Analysis of Varactor Frequency Multipliers for Arbitrary Capacitance Variation and Drive Level

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A general analysis of varactor frequency multipliers is given. It applies to multipliers with any rational ratio of output to input frequency, any idler configuration, and any voltage-charge relationship for the diode. A computer program was written for evaluating the results up to a ×10 multiplier. Results are given for the doubler, 1-2-3 tripler, 1-2-4 quadrupler, 1-2-3-4 quadrupler, 1-2-4-5 quintupler, 1-2-4-6 sextupler, and 1-2-4-8 octupler, for various junctions, under conditions of nominal drive as well as of overdriving. Some higher-order multipliers without idlers, which use a high-nonlinearity diode, were computed. Some results are given for a model with varying series resistance.

I. INTRODUCTION

Varactor frequency multipliers have found considerable application for generating microwave signals for receiver local oscillators, parametric amplifier pumps, and other applications.

Penfield and Rafuse[1] have analyzed many multipliers under the assumption that a nominally driven abrupt-junction diode is used. For practical multipliers this assumption often does not hold because the junction is not abrupt and is driven into forward conduction. Greenspan[2] has analyzed the nominally driven graded-junction doubler, and Davis[3] has given an analysis of the overdriven doubler for various capacitance functions.

This paper gives a general analysis of lossy varactor frequency multipliers. The theory applies to multipliers with any rational ratio of output to input frequency, any idler configuration, and any capacitance variation of the diode. A computer program was written for evaluating the results up to a ×10 multiplier, and results are given for the doubler, tripler, quadrupler, quintupler, sextupler and octupler for different drive levels and different junction capacitance functions. Thus the work reported in this paper can be considered an extension and generalization of the work of the authors mentioned in the previous paragraph.

Circuit losses are neglected in this analysis,* and it is assumed that the only loss occurs in the diode. This restriction was introduced to reduce the number of parameters. Our analysis therefore gives the maximum efficiency that can be obtained with a given diode at a given frequency. Circuit losses usually can be accounted for either by assuming a lower cutoff frequency than that of the diode alone or by computing them separately.

The present analysis differs from that given by Morrison[4] mainly in two ways:

(a) it assumes a lossy diode, and

(b) it assumes that all the input power is converted to one single output frequency, whereas Ref. 4 is concerned mainly with the maximization of input power under the condition of output power at arbitrary frequencies.

Thus Morrison's results give an upper limit to the power which can be handled by a nonlinear capacitance for prescribed magnitudes of breakdown voltage and forward drive. In addition, he considers the specific examples of graded- and abrupt-junction doublers and triplers, which cases are included in the present results for the limiting lossless condition.

In the analysis in Sections II and III of this paper the diode loss is represented by a constant resistance in series with the variable capacitance. The modification of the analysis due to a variable series resistance is discussed in Section IV. The reader who is interested in the results only is referred to Section V, where all the results are given.

II. ANALYSIS

2.1 *Model and Assumptions*

The varactor model chosen is a variable capacitance in series with a constant resistance R_S (for R_S variable see Section IV) as shown in Fig. 1. If the applied voltage varies between the contact potential Φ and the breakdown voltage $-V_B$, the diode is said to be fully driven. The overdriven case will also be analyzed. It is assumed that during the period of forward conduction the junction voltage stays clamped to Φ, whereas

* The effect of lossy idlers has been included in the analysis and in the computer program; however, no results have been computed for this case.

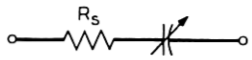

Fig. 1 — Varactor model.

the charge varies. Thus the capacitance in our model becomes infinite, and no correction for the rectified forward current is introduced. This model is suitable if the period of forward conduction is several times shorter than the time in which an appreciable number of minority carriers recombine.

It is assumed that currents in the diode flow only at the input, output, and idler frequencies, and that a suitable external circuit prevents other currents from flowing.

2.2 *Analysis*

The voltage across the variable capacitance can be written as a function of the charge on the capacitance

$$\left(\frac{v_j - \Phi}{V_B - \Phi}\right) = f\left(\frac{q - q_\Phi}{Q_B - q_\Phi}\right) \tag{1}$$

where

v_j : voltage across the capacitance,
Φ : contact potential,
V_B : breakdown voltage,
q : charge on the capacitance,
Q_B : charge at breakdown voltage,
q_Φ : charge at contact potential.

If we introduce the normalized quantities

$$\hat{q} = \frac{q - q_\Phi}{Q_B - q_\Phi}, \tag{2}$$

and

$$\varphi = \frac{v_j - \Phi}{V_B - \Phi}, \tag{3}$$

we can write (1) as

$$\varphi = f(\hat{q}). \tag{4}$$

For the voltage across the diode v_{tot} we have (see Fig. 1)

$$v_{\text{tot}} = v_j + R_S i, \tag{5}$$

where i is the current through the diode. Because of (2), (3) and $i = \partial q/\partial t$ we can write (5) as

$$v_{\text{tot}} = (V_B - \Phi)f(\hat{q}) + \Phi + R_S(Q_B - q_\Phi)(\partial\hat{q}/\partial t) \tag{6}$$

or

$$v_{\text{tot}} - \Phi = (V_B - \Phi)f(\hat{q}) + R_S(Q_B - q_\Phi)(\partial\hat{q}/\partial t). \tag{7}$$

$(v_{\text{tot}} - \Phi)$, φ, $\hat{q}$, and $\partial\hat{q}/\partial t$ can be written as Fourier series

$$(v_{\text{tot}} - \Phi) = \sum_{-\infty}^{+\infty} V_k e^{kj\omega_0 t},$$

$$\varphi = \sum_{-\infty}^{+\infty} \varphi_k e^{jk\omega_0 t},$$

$$\hat{q} = \sum Q_k e^{jk\omega_0 t},$$

$$\partial\hat{q}/\partial t = \sum I_k e^{jk\omega_0 t} = \sum jk\omega_0 Q_k e^{jk\omega_0 t}.$$

The series for $\hat{q}$ and $\partial\hat{q}/\partial t$ have to be summed over all frequencies at which current is flowing in the multiplier.

For the voltage at the load port V_l at frequency $l\omega_0$ we have the relation

$$V_l = (V_B - \Phi)\varphi_l + R_S(Q_B - q_\Phi)I_l = -Z_l(Q_B - q_\Phi)I_l\,, \tag{8}$$

where Z_l is the load impedance.

$$\therefore\ -Z_l/R_S = \frac{(V_B - \Phi)}{R_S(Q_B - q_\Phi)}\frac{\varphi_l}{I_l} + 1,$$

$$-Z_l/R_S = \kappa\frac{\varphi_l}{jl\omega_0 Q_l} + 1, \tag{9}$$

where we have introduced the quantity

$$\kappa = \frac{V_B - \Phi}{(Q_B - q_\Phi)R_S}. \tag{10}$$

κ has a simple relation to the familiar cutoff frequency ω_c of Penfield and Rafuse (Ref. 1, p. 86), which may be obtained as follows. The elastance S is defined as

$$S = \partial v_j/\partial q.$$

Because of (2) and (3)

$$S = \frac{\partial\varphi}{\partial\hat{q}}\frac{(V_B - \Phi)}{(Q_B - q_\Phi)}.$$

Therefore

$$\frac{S}{S_{\max} - S_{\min}} = \frac{\partial \varphi}{\partial \hat{q}} \frac{(V_B - \Phi)}{(Q_B - q_\Phi)R_S} \frac{R_S}{S_{\max} - S_{\min}}$$

$$= \frac{\partial \varphi}{\partial \hat{q}} \frac{\kappa}{\omega_c}, \tag{11}$$

where $S_{\max}$ is the maximum value of the elastance, $S_{\min}$ is the minimum value of the elastance and

$$\omega_c = \frac{S_{\max} - S_{\min}}{R_S}$$

is the cutoff frequency defined by Penfield and Rafuse.

If we set $S = S_{\max}$ in (11) we obtain

$$\frac{S_{\max}}{S_{\max} - S_{\min}} = \left.\frac{\partial \varphi}{\partial \hat{q}}\right|_{\hat{q}=1} \cdot \frac{\kappa}{\omega_c}$$

and

$$\kappa = \frac{\omega_c}{\left.\frac{\partial \varphi}{\partial \hat{q}}\right|_{\hat{q}=1}} \cdot \frac{S_{\max}}{S_{\max} - S_{\min}}. \tag{12}$$

This relation between κ and ω_c is dependent on the nonlinearity of the diode. E.g., if we assume an abrupt junction with $\varphi = \hat{q}^2$ and $S_{\min} = 0$ we obtain $\kappa = 0.5\,\omega_c$.

We now want to solve (9) for the real and imaginary parts of Z_l. Fig. 2 shows the quantities of interest plotted in the complex plane. The input current I_{in} is assumed to be real and positive. With the phase angles as defined we obtain from (9)

$$-R_l/R_S = \kappa \frac{|\varphi_l|}{|Q_l|} \frac{1}{l\omega_0} \sin(-\alpha + \beta_l) + 1.$$

If we solve this for $|Q_l|$ we obtain

$$|Q_l| = \frac{\kappa\,|\varphi_l| \sin(-\alpha + \beta_l)}{(-R_l/R_S - 1)\,l\omega_0}. \tag{13}$$

For the imaginary part X_l we obtain

$$X_l/R_S = \kappa \frac{|\varphi_l|}{|Q_l|} \frac{1}{l\omega_0} \cos(-\alpha + \beta_l). \tag{14}$$

If we define an effective output elastance S_{effout} according to $S_{\text{effout}}/\omega_0 l = X_l$ we obtain

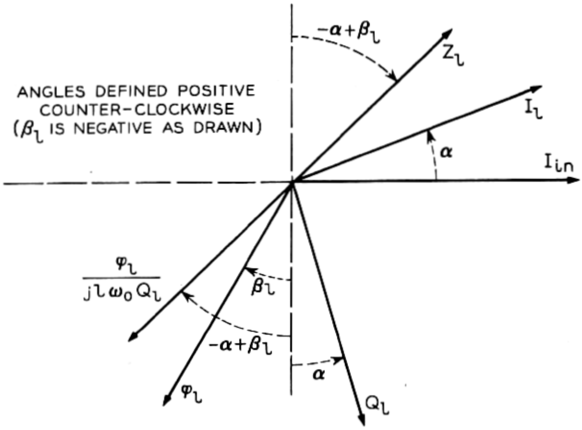

Fig. 2 — Output quantities in complex plane.

$$\frac{S_{\text{effout}}}{S_{\max} - S_{\min}} = \frac{\omega_0 l X_l}{S_{\max} - S_{\min}} = \frac{R_S \kappa}{S_{\max} - S_{\min}} \frac{|\varphi_l|}{|Q_l|} \cos(-\alpha + \beta_l)$$

$$\frac{S_{\text{effout}}}{S_{\max} - S_{\min}} = \frac{\kappa}{\omega_c} \frac{|\varphi_l|}{|Q_l|} \cos(-\alpha + \beta_l). \tag{15}$$

Idler ports: The idler impedance Z_i at the frequency $i\omega_0$ is

$$-Z_i/R_S = \kappa \frac{\varphi_i}{j \cdot i\omega_0 Q_i} + 1. \tag{16}$$

We now assume that all the idler ports are tuned,* i.e., we assume the idler currents to be in phase with I_{in}. We then obtain

$$-R_i/R_S = \kappa \frac{|\varphi_i|}{|Q_i|} \frac{1}{i\omega_0} \sin \beta_i + 1 \tag{17}$$

and

$$|Q_i| = \frac{\kappa |\varphi_i| \sin \beta_i}{(-R_i/R_S - 1)\, i\omega_0}, \tag{18}$$

where R_i is the idler resistance and β_i is defined analogous to β_l. For the effective idler elastance S_{effidl} we obtain

* Penfield and Rafuse[1] find that tuning the output circuit gives near optimum efficiency for the nominally driven abrupt-junction doubler. Davis[3] obtains the same result for a variety of nominally driven as well as overdriven abrupt- and graded-junction doublers. Judging from these results, it was felt that tuning the idlers leads to near optimum efficiency, and the above-mentioned restriction was introduced.

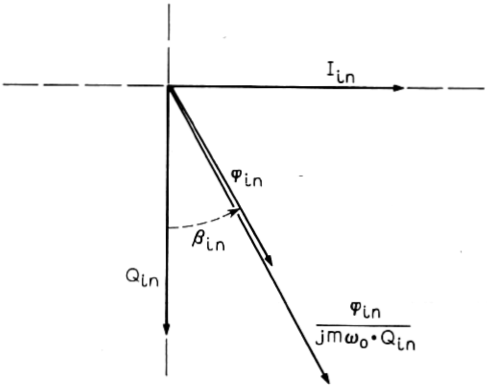

Fig. 3 — Input quantities in complex plane.

$$\frac{S_{\text{effidl}}}{S_{\max} - S_{\min}} = \frac{\kappa}{\omega_c} \frac{|\varphi_i|}{|Q_i|} \cos \beta_i . \tag{19}$$

Input port: The input impedance Z_{in} at $m\omega_0$ is

$$Z_{\text{in}} = \frac{\text{input voltage}}{\text{input current}}$$

$$Z_{\text{in}} = \frac{(V_B - \Phi)\varphi_{\text{in}}}{I_{\text{in}}(Q_B - q_\Phi)} + R_S ,$$

$$Z_{\text{in}}/R_S = \frac{\kappa\varphi_{\text{in}}}{jm\omega_0 Q_{\text{in}}} + 1. \tag{20}$$

The input quantities are plotted in Fig. 3. For the input resistance R_{in} we obtain

$$R_{\text{in}}/R_S = \frac{\kappa |\varphi_{\text{in}}|}{m\omega_0 |Q_{\text{in}}|} \sin \beta_{\text{in}} + 1 \tag{21}$$

and for the imaginary part X_{in}

$$X_{\text{in}}/R_S = -\frac{\kappa |\varphi_{\text{in}}|}{m\omega_0 |Q_{\text{in}}|} \cos \beta_{\text{in}} .$$

$$\frac{S_{\text{effin}}}{S_{\max} - S_{\min}} = \frac{\kappa}{\omega_c} \frac{|\varphi_{\text{in}}|}{|Q_{\text{in}}|} \cos \beta_{\text{in}} . \tag{22}$$

Power relations and efficiency: The input power P_{in} is

$$P_{\text{in}} = 2 |I_{\text{in}}|^2 R_{\text{in}} (Q_B - q_\Phi)^2$$

(I_{in} is a half-amplitude). Introducing the normalization power P_{norm}

$$P_{\text{norm}} = \frac{(V_B - \Phi)^2}{R_S}$$

we can write

$$P_{\text{in}}/P_{\text{norm}} = 2m^2\omega_0^2 \mid Q_{\text{in}} \mid^2 R_{\text{in}} \frac{(Q_B - q_\Phi)^2}{(V_B - \Phi)^2} R_S ,$$

$$P_{\text{in}}/P_{\text{norm}} = \frac{2m^2\omega_0^2 \mid Q_{\text{in}} \mid^2}{\kappa^2} (R_{\text{in}}/R_S). \tag{23}$$

Similarly, we obtain for the normalized output power $P_{\text{out}}/P_{\text{norm}}$

$$P_{\text{out}}/P_{\text{norm}} = \frac{2l^2\omega_0^2 \mid Q_l \mid^2}{\kappa^2} (R_l/R_S). \tag{24}$$

For the normalized power dissipated in the idler resistances $P_{\text{diss1}}/P_{\text{norm}}$ we obtain

$$P_{\text{diss1}}/P_{\text{norm}} = 2\frac{\omega_0^2}{\kappa^2} \sum_i i^2 \mid Q_i \mid^2 (R_i/R_S), \tag{25}$$

where the sum has to be extended over all frequencies at which there are idlers. The normalized power dissipated in the series resistance R_S of the diode, $P_{\text{diss2}}/P_{\text{norm}}$ is

$$P_{\text{diss2}}/P_{\text{norm}} = 2\frac{\omega_0^2}{\kappa^2} \sum_k k^2 \mid Q_k \mid^2, \tag{26}$$

with the sum extended over all frequencies at which there are currents flowing. The total power dissipated P_{diss} is

$$P_{\text{diss}} = P_{\text{diss1}} + P_{\text{diss2}} . \tag{27}$$

For the efficiency ϵ we obtain

$$\epsilon = 1 - (P_{\text{diss}}/P_{\text{in}}) \tag{28}$$

$$= P_{\text{out}}/P_{\text{in}} . \tag{29}$$

Bias voltage: The normalized bias voltage

$$\varphi_0 = \left(\frac{V_0 - \Phi}{V_B - \Phi}\right)$$

is equal to the constant coefficient in the Fourier series for φ.

Evidently the foregoing analysis holds for any integrally related input, output, and idler frequencies and therefore can be applied to any corresponding multiplier configuration.

III. TECHNIQUE OF SOLUTION AND OUTLINE OF THE COMPUTER PROGRAM

For a multiplier the following quantities are usually given physically:

(a) input, output, and idler frequencies,

(b) the voltage-charge relation of the diode and its series resistance R_S, and

(c) the idler resistances R_i.

The following quantities will be treated as parameters, i.e., they will be set constant for one evaluation and will be varied later in order to optimize a certain quantity, usually the efficiency. These parameters are:

(d) the minimum charge $Q_{\min}$ and the maximum charge $Q_{\max}$ between which the diode is driven,

(e) the load resistance R_l of the multiplier, and

(f) the angle of output detuning α.*

The formulas of Section 2.2 now have to be evaluated subject to these constraints. The solution proceeds in a way suggested physically. A current is applied at the input port. This current gives rise to certain voltages at the idler and output ports which in turn cause certain currents at these ports. The calculation of the voltages at the idler and output ports is repeated assuming the diode to be driven by the sum of the currents computed previously. The computation is repeated until the values of all the currents have reached their asymptotic values.

Fig. 4 shows a simplified block diagram of the computer program. First, a value for the input charge coefficient $| Q_{\text{in}} |$ and a value for the load resistance R_l are assumed. The normalized charge $\hat{q}$ is computed at n equidistant points lying in one period of the first harmonic. The maximum and minimum values of $\hat{q}$ are determined† and compared with the values of $\hat{q}_{\max}$ and $\hat{q}_{\min}$ prescribed. The values of $| Q_k |$ are corrected‡ in order to give a variation of $\hat{q}$ between $\hat{q}_{\max}$ and $\hat{q}_{\min}$ and $\hat{q}$ is computed again. From this and the voltage charge law of the diode (4) the normalized voltage φ is computed. The Fourier coefficients φ_k of φ are then evaluated by a subprogram as described in Ref. 5. From the values of

* For all the computations in this paper $\alpha = 0°$ for the same reason given in the footnote to Section 2.2.

† The maximum and minimum values of the n values of $\hat{q}$ are obtained by a sorting routine. The maximum and minimum of $\hat{q}$ are found by quadratic interpolation.

‡ Shift of $| Q_0 |$; compression or expansion of $| Q_k |$, $k \geqq 1$.

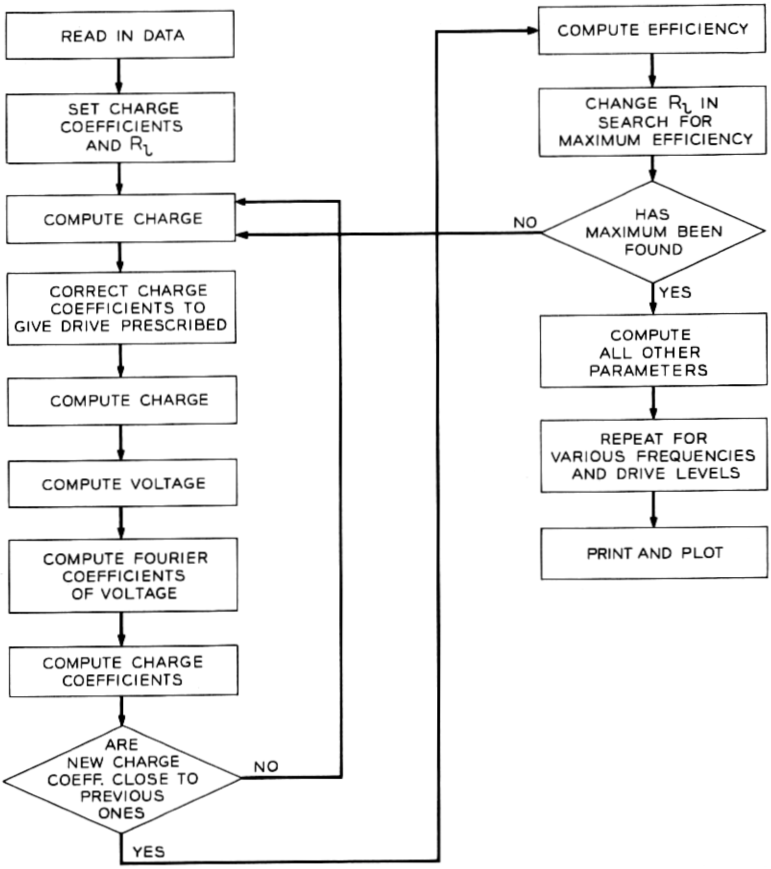

Fig. 4 — Simplified flow diagram of computer program.

φ_k and (13) and (18) new values can be computed for the charge coefficients. The computation is repeated until all the charge coefficients computed in step $(n + 1)$ are very close to those computed in step n. The computation as described does not converge if the multiplier has idlers but rather gives an oscillatory solution for the charge coefficients. It can be stabilized in the following way. Suppose $| Q_i |_{n+1}'$ is the idler charge coefficient computed in step $(n + 1)$ according to (18). One then computes $| Q_i |_{n+1}$ as a weighted average of $| Q_i |_{n+1}'$ and the charge coefficient $| Q_i |_n$ computed in the previous step,

$$| Q_i |_{n+1} = \frac{w | Q_i |_n + | Q_i |_{n+1}'}{w + 1}. \tag{30}$$

The problem now is to find a suitable value for w. If w is too small the solution does not converge; if it is too big the convergence is very slow. Numerous experiments yielded the following (heuristic) formula for w,

$$w = \frac{0.15(m + 1)\omega_c \sqrt{n}}{\omega_0} \tag{31}$$

where m is the number of idlers at higher frequency than the idler frequency at which $| Q_i |$ is being computed; n is the total number of idlers in the multiplier. Equation (31) gave a good initial choice for the values of w. In some cases these values had to be modified slightly, either to speed up convergence or to prevent oscillation. The charge coefficient $| Q_l |$ at the output frequency can be computed according to (13).

When the final values of the $| Q_k |$'s have been reached, the efficiency is computed using (28). For small values of ϵ, (29) is used because it gives more accurate results. R_l is then changed, and the computation is repeated. R_l is changed until a point of maximum efficiency is reached. The optimum value of R_l is then found by quadratic interpolation, and the computation is carried out a last time with this optimum value. After this the values of S_{eff} are computed. The computation is carried out for a number of ratios ω_0/ω_c and a number of drive levels.

The accuracy of the computation was checked by computing the nominally driven abrupt-junction doubler, 1-2-3 tripler, 1-2-4-8 octupler, and the graded-junction doubler. All the results agreed with known results[1,2] within plotting accuracy.

IV. EFFECT OF A VARYING SERIES RESISTANCE R_S

For point contact varactors the model of Fig. 1 is a good representation. For epitaxial varactors, however, R_S depends significantly on bias voltage,[6] and R_S should be considered a function of $\hat{q}$ in our computation. Equation (7) then becomes

$$v_{\text{tot}} - \Phi = (V_B - \Phi)f(\hat{q}) + (Q_B - q_\Phi)(\partial\hat{q}/\partial t)R_S(\hat{q}). \tag{32}$$

Because (32) seemed to be too difficult to be solved exactly a *semi-rigid* approach was made. From the results for the constant series resistance model we know that, for low and moderately high frequencies, input and load resistances of a multiplier are several times bigger than R_S. We conclude that the second term in (32) is small compared with the first one. We normalize $R_S(\hat{q})$ with respect to some value R_{S0} chosen arbitrarily. The computation proceeds as in Section II and *only* the computation of the power dissipated $P_{\text{diss}2}$ in $R_S(\hat{q})$ is *modified* as follows

$$P_{\text{diss2}} = \frac{1}{2\pi} \int_{w_0 t=0}^{2\pi} R_S(\hat{q}) i^2 d(\omega_0 t), \tag{33}$$

where i is the instantaneous current in R_S. With

$$\hat{q} = \sum 2 \mid Q_k \mid \sin k\omega_0 t$$

one obtains

$$P_{\text{diss2}}/P_{\text{norm}} = \frac{\omega_0^2}{2\pi\kappa^2} \int_{w_0 t=0}^{2\pi} \frac{R_S(\hat{q})}{R_{S0}} [\sum 2 \mid Q_k \mid k \cos k\omega_0 t]^2 \, d(\omega_0 t). \tag{34}$$

Equation (34) was substituted for (26) in the program. The integral in (34) was evaluated numerically.

V. RESULTS

5.1 *Computations with R_S Constant*

The results reported in this section were computed for a constant series resistance model and a voltage-charge relation of the form

$$\frac{v - \Phi}{V_B - \Phi} = \left(\frac{q - q_\Phi}{Q_B - q_\Phi}\right)^{1/(1-\gamma)}. \tag{35}$$

Values of $\gamma = 0.5$ (abrupt), $\gamma = 0.4$, $\gamma = 0.333$ (graded), and $\gamma = 0$ were used.* Results were computed for different drive levels.

Plots of all the quantities versus ω_0/ω_c were obtained using the IBM 7094 computer and the SC-4020 microfilm printer. Lack of space prohibits the reproduction of these plots. Instead, the constants occurring in the low-frequency approximations used by Rafuse[7] will be given. These low-frequency approximations are

$$\begin{aligned} \epsilon &= \exp\,(-\alpha\omega_{\text{out}}/\omega_c),\dagger \\ P_{\text{out}}/P_{\text{norm}} &= \beta(\omega_0/\omega_c), \\ R_{\text{in}}/R_S &= A\,(\omega_c/\omega_0), \\ R_l/R_S &= B\,(\omega_c/\omega_0). \end{aligned}$$

The normalized elastances as well as the normalized bias voltage do not depend on ω_0/ω_c at low frequencies. For the tripler and higher-order multipliers the output power versus frequency shows a maximum. This value, designated $P_{\max}/P_{\text{norm}}$, and the corresponding frequency, desig-

* $\gamma = 0$ corresponds to a step in elastance at $\hat{q} = 0$. $S = 0$ for $\hat{q} \leq 0$ and $S =$ a constant for $\hat{q} \geq 0$.

† α was computed from the value of ϵ at $\omega_0 = 10^{-2}\omega_c$.

TABLE I — DOUBLER

	$\gamma = 0.0$		$\gamma = 0.333$			$\gamma = 0.4$			$\gamma = 0.5$	
Drive →	1.5	2.0	1.0	1.3	1.6	1.0	1.3	1.6	1.3	1.6
α	6.7	4.7	12.6	8.0	6.9	11.1	8.0	7.2	8.3	8.3
β	0.0222	0.0626	0.0118	0.0329	0.0587	0.0168	0.0406	0.0678	0.0556	0.0835
A	0.117	0.213	0.0636	0.101	0.126	0.0730	0.102	0.118	0.0980	0.0977
B	0.204	0.211	0.0976	0.158	0.172	0.112	0.157	0.161	0.151	0.151
S_{01}/S_{max}	0.73	0.50	0.68	0.52	0.40	0.61	0.45	0.35	0.37	0.28
S_{02}/S_{max}	0.60	0.50	0.66	0.48	0.41	0.59	0.44	0.38	0.40	0.34
V_{0norm}	0.35	0.25	0.41	0.33	0.27	0.39	0.31	0.26	0.28	0.24

nated ω_{0max}/ω_c, also will be given. These values will help to give a better reconstruction of the actual curves.

The results are contained in Tables I–VIII. The notation 1-2-3-4 quadrupler means that there are idlers at $2\omega_0$ and $3\omega_0$. A 1-8 octupler has no idlers.

The drive is defined as

$$\text{drive} = \frac{Q_{max} - Q_{min}}{Q_B - q_\Phi}. \tag{36}$$

Thus drive $= 1$ corresponds to nominal drive and drive > 1 corresponds to overdriving the junction. For all drive levels it was assumed that the junction was driven up to the charge corresponding to breakdown in the reverse direction.

Tables I–VII show that the power handling capability as well as the efficiency can be increased by overdriving the diode. This increase in efficiency is most pronounced for diodes having low values of γ. For the same cutoff frequency the highest efficiency is obtained for a junction with $\gamma = 0$ and drive $= 2.0$, as is seen in Tables I and III for the doubler and the 1-2-4 quadrupler.

Sometimes the question is asked: What efficiency can one obtain for a multiplier without idlers and a high-nonlinearity diode, and do idlers improve the performance of such a multiplier? To answer this question, a quadrupler, sextupler, and octupler without any idlers were computed for a diode with $\gamma = 0$. The highest efficiency was obtained for drive $=$ 2.0 for all these multipliers. The results of Table VIII show that the efficiencies of these multipliers are about as high as those of the nominally driven abrupt-junction 1-2-3-4 quadrupler,[8] 1-2-4-6 sextupler, and 1-2-4-8 octupler.[1] The power handling capability of the 1-4 quadrupler is lower than that of the abrupt-junction 1-2-3-4 quadrupler, and it compares even less favorably for the sextupler and octupler. Compari-

TABLE II — 1-2-3 TRIPLER

	$\gamma = 0.0$	$\gamma = 0.333$			$\gamma = 0.4$			$\gamma = 0.5$	
Drive →	1.5	1.0	1.3	1.6	1.0	1.3	1.6	1.3	1.6
α	7.0	14.2	9.0	8.1	12.5	8.6	8.6	9.4	9.8
β	0.0212	0.0101	0.0281	0.0490	0.0144	0.0345	0.0563	0.0475	0.0700
P_{max}/P_{norm}	$7.5 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	$8 \cdot 10^{-4}$	$1.4 \cdot 10^{-3}$	$3.0 \cdot 10^{-4}$	$9.6 \cdot 10^{-4}$	$1.5 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$
ω_{0max}/ω_c	10^{-1}	$7.0 \cdot 10^{-2}$	10^{-1}	10^{-1}	$8.0 \cdot 10^{-2}$	10^{-1}	10^{-1}	10^{-1}	10^{-1}
A	0.185	0.104	0.170	0.214	0.120	0.172	0.200	0.168	0.172
B	0.0878	0.0471	0.0753	0.0871	0.0542	0.0755	0.0818	0.0728	0.0722
S_{01}/S_{max}	0.80	0.69	0.54	0.41	0.62	0.47	0.35	0.36	0.26
S_{02}/S_{max}	0.54	0.67	0.50	0.40	0.60	0.45	0.37	0.38	0.31
S_{03}/S_{max}	0.72	0.67	0.52	0.42	0.61	0.46	0.37	0.38	0.30
V_{0norm}	0.32	0.39	0.29	0.22	0.37	0.27	0.20	0.24	0.18

TABLE III — 1-2-4 QUADRUPLER

	$\gamma = 0.0$		$\gamma = 0.333$			$\gamma = 0.4$			$\gamma = 0.5$	
Drive →	1.5	2.0	1.0	1.3	1.6	1.0	1.3	1.6	1.3	1.6
α	11.1	10.3	19.3	12.6	12.2	17.1	12.9	12.9	13.6	14.1
β	0.0154	0.0298	0.0082	0.0224	0.0351	0.0116	0.0271	0.0410	0.0368	0.0530
P_{max}/P_{norm}	$1.8 \cdot 10^{-4}$	$4.0 \cdot 10^{-4}$	$6.2 \cdot 10^{-5}$	$2.3 \cdot 10^{-4}$	$4.0 \cdot 10^{-4}$	$1.0 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$4.3 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$	$5.3 \cdot 10^{-4}$
ω_{0max}/ω_c	$3.2 \cdot 10^{-2}$	$3.1 \cdot 10^{-2}$	$2.3 \cdot 10^{-2}$	$3.3 \cdot 10^{-2}$	$3.3 \cdot 10^{-2}$	$2.4 \cdot 10^{-2}$	$3.0 \cdot 10^{-2}$	$3.3 \cdot 10^{-2}$	$3.0 \cdot 10^{-2}$	$2.4 \cdot 10^{-2}$
A	0.230	0.281	0.115	0.188	0.215	0.132	0.188	0.202	0.180	0.176
B	0.0754	0.101	0.0409	0.0623	0.0719	0.0456	0.0627	0.0688	0.0605	0.0613
S_{01}/S_{max}	0.73	0.50	0.68	0.53	0.40	0.61	0.46	0.35	0.36	0.27
S_{02}/S_{max}	0.73	0.50	0.68	0.53	0.40	0.61	0.46	0.35	0.37	0.27
S_{04}/S_{max}	0.87	0.50	0.69	0.56	0.41	0.62	0.48	0.34	0.36	0.24
V_{0norm}	0.33	0.25	0.40	0.31	0.25	0.38	0.29	0.23	0.26	0.21

TABLE IV — 1-2-3-4 QUADRUPLER

	$\gamma = 0.333$			$\gamma = 0.5$	
Drive →	1.0	1.3	1.6	1.3	1.6
α	14.1	8.9	8.1	9.4	9.7
β	0.0094	0.0260	0.0438	0.0439	0.0647
P_{max}/P_{norm}	$1.1\cdot10^{-4}$	$4.8\cdot10^{-4}$	$8.2\cdot10^{-4}$	$7.4\cdot10^{-4}$	$1.1\cdot10^{-3}$
ω_{0max}/ω_c	$3.0\cdot10^{-2}$	$6.4\cdot10^{-2}$	$6.3\cdot10^{-2}$	$5.2\cdot10^{-2}$	$6.0\cdot10^{-2}$
A	0.0719	0.118	0.155	0.120	0.122
B	0.0489	0.0797	0.0927	0.0748	0.0729
S_{01}/S_{max}	0.69	0.55	0.40	0.36	0.25
S_{02}/S_{max}	0.66	0.48	0.41	0.39	0.34
S_{03}/S_{max}	0.67	0.51	0.42	0.38	0.31
S_{04}/S_{max}	0.67	0.50	0.40	0.38	0.30
V_{0norm}	0.40	0.30	0.23	0.25	0.20

TABLE V — 1-2-4-5 QUINTUPLER

	$\gamma = 0.333$			$\gamma = 0.5$	
Drive →	1.0	1.3	1.6	1.3	1.6
α	21.4	14.5	14.8	15.8	16.6
β	0.0072	0.0198	0.0310	0.0326	0.0470
P_{max}/P_{norm}	$4.2\cdot10^{-5}$	$1.6\cdot10^{-4}$	$2.4\cdot10^{-4}$	$2.5\cdot10^{-4}$	$3.4\cdot10^{-4}$
ω_{0max}/ω_c	$1.4\cdot10^{-2}$	$2.2\cdot10^{-2}$	$2.0\cdot10^{-2}$	$2.2\cdot10^{-2}$	$2.2\cdot10^{-2}$
A	0.104	0.170	0.203	0.167	0.163
B	0.0315	0.0524	0.0592	0.0485	0.0470
S_{01}/S_{max}	0.69	0.54	0.39	0.36	0.25
S_{02}/S_{max}	0.69	0.54	0.40	0.36	0.26
S_{04}/S_{max}	0.68	0.53	0.41	0.37	0.28
S_{05}/S_{max}	0.67	0.49	0.40	0.38	0.32
V_{0norm}	0.40	0.29	0.23	0.24	0.19

TABLE VI — 1-2-4-6 SEXTUPLER

	$\gamma = 0.333$			$\gamma = 0.5$	
Drive →	1.0	1.3	1.6	1.3	1.6
α	19.6	13.0	11.3	13.4	13.7
β	0.0086	0.0239	0.0419	0.0405	0.0598
P_{max}/P_{norm}	$4.1\cdot10^{-5}$	$1.7\cdot10^{-4}$	$3.3\cdot10^{-4}$	$2.7\cdot10^{-4}$	$4.0\cdot10^{-4}$
ω_{0max}/ω_c	$1.4.10^{-2}$	$2.3\cdot10^{-2}$	$2.0\cdot10^{-2}$	$2.5\cdot10^{-2}$	$2.3\cdot10^{-2}$
A	0.0877	0.145	0.177	0.140	0.140
B	0.0179	0.0290	0.0314	0.0271	0.0259
S_{01}/S_{max}	0.69	0.54	0.40	0.36	0.26
S_{02}/S_{max}	0.68	0.52	0.40	0.37	0.28
S_{04}/S_{max}	0.69	0.56	0.41	0.36	0.24
S_{06}/S_{max}	0.68	0.53	0.41	0.37	0.28
V_{0norm}	0.40	0.32	0.26	0.27	0.22

TABLE VII — 1-2-4-8 OCTUPLER

	$\gamma = 0.333$			$\gamma = 0.5$	
Drive →	1.0	1.3	1.6	1.3	1.6
α	28.4	17.8	13.9	17.7	17.5
β	0.0071	0.0205	0.0380	0.0355	0.0537
P_{max}/P_{norm}	$1.7 \cdot 10^{-5}$	$7.2 \cdot 10^{-5}$	$1.6 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$
ω_{0max}/ω_c	$5.0 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	$1.1 \cdot 10^{-2}$	$8.0 \cdot 10^{-3}$	$9.0 \cdot 10^{-3}$
A	0.0795	0.129	0.153	0.125	0.124
B	0.0156	0.0220	0.0255	0.0217	0.0212
S_{01}/S_{max}	0.68	0.53	0.40	0.37	0.27
S_{02}/S_{max}	0.68	0.53	0.41	0.37	0.28
S_{04}/S_{max}	0.68	0.52	0.41	0.37	0.28
S_{08}/S_{max}	0.68	0.50	0.39	0.37	0.30
V_{0norm}	0.41	0.33	0.28	0.28	0.24

son of Table VIII and Table III shows that an idler at $2\omega_0$ improves both the efficiency and power handling capability of the quadrupler using a diode with $\gamma = 0$.

5.2 *Computations with R_S Variable*

Computations were carried out for the doubler and 1-2-3 tripler with abrupt and graded junctions. The series resistance was assumed to be that of the semiconductor wafer *only:* i.e., any contribution due to contact resistance etc. was neglected. The normalization resistance R_{S0} was arbitrarily chosen at $\hat{q} = 0.5$.

With this normalization we obtain (Ref. 1, pp. 515–6)

$$R_S/R_{S0} = 2(1 - \hat{q}) \tag{37}$$

for the abrupt junction and

$$R_S/R_{S0} = 2.91 \ln \hat{q}^{-\frac{1}{2}} \tag{38}$$

TABLE VIII — MULTIPLIERS WITHOUT IDLERS
($\gamma = 0$, Drive $= 2.0$)

Multiplier	1-4	1-6	1-8
α	11.8	17.6	21.7
β	0.0144	0.0063	0.0034
P_{max}/P_{norm}	$2.2 \cdot 10^{-4}$	$4.1 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
ω_{0max}/ω_c	$1.0 \cdot 10^{-1}$*	$1.0 \cdot 10^{-1}$*	$3.0 \cdot 10^{-2}$*
A	0.0415	0.0175	0.0098
B	0.0430	0.0189	0.0106
S_{01}/S_{max}	0.50	0.50	0.50
S_{0out}/S_{max}	0.50	0.50	0.50
V_{0norm}	0.27	0.28	0.29

* Broad maxima.

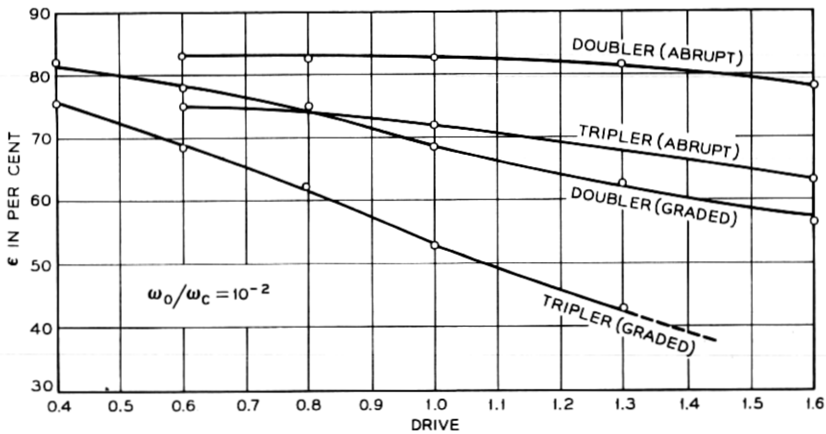

Fig. 5 — Efficiency of variable R_S multipliers vs drive level.

for the graded junction. Equation (38) has a pole at $\hat{q} = 0$ which in reality does not occur because of the nonzero intrinsic conductivity of the material. Arbitrarily, it was assumed that R_S/R_{S0} varied according to (38) for $\hat{q} \geqq 0.005$ and had the value corresponding to $\hat{q} = 0.005$ for $\hat{q} \leqq 0.005$.

Fig. 5 shows the efficiency of the abrupt- and graded-junction doublers and triplers as a function of the drive level at $\omega_0/\omega_c = 10^{-2}$. The efficiencies of the abrupt-junction multipliers decrease slightly; those of the graded-junction multipliers decrease more rapidly with increasing drive level. These results are in contrast to the results for the model with constant series resistance. For the graded junction the values of efficiency for drive > 1, of course, depend on the value of $\hat{q}$ below which R_S/R_{S0} is assumed to be constant. The values for drive < 1, however, do not.

The complete tables for these multipliers are not included here, because all the values can be deduced from the results of the constant series computation as follows:

All the values *except efficiency* are identical to the results for the con-

TABLE IX — VALUES OF k FOR VARIOUS DRIVE VALUES

Drive	1.0	1.3	1.6
Abrupt-junction doubler	1.0	1.2	1.5
Abrupt-junction tripler	1.0	—	1.5
Graded-junction doubler	1.5	3.0	4.0
Graded-junction tripler	1.5	2.8	4.0

stant series resistance computation within 10 per cent. The values for the efficiency can be read from the tables for the constant R_S model. To do so one multiplies the input frequency ω_0 of the variable R_S multiplier by k and computes the efficiency at the frequency

$$\omega_0{}' = k\omega_0$$

using the value of α given in the constant R_S tables. These values agree with the computed values within 3 per cent for efficiencies larger than 50 per cent.

The value of k is given in Table IX.

Needless to say, the correspondence given above applies to our particular definition of R_{S0} only.

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