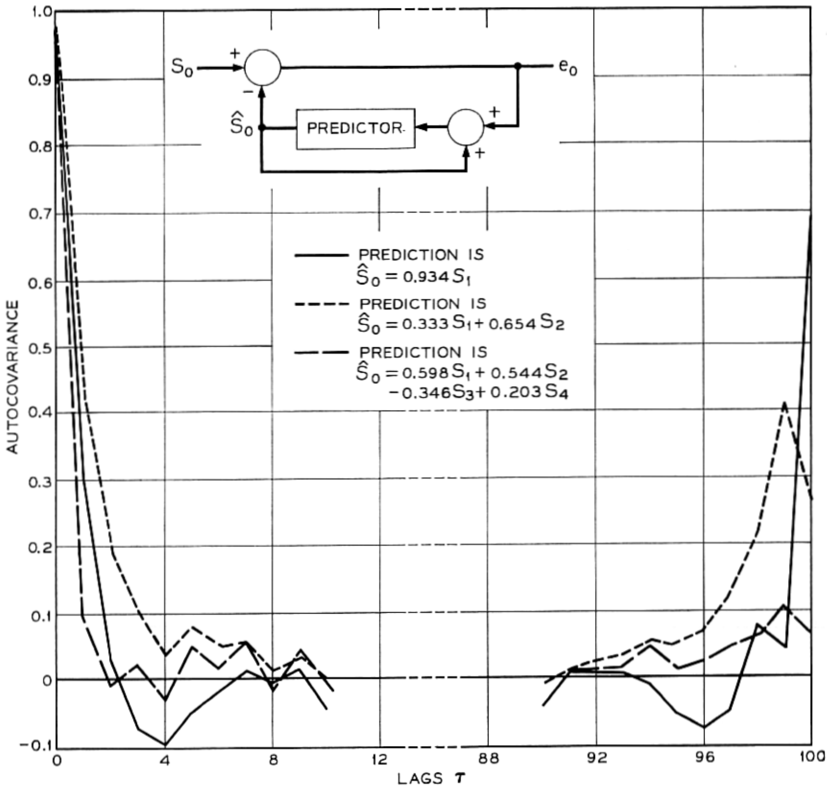

Fig. 8 — Autocovariance of error sequence $\{e_i\}$ of scene C for three linear predictors matched to this scene. (The samples S_1, S_2, S_3, and S_4 are defined in Fig. 3.)

for the three pictures using previous-sample prediction (predictor number 1 in Table I), and line-and-sample prediction (predictor number 3 in Table I), respectively. The shape of these density functions is of foremost importance in designing an optimum quantizer. In both figures the density functions can be approximated reasonably well by Laplacian functions. These amplitude density functions were found by dividing the range $\pm 4\sigma$ into 25 equal intervals and finding the number of sample values in each interval. The points so found were normalized and curves drawn between them.

VI. THE QUANTIZER

In analog systems, it is difficult to evaluate the wisdom in reducing the power by removing the redundancy from a signal. For this process

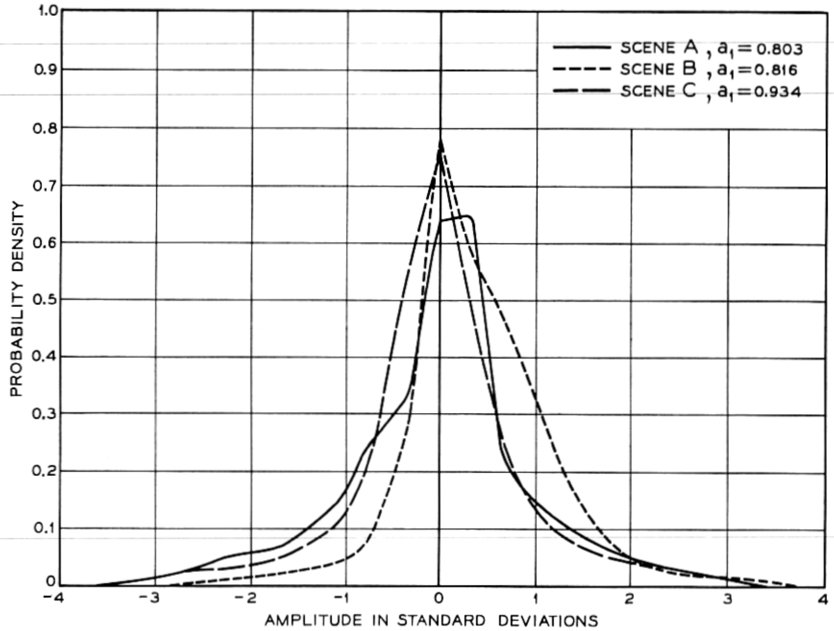

Fig. 9 — Probability density for amplitude of error sequence $\{e_i\}$ for scenes A, B, and C using previous-sample prediction, $\hat{S}_0 = a_1 s_1$. (The value of a_1 is chosen to match each scene.)

automatically makes a signal more susceptible to noise in the transmission medium. While this is still true in digital systems, as will be shown later, we are assured both by logic[16] and by experience[17] that the errors in properly designed digital systems can be made small enough to be neglected. And, if we can ignore this transmission noise, i.e., assume that the probability of error in a digital system can be made as small as we like, there is a dividend in reducing the rms value of the signal to be transmitted. In fact, it will be shown that (for the signals used here, at least) reducing the rms value of the transmitted signal from σ to σ_e decreases the rms value of the quantizing noise by a factor σ/σ_e.

If the input to the quantizer in Fig. 1 is e_0, then its output is $e_0 + q_0$ where q_0 is the quantizing noise. Since the receiver forms the decoded output by adding $e_0 + q_0$ to the estimate $\hat{S}_0$, the quantizing noise in the decoded signal is also q_0. Minimizing the quantizing noise in the decoded output, therefore, is equivalent to minimizing the rms value of the quantizing noise coming out of the quantizer. This method of minimizing the quantizing noise was recognized independently by Nitadori.[18]

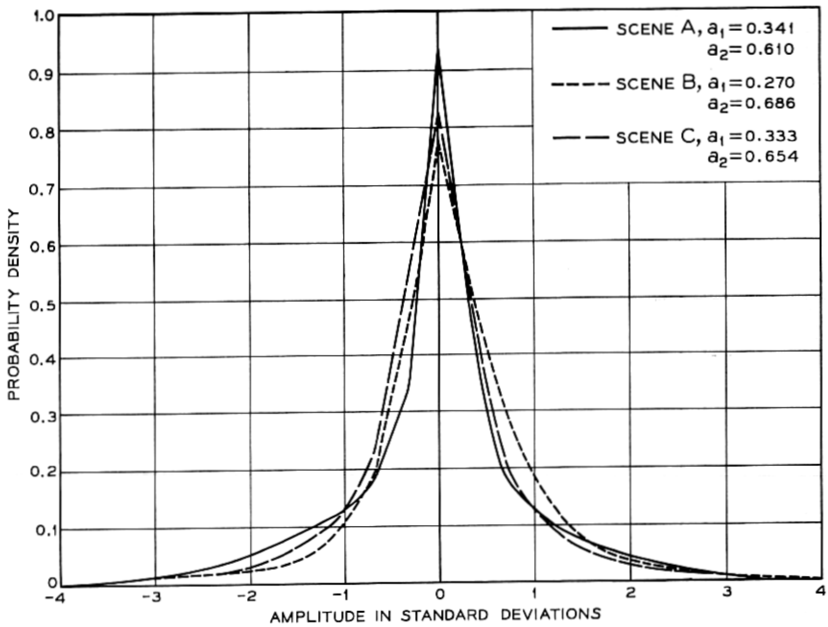

Fig. 10 — Probability density for amplitude of error sequence $\{e_i\}$ for scenes A, B, and C using previous line-and-sample prediction, $\hat{S}_0 = a_1 s_1 + a_2 s_2$. (The values of a_1 and a_2 are chosen to match each scene.)

In what follows, approximations are made which apply when the number of quantizing levels $N = 2^n$ is large. Figs. 12 and 13, to be discussed later, illustrate that the S/N formulas for DPCM systems are accurate for small N as well. It is possible, nevertheless, that inaccuracies may occur under certain conditions when N is too small. If N is less than about 8, the formulas and design procedures presented here should be used with caution.

6.1 *Optimum Quantization*

For n bit quantization, each member of the error sequence is made to assume one of $N = 2^n$ different levels. It has long been known that nonuniform quantization is generally preferable to uniform quantization in DPCM systems. Panter and Dite[10] have shown that the minimum mean square quantizing error is given by

$$\sigma_q^2 = \frac{2}{3N^2}\left[\int_0^V P^{\frac{1}{3}}(e)\, de\right]^3, \tag{13}$$

where $P(e)$ is an even function representing the probability density of the input to the quantizer and $P(e)$ is zero outside the interval $(-V,V)$ which represents the range of the quantizer input.

The curves for $P(e)$ shown in Figs. 9 and 10 may be approximated reasonably well by the Laplacian density function

$$P(e) = \frac{1}{\sqrt{2}\,\sigma_e} \exp\left(-\frac{\sqrt{2}}{\sigma_e} \mid e \mid\right), \tag{14}$$

where σ_e is the rms value of the quantizer input. Since the amplitude density function is different for each scene to be transmitted, the best we can do is to choose some representative density function and match the quantizer to it. We choose this function to be the exponential of (14) and we feel that this will give results which can be expected in practice. Although Figs. 9 and 10 are plots of the error signal without a quantizer in the circuit, computer simulations with the quantizer in the circuit showed that these amplitude density functions are effected very little by the addition of the quantizer as long as the number of levels N was greater than 4. Solving the integral in (13) for this $P(e)$ and taking the limit as V gets large gives, as an approximation for the mean square value of the quantizing noise,

$$\sigma_q^2 = \frac{9}{2N^2}\,\sigma_e^2. \tag{15}$$

Refining this approximation by using the actual value of V changes it very little since, for cases of interest in DPCM, V, which is the peak-to-peak value of the input signal $S(t)$, is always large compared to σ_e. For the three scenes considered here V is about 7 times the rms value σ of the signal $S(t)$, and σ_e is generally much less than σ. For small values of V the density function in (14) must be truncated. This changes and complicates the value of σ_q^2 given in (15).

From (15) we see that if the rms value in the input video signal is σ then the rms S/N ratio in the video (considering only quantizing noise) of a decoded television signal transmitted through a well-designed n digit differential PCM system is (in db)

$$S/N = 10 \log \frac{2N^2\sigma^2}{9\sigma_e^2}$$

$$S/N \cong -6.5 + 6n + 10 \log \frac{\sigma^2}{\sigma_e^2}. \tag{16}$$

Equation (16) gives the ratio of the rms video signal to rms quantizing noise in the video. A bound on the S/N ratio to be presented later differs from (16) only by a constant and suggests that this S/N ratio is within about 5 db of that possible for any encoding system. To convert this S/N to the more useful measure, peak-to-peak composite signal to rms noise in the video, we must add a constant to (16) giving (in db)

$$\mathrm{S/N} \cong -6.5 + C + 6n + 10 \log \frac{\sigma^2}{\sigma_e^2},$$

where C is the ratio in db of peak-to-peak composite signal to the rms value of the video. The value of C is determined by the peak value of the sync pulses. It is also dependent on picture material and upon such apparently extraneous factors as the man or electronic device which regulates the peak values of the video signal. For FCC standard monochrome entertainment television some measurements, as well as some data derived from the flying spot scanner used for these studies, indicate that the rms value of the video is about one tenth the peak value of the composite signal and the value of C is, therefore, about 20 db.* Actual measurements of the signals derived from scenes A, B, and C of Fig. 6 give 20.0, 19.8, and 18.1 db, respectively, for the value of C (assuming FCC sync standards). An approximation, then, for the ratio of peak-to-peak composite signal to rms quantizing noise in the video for a typical FCC standard television signal is (in db)

$$\mathrm{S/N} \cong 13.5 + 6n + 10 \log \frac{\sigma^2}{\sigma_e^2}. \tag{17}$$

Bennett[20] showed that if the input signal is distributed evenly between the quantizing levels, the rms value of the quantizing noise for standard PCM is $E_0/\sqrt{12}$. E_0 is the step size of the uniform quantizer and $E_0 = V_{\text{peak}}/2^n$, where V_{peak} is the peak value of the signal to be encoded and n is the number of quantizing digits. Therefore, the peak-to-peak composite signal to rms noise ratio for standard PCM is (in db)

$$\begin{aligned} \mathrm{S/N} &= 20 \log \sqrt{12} + 20 \log 2^n \\ &\cong 10.8 + 6n. \end{aligned} \tag{18}$$

If the sync pulses could be reconstructed by the decoder, then all the PCM levels could be used for the video and the constant in (18) would be $20 \log (\sqrt{12}/0.072) = 13.6$. In other words, if the sync pulses need not

* Some unpublished studies by J. W. Smith indicate that for systems which have automatic regulation of the peak signal excursions the constant C may be several db less than this. Such systems, in attempting to determine peak white and peak black, introduce a certain amount of clipping.

be transmitted then the ratio of peak-to-peak composite signal to rms noise in the video for standard PCM becomes (in db)

$$S/N \cong 13.6 + 6n. \qquad (19)$$

Transmitting the sync lowers the S/N by 2.8 db for PCM. As one might expect, provided we neglect the sync pulses, the S/N ratios for standard PCM and differential PCM can be approximated by the same expression, namely (17). Since the constant in (17) is somewhat arbitrary, it would be easy to justify making it 13.6 to agree with (19). In standard PCM, there is no feedback loop and the estimate of the sample value S_0 based on previous sample values is simply 0, the mean value of the input sequence. In this trivial case, since $\sigma_e = \sigma$, the DPCM system becomes identical to standard PCM and (17) reduces approximately to (19). Therefore, we may consider standard PCM to be a special case of DPCM which is optimum when all the covariances R_{ij} for $i \neq j$, are zero.

When the feedback loop exists and when the amplifier gain(s) are reasonably large, then the DPCM system can adequately encode the sync pulses as well as the video. However, when the amplifier gain(s) are too small, or when the feedback loop is not provided at all, as in standard PCM, then either the decoder must be arranged to reconstruct the sync pulses, or the range of the quantizer must be increased beyond what is required for the video in order to accommodate the sync pulses.

From (5) and (7), we can express σ_e in terms of the covariances R_{ij}. For the simplest case, the previous-sample feedback system, the peak-to-peak composite signal to rms quantizing noise in the video S/N ratio can be expressed as (in db)

$$S/N \cong -6.5 + C + 6n + 10 \log \frac{\sigma^2}{\sigma^2 - R_{01}^2/\sigma^2}. \qquad (20)$$

This equation illustrates that when R_{01}/σ^2 is close to 1, doubling the bandwidth and the sampling rate (this doubles the horizontal resolution), which is roughly equivalent to halving the value of $\sigma^2 - R_{01}/\sigma^2$, increases the S/N ratio by about 3 db.

6.2 *Design of the Quantizer*

One way to obtain the proper quantizer levels for minimizing the rms quantizing noise is to form a function $y(z)$ such that when z takes on uniformly spaced levels between $-V$ and V, y assumes the proper quantizing levels. Smith[21] shows that, when the probability density of the signal to be quantized is that of (14), the function $y(z)$ is given by

$$y(z) = -\frac{V}{m} \ln \left[1 - \frac{z}{V} (1 - \exp(-m)) \right], \qquad 0 \leqq z \qquad (21)^*$$
$$y(-z) = y(z),$$

where

$$m = \sqrt{2}\, V/3\sigma_e .$$

There are more elegant and exact ways for finding the quantizing levels,[22,23] but it is doubtful if they can be incorporated into practical systems. Furthermore, it is unlikely that these more sophisticated techniques offer a significant decrease in the quantizing noise over what can be obtained by the simple quantizers described here.

Smith studied quantizers with the characteristic of (21) in some detail for the application to standard PCM systems for speech. His rejection of this characteristic in favor of another results primarily from the wide variation of talker volumes present in speech channels. This objection does not apply to television channels whose signal levels are relatively constant.

A typical 8 level quantizer designed by using the characteristic of (21) is shown in Fig. 11. The case shown is for $V = 7$ and $m = 5.5$. The output signal always assumes the quantizing level nearest to the input signal. Overload noise, which occurs when the signal to be quantized is outside the range of the quantizer (± 2.61 in Fig. 11), is a part of the quantizing noise which is minimized here. It must be considered separately only when the range of the quantizer is so small that overload causes a significant alteration in the probability density function of (14).

VII. COMPUTER SIMULATION OF DPCM SYSTEMS

The results of computer simulations verify that systems designed by the procedures presented here do function as predicted.

By applying the principles outlined herein, the parameters for some DPCM systems were determined and these systems were simulated on the IBM 7094 digital computer The input signals used were the 100 line, 100 samples per line television pictures obtained from the scenes of Fig. 6. Section 5.3 contains a description of how these signals were obtained. The results of the simulation are shown in Figs. 12 and 13. The S/N ratios in these figures are ratios of rms video signal to rms

* This is the inversion of (A-6) of Ref. 21.

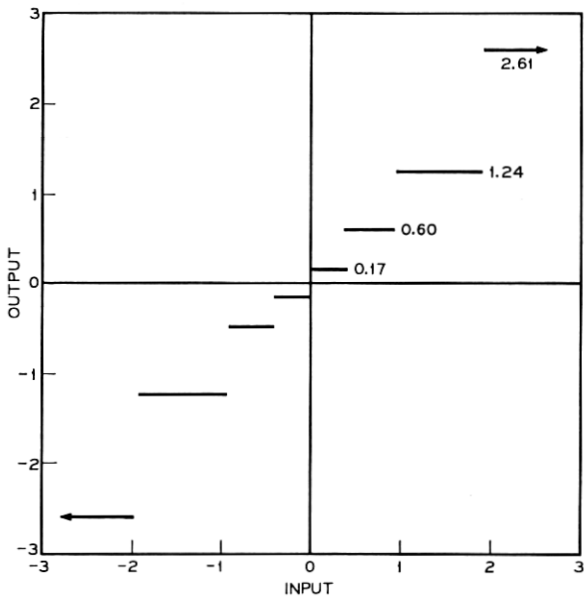

Fig. 11 — Typical 8-level exponential quantizer characteristic obtained from (21). (Case shown is for $m = 5.5$ and $V = 7$.)

noise in the video. To get the ratio of peak-to-peak composite signal to rms noise in the video from these curves, we must add C, the ratio in db of peak-to-peak composite signal to rms video. For scenes A, B, and C this value of C is 20.0, 19.8, and 18.1, respectively (assuming FCC sync standards). Also plotted in these figures are curves of S/N ratio which were predicted for these systems using (16). Fig. 12 gives the results for previous-sample feedback systems (predictor number 1 in Table I), and Fig. 13 presents results for line-and-sample feedback systems (predictor number 3 in Table I). Both figures illustrate the performance of systems whose predictors are tailored to the incoming signal. Table I illustrates that the use of more complicated predictive systems, using samples S_3, S_4, and S_5 in addition to S_1 and S_2, does not significantly lower the rms error in the prediction. Furthermore, the optimum designs of predictors 6, 7, and 8, which give only a slight decrease in the rms error, are radically different for scenes A, B, and C. A system using predictor 6, 7, or 8 designed to give good performance for scene B is likely to give poor performance for scenes A and C. This is not the case with predictors 1 and 3 which were simulated. To verify this, previous-sample feedback DPCM systems were simulated for 4, 5,

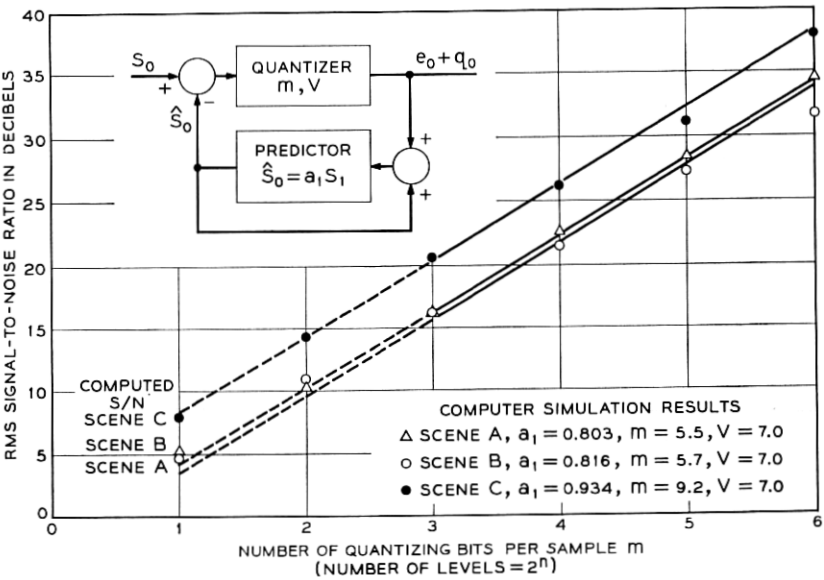

Fig. 12 — Ratio of rms video to rms quantizing noise in the video for systems matched to each scene using previous-sample feedback DPCM. (Theoretical results from (16) [straight lines] are compared with results of computer simulation.)

and 6-bit encoding with $a_1 = 0.815$, $m = 6$ and $V = 7$. The signals from all three scenes were used as inputs and the results were almost identical to those shown in Fig. 12. Similarly, line-and-sample feedback DPCM systems were simulated for 4, 5, and 6-bit encoding with $a_1 = 0.315$, $a_2 = 0.650$, $m = 8$ and $V = 7$, and the three signals all produced S/N ratios essentially the same as those in Fig. 13. The parameters in these two DPCM systems are not critical and need not be exactly matched to the picture material from which the incoming signals were obtained.

VIII. THE MARGINAL UTILITY OF LINE FEEDBACK

For the 100 by 100 matrix pictures used in this study, the use of line feedback increased the S/N ratio by only about 2 or 3 db. This increase is small because the sample values S_1 and S_2 contain substantially the same information about S_0, the sample value to be predicted. And, once S_1 has been used in the prediction, there is only a 2 or 3 db advantage in simultaneously using S_2 in the prediction.

To illustrate this point, consider a scene whose contours of equal

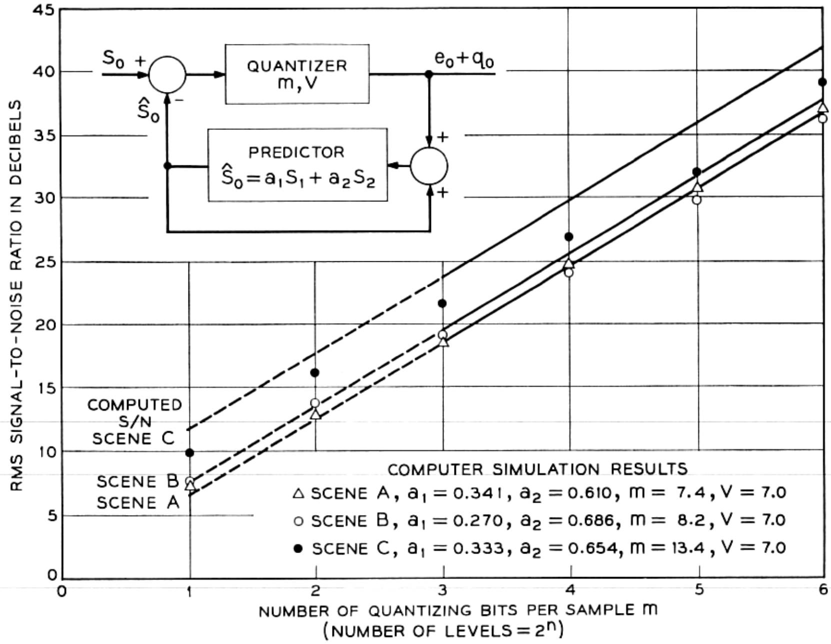

Fig. 13 — Ratio of rms video to rms quantizing noise in the video for systems matched to each scene using line-and-sample feedback DPCM. (Theoretical results from (16) [straight lines] are compared with results of computer simulation.)

autocovariance are circles. Further assume that the autocovariance between any two points, S_i and S_j, separated by a distance D can be expressed as $R_{ij} = \sigma^2 e^{-\alpha D}$. Both of these assumptions are reasonable ones for television picture material. If S_1 (the previous sample) and S_2 (the adjacent sample on the previous line) are equidistant from S_0, then $R_{01} = R_{02}$ and $R_{21} = \sigma^2(R_{01}/\sigma^2)^{\sqrt{2}}$. From (5) the values of the coefficients a_1 and a_2 are

$$a_1 = a_2 = \frac{R_{01}}{\sigma^2 + \sigma^2(R_{01}/\sigma^2)^{\sqrt{2}}} \tag{22}$$

$$\sigma_e^2 = \sigma^2\left[1 - \frac{2(R_{01}/\sigma^2)^2}{1 + (R_{01}/\sigma^2)^{\sqrt{2}}}\right]. \tag{23}$$

Compare this with the mean square value of the prediction error when only S_1 is used in the prediction

$$\sigma_e^2 = \sigma^2 - R_{01}^2/\sigma^2. \tag{24}$$

In Fig. 14, the advantage to be gained in a DPCM system by pro-

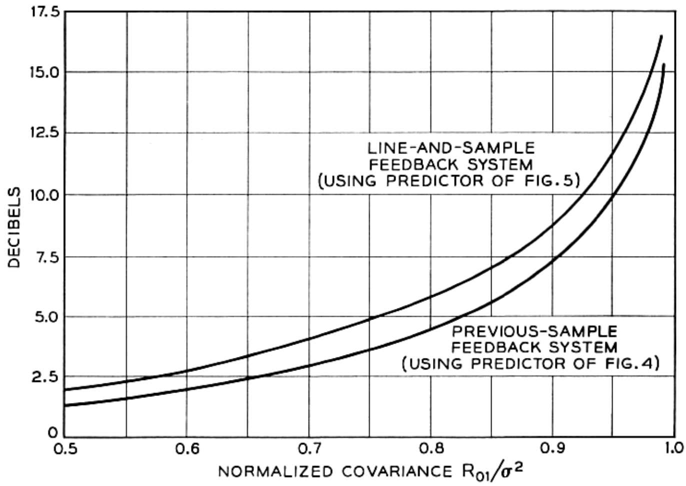

Fig. 14 — Comparison of line-and-sample feedback system and previous-sample feedback system when $R_{01} = R_{02}$. (To get actual peak composite signal to rms noise ratios in an n-digit television system, add $13.5 + 6n$ db.)

viding a line-and-sample feedback predictor is compared with that of a simple previous-sample predictor. This figure applies to sequentially scanned television systems in which the covariance between adjacent samples on the same line is equal to the covariance between neighboring samples on adjacent lines, i.e., $R_{01} = R_{02}$. In television systems using interlace, the S/N ratio improvement provided by using line feedback in addition to sample feedback will be even less. The two curves in this figure are simply plots of $10 \log \sigma^2/\sigma_e^2$ where the value of σ_e^2 is given by (23) for the line-and-sample feedback system, and by (24) for the previous-sample feedback system. From (17) we see that the term $\log \sigma^2/\sigma_e^2$ represents the S/N improvement to be expected from the feedback loop. The two curves in Fig. 14 then show the value of the feedback loops for the two predictors of interest. The distance between the two curves is the improvement provided by the line-and-sample feedback system over the previous-sample feedback system. It can be shown that the maximum value of this improvement approaches about 1.9 db and this occurs as R_{01}/σ^2 approaches 1. In other words, in television signals whose samples have the same covariance in the horizontal direction as in the vertical direction, a line-and-sample feedback system can provide, at best, only 1.9-db improvement in S/N ratio over a simple previous-sample feedback system.

For the scenes used in this simulation, the line feedback loop provided S/N ratio improvements between 2 and 3 db. This was more than the

1.9 db maximum because the scenes used had higher vertical than horizontal correlation. When line-and-sample feedback DPCM is used, sequentially scanning a scene can provide as much as, but no more than, 3-db improvement in S/N ratio over 2:1 interlaced scanning with the same number of lines. This is true because the value of $1 - (R_{02}/\sigma^2)$ for the sequential scanning is about half of what it would be for interlaced scanning. Exactly how much improvement is afforded by sequential scanning depends on the values of R_{01}, R_{02}, and R_{12} which are determined by the scene scanned as well as by the television standards used.

The lower curve in Fig. 14 can also be used to predict the advantage to be gained by frame feedback DPCM. In this case, the abscissa would be R_{0F}/σ^2 where R_{0F} is the covariance between equivalent points on adjacent frames. The S/N ratio for a frame feedback system is given by (20) if R_{01} is replaced by R_{0F}. Some measurements by Kretzmer[5] and Deriugin[15] suggest that R_{0F}/σ^2 may, in general, be less than R_{01}/σ^2. This implies that frame feedback may be of little value in reducing the S/N ratio in DPCM systems.

IX. DPCM FOR MONOCHROME ENTERTAINMENT TELEVISION

In 4.5-Mc/s entertainment black and white television there is little advantage in basing the prediction on any sample values except the previous one, unless, of course, sample values from previous fields are available. For this previous-sample feedback system the approximate value of the S/N ratio to be expected is given in (20). In a 525-line picture at a frame rate of 30 per second, sampling at twice the bandwidth or 9 Mc/s means that there are about 571 samples per line. Only 83 per cent of these, or 474, occur in the video while the others occur during the horizontal and vertical sync pulses.

Using simple linear interpolation* we see that, if scene A of Fig. 6 were sampled at 9 Mc/s, the covariance between adjacent points would be $R_{01} \cong 0.958$. For scene C, $R_{01} \cong 0.986$. Using these numbers and the appropriate values of the constant C in (20), and remembering that $\sigma^2 = 1$ for these signals, the S/N ratios to be expected for transmitting these scenes over a DPCM channel can be found. The S/N ratio for scene B would fall somewhere between those of scenes A and C. These S/N ratios are compared with standard PCM and delta modulation in Fig. 15. For the lower bit rates, these curves must be used with discretion. The line representing the PCM performance is simply a plot of (18) which assumes that the sync is transmitted. The PCM S/N

* Since the aspect ratio is 4:3 we could not transmit these pictures as they are. We assume here that either the top or bottom $\frac{1}{4}$ of the pictures is not transmitted.

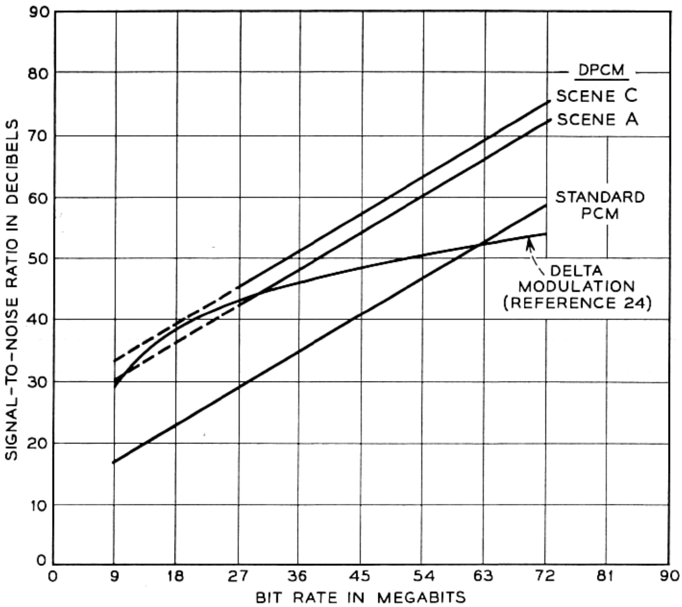

Fig. 15 — Comparison of previous-sample feedback DPCM with standard PCM and delta modulation for monochrome 4.5-Mc/s entertainment television. (Peak-to-peak composite signal to rms quantizing noise in the video is the signal-to-noise ratio shown. The sampling rate is 9 Mc/s.)

ratios can be increased by 2.8 db if the sync is not transmitted. The delta modulation S/N ratios were found by an entirely different technique[24] and it is gratifying that they are reasonably consistent with the results found here for 1 digit DPCM, which of course, is identical to delta modulation with a sampling rate of twice the bandwidth.

Fig. 15 shows that, for a fixed bit rate, DPCM would give a 14-db improvement over standard PCM for scene A and an 16.8-db improvement for scene C. The advantage of DPCM can also be expressed in terms of bit rate. For a given S/N ratio, DPCM gives a reduction in bit rate over standard PCM of about 18 megabits (2 bits/sample). Since the sampling rate for DPCM and PCM is assumed to be twice the bandwidth or 9 Mc/s, these curves in Fig. 15 are actually defined only at multiples of 9 megabits.

X. THE CHARACTER OF THE QUANTIZING NOISE

Fig. 16 illustrates the autocovariance for the quantizing noise in a previous-sample feedback system. The cases shown are for scene B but these curves are typical of all three scenes. The exact autocovariance

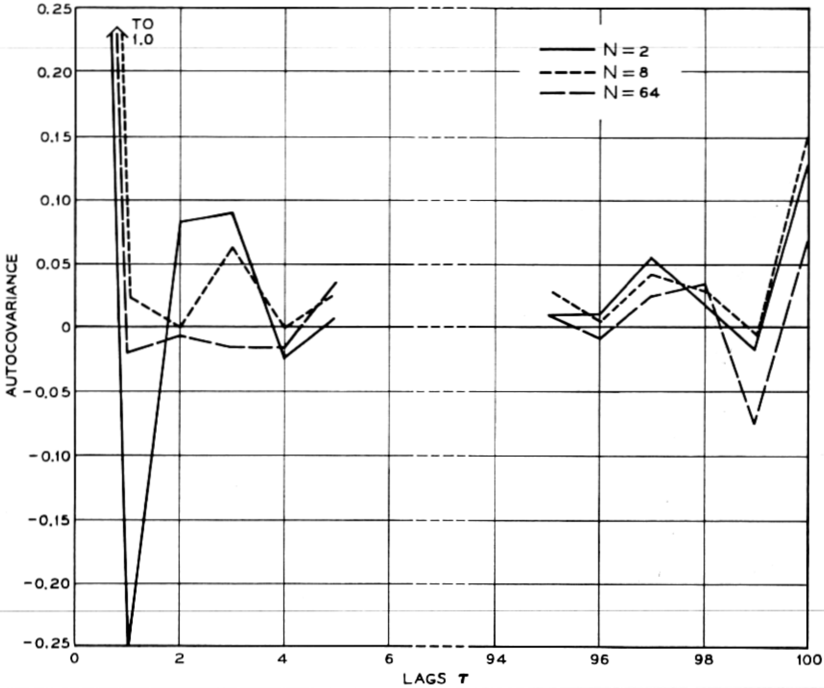

Fig. 16 — Autocovariance of quantizing noise for $N = 2$, 8, and 64 level previous-sample feedback DPCM. (Case shown is for scene B with the DPCM system optimized for this scene.)

function of the quantizing noise depends on the picture material of the incoming signal. These autocovariance functions were essentially the same when the parameters of the DPCM system did not exactly match the statistics of the incoming signal. The spectra of the quantizing noise in the three scenes is found by taking the Fourier transforms of their autocovariance functions. These spectra were found to be relatively flat for both previous-sample feedback and line-and-sample feedback systems. In both cases, there were erratic peaks and valleys at multiples of the line rate but the peaks and valleys differed from each other only by about 2 to 4 db, these differences being slightly greater for the previous-sample feedback system than for the line-and-sample feedback systems. In neither case did the spectra show any general tendency to increase or decrease for higher frequencies. Fig. 16 illustrates that the correlations between sample values is quite weak. The usual assumption of flat quantizing noise in DPCM systems is probably a good one for most purposes.

Fig. 17 shows a plot of the amplitude density function of the quantizing noise for scene B. It is typical of all the scenes that the amplitude density is relatively flat for a small number of quantizing levels N and becomes more Gaussian shaped as N gets large. In all cases, however, even when N was 64, the amplitude density function was flatter than Gaussian. For all three scenes with $N = 2$ the quantizing noise amplitude density function had a dip near zero.

XI. THE PENALTY

Removing the redundancy from the transmitted signal has the disadvantage that the signal becomes more vulnerable to noise introduced in the medium of transmission. This is true of predictive systems, in general, whether or not they are digital. A technique for reducing the redundancy in analog television signals by linear filtering has been proposed by Franks.[14] The similarity between this analog technique and DPCM is apparent. The utility of digital transmission itself is simply that it provides a desirable trade of bandwidth for noise immunity in the transmission medium. We may think of DPCM as a counter-trade. For a given amount of quantizing noise, DPCM allows transmission at a lower bit rate (and therefore bandwidth) than standard PCM. Errors in the transmission channel, however, degrade the decoded DPCM signal more than they would in standard PCM.

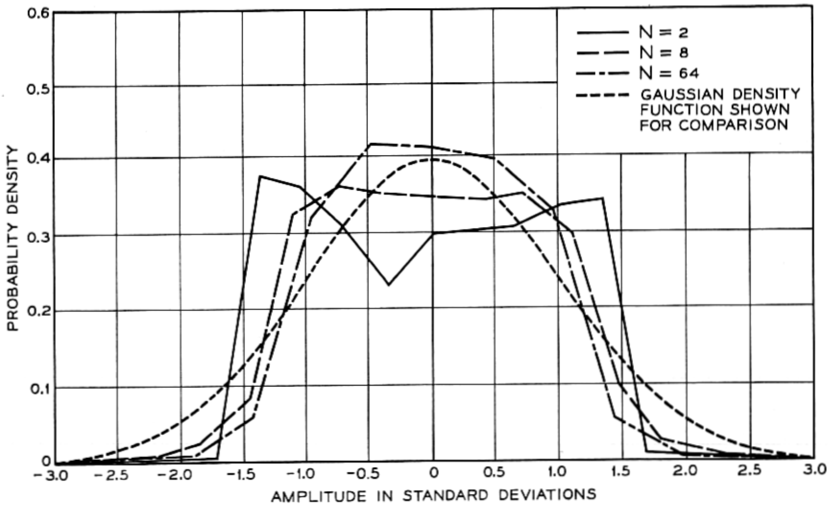

Fig. 17 — Quantizing noise amplitude density functions for $N = 2$, 8, and 64 level previous-sample feedback DPCM. (Case shown is for scene B with the DPCM system optimized for this scene.)

The decoder in a DPCM system is a linear device which operates on an incoming sequence with rms value σ_e to produce a decoded output with rms value σ. Just as the decoder increases the level of the incoming signal by $20 \log \sigma/\sigma_e = k$ db, it will also increase the level of accompanying noise (caused by digit errors in the transmission channel) by k db.

This is easily illustrated by considering the decoder of the previous-sample feedback system with the predictor shown in Fig. 4. Noise η, caused by a digit error, on a member of the incoming sequence is fed through the feedback loop and occurs on all subsequent samples. Such a noise on a single member of the incoming sequence causes the error sequence $\eta, a_1\eta, a_1^2\eta, a_1^3\eta, \ldots,$ in the decoded output sequence. The noise energy in the decoded signal is, therefore

$$\eta^2 + (a_1\eta)^2 + (a_1^2\eta)^2 + \cdots = \eta^2 \left(\frac{1}{1 - a_1^2}\right).$$

For the properly designed previous-sample feedback system $a_1 = R_{01}/\sigma^2$ and $1/(1 - R_{01}^2/\sigma^4) = \sigma^2/\sigma_e^2$. Therefore, a noise of energy η^2 in the transmission channel appears in the decoded signal as noise with energy $\eta^2(\sigma^2/\sigma_e^2)$, a gain of $10 \log \sigma^2/\sigma_e^2$ db.

We have already shown that DPCM provides a decrease in quantizing noise of about $10 \log \sigma^2/\sigma_e^2$ db over standard PCM (assuming the sync pulses are not transmitted). The penalty paid for this decrease in quantizing noise is that the noise in the decoded signal introduced in the transmission medium is increased by exactly that same amount. This does not mean that DPCM provides no advantage. For, in digital systems, noise introduced in the transmission medium can be made extremely small and the limiting degradation in DPCM systems is generally quantizing noise. Decreasing the quantizing noise by k db may be desirable even if the noise introduced in the channel is increased by this amount.

When the probability P of a digit error in the transmission medium is small enough so that the probability of getting two errors in the same word may be neglected, then the noise power N_t in the decoded output introduced by the transmission medium is directly proportional to P and we can express N_t (in db) as

$$N_t = K_1 + 10 \log P + 10 \log \sigma^2/\sigma_e^2. \tag{25}$$

From (17) the quantizing noise N_q can be expressed (in db) as

$$N_q = K_2 - 10 \log \sigma^2/\sigma_e^2. \tag{26}$$

The constants K_1 and K_2 are both dependent on the number of quantizing levels as well as other parameters. The term $10 \log \sigma^2/\sigma_e^2$ represents the effect of DPCM in both equations. Reducing the quantizing noise N_q by k db through DPCM requires increasing 10 log σ^2/σ_e^2 by this amount and this increases the noise N_t introduced in the medium of transmission by k db. Whether or not DPCM can be used to advantage depends on the relative importance of N_t and N_q in limiting the performance of the system. From (25) we see that if we require N_t to remain constant while reducing the quantizing noise by k db, we must reduce the term 10 log P by k db. This requires reducing the value of P by a factor of $10^{0.1\,k} \cong (1.26)^k$.

XII. ACKNOWLEDGMENT

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