The Use of Solar Radio Emission for the Measurement of Radar Angle Errors

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(Manuscript received July 20, 1962)

Space guidance and instrumentation have placed stringent demands on the pointing accuracy of tracking systems. One of the basic problems encountered is the calibration of the angle indicators of the tracking antenna to the true direction of the radio line of sight. A method of calibration is discussed which uses the sun as a primary directional reference.

I. INTRODUCTION

Historically, celestial bodies have been used as primary directional references for optical instruments such as navigational sextants, surveying theodolites, etc. Since celestial bodies are also sources of radio emission, they may be used directly as primary radio directional references. "Radio sextants" use the sun and moon as microwave directional references for all-weather marine navigation.[1] Conventional microwave tracking systems can also track these sources. For example, an X-band monopulse radar having an 8-foot antenna and crystal mixer receiver tracks the sun with an accuracy limited only by atmospheric effects. Moon tracking is of poorer quality because of the lower signal-to-noise ratio, but improvements in noise figure and a larger time-bandwidth product could make moon tracking competitive with present sun tracking.

The major limitation in using the sun as a precise reference has been the uncertainty of the position of its "radio center." This uncertainty is caused by regions of enhanced radio emission associated with sunspots. A method and experimental results will be described which overcome this limitation by taking advantage of the apparent rotation of the solar disk. This makes possible highly accurate alignment and zero setting without the usual optical aids.

II. SUN TRACKING PROCEDURE

Fig. 1 shows the comparison of the spectrum of solar radio emission with those of the moon and the brightest "radio stars." The plot is flux density vs wavelength. At 3 cm, for example, the power from the quiet sun is −164 dbm per square meter of effective antenna area in each cycle per second of bandwidth. This is the level of thermal noise which would be received at the earth if the sun were a black body at 18,000°K. We may say therefore that the sun has an equivalent radio temperature of 18,000°K at 3 cm. The average radio temperature of the moon at this wavelength is 180°K, so that the received power is 20 db less than the power from the sun. Even the brightest "radio stars" at 3-cm wavelength are extremely weak, the received power being about 20 db less than that from the moon.

In order to use celestial sources as directional references, the tracking system must be capable of determining the angle of arrival of the noise signal. Manasse[2] has shown that the optimum procedure is to perform

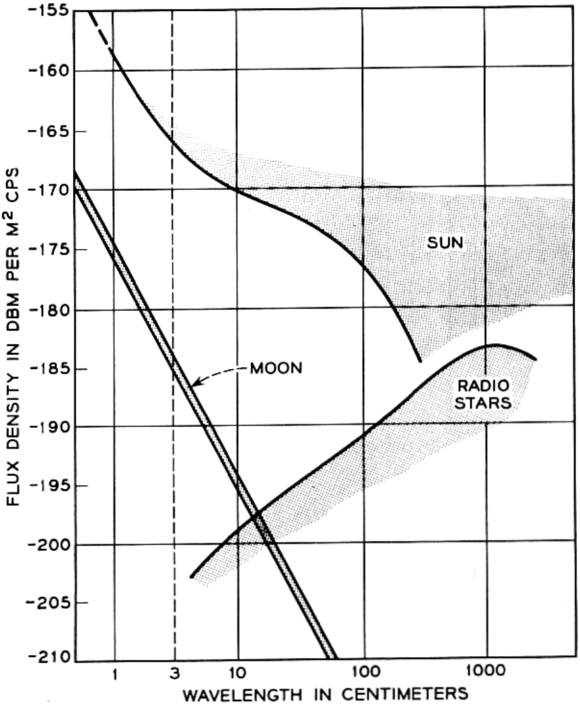

Fig. 1 — Radio emission from celestial bodies.

a simple correlation of the odd and even components of the antenna output. Since this is the technique of angle determination used in monopulse radar,* a monopulse system is well suited to the tracking of celestial noise sources. This was demonstrated in tracking experiments using an X-band monopulse radar. The theoretical sun and moon tracking performance are given in Appendix A. The calculated performance indicates that this system is theoretically capable of tracking the sun with a precision of better than 5 microradians.

To understand how an extended source such as the sun can be used as a precise reference, consider the simple case of more than one point source present in the antenna beam. These sources are not resolved and therefore appear as a single source located at the intensity centroid. Two point sources of equal intensity will appear as a single source midway between the two. Consider the typical difference pattern response of a monopulse antenna as in Fig. 2. A signal source to the left of the antenna null axis produces a positive error signal, a source to the right a negative signal. Zero error signal is obtained in autotracking multiple sources when the positive and negative signal contributions cancel each other. An extended source such as the sun can be considered as a collection of point sources. In this case, regions to the left will contribute a positive signal, regions to the right a negative signal.

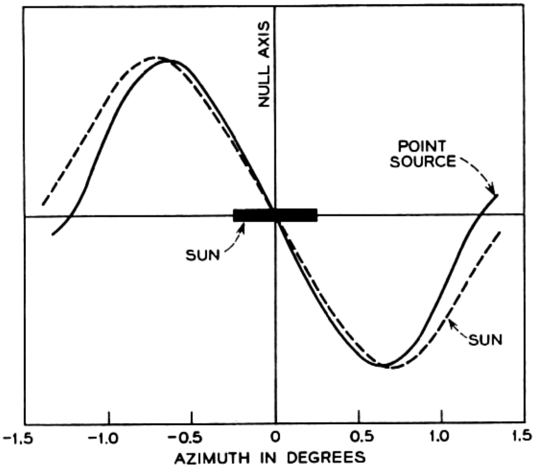

Fig. 2 — Typical monopulse difference patterns.

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* In a monopulse system, the sum (even) and difference (odd) signals are correlated in the angle error detector.

Thus, when the radar is used passively to track the sun, the radio line of sight follows the point which corresponds to the centroid of radio emission, or "radio center." Although extended sources tend to broaden the difference pattern, the effect is surprisingly small for sources less than a beamwidth in extent. The effect is shown exaggerated in the dashed curve in Fig. 2. Actually, the effect of the $\frac{1}{2}$-degree sun within a one-degree beamwidth is barely discernible.

Over the surface of the sun, the radio emission is not uniform, and for this reason the "radio center" is displaced from the geometric center. Fig. 3 is a "radio picture"[3] of the sun which shows regions of enhanced radio emission superimposed on a background level. The background level is constant with time, whereas the enhancements evidence growth, decay and movement much like the sunspots with which they are associated. In the presence of regions of enhanced emission, the radio center is displaced from the visible center. Each region has the effect of pulling the radio center in its direction by an amount proportional

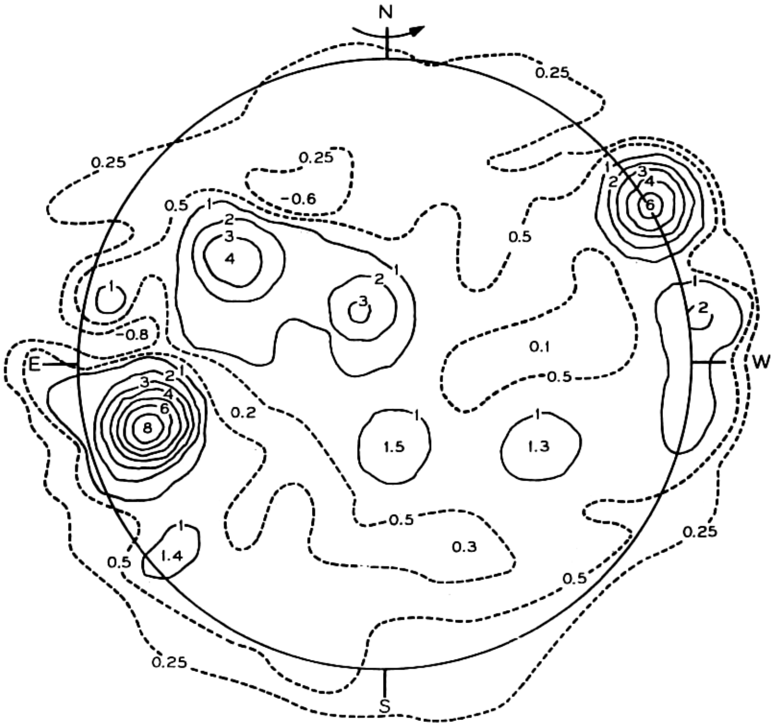

Fig. 3 — Radio picture of the sun.

to the amplitude and area of the enhancement, and its distance from the center. Because of the rotation of the sun about its axis in a period of 28 days, these regions move from left to right in about 14 days. The intensity of the enhancements is wavelength dependent, being most prominent at about 10 cm. For this reason, the resulting displacement of the radio center depends on wavelength. Measurements have been reported[4] which give in one case a maximum displacement of 1.2 mils at 3.2 cm and 0.9 mil at 2.0 cm during the peak of sunspot activity in 1957–58, and in another case 1.05 mils at 3.2 cm and 0.57 mil at 1.6 cm.

Some radio center observations made during the early part of this study are shown in Fig. 4. This is a plot of radio center displacement caused by sunspot activity during a two-week period in November 1960. In order to make these measurements, the antenna was first accurately aligned. Movement of the radio center was associated with a large sunspot which appeared on the eastern limb of the sun on November 5. Then, because of the rotation of the sun on its axis, the sunspot moved across the central meridian of the sun on November 12 and subsequently passed from view off the western limb on November 18. It can be seen that the radio center position changes slowly because of the long life-

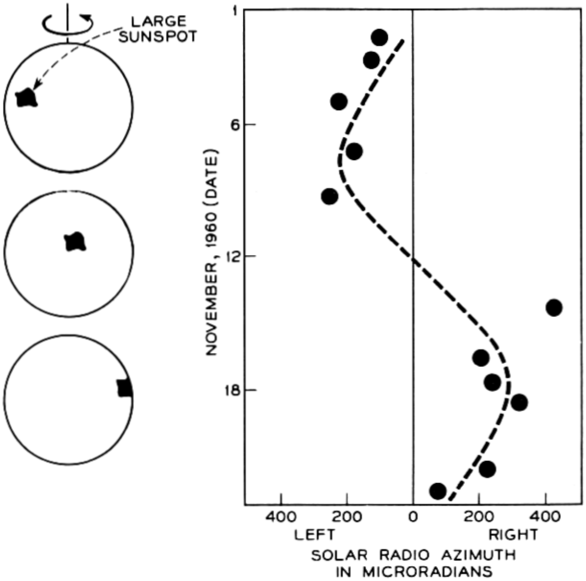

Fig. 4 — Radio center movement.

time of the enhanced regions and their slow movement with the solar rotation. Actually, the variation over a period of several hours is small enough to be neglected.

Unless corrected, this pulling effect imposes a major limitation on the accuracy of radio direction finding using the sun. However, a method has been found which makes it possible to determine the radio center displacement and also antenna alignment errors from data obtained in tracking the sun over a period of a few hours. That is, without previous antenna alignment, it is possible to determine not only the radio center displacement, but also the antenna alignment errors. The method makes use of the apparent motion of celestial bodies caused by the rotation of the earth. The effect is perhaps most easily visualized in the case of the stars. For an observer in northern mid-latitudes, stars which rise in the east reach a maximum elevation angle when they cross the observer's meridian to the south and then set in the west (Fig. 5). Consider two stars which rise one after the other at the same point on the horizon. As they rise, the later one will be below and left of the earlier one. At meridian crossing, the two stars are side by side. When they are setting, the later star is above and to the left of the earlier one. To the observer, the later star has moved clockwise relative to the earlier one. A quantitative description of this rotation is given in Appendix B.

This same rotational effect is also observed in the case of the sun. That is, any point displaced from the center of the sun will appear to rotate around the center. This suggests that if angle measurements on the radio sun are compared to the azimuth and elevation of the geo-

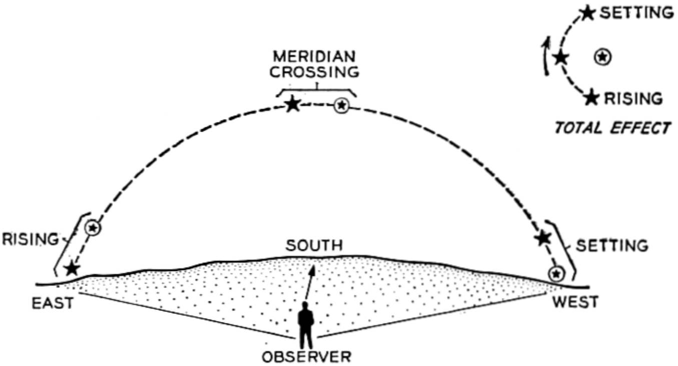

Fig. 5 — Apparent rotation of celestial bodies.

metric center of the sun,* the differences between the measured and calculated values will show the motion of the displaced radio center. An elevation versus azimuth plot of these differences is made to look for this rotational effect. Typical data obtained in sun tracking on August 8, 1961, are shown in Fig. 6. The abscissa is actually the azimuth differences times the cosine of the elevation to refer the azimuth up to the viewing plane. Each point is the mean difference in a 24-second sample. Since the antenna had not been previously aligned, an initial alignment is made on the first observation, which results in zero differences for the first data point. The rotation with time apparent in the curved pattern of the data points is attributable to the apparent rotation of the radio center. Data points for 11 AM, noon, and 1 PM are indicated.

A simple graphical method was used to examine these data. An overlay template was constructed for this particular day and latitude and fitted to the data as shown in the shaded area. Four independent de-

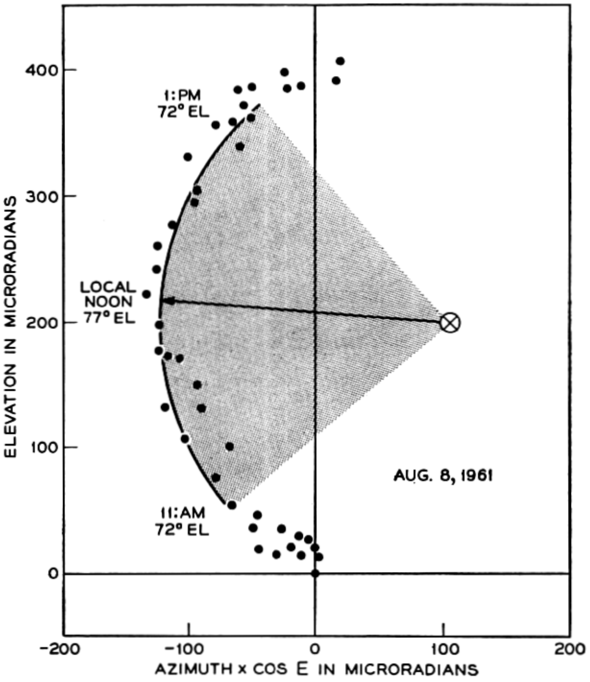

Fig. 6 — Typical sun tracking data, August 8, 1961, with initial zero set.

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* The position of the sun can be calculated accurately from the ephemeris, as described in Appendix C.

terminations were averaged to obtain the "best" estimate of the radio center displacement and error in the initial zero setting. If the initial zero set had been correct, the center of curvature would have been at the origin. Therefore the final zero settings can be determined from the offset in Fig. 6. Fig. 7 shows the same data after making the final zero set adjustment.

The radio center displacement may be described in a coordinate system which is independent of time by resolving the displacement vector at noon into horizontal and vertical components. The horizontal component becomes the displacement in hour angle and the vertical component the displacement in declination. A summary of the values of radio center displacements obtained is given in Table I. The last column in Table I shows the probable error in the radio center determination and therefore indicates the accuracy attainable in using the sun as an X-band directional reference.

Radio center displacement is most easily determined around local noon when the solar disk rotation rate is maximum, making the effect

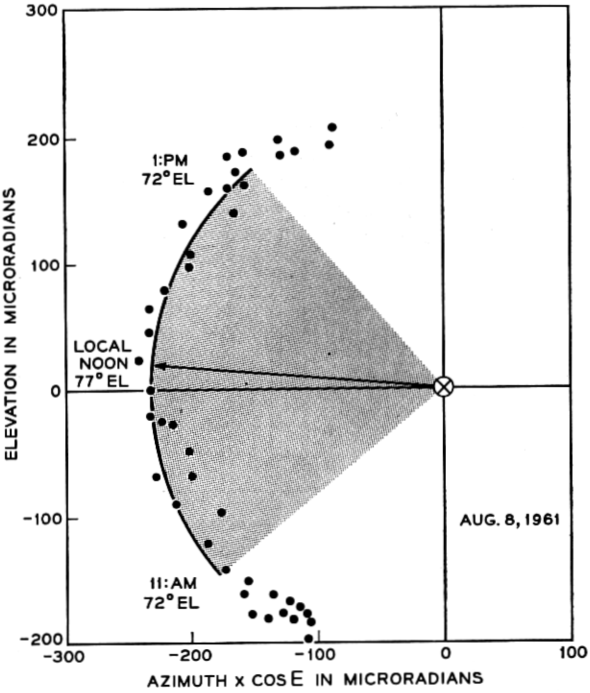

Fig. 7 — Data of Fig. 6 after final zero set adjustment.

TABLE I — RADIO CENTER DISPLACEMENT AT LOCAL NOON

Date	Displacement		Standard Deviation
	Hour Angle	Declination	
	(Microradians)	(Microradians)	(Microradians)
April 24, 1961	−125	+50	14
May 25, 1961	+85	+40	8
June 1, 1961	+95	+20	12
June 15, 1961	−95	+15	9
June 21, 1961	+135	+40	16
July 25, 1961	+95	+50	30
August 8, 1961	−235	+20	26

more easily recognized than at any other time of day. Also, since the sun is at its maximum elevation angle at this time, atmospheric effects are minimized. Another important advantage of noontime observations is that the elevation angle is changing very slowly, which means that elevation-dependent systematic errors can be considered essentially constant.

Azimuth zero set errors are compressed at high elevation angles by the cosine of the elevation angle. For this reason, azimuth zero set accuracy is better at lower elevation angles.

The most important advantage in using celestial microwave directional references is the direct measurement of the radio axis to angle indicator relationship without going through the involved intermediate steps of determining systematic errors between the radio axis and the optical line-of-sight and the systematic errors between the optical line-of-sight and the angle indicators. These experiments have demonstrated that the limitations of using the sun can be overcome to make direct measurements possible.

III. APPLICATIONS AND LIMITATIONS

The sun serves as a useful tracking source in investigating atmospheric refraction and other low-angle effects on angle tracking.

Directional measurements of celestial bodies is the classical method of position determination in celestial navigation. The accuracy obtained in high-angle sun tracking enables the geodetic position of the tracking antenna to be determined to an accuracy of about 600 feet.

Some of the limitations in using the sun as a precise reference are given below.

(a) *Solar flares:* Although the level of solar radio emission can generally be considered substantially constant over a period of several hours, intense outbursts are sometimes observed at the time of large solar flares. These outbursts can exert a strong pulling effect on the radio center, but fortunately they are generally of short duration (several minutes) and are easily recognizable in tracking data. Of the approximately 200 data points in seven days tracking, only one was affected by a suspected outburst. This occurred on June 15, 1961, at 1641 Universal Time at the same time as an "Outstanding occurrence" reported by Ottawa[5] on 10-cm wavelength. The effect was a brief displacement of the radio center amounting to about 150 microradians.

(b) *Solar disk rotation:* Because of the geometry of the solar disk rotation, the determination of radio center displacement is more easily accomplished at the higher elevation angles of the sun. For this reason, radio center displacement is more easily determined in the summer at low latitudes. Longer observing times are required at higher latitudes in the winter.

(c) *Antenna beamwidth:* Sun or moon tracking with antenna beam widths of less than $\frac{1}{2}$ degree will suffer from decreased angle sensitivity caused by the large source distribution. In the extreme case of beamwidths of the order of $\frac{1}{10}$ degree, less extended sources such as the "radio stars" would be more attractive.

IV. SUMMARY

Celestial radio sources are attractive as microwave directional references; however, two major aspects must be investigated in considering them. The first aspect is the ability to track the source. The second aspect deals with the precise knowledge of the position of the celestial radio source. For a conventional X-band radar receiver utilizing a crystal mixer, the sun is most suitable from the standpoint of "noise signal"-to-noise ratio. The moon presents a marginal condition. By tracking the sun in a time-continuous mode (i.e., range gate disabled), the tracking quality becomes limited only by the atmospheric noise.

In examining the second aspect, the radio center of the sun is not coincident with the actual center, but is displaced by local regions of enhanced radio emission associated with sunspot activity. Although the sunspot activity is random on a day-to-day basis, the effect upon the radio center displacement is small enough to be considered constant over a period of several hours. A technique was developed to utilize the diurnal motion of the earth to enable the radio displacement to be determined independently of other sources of error.

V. ACKNOWLEDGMENT

This work was sponsored by the U. S. Air Force.

APPENDIX A

System Description and Tracking Performance

A conventional monopulse tracking system was used for the experiments which have been described. Normally, the radar receiver samples the input only during the very short interval of time when the returning pulse is expected. This is, of course, not the optimum procedure to use in the case of a continuous, low-level signal. However, by means of a simple modification, the receiver can be kept open to optimize system performance with signals which are present continuously.

Since the microwave signals from celestial radio sources are quite constant in level and cover a broad spectrum, neither automatic gain nor frequency control are required.

With these modifications, the performance of the tracking system can be expressed as a function of the system parameters as follows:

$$\delta\Theta = \Theta_b \frac{\sqrt{1 + S/N}}{2S/N\sqrt{2B\tau}}$$

where $\delta\Theta$ = rms angle fluctuations
Θ_b = antenna beamwidth
S/N = input signal-to-noise power ratio
B = receiver bandwidth
τ = post detection integration time.

This result is based on the assumption of a point source of noise in the presence of external background and internal receiver noise, but does not include the effects of transmission through the atmosphere.

The noise power, N, received from the sun is the product of the solar flux density, S, and the effective antenna area, A, thus:

$$N = S \times A \times \tfrac{1}{2} = 4.37 \times 10^{-20} \text{ watts/cps}$$

where the factor $\frac{1}{2}$ accounts for the fact that the antenna accepts only the vertically polarized component of the randomly polarized solar radio emission. It is useful to consider the temperature, T_{eq}, of an equivalent network, replacing the antenna, which would have an available noise power equal to that from the sun. In this case,

$$kT_{\text{eq}} = N$$

where k is Boltzmann's constant, 1.38×10^{-23} watts/°K cps. The resulting effective antenna temperature from the sun is thus 3150°K.

The receiver noise temperature, assuming a nominal 11-db noise figure, is:

$$T_R = (F - 1)T_0 = 11.6 \times 290°\text{K} = 3360°\text{K}$$

where F is the system noise figure expressed as a power ratio and T_0 is the reference temperature, 290°K. Thus, the signal-to-noise ratio is about unity. Assuming a typical 1° beamwidth, 10-mc bandwidth, and $\frac{1}{2}$-second time constant, the resulting tracking performance is about 5 microradians. In actual tracking however, this performance cannot be realized because of atmospheric limitations which are believed to be in the order of 20 to 50 microradians.

When this system is used with a three-second time constant to track the moon, the short-term angle uncertainties are about 400 microradians. This agrees essentially with the theoretical value, indicating that the system performance when tracking the moon is limited by receiver noise rather than by atmospheric effects.

APPENDIX B

Apparent Solar Disk Rotation

The apparent rotation of the solar disk can be derived from the spherical triangle shown in Fig. 8. The orientation of the sun remains fixed with respect to the great circle passing through the sun and the celestial pole, whereas the orientation to an observer in azimuth-elevation coordinates is always referred to the great circle passing through the sun and the zenith. The apparent rotation of the solar disk is described by the variable angle, p. From the Law of Sines:

$$\sin p = \frac{\cos \phi}{\cos \delta} \sin A .$$

For a given set of observations, the observer's latitude, ϕ, and the declination angle of the sun, δ, remain constant; therefore the apparent rotation of the sun is a simple function of the azimuth angle, A. Fig. 8 and the above equation show that the displaced radio center of the sun will trace through the arc of a circle whose radius is the magnitude of the radio center displacement.

The apparent solar disk rotation can also be expressed in terms of the local hour angle, H, which changes with earth's rotation uniformly at about 15° per hour. The expression is:

$$\tan p = \frac{\sin H}{\sin \delta \cos H - \cos \delta \tan \phi} .$$

A plot of this equation is given for 28.5 degrees North latitude in Fig. 9 for several values of declination. From these curves, the maximum

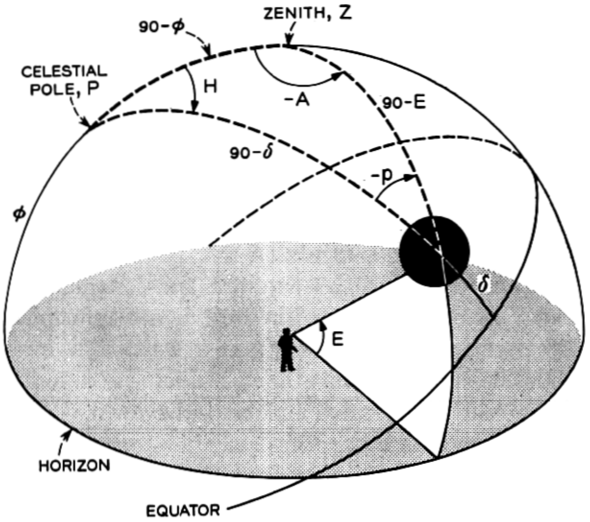

Fig. 8 — Spherical geometry used to derive apparent rotation of solar disk.

rate of rotation occurs at local noon; hence the optimum time to observe the radio center displacement is around local noon.

The latter equation is used to make the overlay template which is fitted to the plotted data to locate the center of rotation of the observed data points.

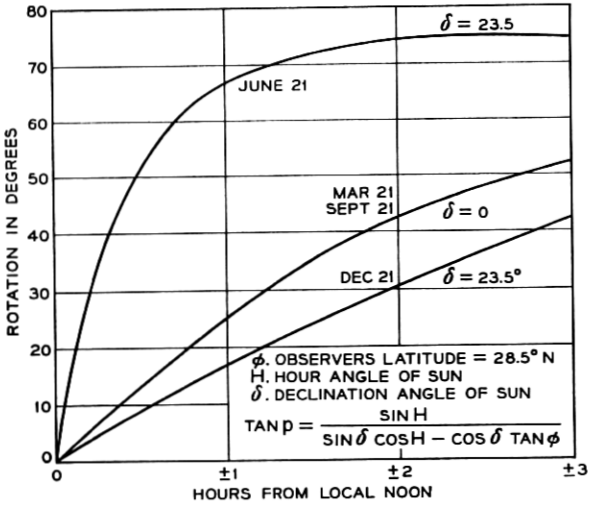

Fig. 9 — Apparent solar disk rotation.

APPENDIX C

Determination of the True Angular Position of the Sun

In the experiment, the tracking system measures the azimuth and elevation angles and records these data along with the time of measurement on magnetic tape. This tape becomes the input to a digital computer which performs the computations described below and compares the measured values with the calculated true position of the sun.

Primary position data for the sun are taken from the *American Ephemeris and Nautical Almanac*. These data are referred to the true equinox and equator of date and contain a correction for aberration. Corrections must be computed for solar parallax, the difference between Ephemeris Time and Universal Time, and the local deflection of the vertical. These corrections are applied in the spherical coordinates of hour angle and declination. The estimated accuracies in these coordinates are 0.02 second of time in hour angle and 0.2 second of arc in declination. During the experiments, time is recorded for each of the angle observations to an accuracy of about 0.05 second. This brings the total accuracy of the computed position of the sun to about 4 microradians.

The hour angle and declination, together with the latitude, are used to calculate the azimuth and elevation angles.

In order to compare the observed values with the computed angular positions of the sun, it is necessary to correct the observed values of elevation for atmospheric refraction. The correction used is

$$\Delta E = -N_0 \cot E$$

where N_0 is the index of refraction determined from observations at the tracking site and E is the observed elevation angle. This equation is adequate for elevation angles above 10 degrees.

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