Some Effects of Laminar and Turbulent Flow on Breakdown in Gases*

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The conservation equation for electrons in a laminar or turbulent flow has been used to determine a criterion for breakdown. The results can be used to define a characteristic length, L_s, and time, t_s, which determine the affects of the flow on the power required to break down the gas. The theory has been compared to the experimental results of Buchsbaum and Cottingham in hydrogen with reasonable agreement in the laminar region. In the presence of turbulence, the results will depend on the velocity dependence of the turbulent diffusion coefficient for the electrons.

I. INTRODUCTION

The physics governing the breakdown of a stationary gas (no flow) has been discussed at great length by a number of authors. For some recent studies the interested reader is referred to a text by Brown[1] and papers by Buchsbaum[2] and by McDonald.[3] The effects of gas flow on breakdown has been studied experimentally for a laminar flow of less than 100 meters per second by Skinner and Brady[5] and for the case of laminar and turbulent flows by Buchsbaum and Cottingham.[4]

Theoretically, the effects of gas flow can be studied by noting that the transport of electrons and ions will depend on the flow as well as the laminar or turbulent diffusion. If the flow is laminar and the electron density such that the diffusion is ambipolar, then the average drift velocity of the electrons and ions are equal and given by

$$\bar{\mathbf{v}} = -D_a \ (\nabla n/n) + \mathbf{V} \qquad (1)$$

where D_a is the ambipolar diffusion coefficient, n, the electron density and $\mathbf{V}$, the flow velocity. The ambipolar diffusion coefficient may, in

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turn, be related to the diffusion coefficient for the ions, D_i, and the temperature of the ions, T_i, and electrons, T_e, by

$$D_a = D_i \{1 + T_e/T_i\} \tag{2}$$

provided the mobility of the electrons is much larger than that of the ions.

If the flow is turbulent, then the ambipolar diffusion coefficient must be replaced by the turbulent diffusion coefficient, D_T.

The theory to follow will contain ratios of the form

$$\bar{V}/2D \quad \text{and} \quad \bar{V}^2/2D,$$

where $\bar{V}$ is the average velocity in a turbulent flow or simply the velocity in a laminar flow. The first can be used to define an inverse length, L_s, such that

$$\bar{V}/2D \equiv L_s^{-1}, \tag{3}$$

and the second an inverse time, t_s, such that

$$\bar{V}^2/2D \equiv t_s^{-1}. \tag{4}$$

The characteristic time, t_s, and length, L_s, are a measure of the extent to which electrons are removed by the flow. These may be compared to the removal of electrons by ordinary diffusion to the walls with the characteristic time, t_D, given by

$$D/\Lambda^2 \equiv t_D^{-1}, \tag{5}$$

where Λ is the size of the container. If

$$t_s \gg t_D,$$

then it is reasonable to expect that the effects of the flow on breakdown are negligible. If, in turn,

$$t_D \gg t_s,$$

it is reasonable to expect that the effect of ordinary diffusion to the walls is negligible as compared to "sweeping" as an electron removal process. In this case, breakdown will be controlled by the flow.

II. GENERAL THEORY

The effects of gas flow on breakdown can be considered theoretically by noting that the transport of electrons, on the average, is due to the mean flow as well as the laminar or the turbulent diffusion. In the absence of secondary emission from the walls, the appropriate conservation equation for the electrons is given by[6]

$$\frac{\partial \bar{n}}{\partial t} + \nabla \cdot (\bar{n}\bar{\mathbf{V}}) \cong (\nu_i - \nu_a)\bar{n} + \alpha\bar{n}^2 + \nabla \cdot (D\nabla\bar{n}) \tag{6}$$

where

$\bar{n}$ = the electron density in a laminar flow or the ensemble average of the electron density (at any given time) in a turbulent flow.

$\bar{\mathbf{V}}$ = the velocity in a laminar flow or the ensemble average of the gas velocity in a turbulent flow. (Experimental conditions usually justify a time average in this case.)

ν_i = the ionization frequency

ν_a = the attachment frequency

α = the recombination coefficient

D = the diffusion coefficient for the electrons (laminar or turbulent).

Consider the geometry of Fig. 1. The originally ambient gas is flowing between two parallel planes separated by a distance d. Two grids are placed normal to the direction of flow and separated by a distance l. For this geometry, (6) reduces to,

$$\frac{\partial \bar{n}}{\partial t} + \bar{n}\nabla \cdot \bar{V} + \bar{V}_y \frac{\partial \bar{n}}{\partial y} = (\nu_i - \nu_a)\bar{n} + \alpha\bar{n}^2 + D\left\{\frac{\partial^2 \bar{n}}{\partial_y{}^2} + \frac{\partial^2 \bar{n}}{\partial_z{}^2}\right\}. \tag{7}$$

If it is further assumed that the distance l in the direction of flow is not excessive, such that,

$$\frac{\partial \bar{V}_y}{\partial_y} = 0 \quad \text{for} \quad 0 \leqq y \leqq l,$$

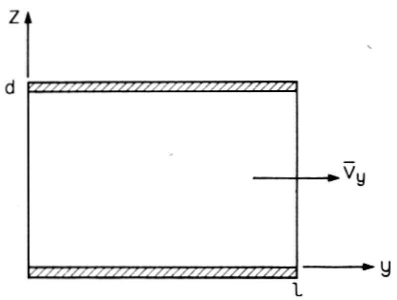

Fig. 1 — Cross section of flow geometry.

$$t_s = \frac{2}{3}\left[\frac{\langle v_i \rangle^2}{V_y^2}\right]\{1 + T_e/T_i\}\frac{1}{\nu_{c_i}} \tag{17}$$

or by

$$t_s = \frac{2}{3}\left[\frac{\langle v_i \rangle}{V_y}\right]\{1 + T_e/T_i\}\frac{l_i}{V_y}. \tag{18}$$

As expected, both L_s and t_s approach infinity as the flow velocity, V_y, approaches zero.

The magnitude of the characteristic sweeping length, L_s, must be compared to the length of the discharge, l, in the direction of flow, y. If $L_s \gg l$, then the term $(V_y/2D)y$, may be neglected.

The magnitude of the characteristic sweeping time, t_s, must be compared to the characteristic time for electron loss by diffusion to the walls, t_D. If $t_D \gg t_s$, then diffusion to the walls may be neglected as an electron removal process. If $t_s \gg t_D$, then sweeping may be neglected. It is evident that since

$$t_s \sim V_y^{-2} \tag{19}$$

then, $t_s \gg t_D$, for low flow velocities. As expected, sweeping may be neglected as an electron removal process at low velocities.

2.2 *Turbulent Flow*

When the flow of the gas is turbulent, it is only reasonable to expect that the diffusion coefficient, D, will increase provided the electron density is sufficient to insure ambipolar or near ambipolar diffusion in the absence of turbulence. While this may be confusing, it is of importance to note that electrons in *free* diffusion are probably unaffected by turbulence.

As in the case of laminar flow, it is possible to define a characteristic length, L_s, and time, t_s, for the sweeping due to the mean flow. Using (12), it follows that

$$L_s^{-1} \equiv \bar{V}_y/2D_{T_e}, \qquad t_s^{-1} \equiv \bar{V}_y^2/2D_{T_e}, \tag{20}$$

where D_{T_e} is the turbulent diffusion coefficient for the electrons. It can be shown that

$$D_{T_e} = D_{i_T}\{1 + T_e/T_i\}, \tag{21}$$

where D_{i_T} is the turbulent diffusion coefficient for the ions. D_{T_e} will simply be written as D_T for the remainder of this paper.

The characteristic time for electron loss by diffusion to the walls will also be modified by the turbulent flow. It follows that

$$t_{D_{mn}}^{-1} = D_T/\Lambda_{mn}^2. \tag{22}$$

III. COMPARISON WITH EXPERIMENT

While the theoretical geometry is not duplicated by any existing experiment, some verification of the theory can be obtained from the results of Buchsbaum and Cottingham[4] in hydrogen. Their experimental set-up has been sketched in Fig. 2. The results for breakdown in hydrogen (H_2) has been plotted as a function of gas velocity in Fig. 3. The peak power required to produce breakdown varied from about 1.7 to 3.2 kw.

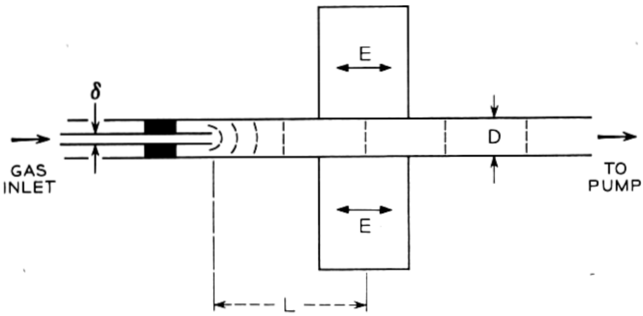

Fig. 2 — Geometry of the experiment of Buchsbaum and Cottingham (Ref. 4).

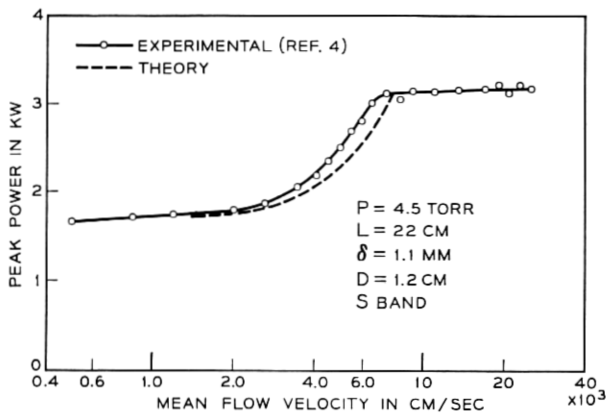

Fig. 3 — Comparison of the theoretical and experimental results for peak power as a function of flow velocity in hydrogen (experimental results of Ref. 4).

These results were used to calculate the effective electric field intensity required to produce breakdown as a function of gas velocity. In this calculation, the effective frequency for electron-molecular elastic collisions was taken as[2]

$$\nu_c = 4.8 \times 10^{-9} p, \ \mathrm{sec}^{-1}. \tag{23}$$

An approximate criteria for breakdown can be obtained from the theory as given by (11). (See Appendix B.) It can be shown that

$$\nu_i t_{\mathrm{on}} = \left[\frac{\bar{V}^2}{2D} + \frac{D}{\Lambda^2}\right] t_{\mathrm{off}}, \tag{25}$$

where ν_i in H_2 has been measured and compared to dc data.[2] For this particular experiment,[4] the geometry was cylindrical with

$$t_{\mathrm{on}} = 1 \times 10^{-6} \ \mathrm{sec}, \qquad t_{\mathrm{off}} = 1 \times 10^{-3} \ \mathrm{sec},$$

and

$$\frac{1}{\Lambda^2} = \left(\frac{\pi}{3.8}\right)^2 + \left(\frac{2.405}{0.6}\right)^2 = 16.7$$

or

$$\Lambda = 0.245 \ \mathrm{cm}.$$

It should be noted that the length of the discharge in the direction of flow has been taken as 3.8 cm even though grids were not used (at $y = 0$ and $y = 3.8$ cm) in the experiment.

3.1 *Laminar Flow*

When the flow of the gas is laminar, the diffusion coefficient for the electrons should be independent of the velocity of the flow. The magnitude of the effective diffusion coefficient, D, can be calculated from the experimental data by noting that, for small $\bar{V}$,

$$D/\Lambda^2 \gg \bar{V}^2/2D \quad \text{for} \quad \bar{V} = 500 \ \mathrm{cm\ sec}^{-1}. \tag{25}$$

(This is the smallest velocity for which data is available in this experiment. See Fig. 3.) This, in turn, implies that the characteristic time for electron removal by diffusion to the walls is much less than the characteristic time associated with the sweeping effect at this velocity of flow. For the laminar case, t_s is given by (17) or (18) provided the diffusion is ambipolar or nearly so.

If the electron density, during the pulse off time, is reduced to a point where the electrons are in free diffusion, then $D \rightarrow D_-$, where[2]

$$D_{-}p \cong 1 \times 10^6 \text{ cm}^2 \text{ sec}^{-1} \text{ torr.}$$

As the electron density increases, D decreases. The resulting effective diffusion coefficient, D_s, has been calculated by Allis and Rose.[7] Using their results and the geometry of this experiment, it follows that

$$D_s \cong 0.2 D_{-} \quad \text{for} \quad \bar{n} = 10^6 \text{ elec/cm}^3,$$

and

$$D_s \cong 0.04\, D_{-} \quad \text{for} \quad \bar{n} = 10^8 \text{ elec/cm}^3.$$

Returning to the question posed by (44) of Appendix B, we note that the ratio

$$\frac{\nu_i \Lambda^2}{D'} \cong 5 \quad \text{for} \quad \bar{n} \text{ small},$$

$$\frac{\nu_i \Lambda^2}{D'} \cong 25 \quad \text{for} \quad \bar{n} \cong 10^6 \text{ elec/cm}^3,$$

$$\frac{\nu_i \Lambda^2}{D'} \cong 125 \quad \text{for} \quad \bar{n} \cong 10^8 \text{ elec/cm}^3.$$

Consequently, diffusion can only be neglected during the pulse on time when the electron density is larger than 10^6 elec/cm^3. This would appear to be the case during the 1-μsec, 3-gc pulse used in this experiment.

Since the incident microwave pulse was relatively flat, (see Fig. 2 of Ref. 4), the peak power can be used to calculate the effective electric field strength and, in turn, the ionization frequency. For a flow velocity of 500 cm sec^{-1}, the peak power required to produce breakdown at the end of one microsecond was 1.7×10^3 watts. This power level corresponds to an effective field strength of about 200 volts per cm or an ionization frequency of 1.5×10^7 radians per sec. Using (24), it follows that

$$D/\Lambda^2 \cong 1.5 \times 10^4 \text{ sec}^{-1}. \tag{26}$$

This corresponds to an effective diffusion coefficient for the electrons of 920 cm^2 sec^{-1} or

$$D_e p \cong 4 \times 10^3 \text{ cm}^2 \text{ sec}^{-1} - \text{torr.} \tag{27}$$

Using this value for D, it follows that (25) is satisfied at 500 cm sec^{-1}, since

$$\bar{V}^2/2D \cong 120 \text{ sec}^{-1}.$$

The magnitude of the effective diffusion coefficient for the electrons can be compared to the ambipolar diffusion coefficient for hydrogen as measured by Persson and Brown[8] and to that calculated from the mobility measurements of Rose.[9] The results of Persson and Brown,[8] derived from afterglow data yield an ambipolar diffusion coefficient of

$$D_a p = 700 \pm 50 \text{ cm}^2 \text{ sec}^{-1} - \text{torr}. \tag{28}$$

This is a factor of six less than the effective diffusion coefficient as given by (27). The difference could be due to the fact that the diffusion in the afterglow was not ambipolar for all t, or to the presence of higher order modes, or to an elevated temperature for the electrons. Since the collision frequency was on the order of 10^9 radians per sec, it would appear that the plasma was rapidly reduced to thermal equilibrium during the pulse off time of one millisecond. It would appear reasonable to conclude that either or both of the first two possibilities are likely.

The mobility data of Rose[9] yields a value for $D_a p$ which is *less* then that measured by Persson and Brown[8] in the afterglow.

At higher flow velocities, the effect of the flow can no longer be neglected. Equation (24) can be used together with (23, 26, 27) and Fig. 3 of Ref. 2 to calculate the peak power required to produce breakdown in one microsecond. The results have been plotted in Fig. 3 for flow velocities of less than 8000 cm/sec, together with the experimental results of Buchsbaum and Cottingham.[4]

While the agreement between theory and experiment is excellent, it should be noted that (24) is based on the condition

$$\nu_i \gg \frac{D'}{\Lambda^2} + \frac{\bar{V}^2}{2D'}, \quad \text{pulse on.}$$

From the results, it is evident that the condition is reasonable provided the electron density is such that the diffusion of electrons is never free, or provided any free diffusion mode is restricted to a negligible part of the pulse on time.

IV. CONCLUSIONS AND RECOMMENDATIONS

The conservation equation for the electron density in a laminar or turbulent flow (2) has been solved for the parallel plane geometry of Fig. 1 (11). Extension to other geometries is straightforward.

For cases where

$$\nu = \nu_i - \nu_a \gg D/\Lambda^2, \tag{29}$$

it was shown that pulsed breakdown in a laminar or turbulent gas may be controlled by the rather simple criterion,

$$\nu \times t_{\text{on}} = \left\{\frac{\bar{V}^2}{2D} + \frac{D}{\Lambda^2}\right\} t_{\text{off}}, \tag{30}$$

where $\bar{V}$ is the mean velocity of the flow.

The power required to produce breakdown in a turbulent gas may, or may not, be a function of the mean velocity of the flow. A velocity independent result is obtained when

$$D_T/\Lambda^2 \gg \bar{V}^2/2D_T \tag{31}$$

provided $D_T \sim \bar{V}l_e$, where l_e is the mixing length for the plasma, and provided l_e is inversely proportional to the Reynold's number of the flow.

In order to test the present theory further, future experiments should be designed such that

$$\nu_i \gg D/\Lambda \quad \text{for all } t.$$

It would be of interest to measure the electron density in the discharge at a number of positions in the direction of flow. This result could be used to check the predicted theoretical variation as well as insure the presence or absence of turbulence.

V. ACKNOWLEDGMENTS

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APPENDIX A

Equation (5) can be solved by the method of separation of variables. Let

$$n(y,z,t) = Y(y)\cdot Z(z)\cdot T(t). \tag{32}$$

It follows that

$$\frac{1}{T}\frac{dT}{dt} + \bar{V}_y \frac{1}{Y}\frac{dY}{dy} = (\nu_i - \nu_a) + D\left\{\frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2}\right\}, \tag{33}$$

or

$$\frac{1}{DT}\frac{dT}{dt} = \frac{1}{D}(\nu_i - \nu_a) + \frac{1}{Y}\left\{\frac{d^2Y}{dy^2} - \frac{\bar{V}_y}{D}\frac{dY}{dy}\right\} + \frac{1}{Z}\frac{d^2Z}{dz^2}. \tag{34}$$

Taking

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -\alpha^2$$

and using the appropriate boundary condition yields

$$Z(z) = A_m \sin \frac{m\pi}{d} z \tag{35}$$

where

$$\alpha^2 = \left(\frac{m\pi}{d}\right)^2.$$

Taking

$$\frac{1}{Y}\left\{\frac{d^2Y}{dy^2} - \frac{\bar{V}_y}{D}\frac{dY}{dy}\right\} = -\beta^2,$$

yields

$$\frac{d^2Y}{dy^2} - \frac{\bar{V}_y}{D}\frac{dY}{dy} + \beta^2 Y = 0$$

with the solution $Y(y) = \epsilon^{+uy}$, where

$$u^2 - \frac{V_y}{D} u + \beta^2 u = 0.$$

Hence,

$$u = \frac{\bar{V}_y}{2D} \pm \frac{1}{2}\sqrt{\left(\frac{\bar{V}_y}{D}\right)^2 - 4\beta^2}.$$

If the solution is to satisfy the boundary conditions at $y = 0$ and $y = l$, it follows that $Y(y)$ must be periodic in y; hence, $4\beta^2 > (V_y/D)^2$. Consequently,

$$u = \frac{V_y}{2D} \pm j\sqrt{\beta^2 - \left(\frac{\bar{V}_y}{2D}\right)^2}$$

with the solution

$$Y(y) = B_n \epsilon^{(\bar{V}_y/2D)y} \sin \frac{n\pi}{l} y \tag{36}$$

where

$$\beta^2 - \left(\frac{\bar{V}_y}{2D}\right)^2 = \left(\frac{n\pi}{l}\right)^2.$$

Finally, the differential equation in t, reduces to

$$\frac{1}{T}\frac{dT}{dt} = (\nu_i - \nu_a) - D\left\{\left(\frac{m\pi}{d}\right)^2 + \left(\frac{n\pi}{l}\right)^2 + \left(\frac{\bar{V}_y}{2D}\right)^2\right\},$$

with the solution

$$T(t) = \exp\left\{(\nu_i - \nu_a) - D\left[\left(\frac{m\pi}{d}\right)^2 + \left(\frac{n\pi}{l}\right)^2 + \left(\frac{\bar{V}_y}{2D}\right)^2\right]\right\} t. \quad (37)$$

It follows that the solution of (5) satisfying the boundary conditions given in (4) is

$$\begin{aligned} n(x,y,z,t) &= \epsilon^{(V_y/2D)y} \sum_m \sum_n A_{mn} \sin\frac{n\pi}{l} y \sin\frac{m\pi}{d} z \\ &\quad \times \exp\left\{(\nu_i - \nu_a) - D\left[\left(\frac{m\pi}{d}\right)^2 + \left(\frac{n\pi}{l}\right)^2 + \left(\frac{\bar{V}_y}{2D}\right)^2\right]\right\} t. \end{aligned} \quad (38)$$

It is apparent at the offset that the usual method for evaluating the constant A_{mn} cannot be used for the general case since

$$\int_0^l \sin\frac{m\pi}{l} y \sin\frac{n\pi}{l} y\, \epsilon^{(V_y/2D)y}\, dy \quad (39)$$

does not vanish for $m \neq n$. In view of this, it is apparent that the usual modes *do not exist.*

Fortunately, a separation into modes is still possible, provided the electron density, at $t = 0$, is given by

$$f(x,y,z)\epsilon^{(V_y/2D)y}. \quad (40)$$

This would appear to be reasonable for the steady state, pulse on pulse off, conditions of the usual pulsed experiment.

Let us assume that the density at $t = 0$, (end of the off cycle) is given by

$$n(x,y,z,0) = n_0 \times \epsilon^{(V_y/2D)y}. \quad (41)$$

For this case,

$$A_{mn} = n_0 \frac{4}{\pi^2}\frac{1}{mn}[1 - \cos n\pi][1 - \cos m\pi] \quad (42)$$

from which it is evident that $A_{mn} = 0$ for m or n even. It is of interest that the 1st "mode" for this case is

$$\begin{aligned} n(x,y,z,t) &= n_0 \frac{16}{\pi^2} \sin\frac{\pi}{l} y \sin\frac{\pi}{d} z\, \epsilon^{(\bar{V}_y/2D)y} \\ &\quad \times \exp\left\{(\nu_i - \nu_a) - D\left[\left(\frac{\pi}{d}\right)^2 + \left(\frac{\pi}{l}\right)^2 + \left(\frac{\bar{V}_y}{2D}\right)^2\right]\right\} t. \end{aligned} \quad (43)$$

In practice, the ratio $\bar{V}_y/2D$ at any velocity will depend on the particular gas used.

For example, in the experiment of Ref. 1,

$$(\bar{V}/2D)y < 11 \quad \text{for} \quad 0 \leqq y \leqq 3.5 \text{ cm}$$

in hydrogen, for $\bar{V} < 7 \times 10^3$ cm/sec, the laminar region. In the turbulent region, the ratio decreases since D increases.

APPENDIX B

The direct application of the theory to any particular experiment may be complicated by the variation of the diffusion coefficient with electron density and, in turn, time. The interested reader is referred to a recent paper by Buchsbaum and Cottingham[2] for a discussion of this problem. Following their example, the effects of electron attachment in H_2 will be neglected.

Let the flow conditions be such that the initial density, n_0, is sufficient to insure ambipolar diffusion or near ambipolar diffusion of the electrons for all or nearly all time. This assumption will be justified for the particular experiment.

During the pulse on time the appropriate equation for the electron density is given by (43) with $D = D'$ (hot electrons) and $\nu_a = 0$. Let D' and $\bar{V}$ be such that

$$\nu_i \gg \frac{D'}{\Lambda^2} + \frac{\bar{V}^2}{2D'} \quad \text{for all} \quad 0 \leqq t \leqq t_{\text{on}} . \tag{44}$$

This implies that

$$\nu_i \gg t_D{'}^{-1} \quad \text{and} \quad t_s{'}^{-1}$$

for all flow velocities during the pulse on time. It follows that (43) for the first "mode" reduces to

$$n(x,y,z,t) \cong n_0 \cdot \frac{16}{\pi^2} \sin \frac{\pi}{l} y \sin \frac{\pi}{d} z \, \epsilon^{(\bar{V}_y/2D)y} \, \epsilon^{\nu_i t}. \tag{45}$$

This result can be applied directly to the experiment of Buchsbaum and Cottingham.[4]

In this experiment, electron density was monitored indirectly by observing the intensity of the power reflected from the discharge region. Since the electron density and, in turn, the reflection coefficient was a function of position within the discharge it is evident that the experimental results can be related to (45) by choosing (x,y,z) such that the reflection coefficient is a maximum. Hence,

$$n(t) \cong n_0^* \exp(\nu_i t), \quad \text{for} \quad 0 \leqq t \leqq t_{\text{on}} . \tag{46}$$

The appropriate equation for the pulse off time may be derived from (43) in a similar manner. It can be shown that

$$n(t) = n_{\max}{}^* \exp - \left\{\frac{D}{\Lambda^2} + \frac{\bar{V}^2}{2D}\right\} t \quad \text{for} \quad 0 \leqq t \leqq t_{\text{off}}. \tag{47}$$

Equations (46) and (47) can be solved to yield the approximate criteria for breakdown.

$$\nu_i t_{\text{on}} = \left\{\frac{D}{\Lambda^2} + \frac{\bar{V}^2}{2D}\right\} t_{\text{off}}\,. \tag{48}$$

LIST OF SYMBOLS

d = distance between the parallel planes in the z direction.
l = distance between the grids in the direction of flow.
l_e = effective mixing length for the electrons in a turbulent flow.
l_i = mean free path for the ions.
$\bar{n}$ = electron density in a laminar flow or the ensemble average of the electron density (at a given time) in a turbulent flow.
t_D = characteristic time for electron removal by diffusion to the walls.
t_S = characteristic time for electron removal by the flow of the gas.
$\bar{\mathbf{v}}$ = average velocity of the electrons or ions resulting from ambipolar diffusion and the mean flow.
$\langle v_i \rangle$ = mean thermal velocity of the ions.
D_a = coefficient of ambipolar diffusion for the electrons in a laminar flow.
D_i = diffusion coefficient for the ions in a laminar flow.
D_{i_T} = diffusion coefficient for the ions in a turbulent flow.
D_T = diffusion coefficient for the electrons in a turbulent flow.
L_S = characteristic "sweeping" length for electron removal by the flow of the gas.
R_e = Reynold's number of the flow.
T_e = temperature of the electrons.
T_i = temperature of the ions.
$\bar{\mathbf{V}}$ = velocity of the gas in a laminar flow or the ensemble average of the gas velocity (at a given time) in a turbulent flow. Experimental conditions usually permit a time average in this case.
α = recombination coefficient for the electrons.
Λ_{mn} = effective diffusion distance for the mn mode.
ν_a = attachment frequency for the electrons to H^+ and $H_2{}^+$.
ν_c = elastic collison frequency for the electrons.
ν_{c_i} = elastic collision frequency for the ions.
ν_i = ionization frequency for the electrons.

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