A General Statistical Determination of Transmission Characteristics Applied to L Multiplex

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There is a growing demand for modern communication systems capable of transmitting high-speed data signals over Bell System facilities designed primarily to provide telephone service. Much of this data traffic can be handled within telephone channels in the switched network. Somewhat higher data rates are feasible on private lines by selection of facilities and special treatment of the voice-grade circuits. The need for still higher data bit rates requires bandwidths equivalent to many message channels, e.g., the 12-channel group, the 60-channel supergroup, and the 600-channel mastergroup.

Presentation of basic measured data to statistically characterize transmission in the broader bands in terms of frequency domain specifications is the object of this paper. Data reduction is covered in detail. Characteristics of built-up connections are predicted from knowledge of the characteristics of subunits, including inherent variability. Such variability is a basic limitation on the degree of equalization that can be achieved with a small set of fixed networks.

I. INTRODUCTION

The demand for new services offered by the Bell System is increasing rapidly. Many of the services have performance requirements that are more critical than those of voice message telephone. They include data services over voice-band private lines and several types of DATA-PHONE service over the switched network, and wideband data services over groups (48-kc wide) and supergroups (240-kc wide) provided by L-multiplexed carrier facilities of the L-carrier plant. Services covering even wider bandwidths are under development. They are more critical in that they tolerate considerably less impairment from a number of sources. Two such impairments are amplitude and envelope delay distortion that are present in the L-type terminals which provide our long-haul facilities. Equalization of the terminals carrying these services seems the obvious solution for reducing distortion to tolerable limits.

Equalization in the frequency domain is the process of designing networks which introduce distortions of equal magnitude but in the opposite sense to those of the system characteristic to be equalized. The success of such equalization hinges critically on the precise statistical knowledge of existing characteristics actually encountered in the plant and is limited by their variability.

Data acquision on transmission of a network as vast and complex as the L carrier plant can be accomplished only by having a judiciously chosen sampling plan. The plan discussed here consists of dividing the plant into representative subunits; many combinations of types of subunits in tandem make up a complete transmission system. Data have been obtained by way of accurate measurements of loss and envelope delay of a random sample of each category of subunits. A general method for processing and statistically determining transmission characteristics from the measurements is discussed. The method includes the problem of synthesizing the over-all system characteristic in statistical terms from separate knowledge of subunit characteristics, given any system make-up. However, the statistics of system make-up would require another study of comparable complexity.

The subunits chosen are back-to-back group, supergroup, and mastergroup modems* and interconnecting equipment of the LMX 1 carrier plant. Examples are shown of both predicted and measured multiple-link characteristics of a complex system make-up.

Comparable work is planned on data acquisition and statistical presentation for the newly designed, transistorized LMX 2 carrier terminals.[1] The methods described in this paper are therefore believed to be of general usage for presentation of transmission characteristics.

II. BASIC DATA AND THE METHOD OF ANALYSIS

2.1 *Transmission Characteristics*

Transmission through any equipment unit can be characterized in general terms by its frequency transfer function in complex form:

$$H(\omega) = \exp \{\alpha(\omega) - j\beta(\omega)\}$$

with α representing amplitude and β phase.

* The word "modem" stands for modulator/demodulator and is used here for equipment being interconnected at modulated frequencies; thus, input and output of a modem are at identical frequencies.

For the purpose of this study we are concerned with accurate knowledge of α in the frequency band of interest. For measurements at like level points or measurements corrected to such points, α is ideally equal to 0. This value is arbitrarily assigned to some reference frequency, generally near the center of the band. The amplitude characteristic of interest is then given in terms of deviation from 0 for all other frequencies. Such deviations are expressed in db, with 0 db at the reference frequency.

For distortionless transmission, β should be a linear function of angular frequency ω. In other words, we are concerned with accurate knowledge of $d\beta/d\omega$ versus frequency. $d\beta/d\omega$ is called envelope delay, and the frequency at which there is minimum envelope delay is generally used as reference. The time units of envelope delay are typically microseconds or milliseconds. Test sets measure envelope delay over a *finite* frequency difference Δf, and results are not precisely $d\beta/d\omega$; but the error is negligible for low-order distortion.

2.2 *Data Acquisition*

The quantities amplitude and envelope delay should be known continuously over the frequency bandwidth to be transmitted. For practical reasons, however, one needs only to measure at discrete intervals such that the actual variation between measurement points is of the same order of magnitude as measurement accuracy. From previous experience with or knowledge of the physics of the equipment, the necessary number and frequency spacing of measurements can be determined. Frequency spacing for group modems was 5 kc, for supergroup modems 20 kc,* and for mastergroup modems 100 kc.

Equipment of a certain type, such as group, supergroup, or mastergroup equipment of the L-carrier terminal, will not exhibit identical transmission characteristics at each installation. Manufacturing tolerances and cabling between the actual equipment and access points are the main contributors to variability. The measurements have shown, however, that this variability is substantially less than that of the quantity of interest to be estimated. In statistical terms, we may assume the coefficient of variation σ_μ/μ to be $\ll 1$, where σ_μ is the standard deviation of the quantity of interest, μ.

Basic data are thus comprised of a representative sample of characteristics of like equipments measured point by point at successive frequencies. In general, we need to make n measurements for each quantity

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* Except for supergroups 1 and 3, where ripples at the band edges proved of generally higher order and intervals smaller than 20 kc where measured.

(loss or envelope delay) across the frequency band of interest and repeat them on a number of like equipments, k, to be determined by the desired degree of statistical accuracy. For instance, supergroup modem characteristics have been measured at 20-kc intervals, if variability was low enough to allow that wide a spacing. Thus, for the 240-kc band, n would generally be 12, and k was selected to equal 10.*

Since the quantity of interest for each type of equipment is the amount of distortion relative to some frequency in the band of interest, all measurements at other frequencies are normalized in terms of deviation from the value at that frequency. For amplitude, the normalizing frequency is usually the lineup or maintenance frequency. For envelope delay, it is usually the frequency of minimum envelope delay.

Measurement accuracy has an important bearing on the results and should be an order of magnitude better than the quantity to be measured. Equally important is the fact that the measurement error is random with zero mean, so that it will not bias the outcome of the experiments. By using laboratory-type equipment and exercising care during measurement and calibration procedures, we can usually fulfill these requirements. For the data reported here, a frequency accuracy of 1 part in 10^6 was ensured. Loss measurements were accurate to within 0.03 db, and for envelope delay measurements the accuracy was ± 1 microsecond.

2.3 *Data Reduction*

The statistician R. A. Fisher wrote that the object of statistical methods is the reduction of data.† In the problem at hand, some 15,000 measurements were taken to represent only 20 pairs of characteristics on amplitude and envelope delay distortion.

For each type of equipment (subunit), the quantity of interest (loss or envelope delay) was measured at a number of selected frequencies. The average and standard deviation at each frequency was then computed. The data, when suitably corrected for the mean, are assumed normal with $\mu = 0$. The validity was proven by plotting the residuals on probability paper, an example of which is shown in Section V.

Where possible, data were pooled to obtain a more precise estimate of the standard deviations. For instance, 5 group modems are numbered

* A sample of 10 generally is not regarded to yield high accuracy, but as will be explained later, an estimate of the standard deviation in this case was obtained on a "pooled" basis, increasing its accuracy by about a factor 3.

† R. A. Fisher, *On the Mathematical Foundations of Theoretical Statistics*, Phil. Trans. Royal Soc., April 1922.

1 through 5. The average loss at some frequency of a sample of each numbered group in general differs from that of another numbered group. However, the variability within each sample proves the same, which allows pooling the estimates of the variance. In the case of supergroups, 10 separate estimates could be pooled at each frequency. Since the data were taken in such a methodical and even manner, the "gamma-plot routine" was used to ensure the validity of pooling.[2] This routine compares an ordered sample of variances obtained from the (normal) data with a mathematically expected set of values. If they are plotted against each other on linear coordinates, one expects a more or less straight line for a good fit of the assumption of equality. The word "gamma" is related to the function of that name because equal-sample variances for Gaussian data are distributed as

$$f(s^2,n)\,ds^2 = \frac{\left\{\dfrac{s^2(n-1)}{\sigma^2}\right\}^{[(n-1)/2]-1}}{\Gamma\left(\dfrac{n-1}{2}\right)\cdot 2^{(n-1)/2}} \cdot \exp\left[-\frac{1}{2}\left(\frac{s}{\sigma}\right)^2(n-1)\right] d\left\{\frac{s^2(n-1)}{\sigma^2}\right\} \qquad (1)$$

which is a gamma density function for the variable $\{[(n-1)s^2]/\sigma^2\}$, also known as a χ^2 variate with $n-1$ degrees of freedom. In (1), s^2 is the sample variance and n the sample size. An example of the use of the "gamma-plot routine" in relation with this distribution is illustrated in Section V.

Cross products of sets of data at any two frequencies were computed for any subunit to obtain the sample covariance between the sets. As will be shown in Section 2.4, covariance computations are essential for unambiguous interpretations of presented graphs.

2.4 *Statistical Characterization*

Having acquired the data on amplitude and envelope delay and having performed calculations to reduce them to a few significant numbers, we are now in a position to statistically characterize transmission of the equipment subunits under study.

Over the bandwidth of interest, we have the average loss or envelope delay at a number of frequencies. A smooth curve connecting these points represents the regression of the quantity of interest on frequency, or to express this another way, the curves shown are "least squared" estimates of loss or envelope delay versus frequency. This curve is the "most likely" characteristic to be encountered and should be used as basic data for trend equalization. However, the degree of variability calculated from the standard deviation indicates the residual variation to be expected after fixed trend equalization.

This variability is expressed in terms of population percentile. Using standard tables, limits are calculated within which a certain percentage of all probable curves is expected to fall for a certain degree of confidence. The coherence of the loss measurements and their limits will now be discussed.

Consider Fig. 1, which shows an average amplitude characteristic of a supergroup modem of the L carrier. Measurements were made at 20-kc

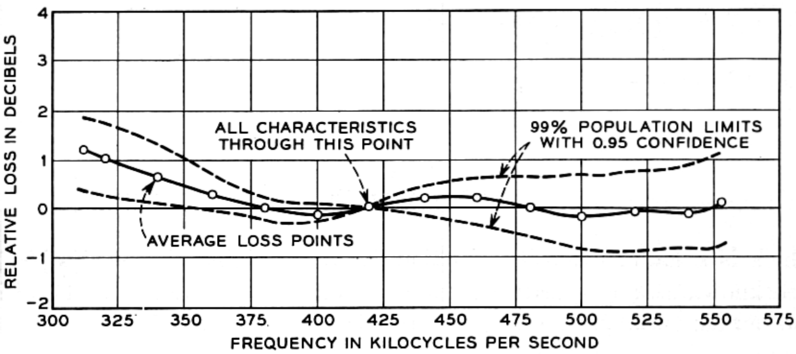


Fig. 1 — Example of average supergroup amplitude characteristic (solid line) and 99 per cent population limits (0.95 confidence).

intervals, and at each measurement frequency the average loss and 99 per cent population limits are indicated. The essential idea here is that the curve connecting the average loss points is representative of the general *shape* of the characteristic. To clarify this point statistically, one should consider sets (x_i, x_j) of pairs of data at two test frequencies, f_i and f_j for any numbered supergroup. *Any* such set may be considered a sample of a bivariate normal distribution whose general density function is

$$f(x_i, x_j)\, dx_i dx_j = \frac{dx_i dx_j}{2\pi\sigma_i\sigma_j\sqrt{1-\rho^2}} \exp - \left\{\frac{1}{2(1-\rho^2)} \cdot\left[\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2 - 2\rho\,\frac{(x_i-\mu_i)(x_j-\mu_j)}{\sigma_i\sigma_j} + \left(\frac{x_j-\mu_j}{\sigma_j}\right)^2\right]\right\}, \tag{2}$$

where

$$\rho^{(i,j)} = \frac{\text{cov }(x_i\,, x_j)}{\sqrt{\text{var }(x_i)\ \text{var }(x_j)}}\,, \tag{3}$$

the correlation coefficient of the variates x_i and x_j. For uncorrelated variates $\rho^{(i,j)} = 0$, and for perfect correlation $\rho^{(i,j)} = \pm 1$. The general form of this distribution is a bell-shaped surface, more oblong as ρ becomes closer to ± 1 (see Fig. 2). The probability of the variates lying in some specified *area* is given by the volume integral of $f(x_i\,, x_j)$ over that area. Tables of the bivariate normal distribution are published in Ref. 3.

The value of ρ can be estimated from the data at pairs of *adjacent* test frequencies and at pairs successively *further* apart. A typical plot of $\hat{\rho}$, an estimate of ρ, versus measurement interval is shown for supergroups in Fig. 3(*a*) and for groups in Fig. 3(*b*).

This "$\hat{\rho}$-function" illustrates very clearly two aspects of the graphs. One is that adjacent test frequencies have high *positive* correlation. This means that if loss at one frequency goes up, so does the loss in the range of ± 20 kc (for supergroups) or ± 10 kc (for groups) about this frequency. The other aspect is that losses at test frequencies more than 120 kc apart for supergroups and more than 20 kc apart for groups have very little correlation. The "threshold" of signifiance taken from significance tables of ρ (see Ref. 4), is given by the straight horizontal dotted lines in Fig. 3(a) and (b).

We are now in a firm position to make the following statements about the presented graphs: (a) each average curve is representative of the universe of like curves, and (b) ripples of higher order than those shown are unlikely. However, pivoting of the curve around the zero reference

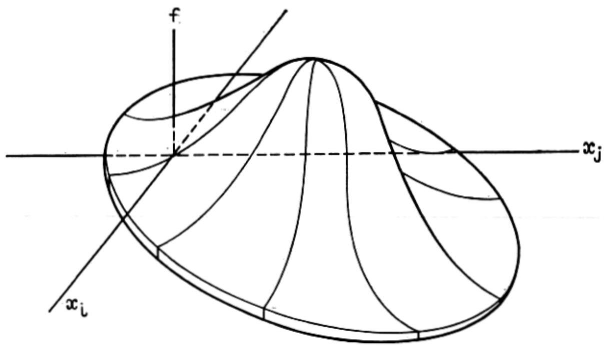
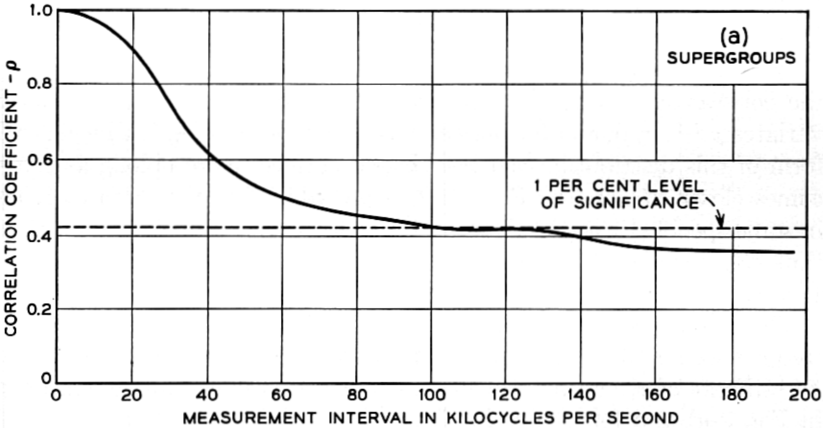


Fig. 2 — General shape of bivariate normal distribution.



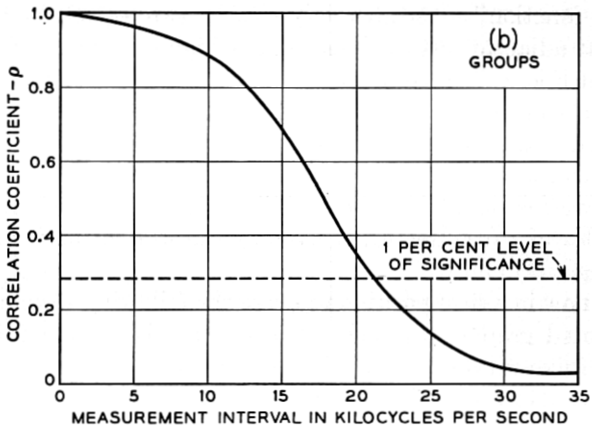


Fig. 3(a) — Correlation coefficient as a function of measurement interval for supergroups.
Fig. 3(b) — Correlation coefficient as a function of measurement interval for groups.

loss point is as likely one way as it is the other. By using the tables of the bivariate normal,[3] one can calculate the likelihood of deviations from the average characteristic.

Although these points may seem intuitively obvious, they are not. For instance, a variably poor impedance match across the band may cause ripples over short-frequency intervals. Yet, an average of such curves may still be smooth and have considerably less ripple. The ρ-function of the type shown in Fig. 3 for such a case, however, would rapidly fall off to statistically insignificant levels as the test frequency spread is increased.

2.5 *Impedance Interactions*

For accurate data, equipments were measured with laboratory-type instruments having precisely known, constant impedances. Actually these equipments are used for operation between less precisely known and controlled impedances. Since the equipments under consideration contain networks sensitive to the value of terminating impedances, departure from design values of these impedances may have an effect on the transmission characteristics.

Thus it may be that what was measured departs from the actual characteristic of the equipment in operation. It is important therefore to have knowledge of this departure, again in statistical form. Usually these departures are small and more or less random. As such, they may be incorporated in measurement errors.

Impedance interaction effects were determined by measuring amplitude characteristics of a built-up tandem connection of equipments, and then comparing it with a synthesized characteristic from appropriate addition of those measured for individual equipments. The results shown in Fig. 4 indicate a good match when the measurement accuracy is noted. Only at the band edges are some departures noticeable, but for wide-band equalization purposes these regions are in general precluded due to excessive delay distortion.

III. MULTILINK PREDICTION

3.1 *General*

The problem considered here is the synthesis of a transmission characteristic of a circuit comprised of several subunits in tandem. The individual characteristics of these subunits are known in terms discussed in Section II. Again, the result would be in the form of an average and expected variation for a given degree of confidence.

In general, transmission characteristics of subunits in tandem which behave like independent variables are additive if expressed in db and if their impedance levels are identical at the interfaces. Statistically this means that the best estimate of the average characteristic of a sum of subunits is the sum of the subunit averages. Thus if the estimated average of the quantity of interest for the ith subunit at some frequency f is $\hat{\mu}_i$, then for n subunits in tandem

$$\hat{\mu} = \sum_{i=1}^{n} \hat{\mu}_i .$$

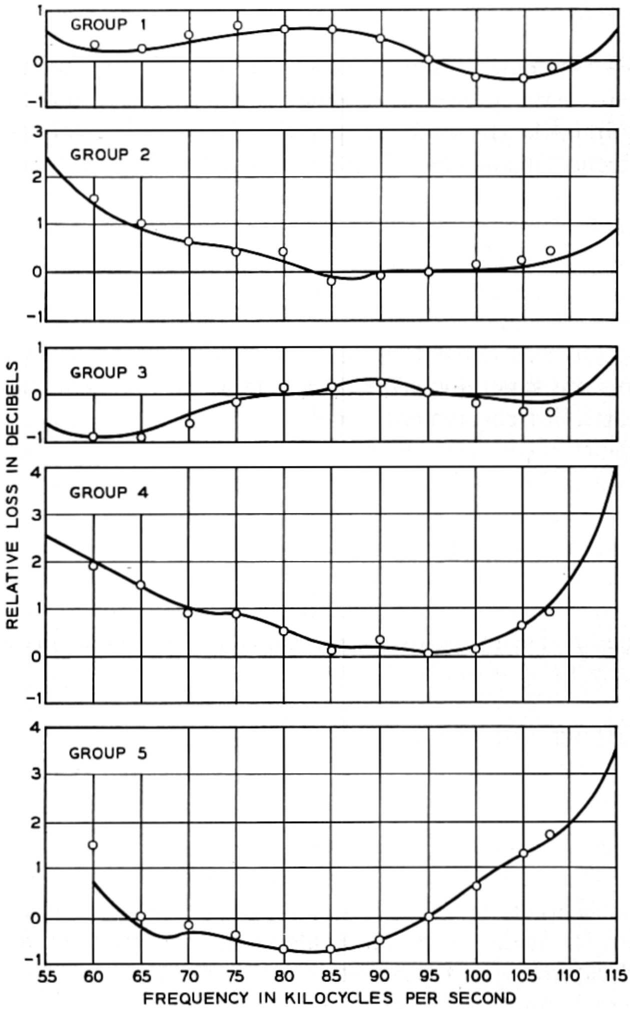


Fig. 4 — Typical example of added versus measured group characteristics. Solid lines are measured.

Similarly, for the variances (σ_i^2)

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \hat{\sigma}_i^2.$$

These expressions should be used with caution if junction loss distortion is appreciable. One may be forced in such cases to actually measure the characteristic, or else appropriately enlarge $\hat{\sigma}^2$ to reflect the effect of impedance interactions. For our purposes we showed in Section 2.4 that impedance interactions resulted in negligible junction loss distortion.

The ρ-function as discussed in Section 2.4 for a composite characteristic can be estimated from knowledge of subunit ρ-functions. The general expression for estimating ρ for n subunits in tandem between two frequencies, f_i and f_j, is

$$\hat{\rho}^{(i,j)} = \frac{\sum_{l=1}^{n} \hat{\rho}_l^{(i,j)}}{n},$$

since we may assume the variance within subunits at a given frequency to be equal. For instance, the estimation of values of ρ for a composite group characteristic traversing supergroups would result in a weighted average of ρ for groups and ρ for supergroups according to the above expression. An example will be shown in Section 3.2 of a ρ-function so derived.

3.2 *Some Applications*

A major portion of the long-distance communication facilities of the Bell System is frequency multiplexed by L-carrier equipment at terminal offices. The technique of frequency-multiplexing employs numerous filters, although the number of different filter designs is relatively few. For instance, to multiplex 600 channels only 27 different filter designs are employed.[5]

To complete transmission from one terminal office to another, a transmitting terminal is necessary at one end and a receiving terminal at the other. To characterize transmission from the transmitting to the receiving offices, one needs transmission characteristics of a transmitting and receiving terminal only, interconnected back to back. The influence of the high-frequency medium, such as radio or coaxial cable, is negligible compared with the terminal over the fraction of bandwidth considered here. That is, a supergroup occupies a 240-kc band, and an equalized coaxial cable has a maximum residual ripple of the order of only 0.2 db per 240 kc.[6]

Having gathered and statistically analyzed back-to-back terminal characteristics, one is in a position to estimate the characteristics of circuits consisting of a given number of links in tandem. Each interconnection may take place at basic group frequencies (60 to 108 kc), basic supergroup frequencies (312 to 552 kc), or basic mastergroup frequencies (564 to 3088 kc). An appropriate connector is used for each interconnection of incoming receiving to outgoing transmitting equipment, and its transmission characteristic should be included in the overall prediction.

Fig. 5 shows the characteristic for an actual group circuit representing a complex layout between New York City and Phoenix, Arizona. This

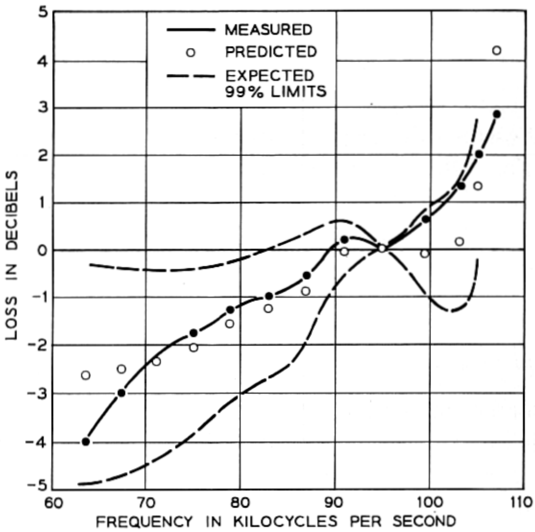


Fig. 5 — New York City-Phoenix group circuit, 8 SG connectors, 4 GR connectors.

group circuit traversed 9 supergroup modems, 8 supergroup connectors, 5 group modems, and 4 group connectors to give a total of 26 subunits. The measured characteristic* is indicated by the solid line, and it will be observed that it falls within the expected limits indicated by the dashed lines. Using the expression given in Section 3.1 for $\hat{\rho}$ and the data in Fig. 3 (*a*) and (*b*), the ρ-function for this kind of a built-up connection was calculated and is shown in Fig. 6.

Another important application of multilink prediction is that of estimating group amplitude slopes. Slope is defined here as loss difference between 63-kc and 103-kc points. From knowledge of the individual characteristics of group and supergroup modems, it is possible to lay out

* Courtesy of the Long Lines Department of the American Telephone and Telegraph Company.

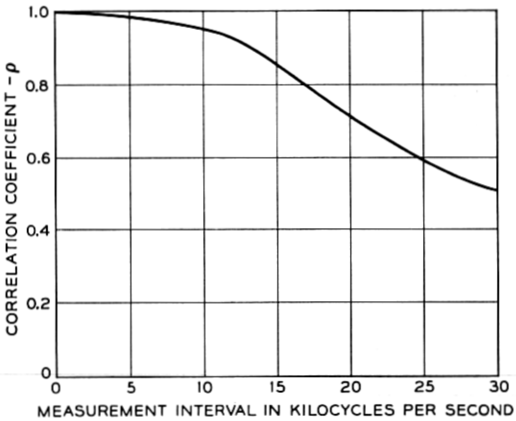


Fig. 6 — Correlation coefficient as function of measurement interval for composite group characteristic of Fig. 5.

all "paths" that a basic group band might take in being transmitted from office A to office B, given the number of links in between. For three links interconnected at basic supergroup frequencies there are 5000* equally probable paths, thus 5000 equally probable slopes. Not all the slopes will have different values, but together they will constitute a *distribution* of average slopes. If the variability within subunits is included, two more distributions of extreme (positive and negative) slopes can be numerically calculated. These three distributions are derived from composite modem characteristics of groups, supergroups, and supergroup connectors and are shown in Fig. 7.

As a third example, consider the problem of lineup of system loss. At present, L multiplex facilities in the Bell System use 92 kc for group circuits, and 424 kc for supergroup circuits as the lineup frequencies. Signals at these frequencies are permanently present for monitoring and adjustment purposes and are called pilots. However, to clear the band of interfering signals for wideband data transmission, the new standard pilot frequencies will become 104.08 kc for the group and 315.92 kc for the supergroup. The new pilot frequencies are thus located approximately 4 kc from the band edge. Maintaining the same system loss as the old pilots at these new frequencies means a general *increase* of load delivered to the high-frequency medium. This comes about because loss at new pilots in general is higher than at old pilots, a fact that could be de-

* In the first link, a set of five possible groups has a "choice" of a set of ten supergroups, and similarly ten choices in the second and third links, totaling 5×10^3.

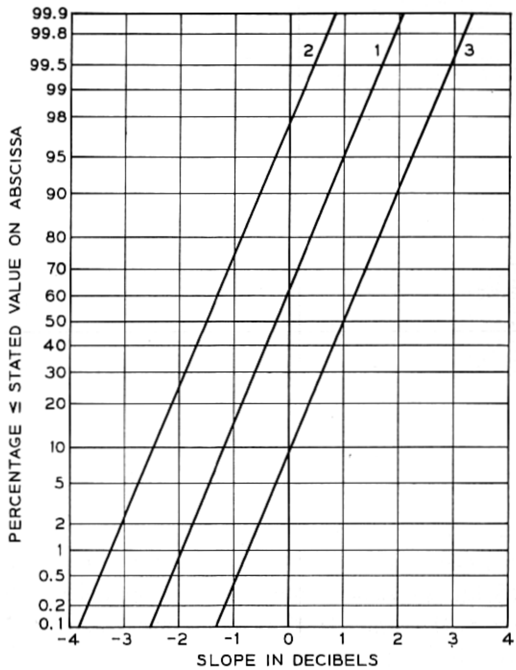


Fig. 7 — Distributions of (1) average slopes, (2) extreme negative slopes, and (3) extreme positive slopes across the group bank in a 3-link system.

termined accurately only from precise knowledge of the amplitude characteristics. On complex multilink circuits, the consequences of change in system misalignment could be determined. This, in turn, has led to work on the design of equalizers to overcome these difficulties, and also has helped to establish requirements controlling the design of new multiplex equipment.

IV. EQUALIZATION

4.1 *Introduction and Requirements*

A "theory of equalization" is very concisely presented in Ref. 6 where the two modes of frequency domain equalization, dynamic and fixed, are considered in detail and a basic set of rules is postulated. Since we assume time-varying distortion to be present only outside the terminals under consideration, it is natural to assume also that dynamic equalization is likewise implemented. Therefore only *fixed* equalization will be considered briefly here.

As was explained in the previous sections, we deal here with characteristics that vary from equipment to equipment, although the frequency band is identical among like units. Equipments of a certain category, such as groups or supergroups, are used interchangeably and very often are operated in tandem. Yet the characteristics of, say, Group No. 4 in Supergroup No. 9* between cities A and B are not exactly the same as the same allocation between cities C and D. Moreover, this particular group circuit may be operated in tandem with, say, Group No. 1 in Supergroup No. 3 interconnected by a group connector equipment unit. Each of these have different distortions to be included in the overall distortion of the particular group circuit.

Thus, the question may be raised of how to equalize a varied plant on a *fixed* basis. The answer to a large degree hinges on the requirements for allowable distortion *after* equalization. Moreover, the requirement for residual distortion is highly dependent on the type of signal to be transmitted. In addition, the choice of equalization depends on the manner in which it is to be administered, to ease the burden of the operating telephone companies.

This paper is not primarily concerned with what plan of equalization should be used under given circumstances. However, the method of data analyzation described here is considered a basic tool to arrive at some plan of equalization with fixed networks. The assessment of residual variability and the method of correlating its limits over the frequency band of interest will subsequently be used to determine the maximum obtainable benefit in applying such networks. Work at Bell Laboratories is in progress to formulate effective equalization plans based on data gathered and analyzed in the way described here, to meet present day service requirements.

V. CHARACTERISTICS OF PRESENT PLANT

5.1 *Purpose*

At present, L-multiplex facilities generally are used for long-distance communication transmission in the Bell System. In order to gain precise knowledge of transmission characteristics, particularly to enable engineering of wideband data communications, numerous point-by-point

* Group and supergroup numbers refer to their frequency allocation *after* modulation or *before* demodulation; see also Ref. 5, p. 34.

amplitude and envelope delay measurements have been made of L-terminal equipment.

The data have been reduced and analyzed in the manner described and are presented in this section for future reference. As the plant grows and new designs like LMX 2[1] are added, more data will be taken, similarly processed, and analyzed to keep a running account of the transmission facilities. The data presented here pertain to LMX 1 terminal equipment.

5.2 *Groups and the Group Connector*

The basic group frequency band is from 60 to 108 kc. Groups numbered 1 to 5, each with different carrier frequencies, are assembled in a group *bank* the output of which, transmitting, constitutes the basic supergroup band. When looped back directly into a receiving group bank, characteristics can be measured on a back-to-back basis for each measured group.

Results of amplitude characteristics of 17 banks are shown in Figs. 8 to 12. Figure 13 shows a typical example of envelope delay for groups. The sample of groups for which envelope delay was measured was rather

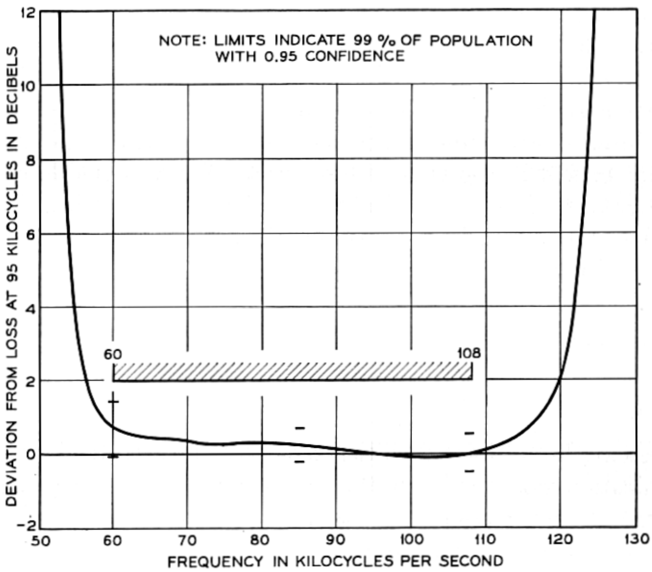


Fig. 8 — Typical modem amplitude-frequency characteristic of group No. 1.

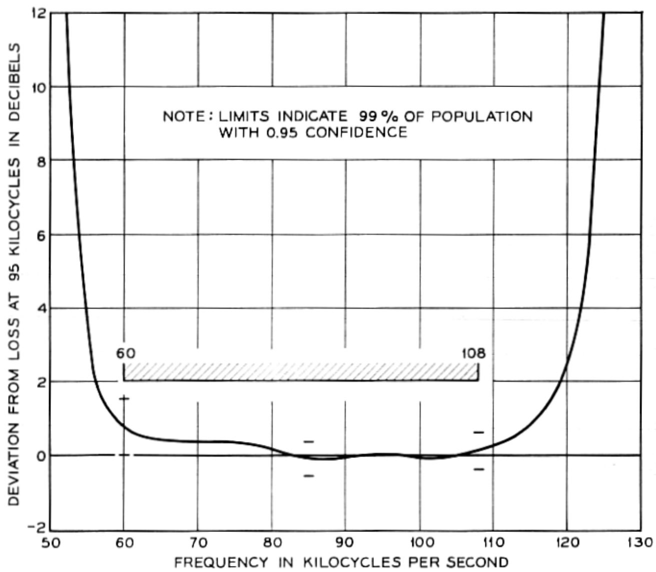


Fig. 9 — Typical modem amplitude-frequency characteristic of group No. 2.

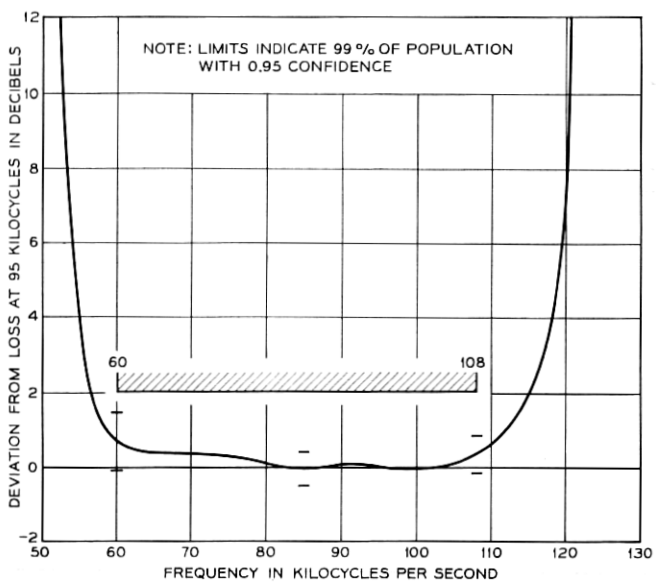


Fig. 10 — Typical modem amplitude-frequency characteristic of group No. 3.

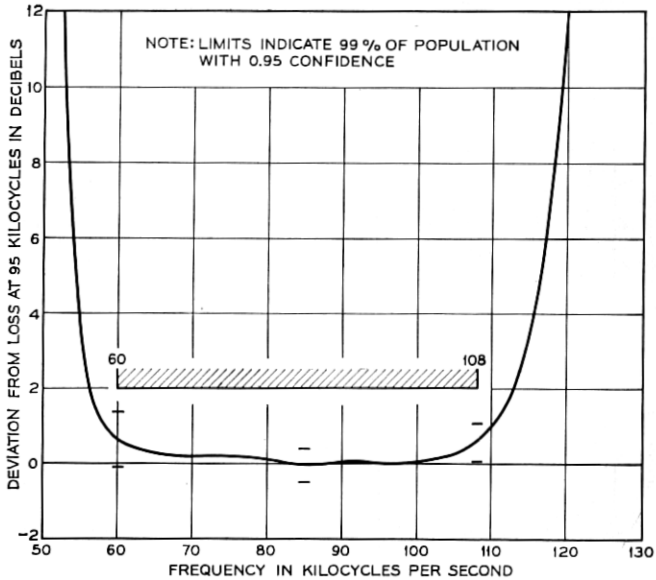


Fig. 11 — Typical modem amplitude-frequency characteristic of group No. 4.

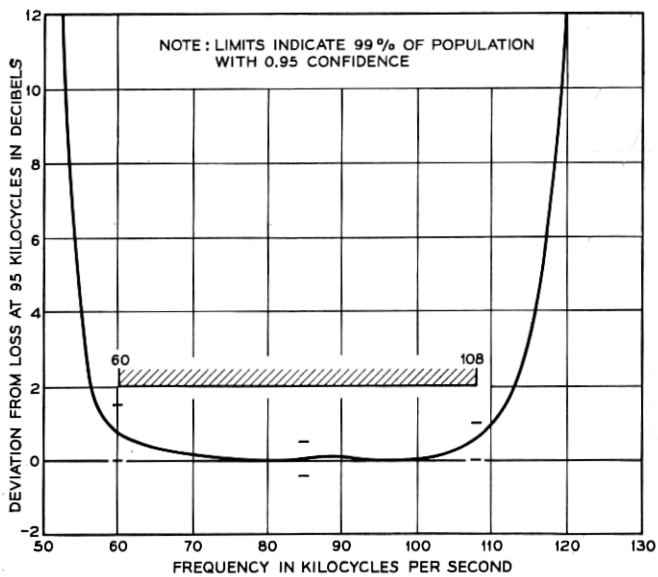


Fig. 12 — Typical modem amplitude-frequency characteristic of group No. 5.

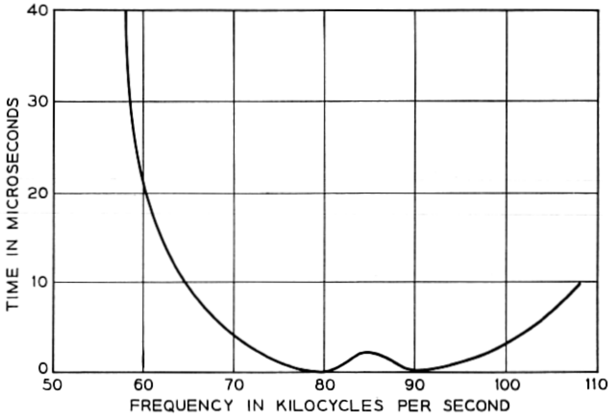


Fig. 13 — Envelope delay distortion for back-to-back group circuits of the LMX 1.

small, and variability within numbered groups proved of the same order of magnitude as variability between groups of a different number, and thus no separate presentation was warranted. The method used for precise checking of this statement is that of the Analysis of Variance where "treatments" are represented by numbered groups. At several frequencies such an analysis was made and the above conclusion confirmed. Population limits are omitted on envelope delay curves because the measurement variability proved to be comparable with equipment variability.

At terminal offices receiving groups are often retransmitted without demodulation to voice frequency. For such cases a *group connector* is used to interconnect the output of one receiving group with the input of a transmitting group. Very sharp cutoff accompanied by severe delay distortion at the band edges is the main characteristic of a group connector. The average and dispersion of nine such characteristics are shown in Fig. 14. As was the case for group bank delay curves, this figure shows no dispersion for the delay characteristics.

5.3 *Supergroups and the Supergroup Connector*

As was done for groups, the average of ten supergroup bank characteristics is presented in Figs. 15 to 24. The basic supergroup frequency band is from 312 to 552 kc, and in LMX 1 carrier systems there are ten numbered supergroups, again each with a different carrier frequency.

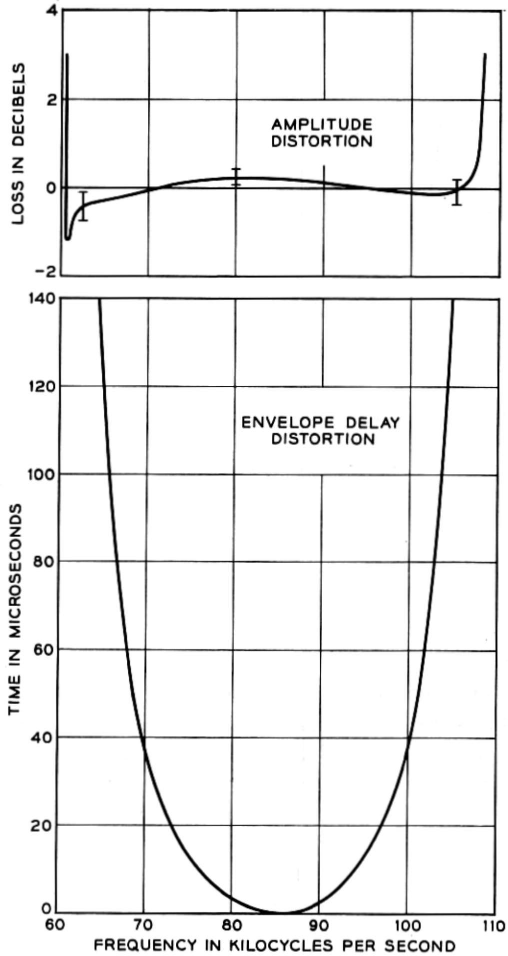


Fig. 14 — Average of nine characteristics of the group connector (L-L).

The envelope delay characteristics cannot all be lumped together as was done for groups. Here, Supergroups No. 1 and 3 are markedly different from each other and from the remaining eight. Thus, Figs. 25 and 26 show the envelope delay for Supergroups No. 1 and 3, respectively. Fig. 27 shows the characteristic for the combined measurements of Supergroups No. 2 and 4 through 10.

Similarly, supergroups are also interconnected at terminal offices, and a supergroup connector is used for this purpose. It is likewise characterized by sharp edge cutoffs, as shown in Fig. 28.

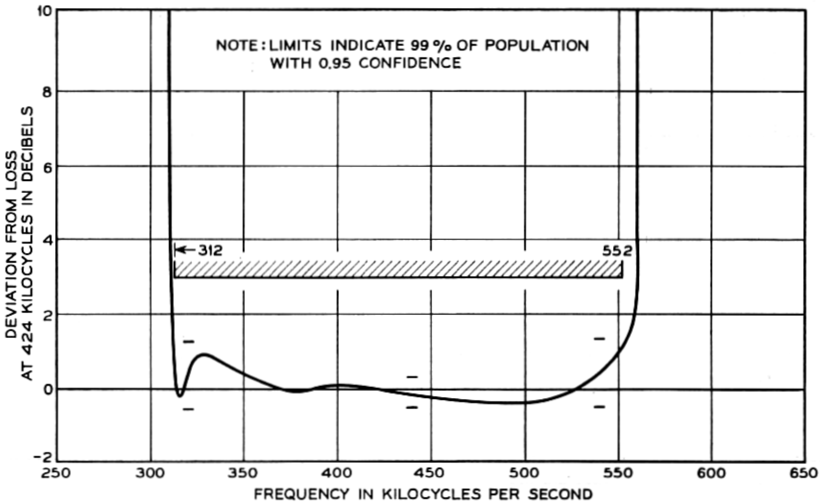


Fig. 15 — Typical modem amplitude-frequency characteristic of supergroup No. 1.

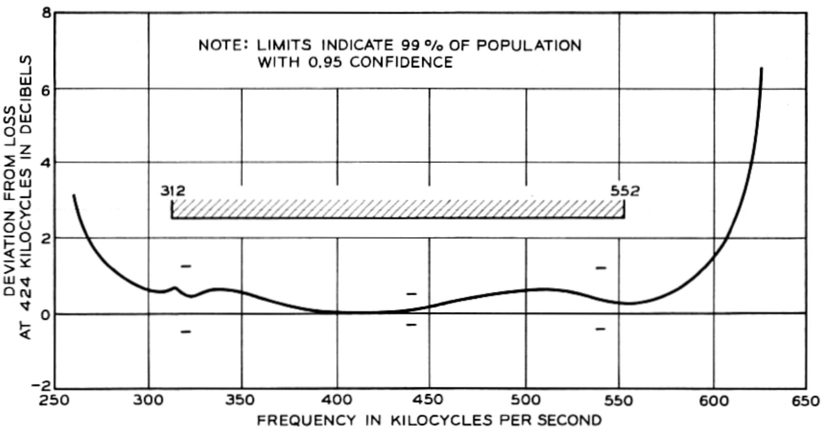


Fig. 16 — Typical modem amplitude-frequency characteristic of supergroup No. 2.

The "gamma-plot routine" mentioned in Section 2.3 was successfully applied to justify pooling of the sample loss variances for each numbered supergroup at each measurement frequency. The ten sample variances at any test frequency for each numbered supergroup multiplied by $(n - 1)/\sigma^2$ form by themsleves a sample of ten of the gamma

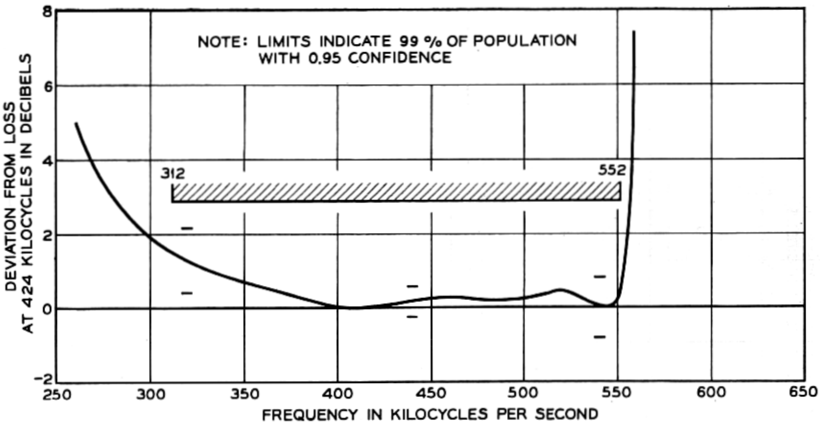


Fig. 17 — Typical modem amplitude-frequency characteristic of supergroup No. 3.

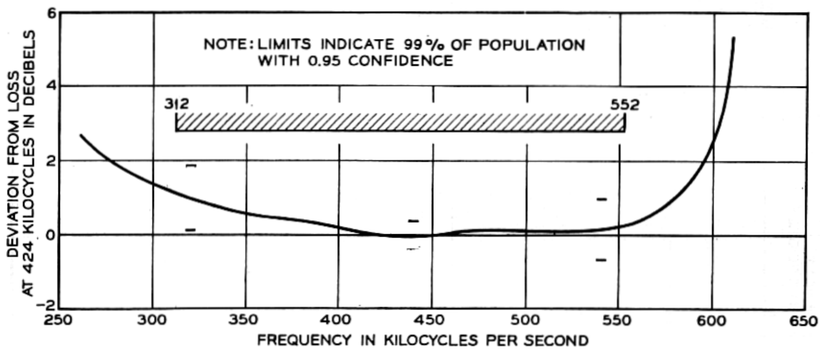


Fig. 18 — Typical modem amplitude-frequency characteristic of supergroup No. 4.

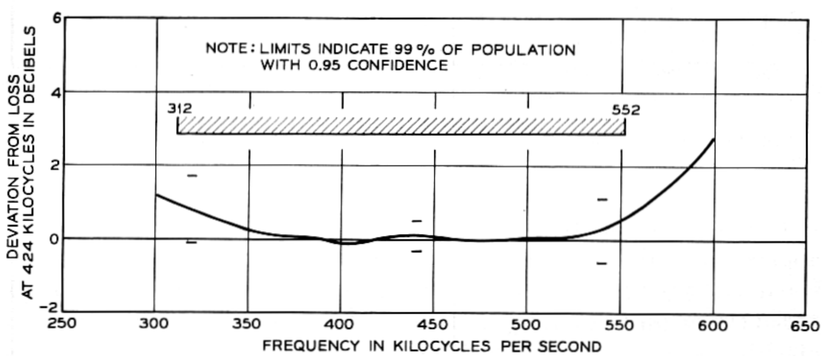


Fig. 19 — Typical modem amplitude-frequency characteristic of supergroup No. 5.

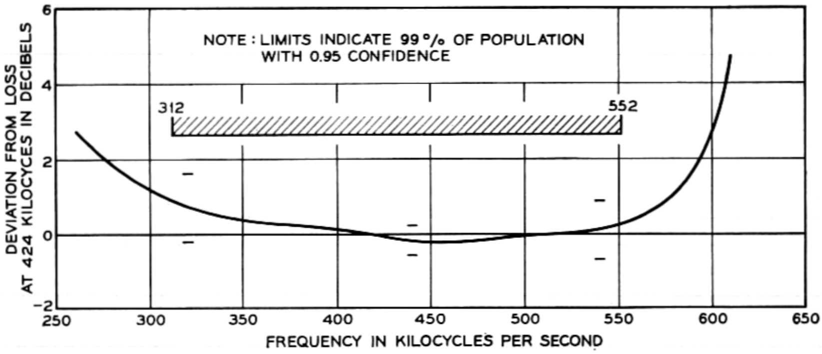


Fig. 20 — Typical modem amplitude-frequency characteristic of supergroup No. 6.

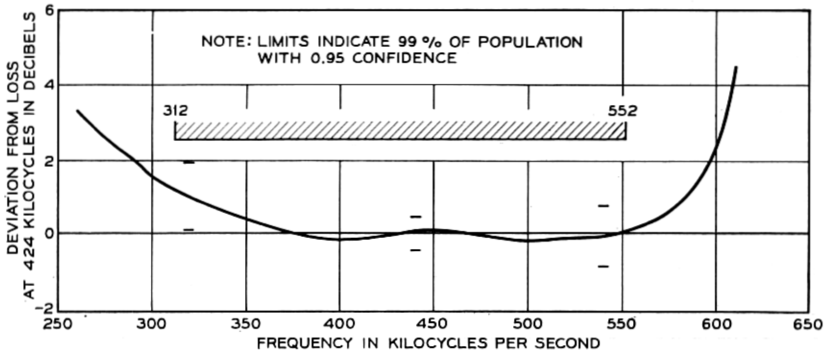


Fig. 21 — Typical modem amplitude-frequency characteristic of supergroup No. 7.

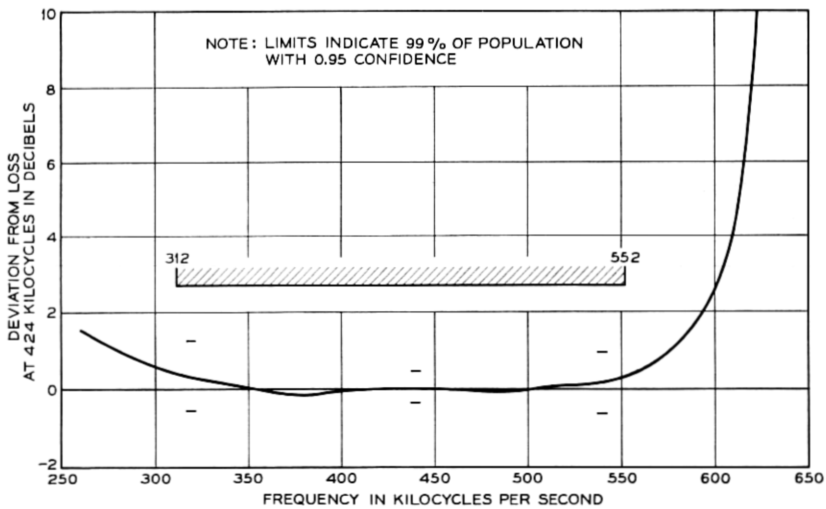


Fig. 22 — Typical modem amplitude-frequency characteristic of supergroup No. 8.

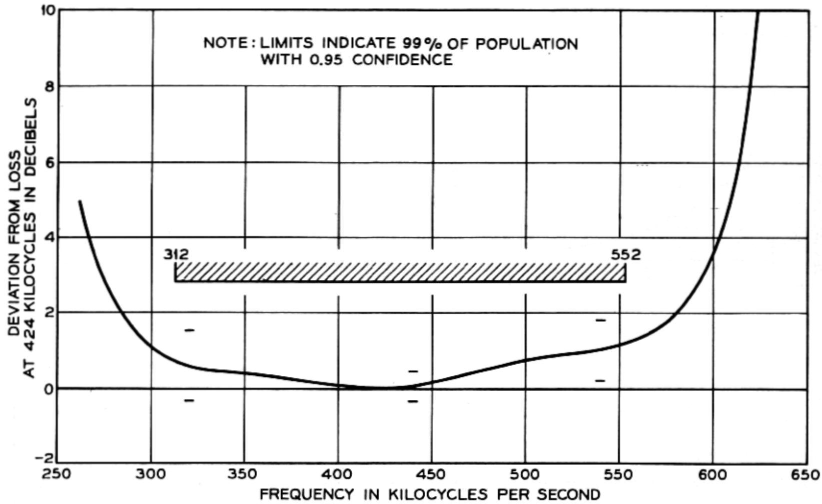


FIG. 23 — Typical modem amplitude-frequency characteristic of supergroup No. 9.

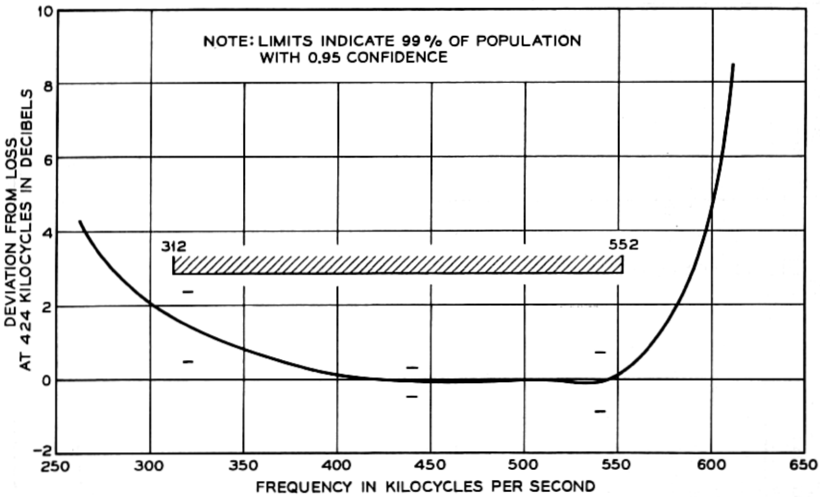


Fig. 24 — Typical modem amplitude-frequency characteristic of supergroup No. 10.

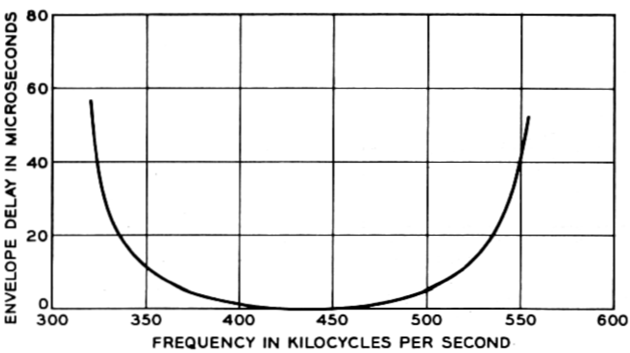


Fig. 25 — Envelope delay distortion for supergroup No. 1 of the LMX 1.

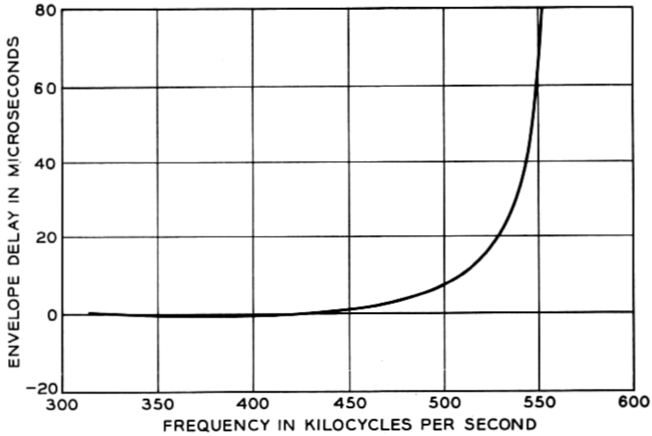


Fig. 26 — Envelope delay distortion for supergroup No. 3 of the LMX 1.

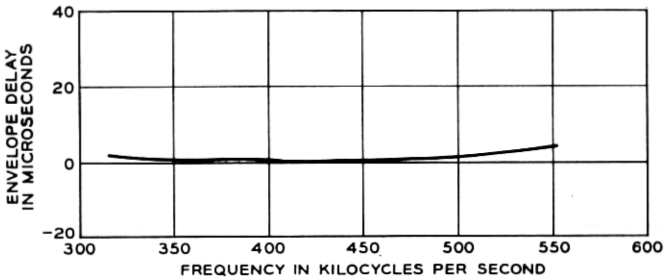


Fig. 27 — Envelope delay distortion for supergroups No. 2 and 4 through 10 of the LMX 1.

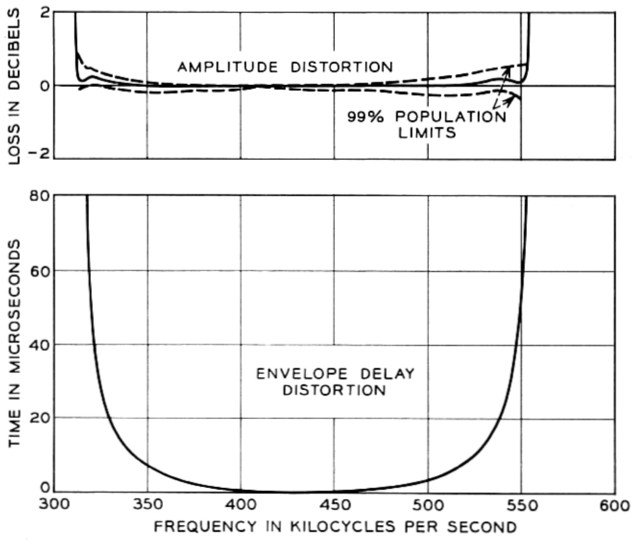


Fig. 28 — Typical amplitude and envelope delay distortion characteristics of the SG connector.

distribution (1), under the assumption that they represent only one variance, σ^2. If these ten points more or less fall on a straight line in the gamma plot, the assumption is adopted. More precisely, if the differences between the straight line values and the observations are less than, say, the 95 per cent confidence intervals of the observations, the fit may be considered good. Fig. 29 shows a typical example of the gamma plot for variances at 400 kc.

In the program of the gamma plot, σ^2 is an unknown constant. In general, it may be taken to equal one. Then the slope of the line fitted through the points would be an estimate of the pooled variance, $\hat{\sigma}^2$. Or vice versa, if another value than one is chosen and the slope of the straight line equals one, that chosen value for σ^2 would prove to be a good estimate. In the example shown in Fig. 29, the value for σ^2 inserted was 0.053 which proved to be a good estimate.

5.4 *Mastergroups and the Mastergroup Connector*

Basic mastergroup frequencies are in the band from 564 to 3084 kc. At present, the L-3 terminal combines three such mastergroups so they occupy line frequencies from 564 to 8284 kc.[7] Again, the three master-

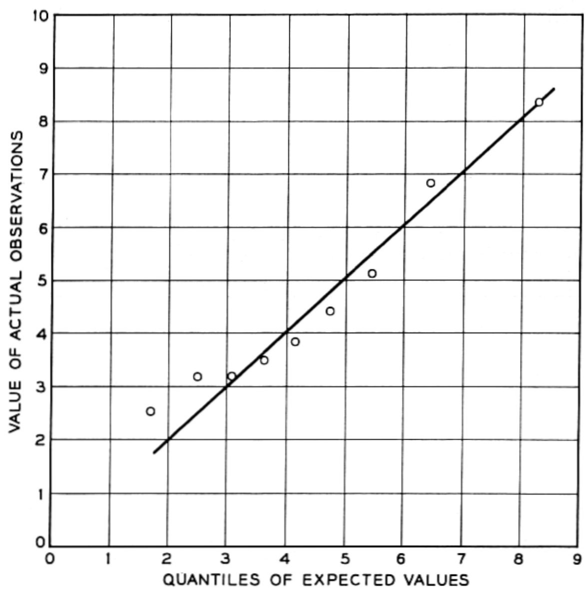


Fig. 29 — Typical gamma plot for supergroup variances at 400 kc.

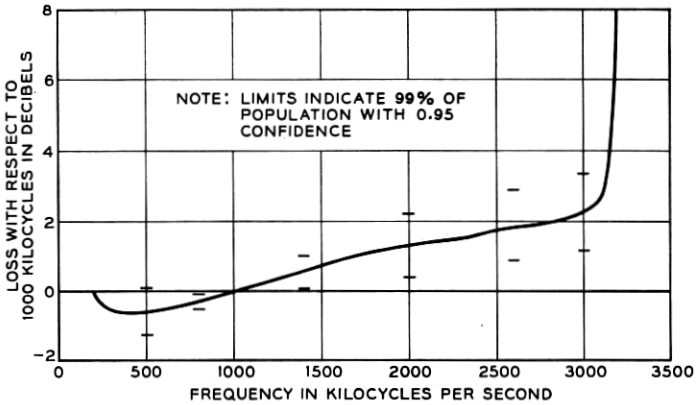


Fig. 30 — Typical modem amplitude-frequency characteristic of mastergroup No. 1.

groups are assembled into a bank, and the average characteristics shown in Figs. 30, 31, and 32 are taken of back-to-back transmitting and receiving mastergroups numbered 1, 2, and 3.

The mastergroup connector serves the same basic purpose as its group and supergroup counterparts. As this connector is relatively new however, with no sample of significant size yet measured, a statistical characterization has not yet been possible.

As was mentioned in Section 2.3, all the data have been assumed

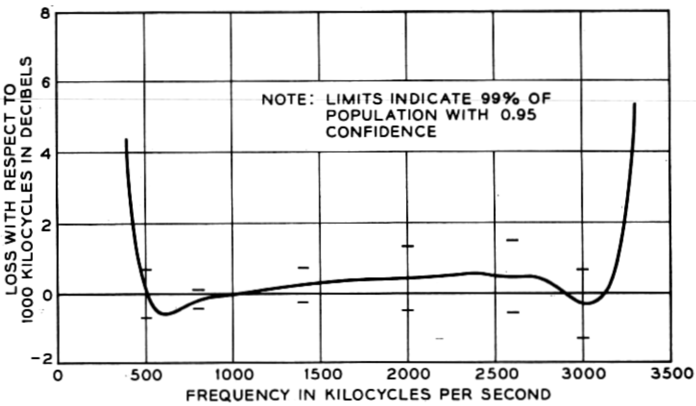


Fig. 31 — Typical modem amplitude-frequency characteristic of mastergroup No. 2.

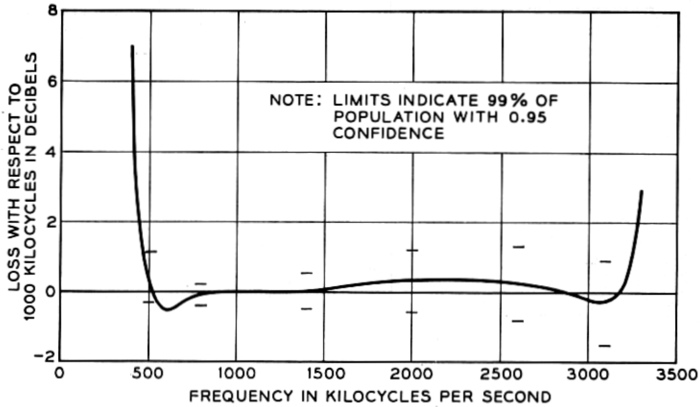


Fig. 32 — Typical modem amplitude-frequency characteristic of mastergroup No. 3.

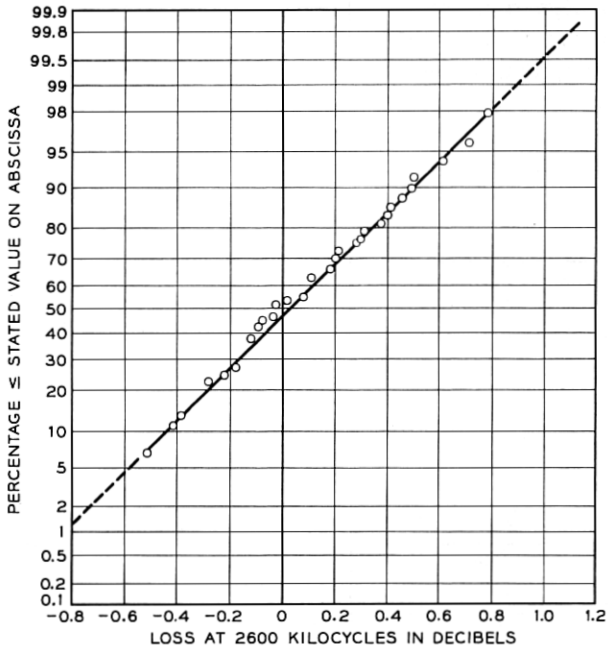


Fig. 33 — Typical example of distribution of back-to-back loss with respect to 1000 kc of a mastergroup (corrected for the mean).

normal. To justify this assumption, the sample distributions of residuals have been plotted on probability paper.* An example is shown in Fig. 33 for mastergroups where residual loss at 2600 kc relative to 1000 kc has been plotted in cumulative form. Similar plots have been made for groups and supergroups with similar results.

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* These residuals are not independent but could be made so by an orthogonal transformation of the observations. See also Ref. 8.