Four-Phase Data Systems in Combined Delay Distortion, Gaussian Noise, and Impulse Noise

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The performance of a four-phase data system over delay- and attenuation-distorted transmission lines in the presence of impulse noise has been simulated on a digital computer previously. This paper describes an extension of that simulation to performance with the additional degradation of additive Gaussian noise. Results are presented for Gaussian noise alone in terms of absolute error rate, and for Gaussian plus impulse noise in terms of conditional error rate given the occurrence of an impulse. Some conclusions on the limiting effects in various situations are given.

I. INTRODUCTION

A previous paper on digital computer simulation of a four-phase data system[1] was concerned with performance in the presence of delay and attenuation distortion, and impulse noise. This paper extends those results to include Gaussian noise. Combinations of Gaussian noise, impulse noise, and delay and attenuation distortion are presented and discussed.

The prime effect of delay and attenuation distortion is to reduce system margin against error. Adding Gaussian noise to the distorted signal can have one or both of two effects. Either errors occur directly due to the Gaussian noise alone, or the margin against other disturbances, in particular impulse noise, is decreased. On most telephone lines it is generally feasible to keep the direct errors produced at a very low level. The generally steep slope of a curve of signal to Gaussian noise (S/N) versus probability of error, (as much as a factor of 10:1 in error rate for one db change in S/N) makes the system particularly sensitive to changes in the amount of Gaussian noise. Thus, at least on land facilities, Gaussian noise is usually kept to a level where it enters basically as a degradation in system margin, i.e., an increase in conditional probability of error given a noise impulse.

II. METHOD

The main problem in straightforward introduction of Gaussian noise into digital simulations is the computation time. The accuracy of the results basically depends upon the number of errors obtained. A sufficiently large number of noise samples to give reliable information about absolute error rates of the order of 10^{-5} or less is extremely time consuming. A rough estimate of the accuracy obtainable can be obtained by considering a test as consisting of N independent trials each with common probability of failure (i.e., of making an error). Call this probability p.

The estimate

$$\hat{p} = \frac{\text{number of errors}}{\text{number of samples}} \tag{1}$$

has expected value p.

The standard deviation of $\hat{p}$ is

$$\sigma(\hat{p}) = \frac{\sqrt{pq}}{\sqrt{n}} \approx \sqrt{\frac{p}{n}} \tag{2}$$

since $q \approx 1$. Thus for $p = 0.001$, $\sigma(\hat{p})$ would be 0.00014 for $n = 50{,}000$.

We emphasize that these results are very rough, since the assumption of independent trials (i.e., independence of successive bit periods) is clearly not too accurate. However, they do give an idea of the number of samples required to obtain accurate results.

When impulse noise is present, this problem is solved by computing the conditional probability of error given the occurrence of an impulse. The long quiet intervals which characterize impulse noise in the telephone plant are in effect taken as having an error probability of zero. For Gaussian noise alone this procedure is not realistic, since the noise amplitudes are not segregated into very large impulses and very low quiet periods.

There are two problems: Gaussian noise alone and Gaussian noise as a degrading factor in the performance of a system with impulse noise. The solution to both of these revolves around a program which uses a tape of approximately 25,000 one-dibit intervals of Gaussian noise bandlimited by the input receiving filter of the data set. A train of

512 dibits is used.* Fifty one-dibit samples of noise are introduced into each signal dibit. To find the effect of Gaussian noise alone, the demodulation is then performed and the errors simply counted. The result is the error rate due to Gaussian noise.

For the second problem, Gaussian noise is introduced and the effect of an impulse is systematically examined in a pseudo-random way. The details of this process are essentially the same as those outlined in Ref. 1 for finding conditional probability of error.

III. SIMULATION ACCURACY

For those facilities in which the effect of Gaussian noise alone was desired, two tests were run for each pattern of delay distortion. Thus, noise was introduced into 50,000 separate dibit intervals and the resulting errors counted. As mentioned above, it is reasonable to trust these results without further evidence only to about an error level of 10^{-3}. At that level about 50 errors are obtained and the accuracy is still quite good.

The curves can be directly extrapolated to perhaps 2×10^{-4}, because determining $P(e)$ to an accuracy of perhaps 3:1 (e.g., from 10^{-4} to 3×10^{-4}) is quite suitable for most applications.

However, we often desire results to error levels of 10^{-6} or lower. To obtain such results we make use of a conjectured property of the $P(e)$ curves which we will call "convexity." If a $P(e)$ versus S/N curve is plotted on log vs db paper then the curve is convex down.

The justification for this conjecture is primarily experience. All experience on laboratory, field test, particular extended computer runs shows this to be so. In addition, a preliminary proof by the author has indicated that the second and third derivatives of the curve with respect to the noise power are both $\leqq 0$. The details of this proof are being clarified, and should be presented in a future paper.

Therefore, an upper bound for any $P(e)$ versus S/N curve can be obtained by extending the curve with a straight line (i.e., second derivative $= 0$; see Fig. 1). Similarly, a lower bound can be obtained by setting the third derivative $= 0$, that is, holding the second derivative constant. The results given sharp bounds, since the curves at $P(e) = 5 \times 10^{-4}$ in general have a severe slope (i.e., 5 or 10 to 1 change in $P(e)$ for 1 db change in S/N). Taking into account that the allowable measured error rate range generally increases, in practice, as the $P(e)$ drops [e.g.,

* This length pattern gives all possible combinations of four dibits, where the first dibit can take any one of the eight phases used in the modem.

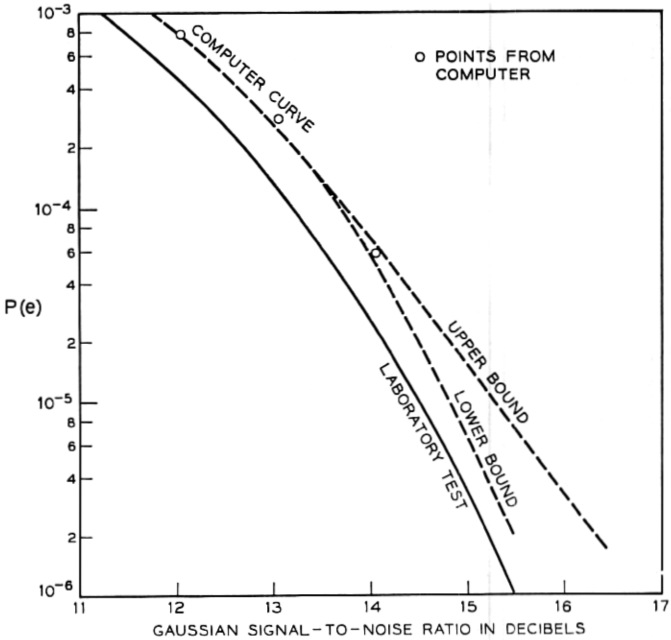

Fig. 1 — Undistorted performance with Gaussian noise, showing upper- and lower-bound convexity projections.

perhaps a 5:1 range in $P(e)$ generally allowable at $P(e) = 10^{-6}$], useful estimates of $P(e)$ can be obtained for $P(e)$ as small as 10^{-6} in most cases. Fig. 1 also shows a laboratory test curve.

In an impulse noise environment, the simulation has inherently much greater accuracy. This is because the aim is to produce conditional probability of error $P_N(e)$ — that is, to count the errors per impulse — and the desired performance range is more in the order of 10^{-2} or perhaps 10^{-3}. Thus, the number of trials, i.e., the number of introductions of Gaussian noise, is sufficiently large to give very good convergence to real conditional probabilities of error.

IV. RESULTS

The four-phase system considered was one using a cycle and a half of carrier per dibit, (e.g., an 1800-cycle carrier with a 1200-dibit or a 2400-bit per second system). In keeping with previous results on the bandwidth in which delay distortion degrades the signal,[1] the delay was specified from the carrier to plus or minus a number of cycles equal to

$\frac{2}{3}$ the dibit speed. For example, for the system considered, delay was introduced between (f_c − 800) and (f_c + 800) cycles. A sequence of peak delays in this range was considered. Because the shape of delay (as well as its peak magnitude) is significant, several delay shapes were considered. These were chosen, based on previous work, to give results which are typical of a wide class of transmission facilities used for data transmission. The delays used were respectively parabolic, centered at the carrier frequency, and a sinusoidal shape with three cycles of sinusoid across the transmission band.

The results are shown in Figs. 2 through 5. Fig. 2 shows the error rates obtained for Gaussian noise alone for the parabolic and sinusoidal lines respectively, with extrapolation obtained by using a compromise between computed points and the derived upper bounds.

Figs. 3, 4 and 5 give the effect of introducing impulse noise and Gaussian noise simultaneously with delay distortion present. Each set

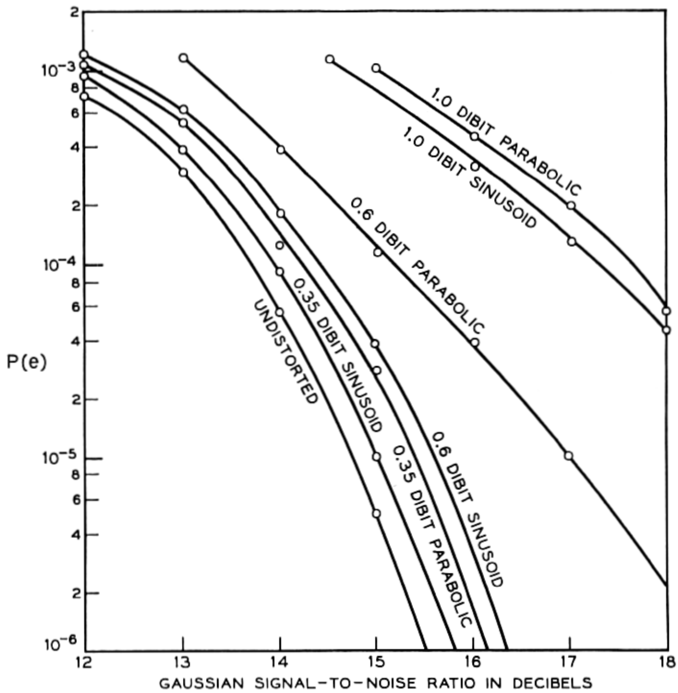

Fig. 2 — Probability of error vs Gaussian noise level for various delays. Parabolic delay specified by delay at $\omega_c \pm 0.7\ \omega_{bit}$; sinusoidal delay specified by delay in passband of peak-to-peak 3-cycle sinusoid.

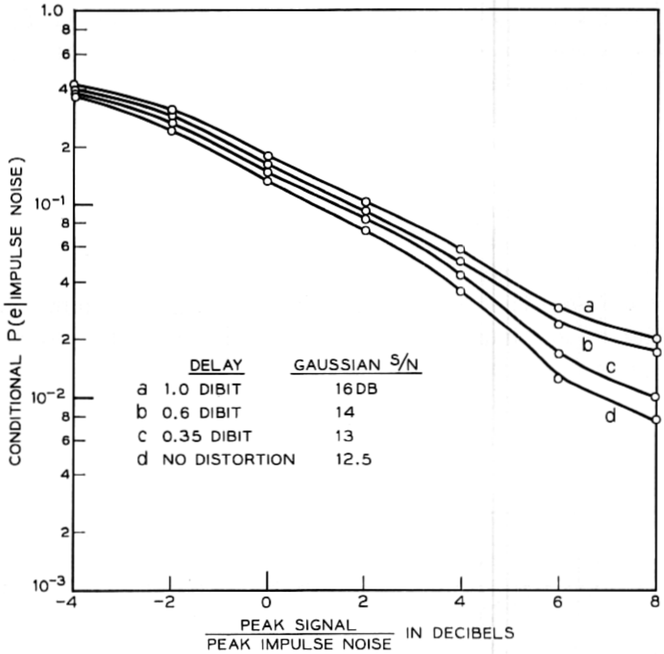

Fig. 3 — Conditional probability of error given an impulse noise for Gaussian noise such that the error rate due to Gaussian noise alone equals 5×10^{-4}. Curves averaged over various impulse shapes and two delay shapes (see Fig. 2).

of curves gives conditional probability of error for a Gaussian noise level chosen to yield some specific error rate. The Gaussian noise was chosen to give error rates of approximately 5×10^{-4}, 10^{-5}, and 5×10^{-7}. Each curve was averaged over a variety of impulse noise shapes. More exact information would have to be known on the noise expected on a given transmission line before precise absolute error rates could be obtained. However, reasonable results should be achieved in most practical situations by specifying an allowable number of counts from the averaged curves.

V. CONCLUSIONS

The curves as given, i.e., the results of the simulation, are primarily useful for design purposes. They can be used to find the trade-off obtainable between Gaussian noise, delay distortion, and number of impulse noise counts. Thus the curves can be used for over-all design of transmission facilities at given error rates with a four-phase data trans-

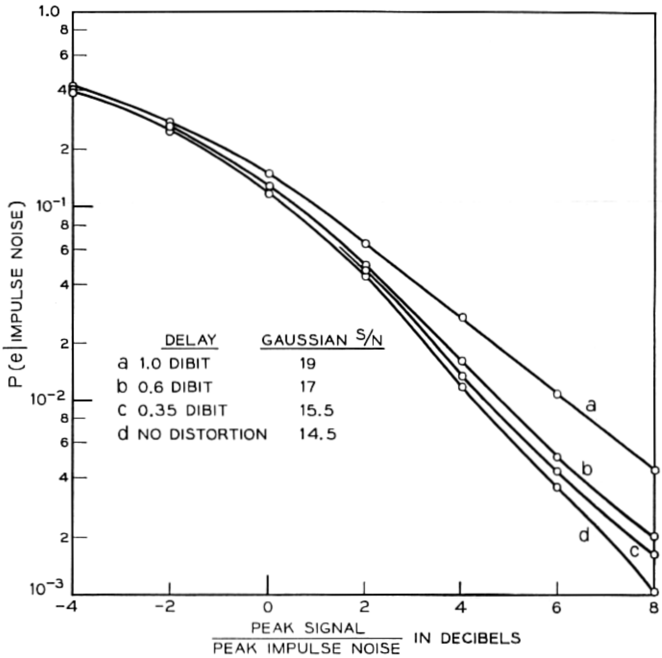

Fig. 4 — Conditional probability of error given an impulse noise for Gaussian noise such that error rate due to Gaussian noise alone equals 10^{-5}. Curves averaged over various impulse shapes and two delay shapes (see Fig. 2).

mission system. Here it is worth noting that the effects of frequency shift on such a system were also investigated as part of the program, and frequency shifts up to ±5 cycles were found to have essentially no effect.

One factor that stands out in the results is the rapid deterioration in performance of the system as delay distortion is increased beyond a certain amount. This is true either for Gaussian noise alone or for the combined effects of Gaussian and impulse noise. It seems that, for the data modem and conditions assumed, signal-to-noise ratios worse than those actually occurring in most of the plant are not harmful for reasonable delay ranges. Thus it seems reasonable that the prime consideration in designing transmission facilities for four-phase data systems must be the minimization of the effect of delay distortion, while the present level of Gaussian noise, at least for land-line systems, does not seem too critical. However, recent data on certain carrier systems show that for data systems using a larger number of phases (e.g., eight- or sixteen-phase systems), but maintaining constant signal element rates, the noise levels might be high enough that the systems would be limited by Gaus-

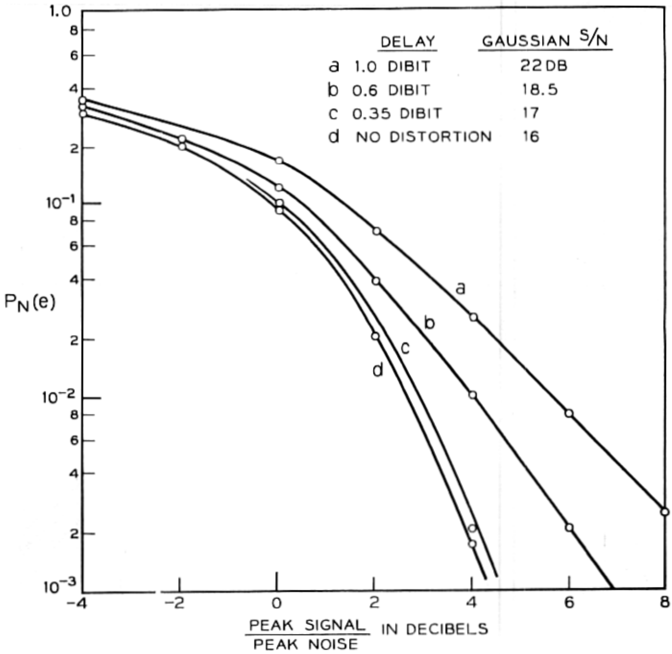

Fig. 5 — Conditional probability of error given an impulse noise for Gaussian noise such that error rate due to Gaussian noise alone equals approximately 5×10^{-7}. Curves averaged over various impulse shapes and two delay shapes (see Fig. 2).

sian noise rather than delay. Thus caution should be used in systems employing more than four phases or levels.

Indeed, this emphasizes that any comparison of multiphase and multilevel systems which is based on their performance under certain degradation (e.g., Gaussian noise) may be misleading. Instead, comparison must be based on the critical factors, which for many systems may well be how they are affected by delay distortion.

REFERENCE

1. Rappeport, M. A., Digital Computer Simulation of a Four-Phase Data Transmission System, B.S.T.J., *43*, May, 1964, p. 927.