Behavior and Applications of Ferrites in the Microwave Region

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(Manuscript received October 4, 1954)

The behavior and applications of ferrites in microwave circuits have been studied at the Holmdel Radio Laboratory with particular emphasis on non-reciprocal properties. There are numerous ways of building non-reciprocal devices which accomplish the same function, and the authors propose at the outset that a terminology based on function be adopted. Definitions of the gyrator, circulator and isolator are given.

A qualitative method of deducing the properties of new ferrite-loaded circuits, termed "point-field" analysis, has been particularly fruitful, in that it permitted the exploration of a wide variety of ferrite-loaded circuits despite the absence of precise mathematical analyses.

Faraday rotation in longitudinally-magnetized ferrite-loaded circuits has been studied; the effects of higher-order modes, the reasons for variation in Faraday rotation as a function of frequency, and the optimum geometry for the ferrite loading are discussed. In transversely-magnetized ferrite-loaded rectangular waveguide it is shown that the single mode medium may be non-reciprocal as to phase constant, attenuation constant, or distribution of field components in the cross-section. The latter effect is called the "field-displacement" effect. In transversely-magnetized ferrite-loaded round waveguide, either reciprocal or non-reciprocal birefringence may be achieved and examples are given. Some non-reciprocal ferrite-loaded dielectric waveguides are discussed. In all of these forms of non-reciprocal transmission media, gyrators, circulators and isolators may be built, and experimental data are reported on some of the possibilities. A brief review of prospective uses for these non-reciprocal elements is given.

CONTENTS

1. INTRODUCTION

In the history of the communication art there has never been available any passive circuit element or waveguide medium which was not reciprocal. After Polder,[1] Beljers,[2] and Roberts[3] demonstrated the gyromagnetic nature of ferrites at microwave frequencies, passive non-reciprocal devices for the first time became possible. Such non-reciprocal devices have great immediate and potential value in the microwave art. Additionally, ferrites have reciprocal properties which are under the control of an applied magnetic field and which lead to new forms of modulators, variable attenuators, and variable reciprocal phase-changers in the microwave range.

Since C. L. Hogan*,[4] demonstrated the practical importance of ferrites to the microwave art by making use of microwave Faraday rotation to build a gyrator, the authors have studied, at the Holmdel Radio Laboratory, the behavior and applications of ferrites with emphasis on their non-reciprocal properties. It soon became apparent that ferrites can manifest non-reciprocal properties in a variety of ways other than by Faraday rotation. Thus, they may produce non-reciprocal birefringence, non-reciprocal phase shift, non-reciprocal field displacement, non-reciprocal coupling through apertures, as well as non-reciprocal rotation of polarization. Any one of these non-reciprocal properties can be used, together with conventional reciprocal circuit elements, to obtain all of the non-reciprocal circuit functions which have so far been devised. It therefore seems desirable, before proceeding further, to identify some of the more important non-reciprocal circuit elements with individual names and circuit symbols irrespective of the detailed means used to obtain these circuit functions.

1-1. *Definitions and Symbols for Non-Reciprocal Devices*

The *gyrator* is defined[5] as a transmission element having no phase shift for wave propagation through it in one direction, but having a phase shift of π radians for propagation in the opposite direction. A single line circuit symbol for this element is shown in Fig. 1. Lines 1 and 2 represent input and output waveguides. The symbol $\overleftarrow{\pi}$ in the box indicates a phase delay of π radians for waves passing in the direction of the arrow, and no phase delay in the reverse direction of propagation. It is of course possible to add any amount of reciprocal phase shift in series with this element so that for one direction of propagation the phase delay may be θ, and for the other direction of propagation the phase delay may be $\pi + \theta$. However, this additional reciprocal phase delay is non-essential, and the function defined by Fig. 1 is the essence of the gyrator behavior. Various practical embodiments are discussed later in this paper.

It may be noted that Tellegen[5] chose to use the *gyrator* as an essential building block, out of which various non-reciprocal circuits could be constructed. For this reason, there has been a tendency for many people to call any non-reciprocal ferrite device a gyrator. We strongly feel that

* Contemporary with Hogan's disclosure, C. H. Luhrs marketed a microwave switch employing Faraday rotation in a ferrite-loaded cavity.

this loose usage should be avoided, since many of the structures which have been called gyrators are in no sense the kind of thing which Tellegen defined. Some of these devices may be of more practical importance than the gyrator and are just as capable of being employed as fundamental building blocks. We shall therefore restrict the usage of the term *gyrator* to that defined above in agreement with Tellegen's original definition.

A second circuit element of considerable fundamental importance is the *isolator*, so named because it can be used to isolate one transmission element from reflections arising from succeeding elements. Fig. 2 is a circuit symbol which demonstrates this behavior. The symbol ←ʌʌʌ in the box indicates absorption of power in an internal load for propagation from 2 to 1, and a transmission with no absorption of power from 1 to 2. This representation reminds one that a device which prevents transmission in only one direction must always do so by absorption rather than by reflection of power, as a consequence of the second law of thermodynamics.

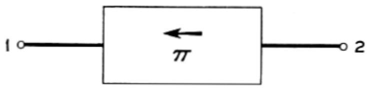

Fig. 1 — Gyrator symbol.

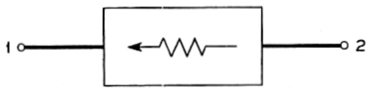

Fig. 2 — Isolator symbol.

Figure 3(a) shows the circuit symbol for a *circulator*. The term *circulator* was chosen to connote a commutation of power from one transmission-mode terminal to another. Thus, power delivered at terminal 1 passes through the circulator in the direction indicated by the curved arrow to the next terminal, 2, where it emerges. Power inserted at 2 proceeds on around to terminal 3; power inserted at terminal 3 proceeds to terminal 4; and power inserted at terminal 4 finally returns to terminal 1. Although the four-arm circulator is displayed in Fig. 3(a), it is also possible to build three-arm circulators as well as various other multi-arm circulators, as illustrated in Fig. 3(b). Practical means for obtaining these devices are described later in this paper.

Finally, one should note that by employing any of the various means available for providing non-reciprocal phase delay, it is possible to obtain an arbitrary number of degrees of non-reciprocal phase shift as shown symbolically in Fig. 4. Such a device can be called a *directional phase shifter*. Clearly, the gyrator is merely a special case of the more general *directional phase shifter*.

By clearly distinguishing between these various important circuit functions, both symbolically and in terminology, much confusion can be avoided.

The above mentioned non-reciprocal functions were first realized by means of Faraday rotation in ferrite-loaded round waveguides.* It became apparent at an early stage that the uniform plane-wave explanation of Faraday rotation is totally inadequate to explain the behavior of ferrites in waveguides except in a qualitative way. Measurements of the reflection and transmission properties of ferrite-loaded waveguides show many anomalies which would not be predicted from the infinite uniform

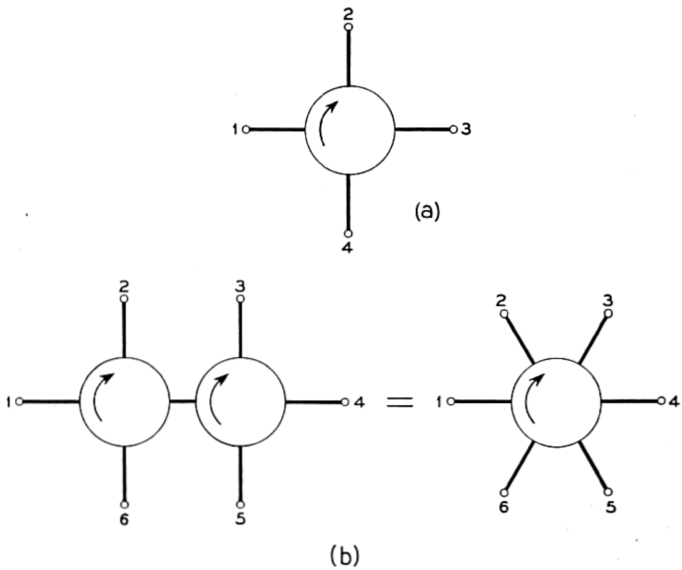

Fig. 3(a) — Circulator symbol.
Fig. 3(b) — Multi-arm circulator symbol.

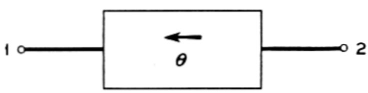

Fig. 4 — Directional phase shifter symbol.

* See Reference 4, C. L. Hogan.

plane-wave theory. On the other hand, a rigorous solution of Maxwell's equations presents formidable mathematical problems which have so far been solved for only a few cases. It is possible, however, to obtain a qualitative explanation of the observed facts together with a considerable amount of physical insight by making use of a "point-field analysis" which is based on some reasonably intelligent guesswork as to the nature of the electromagnetic-wave field patterns. We assume that the ferrite-loaded waveguide contains a perturbed wave bearing a strong resemblance to one or more of the well-known modes in an empty waveguide. We determine the fields which this wave would produce in every part of the ferrite. The behavior of the ferrite at every point can then be predicted on the basis of Polder's tensor permeability theory, or on the basis of the clockwise and counterclockwise permeability theory. Finally, the total behavior can be deduced as a weighted sum of the behavior of all the parts of the ferrite.[6] Not only does this approach give explanations for the observed behavior of Faraday rotation elements, but it has aided greatly in understanding and predicting the behavior of ferrites in radically different geometries making use of transverse applied magnetic fields. It should be emphasized that the non-reciprocal properties of ferrites stem from the gyroscopic nature of the electrons which may be manifested in a variety of ways of which Faraday rotation is but one. In this paper we attempt to build up a physical picture of the way ferrites actually interact with waveguide waves and to show how these same physical concepts have been useful in deriving new non-reciprocal devices.

2. MICROWAVE PROPERTIES OF FERRITES

The magnetic properties of ferrites are essentially the same as those of iron and other ferromagnetic metals. The one outstanding characteristic which distinguishes good microwave ferrites from the magnetic metals and which gives rise to their unique microwave behavior is their extremely high resistivity (typically from 10^{+6} to 10^{+8} ohm-cm for ferrites relative to 10^{-5} for iron). Whereas in the case of iron, a radio wave sees an effective reflector, in the case of ferrite the wave can enter and pass through substantial amounts of the material without excessive reflection or attenuation. In the process, the wave has an opportunity for strong interaction with the spinning electrons which are responsible for the magnetic properties of the material. As a result of this interaction Faraday rotation and other non-reciprocal properties may be manifested.

While the ferrites exhibit a favorably high resistivity, they also have moderately high dielectric constants at microwave frequencies, which are typically in the range from 10 to 20. This gives rise to important problems in applying ferrites. The high dielectric constant can cause appreciable reflection of power at an air-ferrite boundary, and such reflections may be but slightly reduced by the polystyrene matching cones which have sometimes been used. Also, the presence of high dielectric constant material within a normally single-mode waveguide may give rise to mode conversion (conversion of power from the desired to undesired waves in the ferrite-loaded section) even in the absence of an applied magnetic field. Both of these effects can greatly alter the reciprocal and non-reciprocal transmission properties of the ferrite-loaded region, and many of the capricious results obtained in early studies of ferrites are attributable to reflection and mode conversion difficulties.

The vital property of ferrites which makes them so useful in the microwave art is their unique permeability. There are two ways of defining ferrite permeability — one, in terms of a tensor permeability for linearly polarized waves, and the other, in terms of a scalar permeability for circularly polarized waves.

The ferrite's microwave permeability is due to the effects of certain electrons, which behave en mass gyroscopically according to the classical picture of Fig. 5. It is found that the charge, mass and spin of these electrons are associated with an angular momentum and a magnetic moment in the directions shown, which it may be noted are the same as those to be expected for a spinning positive mass and negative charge.

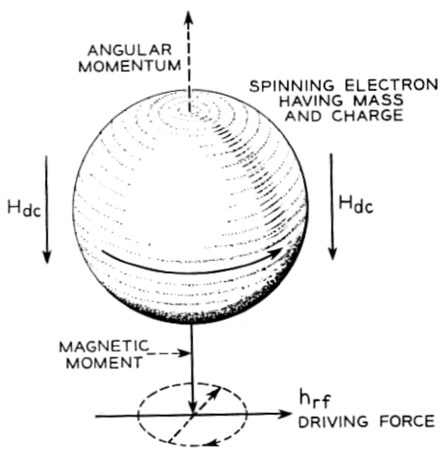

Fig. 5 — Schematic representation of an electron.

With the application of a dc magnetic field (H_{dc}), the axis of the electron spin is aligned with the dc field. If the spin axis is momentarily deflected from parallelism with the dc field, it will not return to its original position immediately but will precess as a gyroscope about H_{dc} at a frequency which is proportional to the magnitude of H_{dc}. This frequency we call the gyromagnetic resonance frequency, and the behavior of ferrites is profoundly dependent upon whether the frequency of applied radio waves is greater or less than the gyromagnetic resonance frequency. If an alternating magnetic field (h_{rf}) is applied perpendicular to H_{dc}, the resulting precession will cause the tip of the magnetic moment vector to describe an elliptical path as indicated by the dashed line in Fig. 5. The direction of rotation around the ellipse will always be clockwise when looking along H_{dc}. As a result, the electron will produce components of magnetic flux perpendicular to both H_{dc} and h_{rf}.*

2-1. *Tensor and scalar permeabilities*

With this classical model as a postulate, Polder[1] has shown that a uniformly magnetized and saturated region of ferrite subjected to uniform rf fields will contain uniform flux densities given by

$$\begin{aligned} b_x &= \mu h_x - jkh_y \\ b_y &= \mu h_y + jkh_x \\ b_z &= \mu_0 h_z \end{aligned} \tag{1}$$

where

$$\mu = \mu_0 - \frac{\gamma M \omega_0}{\omega_0^2 - \omega^2} \tag{2}$$

$$k = \frac{\gamma M \omega}{\omega_0^2 - \omega^2} \tag{3}$$

* This description is a very much simplified explanation of ferrite behavior. For example, most of the electrons in the ferrite are paired with their spins pointing in opposite directions, and these do not contribute to any first order magnetic effects. By virtue of certain forces between them known as "exchange forces," some electrons prefer to line up with their spins in parallel and may be easily oriented en mass by the application of a relatively small magnetic field. This is what makes the material appear "magnetic." At the same time the presence of thermal energy has an opposite effect in that it tends to create disorder in the alignment of electron spins. As a result, the spins never all line up perfectly with the applied field. Nevertheless, we can assume that out of the total number of unpaired electron spins a certain fraction do line up with the applied field and behave in accordance with the simple picture given above for a single electron.

$$\omega_0 = -\gamma H_i \tag{4}$$

μ_0 = permeability of free space

γ = ratio of magnetic moment to angular momentum for the electron

H_i = internal dc field directed along + Z axis

The rationalized MKS system is used here and throughout this paper. It should be understood that H_i is the macroscopic dc H in the medium calculable by standard magnetostatic techniques. It is not the H which would occur in a spherical cavity in the medium. The components h_x, h_y, and h_z are the total internal rf field components in the macroscopic sense. They include not only the externally applied rf field, but also any demagnetizing fields which may be present. M is the magnetic polarization density and is taken as $B - \mu_0 H$. ω_0 may be interpreted as the natural precession frequency of an isolated electron placed in the steady field H_i. Finally, it is important to note that the magnetic moment points in the opposite direction from its angular momentum. Therefore γ is negative, and a minus sign must be used when substituting in the above equations. Consistent with equation (4), γ must have the dimensions of ω/H. It is equal to

$$0.035 \frac{\text{mc}}{\text{amp/meter}}, \text{ (or 2.8 mc/oersted)}$$

The tensor form of the equations (1) indicate that an rf h vector applied along the X axis will give rise to rf components of b along both the X and Y axes. Consequently, the k component of the permeability tensor plays the part of a coupling coefficient which can account for the transfer of rf wave power from one polarization to an orthogonal polarization.

An alternative expression for permeability can be derived from the tensor form. Let us assume that the ferrite is excited by a clockwise circularly polarized field vector as seen looking along the dc magnetic field, and this will be designated as a (+) field vector. Then

$$h_y = -jh_x\} \quad (+) \text{ rf } h \text{ vector} \tag{5}$$

Substitution of these in equation (1) gives:

$$\left.\begin{aligned} b_x &= (\mu - k)h_x \\ b_y &= -j(\mu - k)h_x \end{aligned}\right\} \quad (+) \text{ rf } b \text{ vector} \tag{6}$$

Thus, the resultant b is circularly polarized in a clockwise sense. If we now assume a counterclockwise field vector looking along the dc magnetic field and denoted by (−),

$$h_y = +jh_x\} \; (-) \text{ rf } h \text{ vector} \tag{7}$$

Substitution in equation (1) gives:

$$\left.\begin{aligned} b_x &= (\mu + k)h_x \\ b_y &= +j(\mu + k)h_x \end{aligned}\right\} (-) \text{ rf } b \text{ vector} \tag{8}$$

The resultant b is now polarized in a counterclockwise sense. We can see that for circularly polarized fields the total permeability is a scalar. Thus,

$$\mu_+ = \frac{b_+}{h_+} = (\mu - k) = \mu_0 - \frac{\gamma M}{\omega_0 - \omega} \tag{9}$$

$$\mu_- = \frac{b_-}{h_-} = (\mu + k) = \mu_0 - \frac{\gamma M}{\omega_0 + \omega} \tag{10}$$

Fig. 6 illustrates schematically how the μ_+ and μ_- vary as a function of H_i. The μ_+ has been plotted to the right and the μ_- to the left of the $H_i = 0$ axis, with H_i increasing in both directions. While Polder's equations were developed without any losses included, the permeability curve

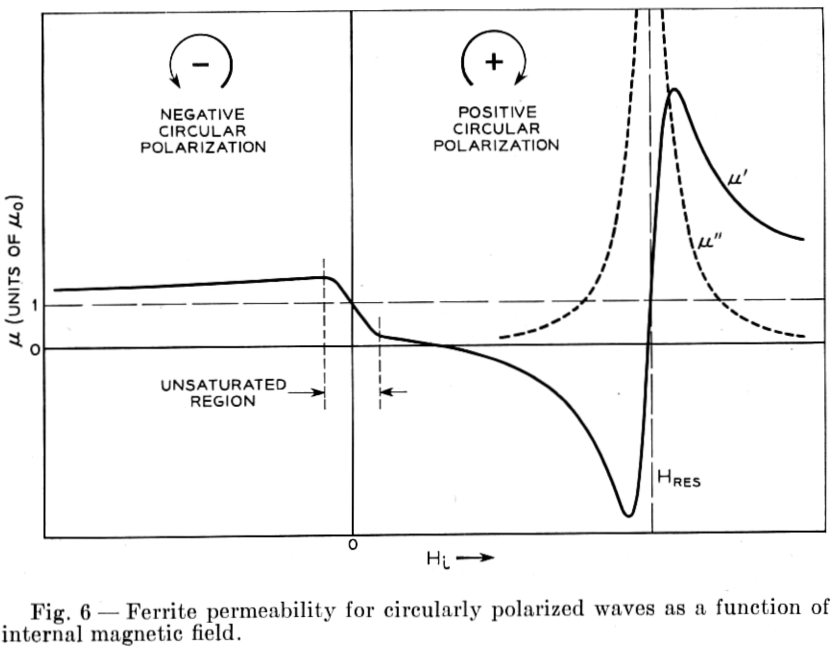

Fig. 6 — Ferrite permeability for circularly polarized waves as a function of internal magnetic field.

is plotted in Fig. 6 as it would be expected to appear if some magnetic loss were present. Thus, the real part (μ') of the permeability remains finite throughout the resonance region. The imaginary or loss component of the permeability (μ'') should rise to a maximum at resonance as shown by the dashed curve. Polder assumed that the ferrite was completely saturated, in which case the magnetization M is a constant. However, it is believed that equations (9) and (10) are valid, at least qualitatively, for magnetizations less than saturation, in which case the permeability curve for low fields goes to 1.0 as shown. Although ferrites may be operated near resonance for some isolator applications where large attenuations are desired, for other applications where small transmission losses are desired it is preferable to operate at fields as far below resonance as possible.

The permeability of Fig. 6 is the rf permeability which appears in Maxwell's equations, and is the ratio of internal b to internal h. It is independent of the shape of the ferrite body. However, the internal h may be quite different from the externally applied h. Since we are frequently more interested in the external h than the internal h, it is convenient to define the ratio of b internal to h external as an "external μ" (μ_{ex}). This external μ will go through resonance not at an internal field given by $\omega/|\gamma|$ but at a field which depends on the shape of the ferrite body. Thus, if the demagnetizing factors in directions normal to H_i are appreciable, a displacement of the total angular momentum vector from its equilibrium position will create restoring torques due to the creation of magnetic poles on the surface of the body. These restoring torques added to the torque produced by H_i will increase the resonance frequency of the body as a whole above $\omega_0 = -\gamma H_i$. Furthermore, anisotropy fields may also change the resonant frequency. It is therefore convenient to postulate a fictitious dc magnetic field "H effective" which takes into account H_i, the rf demagnetizing factors, and the anisotropy fields, and which if operating on an electron in free space would produce the same resonance frequency as is observed for the ferrite body.[7] By reading the abscissa of Fig. 6 as "H effective" and the ordinate as μ_{ex}, Fig. 6 may conveniently be used as a universal curve applicable for any shape of body for which an "H effective" can be defined. Since losses are a maximum at the resonance for μ_{ex} (not at the resonance for μ), the literature usually implies this μ_{ex} resonance when discussing ferromagnetic resonance. In this paper the unqualified term "resonant frequency" refers to the resonant frequency of μ_{ex}.

3. EXPLANATION OF FARADAY ROTATION IN TERMS OF NORMAL MODES OR COUPLED MODES

With a knowledge of the microwave properties of ferrites one can analyze the electromagnetic transmission properties of a ferrite-loaded medium but, in general, such an analysis is quite complex. However, the effect of ferrite loading of a two-mode transmission medium can be understood with relative simplicity using either of the following two viewpoints:

(1) the normal mode approach in which the input and output waves are resolved into two modes which are orthogonal in the presence of the ferrite,

(2) the coupled-mode approach, in which the behavior of the ferrite-loaded region is described in terms of a coupling, introduced by the ferrite, between two modes which are orthogonal in the absence of the biased ferrite.

In the following section both viewpoints are applied to an explanation of Faraday rotation in ferrite-loaded round waveguides.

3-1. *Normal Modes in a Ferrite Medium*

As Polder has pointed out, the fact that the permeability is a scalar quantity for circularly polarized fields means that in an infinite ferrite medium the normal modes of propagation along the Z-axis are $(+)$ and $(-)$ circularly polarized waves. In fact for considerably more complicated cases where, for example, the medium is a round rod of ferrite along the Z-axis in free space propagating dominant guided waves along the rod, it will still be true that circularly polarized waves will be the normal modes for the medium.

For a waveguide comprising a metal tube of circular cross section containing a very slender pencil of ferrite lying along the axis of the tube and having a diameter much less than the tube diameter, we can ascribe to the composite medium inside the metal tube an effective dielectric constant (ϵ_e) and effective permeability (μ_e) which for a homogeneous dielectric across the entire cross section would yield the same characteristic impedance and phase constant as is actually observed. Since the ferrite pencil was said to be slender, ϵ_e would be only slightly greater than one. Since the ferrite will perturb the dominant wave only slightly, circularly polarized dominant waves will produce circularly polarized transverse magnetic fields in the rod. The effective permeability for the waveguide would then be slightly greater or less than one for the $(-)$ and $(+)$ dominant modes respectively. Thus, the two circularly polarized dominant waves which are the "normal modes" for the medium will travel at slightly different velocities. Alternatively, we can say that the index of refraction of the dielectric medium for (+) waves is different from that for (−) waves. In the usual case where the wave frequency is high and the H_i is low so that $\omega_0 < \omega$, the index of refraction for (+) waves will be lower than the index for (−) waves. Therefore, the (+) waves will travel through the tube at a higher phase velocity than (−) waves. As a result, a linearly polarized input wave traveling along H_i may be decomposed into (+) and (−) components which, after passing through the section, may be added to give a new linearly polarized wave whose polarization axis has been rotated in a clockwise sense relative to that of the input. The magnitude of the rotation depends upon the length of the section and also upon the effective dielectric constants and permeabilities. The difference in phase $\Delta\varphi$ between the (+) and (−) components passing through a length z of ferrite loaded waveguide is

$$\Delta\varphi = \ell(\beta_- - \beta_+) = \omega\ell\sqrt{\epsilon_e}(\Phi_-\sqrt{\mu_{e-}} - \Phi_+\sqrt{\mu_{e+}}) \qquad (11)$$

where Φ_+ and Φ_- are cut-off factors for the two polarizations which are determined by the geometry of the conducting boundaries and the μ_e and ϵ_e of the dielectric medium. If the operating frequency is well above cut-off, then to a first approximation Φ_+ and Φ_- equal one.

The angle of rotation for a linearly polarized wave will be

$$\theta = \frac{1}{2}\Delta\varphi = \frac{\omega\ell}{2}\sqrt{\epsilon_e}\,(\Phi_-\sqrt{\mu_{e-}} - \Phi_+\sqrt{\mu_{e+}}) \qquad (12)$$

We see that the Faraday rotation angle is dependent not only upon the difference between the square roots of μ_- and μ_+ but also upon the effective dielectric constant of the medium. Thus, if the medium is loaded dielectrically without altering the effective permeability, the Faraday rotation should increase.

3-2. *Coupled Modes in a Ferrite Medium*

What has just been outlined is the normal mode approach based upon the fact that the ferrite permeability is scalar for the two circularly polarized modes. Alternatively, we can use the coupled mode approach based on the use of linearly polarized modes and the associated tensor components of permeability. We recognize, as stated earlier, that the k part of the permeability tensor is a measure of the ability of the electrons to couple energy from a field of one polarization to a field of orthogonal polarization. We know from the theory of distributed wave coupling[8] that if we have two modes of propagation in a waveguide having the same phase velocity, power can be completely transferred from one mode to the other if there exists uniform distributed coupling between the modes along the waveguide. In particular, if all of the energy is introduced as a wave in one of the modes, it will diminish in amplitude as it travels along the waveguide, while a wave in the other mode will start from zero and grow in amplitude. After a certain distance all of the power will have been transferred from the first to the second mode. Beyond this point the reverse process will take place. The amplitudes (voltages) of the two waves as a function of distance z along the waveguide will be given by

$$V_1 = V_0 \cos (KZ) \; \epsilon^{-j(\beta+K)Z+j\omega t} \tag{13}$$

$$V_2 = -jV_0 \sin (KZ) \; \epsilon^{-j(\beta+K)Z+j\omega t} \tag{14}$$

where K is the rate of change of voltage with distance of one wave produced by unit voltage in the other wave. Thus, the larger the coupling coefficient K, the more rapidly with distance will power cycle back and forth between the two modes.

Let us now apply this point of view to the round waveguide containing an axially magnetized pencil of ferrite, located along the waveguide axis. We may identify vertically and horizontally polarized dominant modes as the coupled modes for the medium. They do have the same phase velocity and they are orthogonal. When the ferrite pencil is magnetized, the spinning electrons produce a uniformly distributed coupling between the modes. If we launch a vertically polarized wave at the input it will decrease in amplitude as it travels along the waveguide and the horizontally polarized wave will increase in amplitude.

Let us assume that the X axis is vertical and that H_i and the direction of propagation are in the plus Z direction. In a thin transverse section of the waveguide, the h_y of the vertically polarized* wave induces through the spin-precession coupling an rf magnetization m_x which, for the usual case where $\omega_0 < \omega$, lags h_y by 90 degrees. This m_x then reradiates a horizontally polarized wave having an h_x which lags m_x by 90°.[9] Thus, h_x lags h_y by 180°, and the induced horizontally polarized wave will always be 180° out of phase with the vertically polarized wave (or in phase if $\omega_0 > \omega$). The resultant wave at any cross section will be linearly polarized and the angle of polarization will increase in a clockwise sense with distance in the direction of propagation. This is, of course, the same result

* As is customary, the direction of the electric field components is taken as the direction of polarization.

as was obtained using the normal mode approach. However, the coupled-wave point of view is very convenient for some problems. In particular, if we consider what will happen if our round waveguide is slightly flattened vertically, we can see immediately that the phase velocities for horizontal and vertical polarization will no longer be equal. Therefore, it will no longer be possible to get complete transfer of power from the vertically to the horizontally polarized modes. At the cross section where maximum power has been transferred to the horizontally polarized mode, the total wave will be elliptically polarized. At twice this distance the resultant wave will again be a pure vertically polarized wave, and the process will repeat cyclically along the axis of propagation.

The coupling coefficient K which is important to the coupled mode argument is obviously related to the effective permeability μ_e used in the normal mode argument, and either one should be derivable from the other. In deriving K directly we must first know the total transverse magnetic field (h_{xy}) at every point in the cross section for one of the coupled modes; and although we frequently assume an unperturbed dominant mode pattern for simplicity, strictly speaking we should use the pattern for the perturbed wave. Knowing the tensor permeability of the ferrite, we can deduce what the transverse b_{xy} at every point in the ferrite will be at a time one quarter period later than the applied field maximum. This induced b_{xy} will be at right angles to the h_{xy} which produced it and hence will couple to a new wave polarized at right angles to the original. However, in order to know how effective this b_{xy} is in generating the new wave, we must know how it conforms in relative magnitude and direction to the h for the pattern of the new wave. Since the coupling of the induced b to the new wave at each point is proportional to $(\vec{m} \times k\vec{h}_1)\cdot\vec{h}_2$, the total coupling coefficient is proportional to the integral of this product over the ferrite cross section.

$$K \approx k \int_A (\vec{m} \times \vec{h}_1)\cdot\vec{h}_2 \, da, \tag{15}$$

where $\vec{m}$ is a unit vector in the direction of the dc magnetization here assumed along the waveguide axis, $\vec{h}_1$ is the rf magnetic intensity produced in the ferrite by the original wave normalized to unit power, and $\vec{h}_2$ is the rf magnetic intensity which would be produced at the same point by the new wave normalized in the same fashion.

For points in the ferrite near the axis of a round waveguide, $\vec{m} \times \vec{h}_1$ for a linearly polarized TE_{11} wave is nearly conformal* with h_2 for the

* As used here, "conformal" does not relate to "conformal transformations," but is a measure of the similarity (in amplitude and direction) of two vector fields.

perpendicular polarization of the TE_{11} wave, and this region is very effective in producing coupling between linearly polarized waves. However for points in the ferrite far from the wave guide axis, the induced field will not be conformal with h_2, and consequently these regions of the ferrite will be less effective in producing coupling. Finally, since the Faraday rotation per unit length is equal to K, which is in turn proportional to the magnitudes of $\vec{h}_1$ and $\vec{h}_2$, it may be seen that anything which increases $|\vec{h}_1|$ and $|\vec{h}_2|$ for unit wave power will increase the Faraday rotation.

This procedure of integrating the product of the induced b with the h of the new wave field pattern is important in the determination of effective coupling between different modes in other instances as will appear later.

4. PRINCIPLES OF FERRITE BEHAVIOR IN FARADAY-ROTATION ELEMENTS

4-1 *Plane-Wave versus Loaded-Waveguide Theory*

The first microwave applications of ferrites made use of a round waveguide containing an axial ferrite pencil in an axial magnetic field. Such "Faraday plates" continue to be of great practical importance. We shall, therefore, endeavor to explain, by means of point-field concepts, the various considerations entering into the design of Faraday plates and to explain the peculiar behavior which they sometimes exhibit.

Let us first consider the case of a uniform plane TEM wave in an unbounded ferrite medium. With magnetization in the direction of propagation, the Faraday-rotation angle is related to distance in the direction of propagation by equation (12) provided we let the cut-off factors Φ_- and Φ_+ equal 1. The effective μ and ϵ will in this case be equal to the μ and ϵ for the ferrite itself. Using equations (9) and (10) for μ_+ and μ_- and assuming that

$$\omega \gg \omega_0 \qquad \text{and} \qquad \omega \gg \frac{\gamma M}{\mu_0}$$

equation (12) reduces to the following:

$$\theta = \frac{\gamma M \ell}{2} \sqrt{\frac{\epsilon}{\mu_0}} \tag{16}$$

This dependence of Faraday rotation on saturation magnetization and dielectric constant can be predicted qualitatively by making use of coupled wave theory. The Faraday rotation of a linearly polarized wave can be described as being due to a distributed coupling between the vertically and horizontally polarized waves. This coupling is produced by the elementary magnetic dipole moments (electron spins) which precess about the direction of dc magnetization. The greater the magnetization, M, the greater the number of dipole moments per unit length of ferrite, thus causing an increase in coupling coefficient and in Faraday rotation. Similarly, if the dielectric constant of the ferrite is increased, the characteristic impedance will decrease, resulting in an increase in the magnitude of the rf h vector in the ferrite, by a factor of $\epsilon^{1/4}$, for a given amount of power in the wave. Since in the plane wave case $|\vec{h}_1|$ is everywhere equal to $|\vec{h}_2|$, the Faraday rotation is proportional to $\vec{h}^2$ (equation 15) and thus is proportional to $\sqrt{\epsilon}$.

Since the angle of rotation is proportional to the dc magnetization of the ferrite, one would expect a curve of rotation vs applied dc field to be similar to the saturation curve of the material. Furthermore, a curve of loss for circularly polarized waves as a function of applied dc field should reveal a high loss only near ferromagnetic resonance for the positive circularly polarized wave, with little loss at all applied fields for the negative wave. The plane wave theory also predicts that the angle of rotation should be independent of frequency so long as the frequency of the wave is much greater than the ferromagnetic resonance frequency.

Although the above picture is simple and explains the basic facts of Faraday rotation, it is quite inadequate for the ferrite-loaded waveguide problem. To choose a particular example, the dominant TE_{11} wave in a circular waveguide has an H pattern which is transverse only at the center, and may be either longitudinal or transverse near the walls of the waveguide, depending on azimuthal location. These different parts of the magnetic field pattern react with any ferrite present in quite different ways, so that the situation becomes much more complicated than for the simple plane wave case; in the latter condition h is everywhere transverse and reacts with the ferrite in the same way at every point These complications may cause various apparently "anomalous" effects which can readily be explained in a qualitative manner by examining the rf magnetic field pattern microscopically and determining the interaction of the wave and the ferrite at each point. We shall therefore discuss some of the problems involved in designing Faraday rotation elements in circular waveguides, emphasizing the above point of view.

4-2. *The Impedance-Match Problem*

One of the first problems confronting the designer of Faraday rotation elements is that of obtaining an impedance match into the ferrite whose dielectric constant is typically in the range from 10 to 20. If no attempt is made to obtain an impedance match, reflections from the ends of the sample of ferrite will occur and will interfere with each other, depending upon the electrical length of the sample. Since the effective permeability of the ferrite is a function of the applied magnetic field, the electrical length of the sample is a function of magnetic field. As a result, end reflections can and do cause serious aberrations or "anomalies" in the observed Faraday rotation and transmission loss as the magnetic field is varied.

The use of quarter-wave impedance-matching transformers or other such simple lumped structures is not an easy solution to the ferrite matching problem for the simple reason that characteristic impedance and the phase velocity for the ferrite will depend upon the magnetic field applied and also upon the direction of rotation of circularly polarized waves. A simple matching transformer can produce a match for the dielectric discontinuity at the ferrite ends, and in cases where this is the dominating effect the quarter-wave transformer may be attractive. However, when the magnetic effects are included there is no length of ferrite-loaded line which is truly a quarter-wave long for both the positive and negative circularly polarized waves which compose any linearly polarized wave, and when this difference in phase constants is large the quarter-wave transformer is not attractive. Continuous impedance tapering, however, appears to be a method which will always work for both directions of polarization as well as for all values of magnetic field. Thus, a ferrite cylinder may have its ends ground down to conical points, or it may be diluted at the ends by mixing with some other dielectric so that the effective dielectric constant tapers to a low value at the ends of the sample. We have found that mechanically tapering the ends of the sample is an effective means of producing impedance match provided that the diameter of the waveguide and of the sample is such as to permit only the dominant wave to be propagated in the ferrite-loaded section.

4-3. *Limitation on Ferrite Diameter*

The relatively high index of refraction of most ferrites also introduces a mode conversion problem in the design of Faraday rotators. Thus, a dominant-mode hollow metal waveguide may, when fully or even partially loaded with a ferrite rod, be capable of propagating a higher order wave. If the ferrite is tapered and placed along the axis of a waveguide propagating the dominant TE_{11} mode, there will in fact be a tendency to convert to the TM_{11} mode, as has been demonstrated experimentally for pencils of high dielectric constant. This mode conversion tendency can be understood qualitatively by referring to the magnetic field patterns of the TE_{11} and TM_{11} modes as shown in Fig. 7. For the TM_{11} mode the h at the center is 180° out of phase with the h at the periphery, whereas for the TE_{11} mode the center and peripheral h lines are in phase. By delaying the center h lines of the TE_{11} mode relative to the h lines near the periphery, the tapered pencil would tend to convert the TE_{11} mode to the TM_{11} mode. From symmetry considerations one can show that the TM_{01}, TE_{21} or TE_{01} modes which have cut-offs between the TE_{11} and TM_{11} modes will not be generated unless the ferrite is inhomogeneous.

This mode conversion problem becomes evident when one makes measurements of rotation, loss, and ellipticity on a large diameter, properly tapered ferrite rod, as a function of magnetic field. Instead of the smooth variations expected from plane wave theory, peculiar large variations of the above quantities are observed. We now believe that these variations can be attributed to a partial conversion of the dominant TE_{11} mode to the TM_{11} mode by the ferrite pencil which is of sufficient diameter to permit the propagation of both modes. As the magnetic field is varied, the effective electrical length of the ferrite changes so that the interference between the TM_{11} and the TE_{11} modes can also vary, thereby causing fluctuations in transmission and rotation. Furthermore, such a sample will show relatively large reflections, of the order of 10 db below the incident wave, notwithstanding the fact that it may be equipped with slender tapers of sufficient length that good impedance

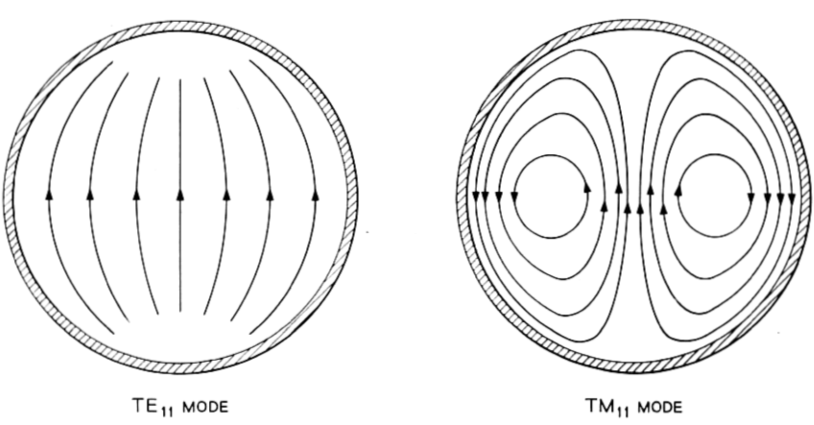

Fig. 7 — Patterns of magnetic intensity for the TE_{11}○ and TM_{11}○ Modes.

match would be expected. This also is to be explained in terms of the presence of a TM_{11} wave within the sample which may be multiply reflected internally and which can give rise to reflected TE_{11} power by reconversion at the input end.

The above mode conversion hypothesis has been checked in the following manner. Let us consider the TE_{11} mode as being composed of two circularly polarized TE_{11} waves rotating in opposite directions. Since the theoretical permeability for the negatively rotating component is larger than the theoretical permeability for the positively rotating component

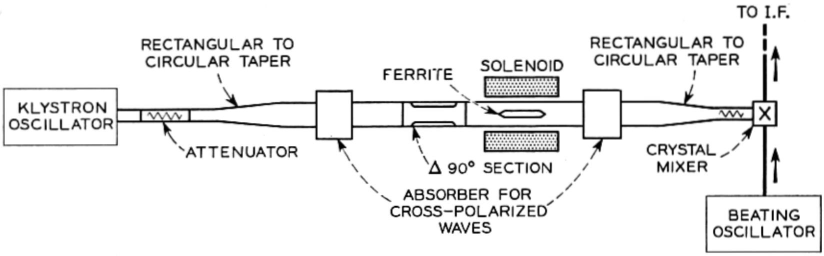

Fig. 8 — Apparatus for the measurement of ferrite-loaded round guide with circularly polarized TE_{11}○ waves.

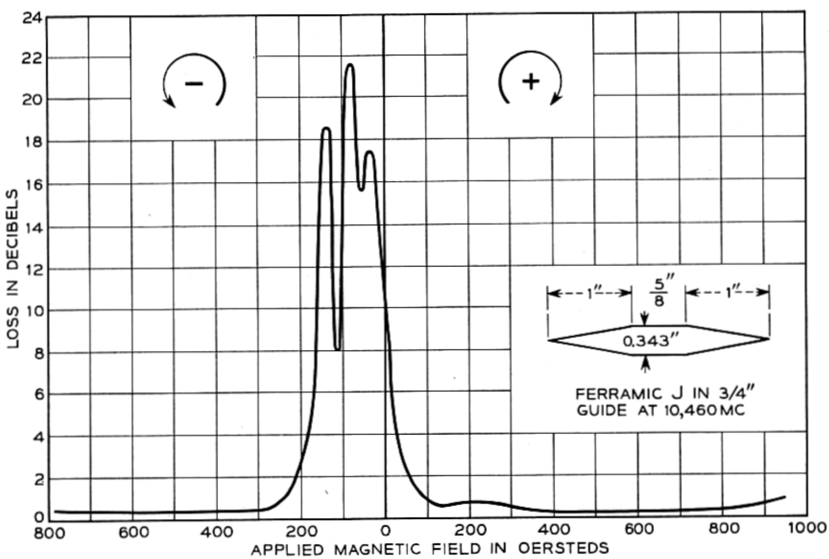

Fig. 9 — Measured curve showing mode conversion effects in ferrite-loaded round guide at low longitudinal fields for negative circularly polarized waves.

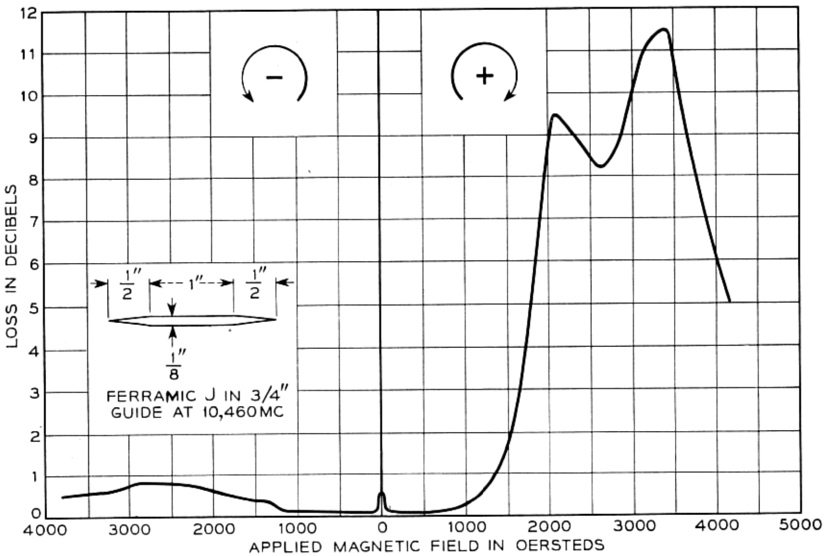

Fig. 10 — Curve showing ferromagnetic resonance in ferrite-loaded round guide.

(see Fig. 6), we might expect that the negatively rotating component would be more readily subject to mode conversion than the positively rotating component. This supposition has been verified experimentally. Circularly polarized waves were generated by means of a 90° "squash" section and were transmitted through samples of ferrite as shown by the circuit of Fig. 8. The resulting curves of transmission loss vs magnetic field showed that for ferrite samples of large diameter there were many violent fluctuations of loss for both positive and negative circularly polarized waves. For a somewhat smaller diameter, fluctuations in loss for positive waves diminished substantially or disappeared entirely while the fluctuations for negative waves remained, as shown in Fig. 9. For still smaller diameters the fine-grain fluctuations disappeared entirely in the zero to 500 oersted region for both positive and negative circularly polarized waves, as shown in Fig. 10. This, we feel, confirms the hypothesis that the loss and rotation fluctuations are caused by mode conversion. Therefore, the diameter of ferrite used in Faraday-rotation devices must be small enough so that no mode conversion takes place.

Even if the dielectric constant of the ferrite were low enough so that no mode conversion problems would arise, one would still confine the ferrite to the axial region of the waveguide, since it is in this region that the ferrite is most effective as a rotator while the peripheral portions may contribute to loss but not to rotation. There are various ways of making the above statement evident, but perhaps the easiest is to consider the rotation as being produced by the elementary electrons which induce an rf magnetic field perpendicular to the exciting rf field. Therefore, near the axis of the guide this induced field will set up a TE_{11} mode perpendicular to the input field and thus cause rotation. Near the periphery, however, the electrons will tend to set up rf magnetic fields normal to the wall of guide which cannot reradiate as a TE_{11} mode since no such field component exists for the TE_{11} mode. Thus, the peripheral portions of the ferrite contribute nothing to the coupling coefficient K given by equation (15). Therefore, the peripheral portions of the ferrite will not be effective as a Faraday rotator but may produce dielectric loss due to the peripheral electric field as well as magnetic loss due to the peripheral magnetic field.

4-4. *Dependence of Rotation and Loss on Geometry of Ferrite*

Realizing the necessity of confining the ferrite to the axial region in order to prevent multimoding and in order to have the ferrite in the most effective portion of the guide for rotation, one is still confronted by the problem of determining the diameter which will yield a maximum ratio of rotation to loss. This ratio may be used as a figure of merit, F, defined as the Faraday rotation in degrees per db transmission loss.

We shall first discuss the variation of rotation as a function of diameter of the ferrite. From the naive plane wave picture, one would expect the rotation to be directly proportional to the number of precessing mangetic moments, and thus to the mass of ferrite or the square of the ferrite diameter. For small diameter ferrite rods, the measurements recorded in Fig. 11 demonstrate that rotation per gram is indeed independent of diameter, for small diameters.

Theoretical guidance on this same problem can be found in the work of L. R. Walker and H. Suhl,[10] who have solved mathematically for the magnitude of Faraday rotation to be obtained from thin pencils of ferrite in a circular waveguide. This mathematical treatment also predicts that for small diameter pencils the rotation should be proportional to the square of the diameter of the ferrite.

For pencils of large diameter, no general mathematical treatment which takes into account the dielectric constant of the ferrite is available. However, for the particular case of a ferrite of dielectric constant 10 (typical for most ferrites) inside a circular waveguide with a diameter to free-space-wavelength ratio of 0.8, Suhl and Walker as well as H.

Seidel[11] of Bell Telephone Laboratories have obtained values of loss and rotation as a function of ferrite diameter. Fig. 12 is a theoretical curve of rotation per mass as a function of ferrite diameter based on calculations of Suhl and Walker, while Fig. 11 shows a typical experimentally obtained curve. For very small ferrite diameters, the rotation per mass is seen to be independent of ferrite diameter as explained previously. With further increase in ferrite diameter, both curves rise sharply, while the theoretical curve reaches a peak and then declines. With most typical ferrites one cannot usually observe this peak because multimoding problems become serious before this large diameter is reached. However, with some ferrites of lower dielectric constant, such rotation per gram peaks have been observed.

The above results can be explained in the following manner. Any increase in diameter of a dielectric rod above a certain minimum value increases the energy concentration in the dielectric much more rapidly than the increase in cross-sectional area of the dielectric. This has been shown to be the case by H. Seidel,[11] and also seems reasonable by analogy from the dielectric waveguide studies of M. C. Gray.[12] Thus, as the

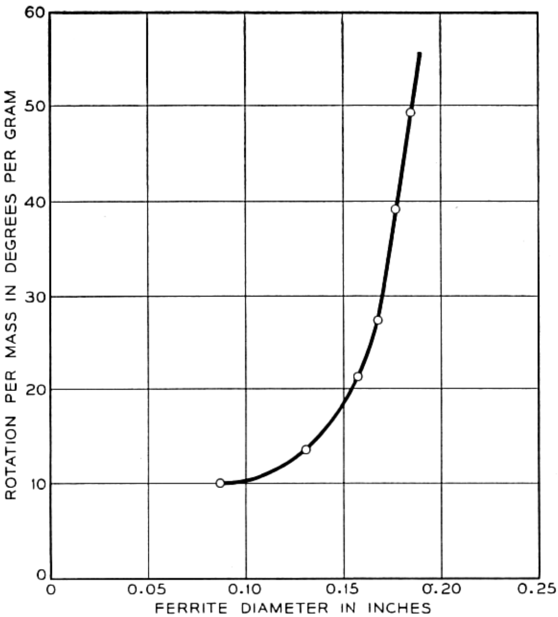

Fig. 11 — Measured rotation per ferrite mass versus ferrite diameter for 0.75″ I.D. guide at 11.2 kmc.

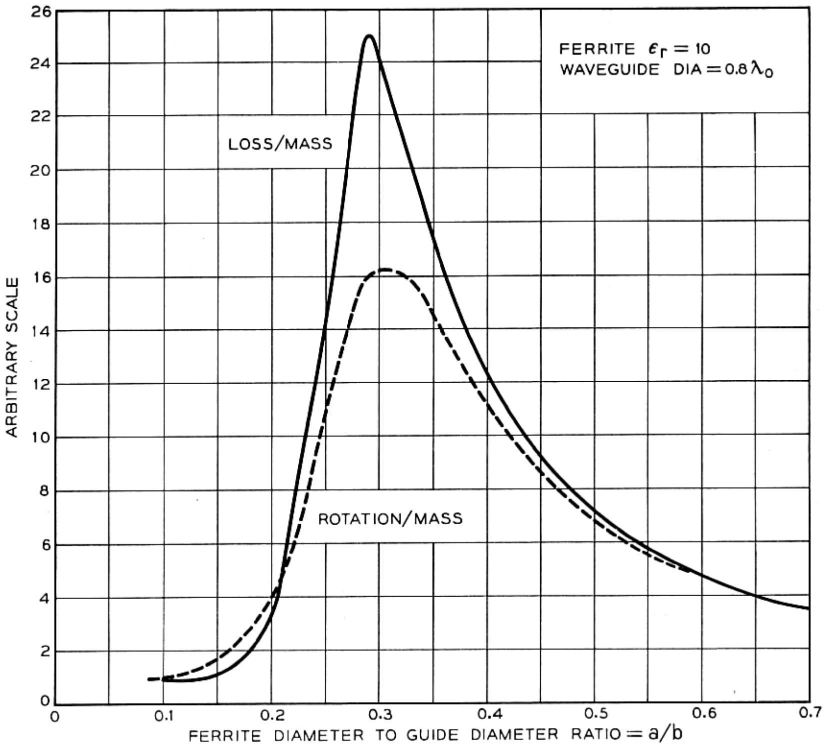

Fig. 12 — Calculated rotation and dielectric loss per ferrite mass versus ferrite diameter.

diameter of the ferrite is increased, the energy concentration in the ferrite increases rapidly, first for the negative circularly polarized wave whose permeability is above unity, and then for the positive wave whose permeability is below unity. Therefore, the rotation per gram as well as the rotation will increase with ferrite diameter in this range of diameters.

Another consequence of an increase of ferrite diameter is an increase in the effective dielectric constant of the composite air-ferrite guide, thus decreasing the characteristic impedance of the guide and thus resulting in an increase in the magnitude of the transverse rf h vector in the ferrite for a given amount of input power. This too can result in an increase of Faraday rotation per gram with increasing ferrite diameter entirely apart from any redistribution of energy in the waveguide's cross section.

The combination of the dielectric waveguide effect and the lowering of the effective characteristic impedance results in a broad maximum value of rotation which occurs at a diameter for which the difference in the propagation constants of the positive and negative rotating components is a maximum. Increasing the diameter in this range will increase the mass of the ferrite without appreciably increasing the rotation, thus decreasing the rotation per mass.

We shall now discuss the variation of loss with diameter so as to be able to determine the figure of merit, F, (rotation/db loss) as a function of diameter. The loss in the ferrite can be due to dielectric loss or to magnetic loss. Let us first consider a ferrite for which the loss is entirely magnetic. This loss will be proportional to the square of the transverse magnetic field since the longitudinal rf magnetic field will cause no losses if the ferrite is saturated in the longitudinal direction.

Except for large ferrite diameters both the loss per mass and the rotation per mass are proportional to the square of the transverse magnetic field so that the figure of merit is independent of ferrite diameter regardless of how transverse h varies with diameter. This constancy of the figure of merit for ferrites with magnetic losses only is confirmed by the Suhl and Walker analysis.*

For large diameter ferrites, some of the transverse h does not contribute to rotation (as discussed in Section 4.3) while all of the transverse h contributes to loss. Therefore, one would expect the figure of merit to decrease with increasing diameter for large-diameter ferrites having magnetic losses. However, it appears from the Suhl and Walker analysis that this effect is small until the waveguide is almost completely filled.

The above discussion assumes that the operating frequency is far above the resonance frequency. The situation is more complicated at low microwave frequencies where, due to proximity to ferromagnetic resonance, the loss for the positive circularly polarized component may be higher than for the negative component.

Let us next consider the case of a ferrite having dielectric loss and no magnetic loss. H. Seidel[11] has made theoretical computations of dielectric loss as a function of ferrite diameter with zero applied magnetic field. This data has been plotted as loss per mass as a function of ferrite diameter in Fig. 12, and is to be compared to the rotation per mass curve of the same figure. It is seen that the loss per mass curve rises sharply with

* See Reference 10, p. 1152. Using their notation, rotation is proportional to $\kappa'Q$ while loss is proportional to $\mu''P + \kappa''Q$. Far from resonance $\kappa'' \ll \mu''$ so that the loss is proportional to $\mu''P$ and the figure of merit is proportional to $\kappa'Q/\mu''P$. But from Fig. 3 in the Suhl and Walker paper, P/Q is approximately unity for most values of ferrite diameter. Therefore, the figure of merit will also remain approximately constant over a wide ferrite diameter range.

increasing ferrite diameter, reaches a peak near $a/b = 0.3$ and then declines. The first sharp rise is due to two causes: first, the increase in energy concentration in the ferrite because of the dielectric waveguide effect discussed previously in connection with the rotation per mass curve; and second, because of the sharp increase in the longitudinal electric field in such a shielded dielectric waveguide. As the ferrite diameter increases beyond the point where most of the electromagnetic energy is in the ferrite, the loss due to the transverse electric field does not increase, while the longitudinal electric field decreases until the latter is zero in a completely filled waveguide. Therefore, the loss will drop with increasing ferrite diameters in the large diameter region, resulting in an even sharper drop in loss per mass in this region.

The figure of merit as a function of ferrite diameter will be given by the ratio of the rotation to the sum of the magnetic and dielectric losses. Since the figure of merit due to magnetic loss is essentially independent of diameter, the variation in the figure of merit with diameter depends on the ratio of rotation to dielectric loss. One is usually interested in ferrite-to-waveguide diameter ratios below about 0.3. In this region both the rotation and dielectric-loss curves rise sharply (Fig. 12), but their ratio first rises to a peak at a ferrite-to-waveguide diameter ratio of about 0.125 for the typical conditions shown in Fig. 12. At larger ferrite diameters the theoretical figure-of-merit drops and then rises again at ferrite-to-waveguide diameter ratios exceeding 0.275. Fig. 13 shows experimental curves obtained by J. P. Schafer for the figure of merit as a function of ferrite diameter for two typical low-loss ferrites. These curves have a shape which agrees with theoretical expectations, and confirm that there is an optimum diameter for maximum figure of merit.*

4-5. *Rotation as a Function of Frequency*

The plane-wave analysis (Section 4-1) shows that the rotation is independent of frequency if the operating frequency is much higher than the resonance frequency. In a waveguide, however, the rotation will increase with increasing frequency for two reasons. First, since the impedance of a TE mode in a waveguide decreases with increasing frequency, the transverse rf magnetic field increases as the frequency gets

* It must be emphasized that the above discussion assumes that the ferrite sample is long compared to its diameter so that changes in diameter will not seriously affect the demagnetizing factors and the uniformity of field in the ferrite. If, with constant applied field, the diameter of a ferrite cylinder is increased so that its length-to-diameter ratio is no longer large, the increase in demagnetizing factor may cause a loss in saturation, thus increasing the low field losses and decreasing the figure of merit.

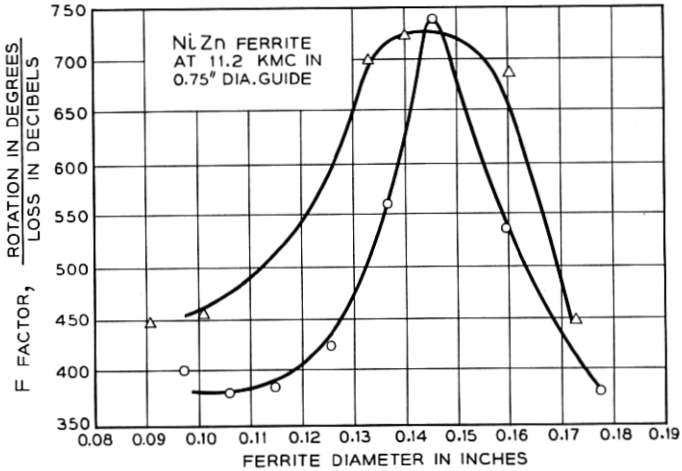

Fig. 13 — Faraday rotation per db loss as a function of ferrite diameter.

farther above cut-off, thus resulting in an increase in rotation. Second, for a waveguide partially loaded with ferrite, an increase in frequency causes the electromagnetic energy to become concentrated more and more in the ferrite rod, which acts as a dielectric waveguide as described in Section 4-4. Since for dielectric waveguides it is the ratio of diameter to wavelength which determines the ratio of energy in the ferrite to the total energy in the guide,[12] a decrease in wavelength should have the same effect on the energy concentration in the ferrite as an increase in diameter. Therefore, for diameters in the steep part of the curve of rotation per gram versus diameter, (for example, Fig. 12), one should expect a rapid increase in rotation with increasing frequency. For large ferrite diameters for which all of the electromagnetic energy is in the ferrite (or for high enough frequencies) the rotation should be independent of a further increase in frequency. Fig. 14 shows an experimental curve taken by J. P. Schafer of the ratio of the rotation at 11.7 kmc to the rotation at 10.7 kmc as a function of diameter in the small diameter range. This curve is in agreement with the above discussion.

4-6. *Dependence of Rotation on Dielectric Constant of Surrounding Medium*

J. P. Schafer and J. H. Rowen[13] have observed an enhancement of rotation for a partially ferrite-loaded guide with a high dielectric constant surrounding medium over that observed with a polyfoam (ϵ = 1.05) surrounding medium. Our own observations have shown both an increase and a decrease of rotation with an increase in the dielectric constant of the surrounding medium, depending upon the diameter of the ferrite. Embedding a small-diameter ferrite pencil in a high dielectric constant support increases the effective dielectric constant of the guide, thus increasing the transverse magnetic field in the ferrite and increasing the rotation. If the loss in the ferrite is primarily dielectric in nature, the figure of merit will also improve because of the decrease in the electric field in the ferrite.

For ferrites of larger diameter, however, the dielectric waveguide effect becomes important and the ratio of energy in the ferrite to the total guide energy decreases with increasing dielectric constant of the surrounding medium. This results in a decrease in the rotation rather than the increase noted by Schafer and Rowen.

4-7. *Dependence of Rotation and Loss on Temperature*

As shown by equation (16), the rotation is directly proportional to the saturation magnetization, M. It is well known, however, that M decreases with increasing temperature, going to zero at the Curie temperature.[14] Therefore Faraday rotation will also decrease with increasing

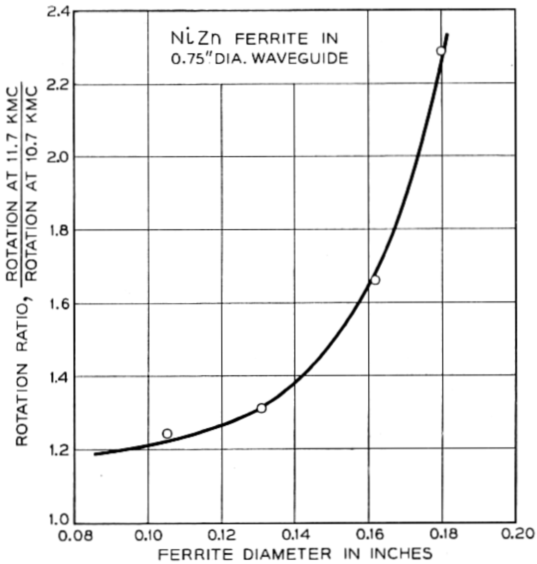

Fig. 14 — Frequency sensitivity of Faraday rotation as a function of ferrite diameter.

temperature. At 11 kmc for a ferrite with a Curie temperature of 150° C, we have measured a variation of 0.4 per cent rotation per °C at room temperature. Ferrites having a much higher Curie temperature show a much smaller variation of rotation with temperature. In fact, measurements have been made with ferrites having a Curie temperature of 500° C, which showed less than 0.03 per cent change in rotation with variations of temperature of 20° C near room temperature.

Magnetic losses are also affected considerably by temperature. It is known* that anisotropy in ferrites drops rapidly as one approaches the Curie temperature. If the magnetic losses are due to anisotropy, then heating the ferrite should also reduce the losses. Experimental results on fairly low Curie temperature ferrites indicates that this is indeed the case. Thus, for a particular sample at 10,000 mc, the loss at saturation dropped from 3 db at room temperature to 1 db at 80° C. For ferrites having high Curie temperatures, the loss does not necessarily decrease with increase in temperature since the anisotropy variation at temperatures far below the Curie temperature may be rather complex.[15]

The variation of figure of merit with temperature depends on whether the loss drops more or less rapidly than the rotation drops with increasing temperature. The figure of merit of some ferrites has been observed to improve and others to drop with increasing temperature.

4-8. *Idiosyncracies of Ferrites Used as Faraday Rotation Elements*

a. Loss Near Zero Field

It has been known for some time that some ferrites have rather large loss at low magnetic fields, and that with the application of moderate magnetic fields in any direction, longitudinal or transverse, this loss disappears. This low-field loss has been explained in a variety of ways but, in general, these all depend upon the presence within the ferrite of domain walls. Consequently, with fields strong enough to saturate the ferrite, the domain walls and the loss disappear. One would expect that if this loss were measured with circularly polarized waves, it should be independent of the direction of circular polarization used. Actually, however, our experiments on small-diameter pencils, which are free of mode conversion effects, have shown that this low-field absorption has a peak for counterclockwise polarized waves as seen in Fig. 15.

This puzzling asymmetry can be explained by the fact that more of the energy will travel within the ferrite for the negative circularly polar-

* C. Kittel, Reference 14, p. 182.

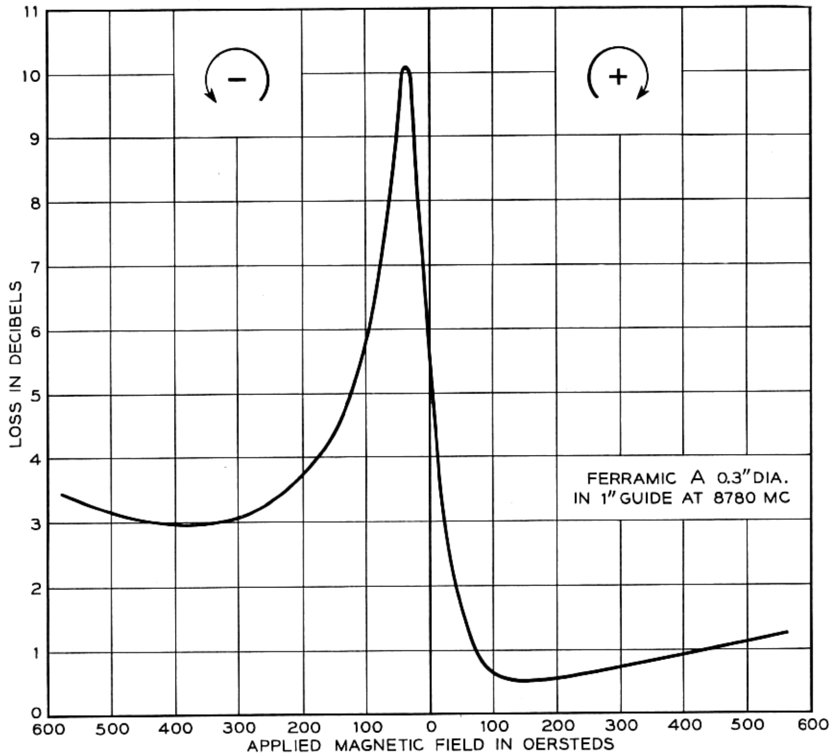

Fig. 15 — Measured curve showing displacement of low field loss peak.

ized wave than for the positive wave because of the higher index of refraction of the former (see Fig. 6) as explained in Section 4-4. As a result, the low-field loss can be expected to be greater for the negative wave as observed experimentally.

b. Ferromagnetic Resonance for Negative Circularly Polarized Waves

Our measurements on slender pencils of ferrite using circularly polarized waves disclosed another wholly unexpected result. One would expect to observe a single loss peak, at fields high enough to produce ferromagnetic resonance, for the positive circularly polarized wave. However, in addition to this expected resonance, another resonance was observed for negative circularly polarized waves at magnetic fields of nearly the same value as that required to produce ferromagnetic resonance for the positive wave as shown in Fig. 16. The magnitude of this high-field peak for the negative wave depends upon the size of ferrite pencil used, being very small for very thin pencils (Fig. 10) and reaching values above the loss peak for the positive wave for some larger diameter samples (Fig. 16).

It is believed that this effect can be explained by analyzing the rf magnetic field at different points in the ferrite. Thus, by reference to a field pattern for the TE_{11} mode (Fig. 7), it can be seen that a rotation of this pattern as a whole, corresponding to a circularly polarized wave, causes the rf magnetic vectors near the center to rotate, indicating that they are circularly polarized. However, the magnetic vectors at the wall of the waveguide are everywhere tangent to the wall, and are linearly polarized even though the wave as a whole is circularly polarized. Since a linearly polarized vector consists of both positive and negative circular polarizations, any ferrite located off the center of the waveguide will see a certain amount of positive polarized field vector with its attendant loss even though the driving wave is negatively polarized. Consequently, one would expect to observe a small loss peak for a negative circularly

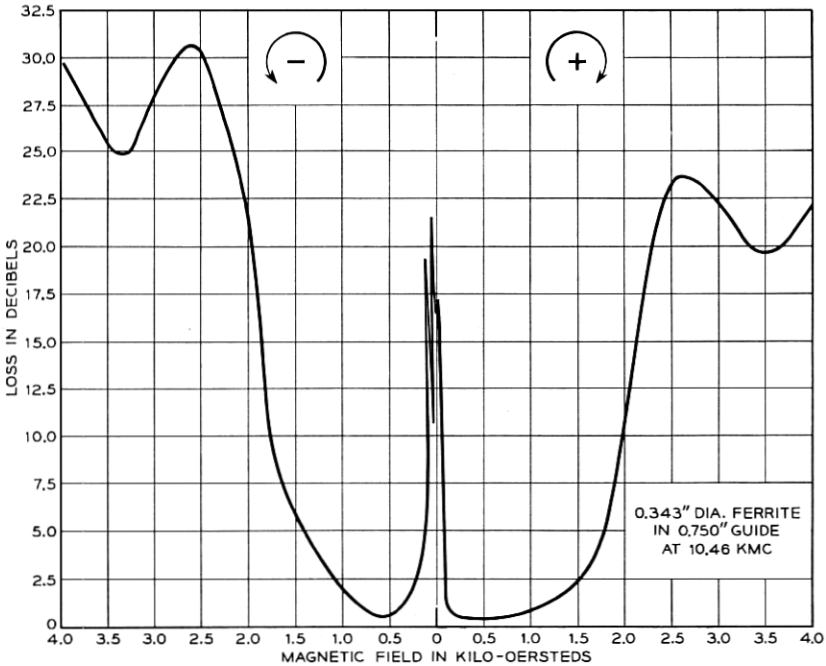

Fig. 16 — Measured curve showing ferromagnetic resonance for both positive and negative circularly polarized waves.

polarized wave, but this alone would not appear to account for the high loss peak obtained with the large diameter ferrites.

The high negative wave loss observed in the larger diameter samples can perhaps be attributed to TM_{11} mode propagation. By examining the transverse magnetic field pattern for this mode, it can be seen that when the rf magnetic field vector is circularly polarized in the negative sense in the center of the guide, it is polarized in the positive sense near the periphery. This is substantiated by the theoretical work of H. Suhl and L. R. Walker[16] which shows that the propagation constant of both positive and negative circularly polarized waves are nearly equal in a ferrite-filled circular waveguide propagating the TM_{11} mode. For this mode, the ferrite cannot distinguish between a positive or negative circularly polarized wave. Therefore, one would expect equal loss for the positive and negative waves at the resonance magnetic field value.

When one has a mixture of TE_{11} and TM_{11} waves in the ferrite, the field at the center may be stronger or weaker than the field near the periphery, depending upon the relative phase of the two waves. Thus, the negative-wave loss might be greater or less than the positive-wave loss, depending on phasing. It is indeed probable that the TM_{11} wave was present for the case of Fig. 16 since the ferrite was sufficiently large to permit anomalies in the low field region which are normally associated with multimoding, and the geometry is one which we know can generate the TM_{11} wave.

It may be noted that the ferromagnetic resonances shown in Fig. 16 have a double humped character. Similar tests made on other ferrites did not reveal any such double hump. In order to determine whether the double hump in Fig. 16 is a characteristic of the ferrite composition and structure, W. A. Yager measured the ferromagnetic resonance of a tiny sphere of this material at 24 kmc using his cavity technique. Yager did not find a double hump, indicating that the geometry and size may be responsible in our observations.

c. Molded Powdered Ferrites

Techniques have been developed for powdering ferrites and then molding them with polystyrene powder under heat and pressure. This process has the advantage of making it easy to obtain samples of any desired size with tapers molded in. Furthermore, it enables one to choose a wide variety of densities so as to obtain a variety of values for effective dielectric constant and for rotation per unit length.

However, measurements at 10 kmc show that the loss is much higher for molded or powdered ferrites than it is for solid ferrites. This is not due to the molding process but can perhaps be explained on the basis of the broadening of resonance in the molded samples due to the wide variety of demagnetizing factors in the little powder particles. At 20 kmc and higher this increase in loss for molded samples should disappear since even a broadened resonance would not have much effect at the comparatively low fields required for saturation. Although this has not been checked in detail, in general it has been observed that molded samples of certain ferrites have extremely low loss at 24 kmc.

4-9. *Faraday Rotation Devices*

We shall now briefly describe some applications of Faraday rotation elements.

The gyrator function is performed by a Faraday rotation of 90° as described by C. L. Hogan.[4]

The Faraday-rotation type of circulator, due to S. E. Miller,[17] is illustrated in Fig. 17. Experimental models have been built for operation at 4, 11 and 24 kmc. The Faraday rotator for the 24-kmc circulator contains a 1/8″ diameter tapered ferrite rod mounted in a polyfoam holder in a 3/8″ round waveguide. The longitudinal magnetic field is supplied by a permanent magnet of such field strength that the ferrite produces 45° rotation. Sections 1 and 2 are rectangular-to-round transitions while sections 3 and 4 are cross-polarization pick-offs of a type designed by A. P. King of Bell Telephone Laboratories. The rectangular guide portions of these latter elements accept all of the wave energy in the round waveguide which is polarized orthogonal to the output in the rectangular-to-round transitions. Thus, an input at 1 is rotated by 45° in passing through the ferrite section and is transmitted to 2. Similarly, an input at 2 will go to 3, at 3 to 4, and at 4 will go back to 1 with a 180° phase reversal.

By terminating terminals 3 and 4 in the above circulator, one has an isolater which will transmit energy from 1 to 2, but any energy entering at 2 will be lost in the termination at 3. However, for this isolator application, there is no need for the cross polarization pickoffs since these can be replaced by diametral resistance-sheet terminations in the circular waveguides near terminals 1 and 2. These sheets are oriented in such a manner as to absorb all energy polarized perpendicular to terminals 1 and 2. Isolators of this kind have already proved very useful in preventing oscillator pulling when operating into mismatched loads.

In the above isolator, the ferrite element must be adjusted for exactly

45° of Faraday rotation in order to be theoretically capable of infinite attenuation in the reverse direction. Since the rotation angle is a function of frequency and temperature, one would expect the reverse attenuation to vary with frequency and temperature, and this will limit the bandwidth of the device.

The frequency and temperature range over which satisfactory isolation is provided can, however, be extended by operating several isolators in tandem as follows: Fig. 18 shows an isometrically exploded view of three isolators operated in tandem. Power is introduced at the left through a rectangular-to-circular waveguide transition and enters the system polarized as shown by e_1. After passing through the three Faraday rotators it emerges polarized as e_4, which is then accepted by a circular-to-rectangular transition. R_1, R_2, R_3 and R_4 are cross polarization absorbers designed to absorb reflected wave components. It can be seen that this assembly is in reality merely three isolators in tandem, where intervening transitions from circular-to-rectangular waveguide have been omitted as unnecessary. If all of the Faraday rotators are designed to produce exactly 45° of rotation at some one temperature, then the return loss for the whole system would be infinite at this temperature, even with a reflectively terminated receiving end. At some other temperature where the return loss for each isolator would be x db,

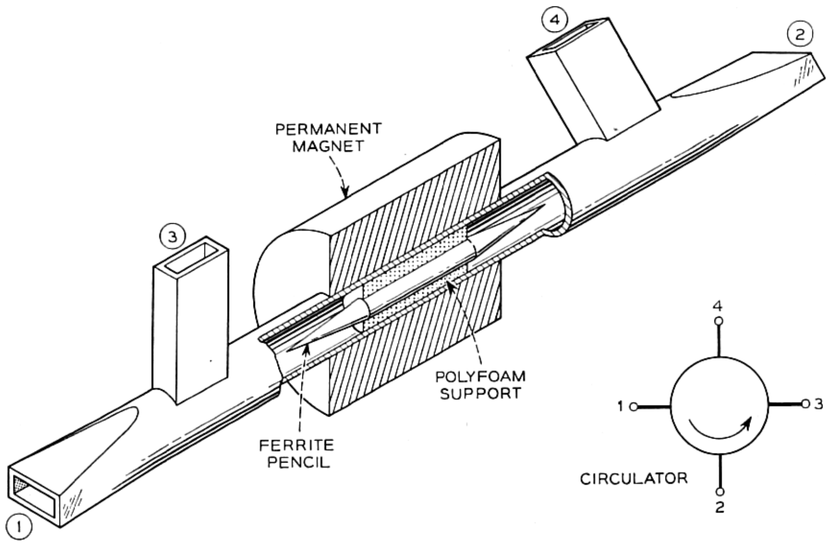

Fig. 17 — Faraday rotation circulator.

the total return loss for the system would now be increased to $3x$ db. Naturally, the forward transmission loss would also be increased by a factor of 3. If, however, this forward transmission loss is sufficiently small for a single isolator, an increase of transmission loss of three times may be a small price to pay for an increase in the return loss of three times. While tandem operation of three isolators is discussed here, any other number may of course be used. J. P. Schafer[18] has developed a two section isolator of this type for 11 kmc, and reports a reverse loss of 53 db and a forward loss of 0.25 db.

The above adjustment of the isolators gives a sort of maximally flat variation with temperature as indicated schematically in Fig. 19(a). A somewhat different type of broadbanding with temperature can be obtained by designing each of the isolators to produce exactly 45° of rotation at 3 different temperatures in the desired operating range. The return loss characteristic as a function of temperature will then look somewhat as shown in Figure 19(b).

The proposals made above for widening the operating temperature range should also be suitable for widening the frequency band of an isolator. In this case the tandem isolators of Fig. 18 might be designed to produce exactly 45° of rotation at the same frequency, or alternatively to produce 45° of rotation at three different frequencies in order to increase the return loss over the band.

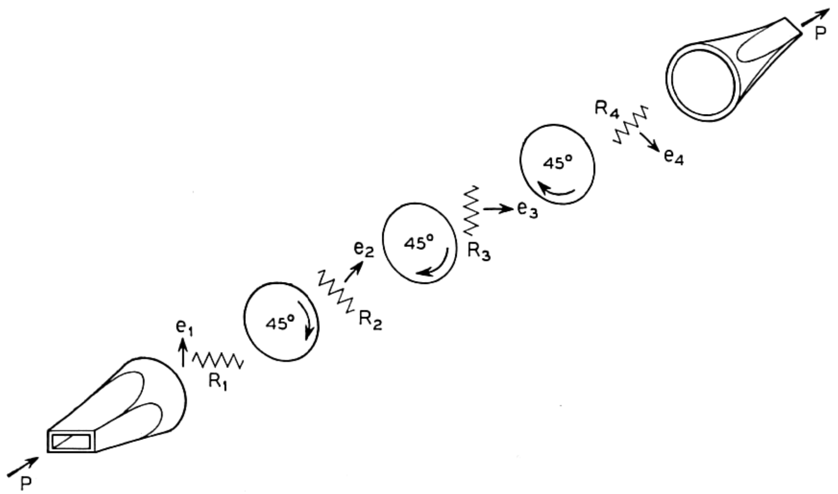

Fig. 18 — Schematic of a broadband isolator.

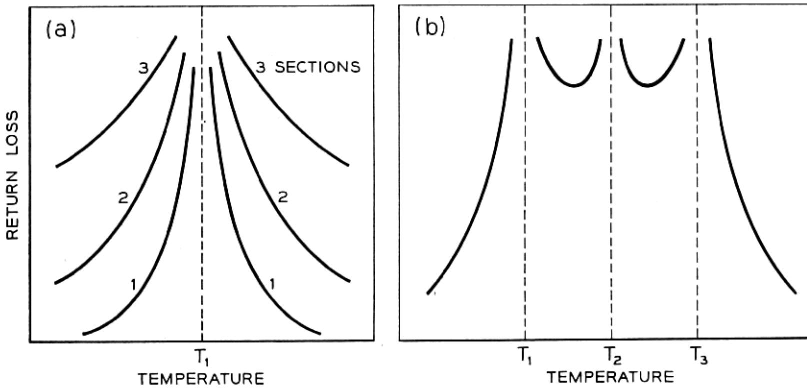

Fig. 19 — Alternative design characteristics for the isolator of Fig. 18.

It has been assumed that the cross-polarization absorbers R_1, R_2, R_3, R_4 in the compound isolator absorb 100 per cent of the power polarized parallel with the card. If they do not absorb all of the incident power, the infinities of the loss curves of Fig. 19 (a) and Fig. 19 (b) become finite. However, other infinities may exist due to cancellation effects. In particular, for the 2-section isolator of Fig. 19(a), a new infinity will appear on either side of the center of symmetry (T_1). The spacing of these infinities from the center will be proportional to the transmission past the absorbers. Schafer[18] has developed a two-section isolator employing broadbanding of this type to obtain a reverse loss in excess of 30 db over the band from 10.7 kmc to 11.7 kmc.

The above circuits utilize 45° of Faraday rotation. For other rotation angles, the devices may have a variety of peculiar properties. For example, if 22½° of Faraday rotation is used with the terminal polarizations shown in Fig. 20, the device becomes a non-reciprocal power divider with the properties indicated. Between certain pairs of terminals the power transmission is the same as in a circulator, while in the reverse direction there is a fifty-fifty power division as in a hybrid.

4-10. *Faraday Rotation in Non-Circular Guides and for Other Modes of Propagation*

Faraday rotation depends upon the existence of two degenerate modes of propagation in the ferrite-loaded waveguide. For the discussion up to this point the two modes were the two polarizations of dominant (TE_{11}°) waves, which are degenerate in the sense that they have identical phase and attenuation constants, and are orthogonal in the sense that separate connections may be made at each end of the ferrite-loaded medium from the two TE_{11} waves to a pair of single-mode waveguides (illustrated in Fig. 17). It is important that the ferrite have a cross-sectional shape which is symmetrical and that the waveguide be circular in order that the two TE_{11}° waves have identical phase constants. If the guide is not round, if the ferrite is not symmetrical (or not homogeneous) in cross-section, or if the ferrite is located off the axis of the cylindrical guide, the phase constants of the two TE_{11} waves will be unequal; then for a linearly polarized input wave the output of the ferrite-loaded section will be elliptically polarized, and it will be impossible (in the absence of phase-correcting appendages) to achieve complete decoupling between the various terminals in an isolator or circulator.

The ferrite-loaded guide may be square instead of round, and for such a structure Faraday rotation of dominant-wave ($TE_{10}^{\square}$) polarization would take place. Applications directly analogous to those for round guides may be drawn.

Faraday rotation may be utilized with other pairs of degenerate modes. For example, in round guide the TE_{12}, TE_{13} $\cdots$ TE_{1m} modes all exist in degenerate pairs and all have a purely transverse magnetic intensity at the guide center-line; for these modes a ferrite rod on the axis may be used to produce Faraday rotation. The other TE_{nm}° modes of round guide* also exist in degenerate pairs, and Faraday rotation may be produced; however, in this case the proper location for the ferrite is not on the waveguide axis because there is no transverse magnetic intensity at that point. However, the transverse magnetic intensity at points off the guide axis is purely radial at certain angular positions,

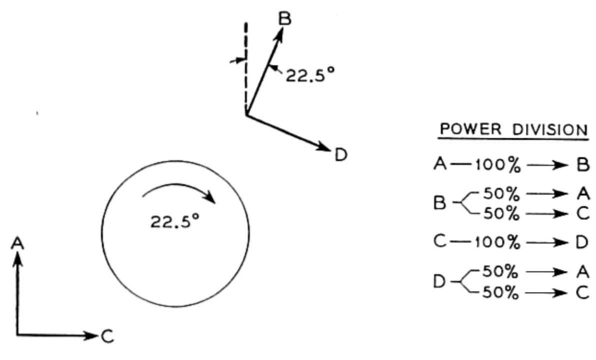

Fig. 20 — Non-reciprocal power divider using Faraday rotation.

* $n \neq 0$.

and purely circumferential at other angular positions. This is illustrated in Fig. 21 for the TE_{21} mode. The coupling which produces Faraday rotation is therefore from the radial magnetic intensity of one TE_{nm} mode to the circumferential magnetic intensity of the orthogonal degenerate mode. This coupling will be a maximum at the fractional guide radius where the product of the radial and circumferential magnetic intensities is a maximum — namely, where the following expression is a maximum:

$$\frac{R}{\rho} J_n' \left(\frac{k_{nm}'\rho}{R}\right) J_n \left(\frac{k_{nm}'\rho}{R}\right) \tag{17}$$

ρ = radial coordinate

R = waveguide radius

k_{mn}' = the m^{th} positive root of the Bessel function derivative $J_n'(x) = 0$

For the TE_{21} mode, the maximum coupling occurs at a fractional guide radius of 0.6. The optimum geometry of ferrite is a thin-walled tube of mean radius $0.6R$, longitudinally magnetized and centered on the waveguide axis.

5. TRANSVERSE FIELD EFFECTS IN RECTANGULAR WAVEGUIDE

We turn now to a consideration of reciprocal and non-reciprocal effects due to ferrite loading of a single-mode waveguide. Because there

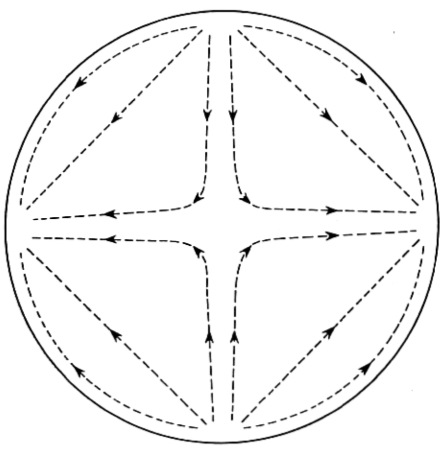

Fig. 21 — Pattern of magnetic intensity for the TE_{21}○ mode.

exists only one mode of propagation in each direction in the ferrite-loaded guide, the circuits using these effects are fundamentally different from those depending on Faraday rotation.

It is familiar to the art that every mode in a waveguide is characterized by three distinctive features;

(1) phase constant
(2) attenuation constant
(3) magnetic and electric field configurations in the waveguide

We will now show that suitable addition of magnetized ferrite may cause non-reciprocal as well as reciprocal changes in any of these distinguishing characteristics. It follows that the ferrite-loaded medium may be non-reciprocal by virtue of differing phase constants for the two directions of transmission, differing attenuation constants for the two directions of transmission, or differing field configuration for the two directions of transmission.

The configuration of magnetic intensity for the dominant wave in empty rectangular guide is sketched in Fig. 22(a). The H-loops lie

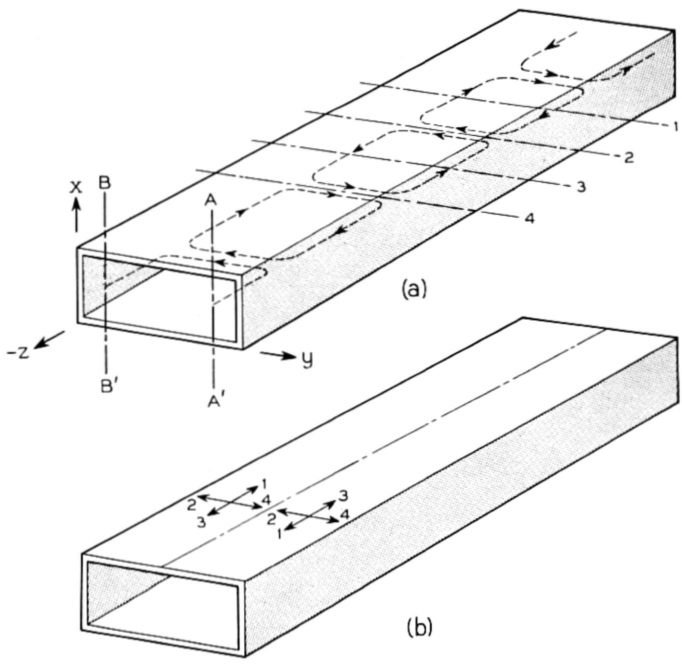

Fig. 22 — Magnetic intensity relations for the TE_{10}□ mode showing the presence of elliptically polarized h.

entirely in planes which are parallel to the wide faces of the guide. At any point other than at the center-line or at the side-walls of the guide, the magnetic intensity vector is elliptically polarized in planes parallel to the wide faces. This may be illustrated qualitatively by the series of vectors intercepted by transverse planes 1, 2, 3, 4 of Fig. 22(a). A viewer looking along the line A to A' will observe, for propagation in the $+z$ direction, the magnetic intensities at planes 1 through 4 in the time sequence 1, 2, 3, 4; the spatial orientation of these vectors is sketched in Fig. 22(b). It is evident that a viewer looking along A–A' will see a clockwise rotating vector for wave propagation in the $+z$ direction. For wave propagation in the $-z$ direction, the viewer will observe the vectors in the time sequence 4, 3, 2, 1, and the viewer looking along A–A' will see a counterclockwise rotating vector.

A viewer looking from B to B' will see a similar (but different) series of vectors. The spatial orientations of the H vectors at the planes 1 through 4 are shown in Fig. 22(b) on the left-hand side of the guide center-line. For wave propagation in the $+z$ direction the vectors appear at B–B' in the time sequence 1, 2, 3, 4 and a counter-clockwise rotating vector is observed. For wave propagation in the $-z$ direction, the vectors appear at B–B' in the time sequence 4, 3, 2, 1, and a clockwise rotating vector is observed.

Let us consider now the effect of introducing a sheet of ferrite, as sketched in Fig. 23, sufficiently thin so that the fields are not appreciably altered. The dielectric constant of the ferrite will perturb equally the phase constants for the two directions of propagation, but the permeability of the ferrite will perturb unequally the phase constant for the two directions. With the direction of magnetization shown in Fig. 23,

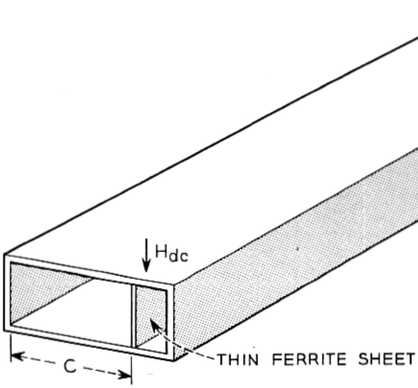

Fig. 23 — Non-reciprocal ferrite-loaded rectangular guide.

the electron spins are lined up perpendicular to the rf magnetic intensity, and thus the change in permeability introduced by the magnetized ferrite may be predicted using the known permeability values for the two senses of circular polarization (Fig. 6). With the ferrite on the right-hand side of the guide as shown in Fig. 23, and for wave propagation in the $+z$ direction, the positive circularly polarized component of rf h is greater than the negative circularly polarized component. Fig. 6 shows that the low-field permeability of the ferrite is less than unity under this condition. For wave propagation in the reverse direction the negative circularly polarized component predominates at the ferrite, and the low-field permeability of the ferrite is greater than unity. Thus, an increase of H_{dc} from zero (for which the ferrite-loaded guide is reciprocal) to values in the vicinity of saturation, introduces a decrease in phase constant for propagation in the $+z$ direction and an increase in the phase constant for propagation in the $-z$ direction. This configuration constitutes a *directional phase shifter*, whose total phase difference is proportional to the length of the ferrite-loaded medium, and when this difference is 180° the device is a *gyrator*.[19]

Suppose we introduce two identical thin ferrite sheets at equal distances from and parallel to the side walls as in Fig. 24. The above description of the fields illustrates that the static biasing field H_{dc} must be in opposite directions in order that the predominating rf h have the same sense of circular polarization at both ferrite elements. When biased oppositely as in Fig. 24, the two ferrite pieces cooperate in producing non-reciprocal phase shift.

If the static biasing field has the same magnitude and direction at

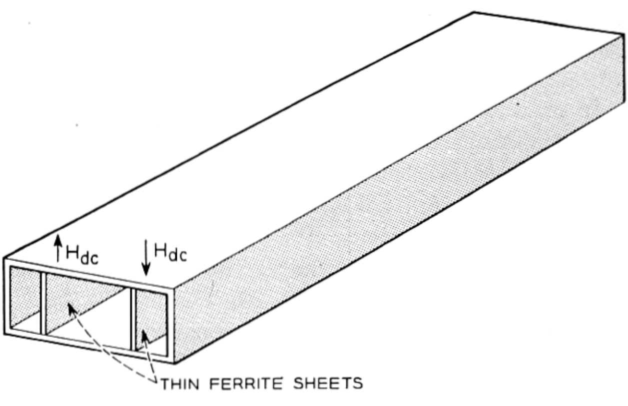

Fig. 24 — Alternative non-reciprocal ferrite loading for rectangular guide.

both ferrite elements of Fig. 24, the non-reciprocal phase contributions of the two elements cancel each other. However, a reciprocal phase change due to the magnetic field remains. This effect is small in the region where μ_+ and μ_- have approximately odd symmetry about the zero field point of Fig. 6. To the extent that the symmetry is perfect, the increase in μ_- should cancel the decrease in μ_+. For large applied fields or for operating frequencies near ferromagnetic resonance, the odd symmetry no longer holds, and the difference between μ_+ and μ_- will leave a reciprocal phase contribution. Calculations which clarify these interrelations are reported in Section 6.

The configuration of Fig. 24 is potentially non-reciprocal in another sense. For both elements biased downward and for wave propagation out of the paper, the right-hand ferrite element presents a permeability greater than unity and the left-hand ferrite element presents a permeability less than unity. As a consequence of this higher index of refraction on the right-hand side of the guide, one would expect a concentration of energy on the right-hand side of the centerline for wave propagation out of the paper. On reversing the direction of wave propagation, the sense of the predominating circularly polarized rf h is reversed at each ferrite element, the index of refraction is larger on the left-hand side of the guide, and one would expect to find the energy concentrated on the left-hand side of the guide. This situation is similar to the explanation for the asymmetry in the low-field loss of longitudinally-biased ferrite rods in circular guide, Section 4-8a. Experiments have shown that this *spatial displacement of the fields* does indeed take place, and we will presently describe how the effect may be incorporated into non-reciprocal devices. It should be noted that this non-reciprocal *field-displacement effect* may be obtained in configurations which have reciprocal attenuation and phase characteristics (such as Fig. 24), or in configurations which have non-reciprocal attenuation and phase characteristics (such as Fig. 23). In the circuit of Fig. 23, the redistribution of fields in the presence of the magnetized ferrite (the field-displacement effect) may be thought of as augmenting the differential phase shift which has been described above soley in terms of the difference between μ_+ and μ_- in an unperturbed $TE_{10}^{\square}$ field distribution.

Non-reciprocal attenuation in the ferrite-loaded medium may be realized using the configuration of Fig. 23. The biasing field H_{dc} is set at the value required for ferromagnetic resonance, H_{res} of Fig. 6, and since this resonance loss occurs only for the positive circularly polarized component, the transmission loss of the medium is larger for propagation in the $+z$ direction than for propagation in the $-z$ direction. The ratio of these two losses is quite high, thereby forming a *resonance isolator*,* when the ferrite is placed at a point where the ratio of the positive to the negative circularly polarized components of h is high. The optimum ferrite location for the resonance isolator and several other quantitative aspects of the ferrite-loaded rectangular waveguide may be made clearer with reference to the mathematical expressions for the fields.

The two linear components of magnetic intensity h_y and h_z for TE_{10} in hollow rectangular guide, Fig. 22(a), are represented by the following relations for propagation in the $+z$ direction:

$$h_y = D \sin\left(\frac{\pi y}{a}\right) \epsilon^{j(\omega t - (2\pi z/\lambda_g))} \tag{18}$$

$$h_z = \frac{D}{\sqrt{\left(\frac{2a}{\lambda_0}\right)^2 - 1}} \cos\left(\frac{\pi y}{a}\right) \epsilon^{j(\omega t - (2\pi z/\lambda_g) - \pi/2)} \tag{19}$$

where

$$D = \sqrt{\frac{2\pi}{\omega\mu\lambda_g ab}} \tag{20}$$

in which a, b = the large and small dimensions of the waveguide respectively

λ_0 = free-space wavelength

λ_g = guide wavelength

These expressions have been normalized for unit power flow in the direction of propagation and the conductivity of the walls is assumed to be infinite. Since the h_z and h_y components are in space and time quadrature, pure circularly polarized h exists when their magnitudes are equal, a condition governed by the relation:

$$\left|\tan\left(\frac{\pi y}{a}\right)\right| = \frac{1}{\sqrt{\left(\frac{2a}{\lambda_0}\right)^2 - 1}} \tag{21}$$

A plot of the magnitudes of the h_y and h_z components as a function of y/a is given in Fig. 25. It is seen that the location of the planes of pure circularly polarized magnetic intensity depends upon the ratio of the operating frequency f to the cut-off frequency f_c, approaches the center line for a waveguide near cut-off, and approaches the side wall for a waveguide remote from cut-off.

* The first published description of this form of isolator is in Reference 19.

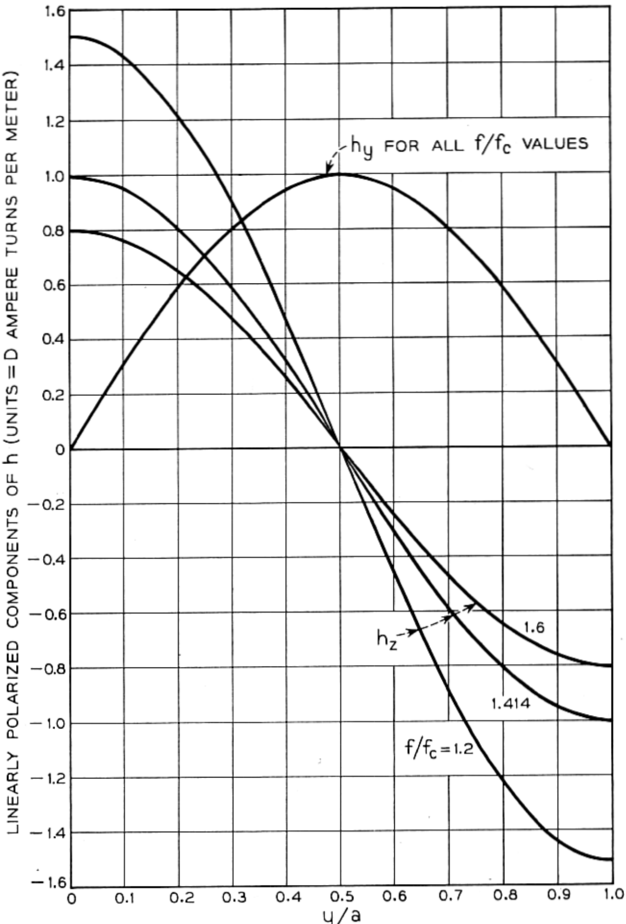

Fig. 25 — Theoretical magnitudes of transverse and longitudinal magnetic intensity of TE_{10} in empty rectangular guide.

For propagation in the $-z$ direction, h_y and h_z of (18) and (19) are altered by replacing

$$e^{-j(2\pi z/\lambda_g)} \qquad \text{by} \qquad e^{+j(2\pi z/\lambda_g)}$$

and by reversing the algebraic sign of h_z.

Since the ferrite presents a scalar permeability to circularly polarized waves, it is preferable in some instances to deal with the circularly polarized components of h in the hollow guide. These circularly polarized components of h in the yz-plane are, for propagation in the $+z$ direction with reference to a magnetic field in the $-x$ direction:

$$h_{yz}^{\pm} = \frac{D}{2}\left[\sin\frac{\pi y}{a} \mp \frac{1}{\sqrt{\left(\frac{2a}{\lambda_0}\right)^2 - 1}}\cos\frac{\pi y}{a}\right] e^{j(\omega t - (2\pi z/\lambda_g))} \qquad (22)$$

For propagation in the $-z$ direction $e^{-j(2\pi z/\lambda_g)}$ is replaced by $e^{+j(2\pi z/\lambda_g)}$ and the sign in front of $\cos(\pi y/a)$ becomes $\pm$ instead of $\mp$. Thus, reversal of the direction of propagation reverses the sense of the circularly polarized component at a given point. Figure 26 shows the $h_{yz}^{\pm}$ components for $+z$ propagation and for several ratios of operating frequency to cut-off frequency, and illustrates that the location of the plane of pure circularly polarized h, as desired in the resonance isolator, depends on operating frequency. It is to be emphasized that the above field relations for the empty guide are valid only in case of small perturbations. In practical ferrite devices, these relations may be profoundly altered by the high dielectric constant of the ferrite.

5-1. *Rectangular Waveguide Pertubations Due to the Presence of Dielectric Material*

In the preceding section it was assumed that the field configuration was not appreciably altered by the addition of the ferrite. For very small pieces of ferrite this approximation is adequate and leads to a correct conclusion as to the general nature of the effects resulting from the addition of biased ferrite. However, when one tries to optimize the performance of a device which utilizes the interaction between the ferrite and the electromagnetic field, it is advantageous to use as much ferrite as is tolerable in order to obtain a large ferrite effect. When too much ferrite is added, the relatively high dielectric constant of the ferrite makes the waveguide medium capable of propagating secondary modes which can seriously interfere with the desired transmission properties. Calculations have been made to determine the conditions for cut-off of the TE_{20} mode in a rectangular waveguide for several locations of the ferrite in the guide. It was assumed that the relative permeability of the ferrite is unity (the field-free condition). The equations used in these calculations can be readily obtained by a technique due to N. H. Frank.[20]

Fig. 27 shows the results for the case of a longitudinal slab of dielectric of thickness d in the center of the guide as well as for dielectric of thickness $d/2$ at both sides of the waveguide. These two distributions of ferrite produce identical TE_{20} cut-offs because the dielectric is inserted

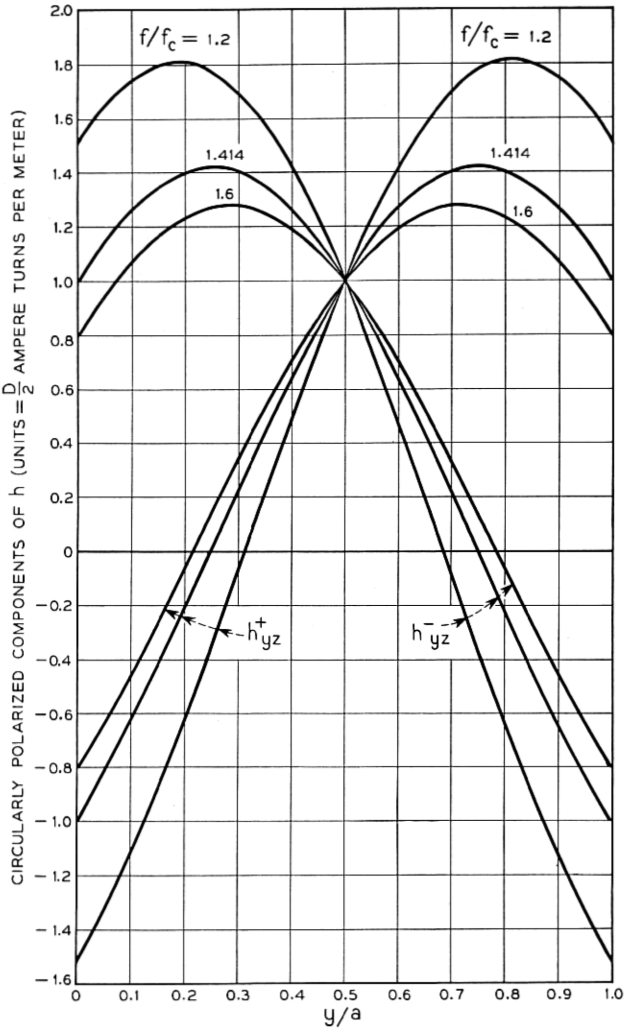

Fig. 26 — Theoretical magnitudes of circularly polarized h in empty rectangular guide.

in corresponding regions of field in both cases. One might presume that the curves would have a maximum slope when the transverse electric field maxima are passing through the interfaces between the dielectric and air. It should be noted, however, that the maximum slope occurs at values of d/a considerably less than 0.5, indicating that the field is

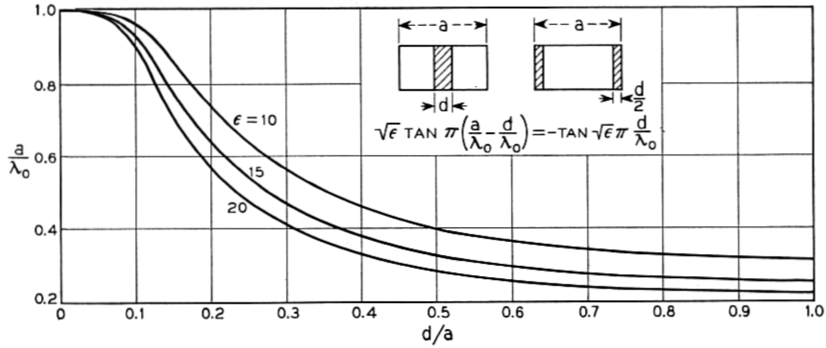

Fig. 27 — TE_{20} cutoff for rectangular guide with dielectric loading at the center or at both side walls.

being pulled into the dielectric. This field concentration in the ferrite is greater for increasing values of ferrite dielectric constant. The cut-off equation for these cases is:

$$\sqrt{\epsilon} \tan \pi \left(\frac{a}{\lambda_0} - \frac{d}{\lambda_0}\right) = -\tan \sqrt{\epsilon}\pi \frac{d}{\lambda_0} \tag{23}$$

Fig. 28 shows the curves of a/λ versus d/a for the case of a dielectric slab at one side of the guide. The equation for cut-off calculations is

$$\sqrt{\epsilon} \tan 2\pi \left(\frac{a}{\lambda_0} - \frac{d}{\lambda_0}\right) = -\tan \sqrt{\epsilon}2\pi \frac{d}{\lambda_0} \tag{24}$$

Here one can observe that the curves go through two high slope regions

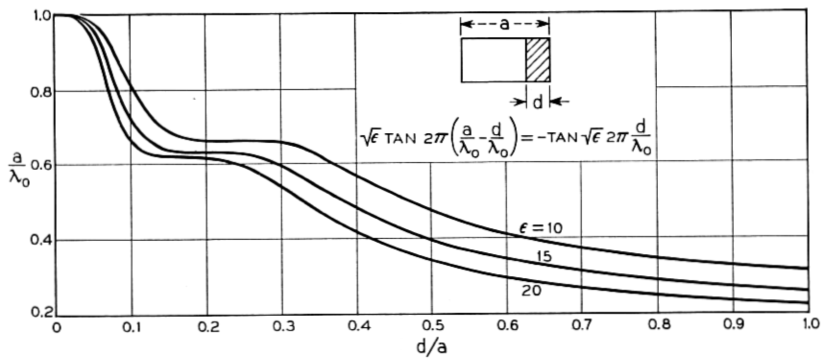

Fig. 28 — TE_{20} cutoff for rectangular guide with dielectric loading at one side wall.

since the transverse electric field of the TE_{20} mode has two maxima which pass through the air dielectric interface at two values of d/a. Again, because of the electric field concentration in the high dielectric constant material, these values of d/a are considerably less than 0.25 and 0.75, the values for maximum field in an empty guide. For small values of d/a, a smaller guide width will propagate the TE_{20} mode for this case of dielectric at one side than for the symmetrically placed dielectric as one might expect from the electric field distribution in the two cases.

Fig. 29 shows the results for a dielectric slab placed one quarter of the way from the side wall, where the dielectric should be most effective in propagating the TE_{20} mode, since it is in the maximum electric field region even at very small thickness. The cut-off equation is given by

$$\sqrt{\epsilon} \tan \pi \left(\frac{3a}{2\lambda_0} - \frac{d}{\lambda_0} \right) + \tan \sqrt{\epsilon} 2\pi \frac{d}{\lambda_0} + \left[\sqrt{\epsilon} \tan \pi \left(\frac{a}{2\lambda_0} - \frac{d}{\lambda_0} \right) \right] \left[1 - \sqrt{\epsilon} \tan \pi \left(\frac{3a}{2\lambda_0} - \frac{d}{\lambda_0} \right) \tan \sqrt{\epsilon} 2\pi \frac{d}{\lambda_0} \right] = 0 \tag{25}$$

Equations for obtaining the phase constant of the TE_{10} mode in the dielectric-loaded waveguide, per Fig. 28, are available in the literature,[20] and the technique referred to above[20] can be employed to determine the TE_{10} phase constants for the configurations of Figs. 27 and 29.

The above theoretical relations are helpful in avoiding multi-mode conditions in ferrite-loaded rectangular waveguide. However, a very

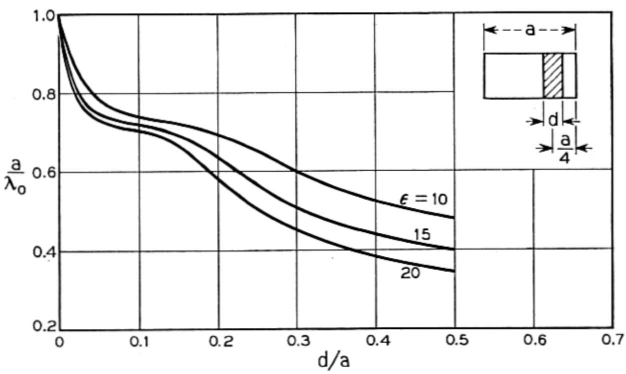

Fig. 29 — TE_{20} cutoff for rectangular guide with dielectric centered midway between the side wall and the center line.

effective means of increasing the available ferrite interaction before the onset of higher modes is to leave an air gap between the wide faces of the guide and the ferrite. Such an air gap introduces appreciable impedance to the displacement currents and allows the use of more ferrite. Unfortunately, the theoretical relations giving TE_{20} cutoff and TE_{10} phase constant under these conditions are not available.

5-2. *Rectangular Waveguide Directional Phase Shifters*

A simple gyrator in rectangular dominant-mode waveguide may be built using the configuration of Fig. 23 or 24, perhaps modified with an air gap between the ferrite sheet and the wide faces of the waveguide. The height of the ferrite sheets may be tapered to a point or stepped in a series of quarter-wave transformers in order to provide a smooth impedance transition from the empty waveguide to the ferrite-loaded region.

Theoretical work by H. Suhl and L. R. Walker[21] has shown that that optimum spacing between the side-wall and a very thin ferrite sheet to produce a maximum of differential phase shift for the two directions of propagation is ¼ of the guide width. A physical explanation for this optimum ferrite location can be given in terms of the point-field concept. The non-reciprocal phase shift due to the ferrite may be thought of as the result of a vector addition of the directly transmitted component of h with the reradiated component of h which is a consequence of the precessional motion of the ferrite's magnetization. From this point of view, the transverse h of the driving wave causes a reradiation via a longitudinal component of b and, as discussed in connection with equation (15), the effect will be proportional to the product of the transverse h and the longitudinal b. Since in this case both the driving and reradiated waves are dominant modes in the rectangular guide, the product of transverse h and longitudinal b is related to the ferrite location at point y by [see equations (18) and (19)].

$$\sin\left(\frac{\pi y}{a}\right)\cos\left(\frac{\pi y}{a}\right)$$

which is a maximum at $y = a/4$ or $3a/4$. Similar consideration of the longitudinal h of the driving wave results in the same conclusion.

For thicker sheets of ferrite, the optimum position for the ferrite is closer to the adjacent waveguide wall, due to a perturbation of the distribution of field components due to the dielectric constant of the ferrite. A plot of the measured difference in phase shift for the two directions of

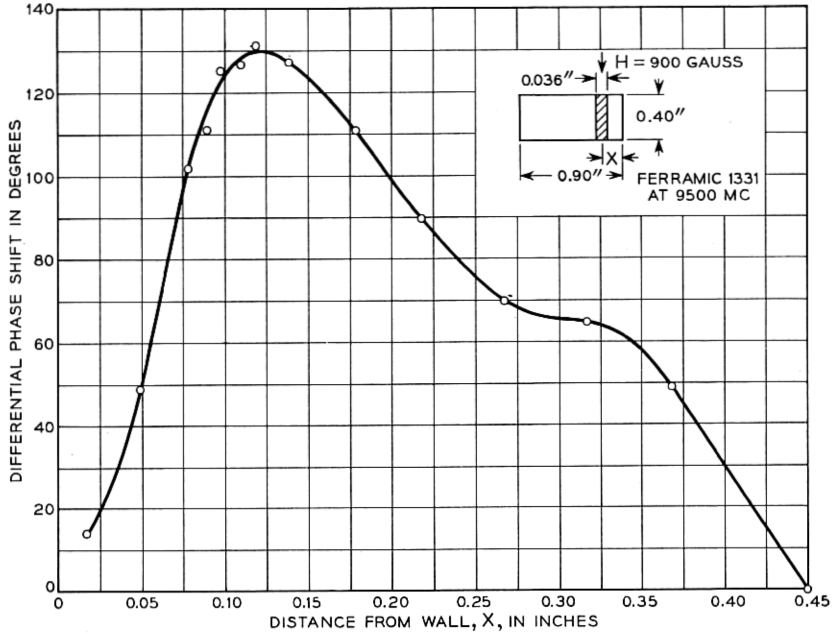

Fig. 30 — Measured non-reciprocal phase shift for a ferrite-loaded rectangular guide.

transmission as a function of the distance between the ferrite and the side wall is given in Fig. 30; even though the ferrite thickness is only 4 per cent of the guide width, the maximum differential phase shift occurs at a spacing of 0.133 times the guide width rather than at 0.25, as predicted by the perturbation theory.

A gyrator of the general form sketched in Fig. 23 has been built in the frequency region near 9,000 mc, and the observed difference in phase shift for the two directions of propagation as a function of frequency is shown in Fig. 31. If a smooth curve is drawn through the points shown, one would conclude that the differential phase shift is within $\pm 10°$ of 180° over a 1,000-mc band. However, we believe the ripples in the curve as drawn are characteristic of the true performance, the accuracy of measurement being about $\pm$ 2°. These ripples cannot be due to multiple reflections of the dominant wave between the ends of the ferrite sheet, because the tapers are gradual enough to prevent reflections of the magnitude this would imply.

The TE_{20} mode is beyond cut-off in the ferrite region (on the presumption that the ferrite dielectric constant is less than 20), so we would not expect mode conversion difficulties. Nevertheless, there is some indication that these ripples are due to higher order modes in the ferrite-loaded region, because a similar gyrator designed for a mid-band frequency near 11 kmc in the same (0.400″ x 0.900″) waveguide showed appreciably larger phase ripples.

An alternative form for the non-reciprocal phase shifter consists of a series of rods of ferrite extending entirely through the waveguide in the plane of the ferrite sheets of Figs. 23 and 24. In this form there need be no air gap between the magnetic structure used to bias the ferrite and the ferrite itself because the rods can extend through the waveguide wall without radio frequency power leakage.

5-3. *Rectangular Waveguide Circulators Using Gyrators*

With the development of the rectangular waveguide directional phase shifter, it becomes possible to build a variety of rectangular waveguide circulators by combining this new circuit element with standard directional couplers.[8,22]

One of the simplest such circulator configurations is shown in Fig. 32. This circulator consists of two 3 db (0.707 amplitude) couplers joined by sections of guide so that one guide has π radians more phase shift for transmission from left to right in the direction of the arrow than for the opposite direction of propagation. This non-reciprocal phase shift is provided by a piece of ferrite of appropriate length near the side wall of the rectangular guide with an appropriate dc magnetic field. The other guide must have a dielectric counterpoise to balance the reciprocal phase shift provided by the ferrite in the bottom guide. One can easily show that the above configuration will give a circulator action.

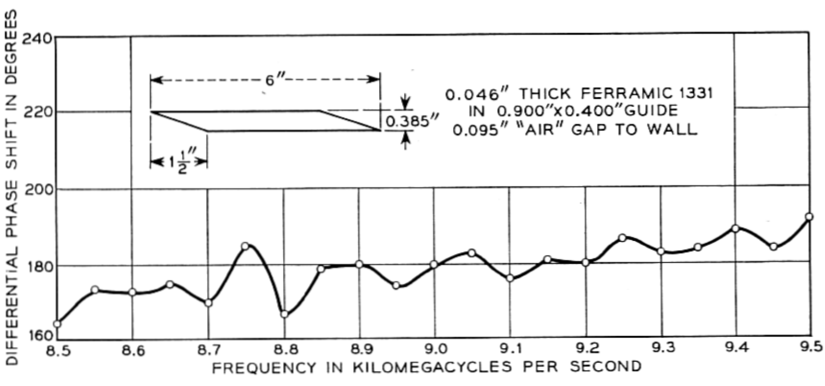

Fig. 31 — Frequency characteristic of a 9,000-mc gyrator.

This type of circulator has been built for operation at 9,000 mcs using two Riblet couplers with the magnetic field of about 1,000 oersteds provided by a permanent magnet. The overall length of the device was 12 inches. The forward loss was about 0.1 db while the signal in the "zero-transmission" terminals was down by at least 30 db. No bandwidth data have as yet been taken.

The above scheme looks very similar to the circulator using two hybrids and a gyrator which has been described by C. L. Hogan. However, this circulator design can be generalized in that a number of different configurations can be designed, some of which may have different and useful properties.

Thus, Fig. 33 shows an alternative circulator scheme which uses two 90° directional phase shifters, one in each line. An extra quarter wavelength reciprocal section is required in one of the guides as shown. This scheme should shorten the overall length of the circulator, but it has been found to be more difficult to adjust because the magnetic field used to balance the non-reciprocal phase shift also affects the reciprocal phase shift.

The circulator shown in Fig. 32 depends for its action on the fact that the ferrite gives a non-reciprocal phase shift of exactly 180°. Any departure from this value will cause some undesirable transmission in the "zero-transmission" direction. However, the amount of phase shift given by the ferrite is dependent on the temperature of the ferrite and the frequency of operation. Therefore, the directional-phase section may have a phase shift differing from π by a small angle, Δ, at a temperature or a frequency differing from the design value.

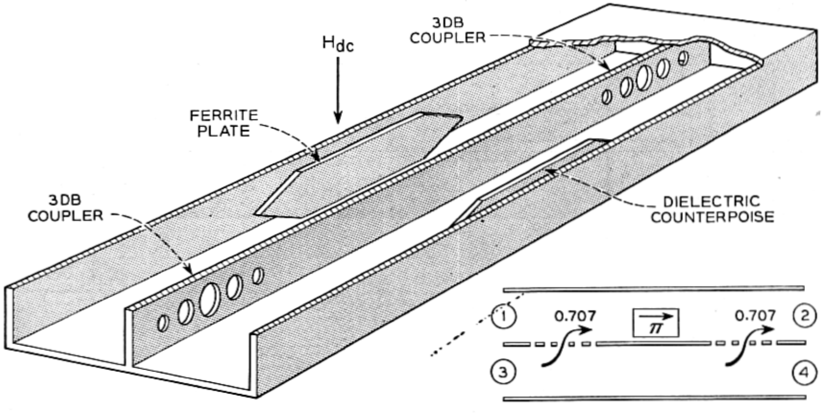

Fig. 32 — Circulator using 3-db couplers and a gyrator.

In order to minimize the effect of this change in the value of the phase shift, we shall show that one can split the coupling sections in a symmetrical and appropriate manner. Fig. 34 shows such a three-section circulator which minimizes the effects of temperature or frequency variations. The necessary values of coupling in this three-section circulator can be determined as follows: The amplitude coupling in the first and last couplers is equal to $\sin\theta$, while the two middle couplers each have a coupling equal to $\sin\varphi$. In order that complete coupling take place in the desired direction, $2\theta + 2\varphi$ must equal 90°.*

From the above it may be shown that $\cos 2\theta = \sin 2\varphi$ and $\sin 2\theta = \cos 2\varphi$. Let us assume that all the $\vec{\pi}$ sections have a phase shift of $\overrightarrow{\pi + \Delta}$. With an input of unit amplitude at (3) in Fig. 34, let us calculate the output at (2). The following table shows how one proceeds, taking into account only differences between the phase shifts occurring in top and bottom guides:

	At a	At b	At c	At d
Upper guide..	1	$\cos\theta$	$\cos\theta$	$\cos\theta\cos\varphi + \sin\theta\sin\varphi\, e^{j\Delta}$
Lower guide..	0	$j\sin\theta$	$-j\sin\theta\, e^{j\Delta}$	$-j\sin\theta\cos\varphi\, e^{j\Delta} + j\sin\varphi\cos\theta$

and so on until we reach the output at (2). This is given by

$$\sin\theta\cos\theta\cos^2\varphi[1 - e^{j3\Delta}] - [\cos^2\theta\sin\varphi\cos\varphi - \sin^2\theta\sin\varphi\cos\varphi - \cos\theta\sin\theta\sin^2\varphi]\,[e^{j\Delta} - e^{j2\Delta}] \quad (26a)$$

which we want to depart from zero at a minimum rate. We thus have an expression which we shall write for simplicity as

$$A[1 - Be^{j\Delta} + Ce^{j2\Delta} - De^{j3\Delta}]$$

It is well known that if 1, B, C, D form a binomial relation, 1-3-3-1,

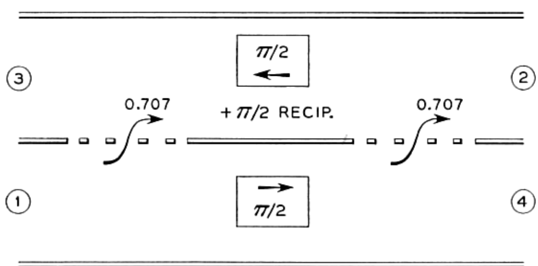

Fig. 33 — Alternative form for circulator of Fig. 32.

* For further details on couplers see Miller, Reference 8, p. 694.

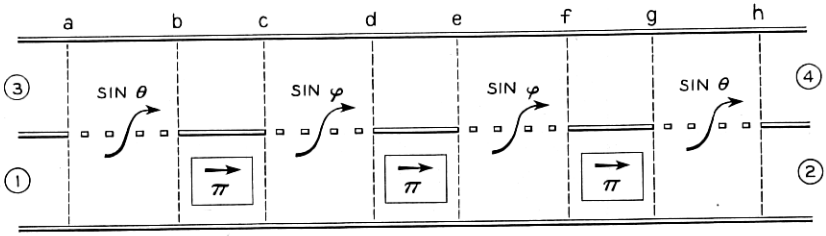

Fig. 34 — Circulator using three gyrators for broad banding.

then the above expression will be equal to $A(1 - e^{j\Delta})^3$, which is equal to $(j\Delta)^3$ + higher order terms since*

$$1 - ne^{j\Delta} + \frac{n(n-1)}{1 \cdot 2} e^{2j\Delta} + \cdots + (-1)^n \frac{n(n-1)\cdots 1}{1 \cdot 2 \cdot 3 \cdots n} e^{nj\Delta} \tag{26b}$$
$$= (1 - e^{j\Delta})^n = (j\Delta)^n + \text{higher order terms}$$

Therefore, by equating corresponding terms in (26a) and (26b) we can solve for θ and φ. The result is $2\theta = 21.466°$ and $2\varphi = 68.534°$. With this design the unwanted output at (2) with input at (3) will therefore be proportional to $(j\Delta)^3$ which is, of course, much smaller than for the single section circulator for which the output at (2) is proportional to $j\Delta$. The above scheme can be used for designing an n section circulator which will be compensated for changes in the $\vec{\pi}$ phase shift to $(j\Delta)^n$.

With the rectangular waveguide gyrator, one can also make a variety of one-way couplers which couple only part of the power over in a non-reciprocal manner. Thus, Fig. 35 shows the rectangular waveguide equivalent of the non-reciprocal power divider described in Section 4-9c.

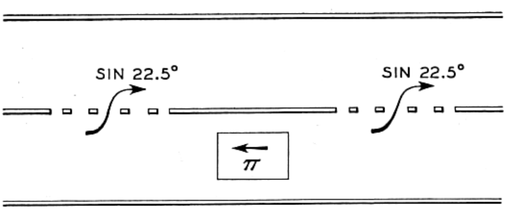

Fig. 35 — Non-reciprocal power divider in rectangular guide (analogue of Fig. 20).

* See Kyhl, Reference 8, p. 882.

5-4. *Circulators Employing Non-Reciprocal Phase Constants*

Two transmission lines having non-reciprocal phase constants may be employed to form a circulator as sketched in Fig. 36. The coupling apertures between guide A and guide B are predetermined in size and number so that complete transfer from one guide to the other will occur when the phase constants are equal.* If the magnetizing field in Fig. 36 were zero and the phase constants of the two guides were equal, then power entering at 1 would emerge entirely at 2, and of course reciprocity would hold. In the circulator application of this configuration the phase constants for the two guides at zero H_{dc} are adjusted to be different through a suitable choice of the waveguide width, thickness of ferrite, or position of ferrite in the two guides. As H_{dc} is increased from zero, the phase constants for the two guides vary in the manner sketched in Fig. 37. The forward waves in guides A and B have the same phase constant for the $H_{dc} = H_1$ whereas the backward waves in the two guides have different phase constants at this same value of H_{dc}. Under this condition, transmission will occur from terminal 1 to terminal 2 (Fig. 36) because the phase constants are equal and the number and size of coupling apertures has been preselected for complete power transfer. Power entering in terminal 2, however, will not return to terminal 1 because the phase constants are unequal. Components of power transferred from guide B to guide A at the various coupling holes add up destructively in guide A when the phase constants are unequal and for a suitably

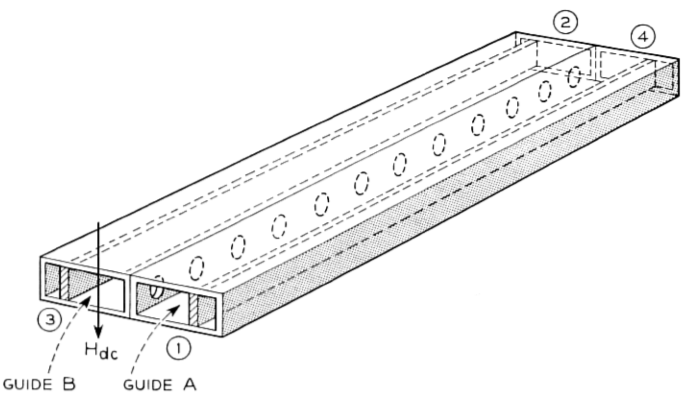

Fig. 36 — Rectangular waveguide circulator using distributed reciprocal coupling between waveguides having non-reciprocal phase constants.

* S. E. Miller, Reference 8.

large ratio of the difference between the phase constants to the coefficient of coupling between the guides, the power transferred from 2 to 1 can be made small. Power entering at terminal 2, therefore, is transferred to terminal 3. From the symmetry of the structure it is apparent that power entering at 3 will also be transferred to terminal 4 and power entering terminal 4 will return to terminal 1. Thus, we have the characteristics previously defined as those of a *circulator*.

The structure of Fig. 36 may be simplified by employing a section of ferrite in only one of the waveguides at the expense of some frequency selectivity due to dielectric asymmetry. In some applications this selectivity may be an advantage.

It may be noted that the equality of phase constants can be adjusted in a completed assembly by adjusting the strength of the magnetizing field.

The coupled-wave circulator may also be adapted to serve as a mode selective device. Fig. 38 shows one such structure, a transducer from single mode rectangular guide to multimode or single mode round waveguide. One may couple either to the dominant (TE_{11}), to the circular electric (TE_{01}), or to any other TE mode of round guide using the configuration sketched in Fig. 38, and by coupling through the wide wall of the rectangular waveguide the TM modes of round guide may be used. Let us assume that the phase constant of the rectangular guide is made equal in one direction to that of the TE_{01} mode in round guide

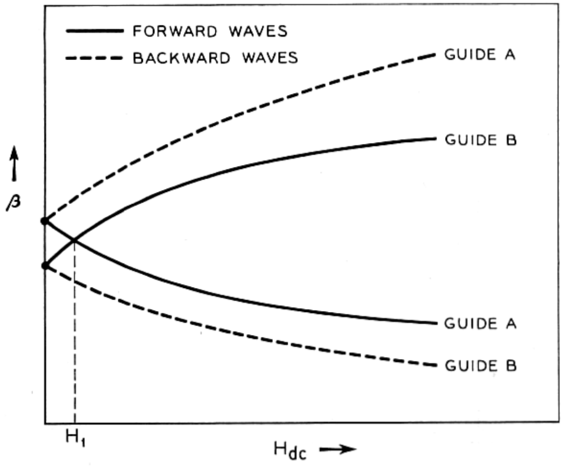

Fig. 37 — Phase constants vs applied field for the guides of Fig. 36.

and that the phase constant for the rectangular guide in the reverse direction is unequal to the phase constant of any one of the modes of the round guide. Then the transmission effects will be as follows:

(1) From terminal 4 to terminal 1 only
(2) From terminal 1 to terminal 2 in the TE_{01} mode only
(3) From terminal 2 in any mode to terminal 3 in the same mode
(4) From terminal 3 in any mode except TE_{01} to terminal 2 in the same mode. From terminal 3 in TE_{01} to terminal 4 only.

The significant feature of the configurations described in this section is the use of reciprocal coupling between transmission lines having non-reciprocal phase constants, and this idea may be generalized to other forms of non-reciprocal transmission line.

5-5. *Resonance Isolators*

We turn now to a device making use of the non-reciprocal attenuation of a ferrite-loaded guide. In Section 5 it was shown that a thin strip of ferrite placed in a rectangular guide, as in Fig. 23, forms a resonance isolator if the applied magnetic field is of the right magnitude to produce ferromagnetic resonance.

We have shown experimentally that for a properly positioned peice of ferrite large differences in transmission loss for the two directions of propagation can indeed be observed. Information on one such isolator designed for 4 kmc is given in Fig. 39. While the dc magnetic field required to produce resonance was of the order of ω/γ, it deviated appreci-

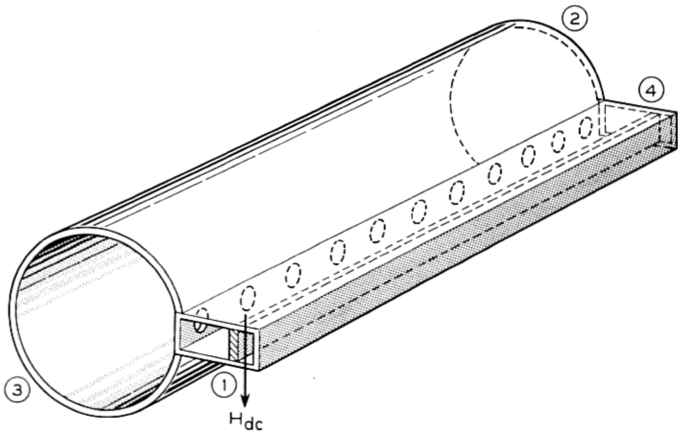

Fig. 38 — Mode-changing circulator analogous to Fig. 36.

ably from this value by virtue of the dc demagnetizing factor of the ferrite strip. Additionally, it was observed that the field required to produce maximum loss was also a function of the distance of the ferrite from the side wall of the waveguide. In other words, the circuit environment of the ferrite plays an important part in determining the resonant frequency.

Since there is no theory available for a ferrite-loaded waveguide operating near or at gyromagnetic resonance, the design of such isolators has been empirical. Many different geometries have been tried, but in general we find that the performance of this type of isolator is limited by an unexpectedly high forward loss. Thus, while enormously high reverse losses are easily obtainable using reasonably short ferrite-loaded sections, the presence of this forward loss has prevented us from observing reverse-loss to forward-loss ratios of greater than 20 to 1 or 25 to 1 in decibels, even for very thin ferrite slabs. This ratio of reverse loss to forward loss is approximately independent of length, and one can elect a forward loss of 1 db with a reverse loss of 20 db, or a forward loss of 4 db with a reverse loss of 80 db. Examination of the loss as a function of applied field for both directions of propagation yields curves of the type shown in Fig. 40. It is seen that for both the plus (forward) and minus (reverse) directions of propagation there is a resonant rise in the loss for particular values of field applied. In general, the loss peaks for the plus and minus directions do not occur at exactly the same value of applied field. The presence of the resonant loss for the minus wave is the reason for the limitation on back-to-front ratios.

This resonant loss for the forward direction is not easily explainable. As seen from Fig. 40, it is present for all positions of the ferrite slab. This forward loss would be expected for a thick ferrite slab, since not all of the ferrite can be at the point of pure circularly polarized h. How-

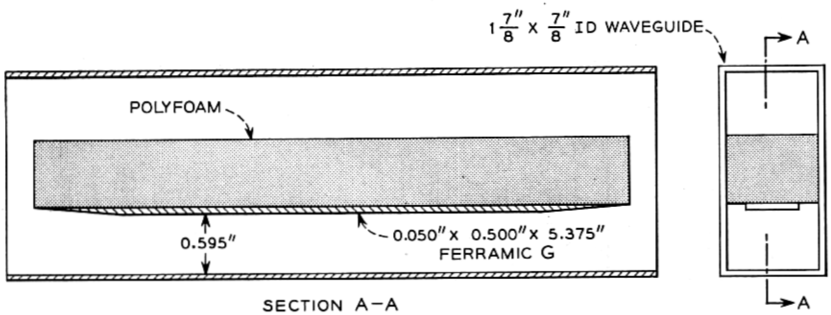

Fig. 39 — 4-kmc resonance isolator.

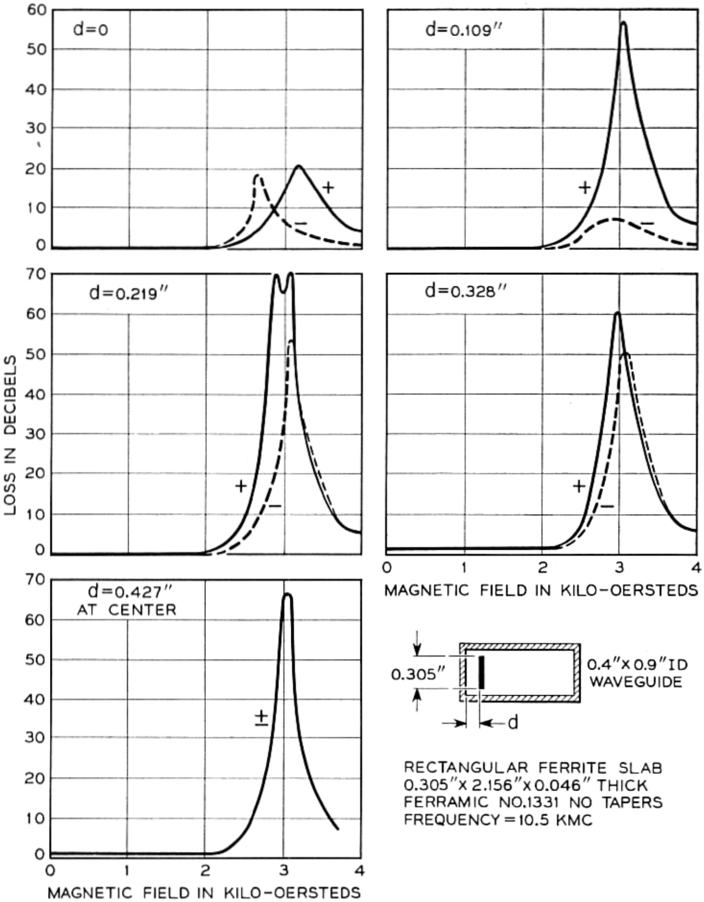

Fig. 40 — Resonance absorption for the forward (+) and reverse (−) directions of propagation as a function of the position of the ferrite element.

ever, if the ferrite is made thin enough and is placed at exactly the right point, one would not expect to find any plus polarized rf field in the ferrite when waves are passing in the minus direction through the waveguide, and the forward loss ought to remain low for all values of dc field. In spite of these considerations, however, it has not been our experience that making the ferrite slab thinner has eliminated the presence of this resonant loss for the minus direction of propagation. If the ferrite slab extends completely from top to bottom of the waveguide, the back-to-front ratio for transmission loss has been observed to be quite poor and of the order of 8 or 10 to 1 in decibels. However, by cutting down the height of the ferrite slab to perhaps ⅔ of the height of the narrow wall of the waveguide, the ratio of 20 or 25 to 1 has been obtained. This strongly suggests that dielectric effects may be playing an important part, since reduction in the height should reduce the displacement current through the ferrite much more rapidly than the interaction of the ferrite with the magnetic field.

In adjusting the position of the ferrite in the waveguide, it has been observed that the position which produces the highest reverse losses is not in general the same as the position which will produce the smallest resonant loss in the forward direction. This also may be seen from Fig. 40. In selecting the best compromise to give a high reverse-to-forward loss ratio, it has been found that the ferrite should always be placed so that the resonant loss in the forward direction is a minimum. In Fig. 40 this corresponds to the $d = 0.109''$ condition. Having so positioned the ferrite, it is also found that the magnetic field which produces the largest reverse loss is not the same as the field which produces the least forward loss. We observe that the best reverse-to-forward loss ratio is obtained when the magnetic field is adjusted for the maximum of reverse loss.

It is interesting to note that as shown by Fig. 40 the fields required to produce resonance in the (+) and (−) directions change as a function of displacement of the ferrite from the sidewall of the waveguide. The resonant frequencies are different for the two directions of propagation except when the ferrite is on the center line, when it becomes perfectly reciprocal.

Fig. 40 also demonstrates the typical behavior of a ferrite with low magnetic loss at zero field strength. With the ferrite slab against the side wall, it is in a position of very low electric field, and any loss observed should be principally due to the rf magnetic field. Fig. 40 shows no observable loss at the sidewall position, and it is believed that for this ferrite the magnetic loss is less than the dielectric loss. Increase in zero field loss as the slab is moved toward the center is believed due to dielectric loss. However, many other ferrites show a substantial loss at the sidewall which decreases as the slab is moved toward the center line. These are believed to have zero field magnetic losses considerably higher than the dielectric losses.

Although a resonance isolator depends upon the adjustment of the magnetic field to produce a resonance condition, this device is not narrow-band if ferrites with broad ferromagnetic resonance lines are used. In Fig. 41 is shown the way in which the forward and reverse losses

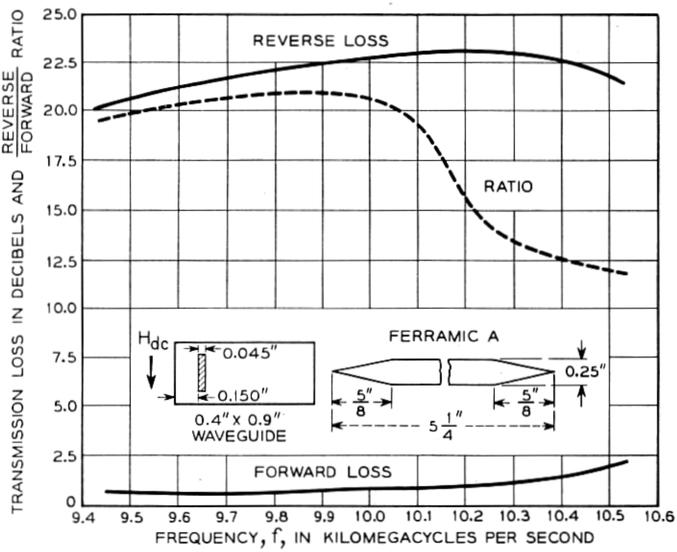

Fig. 41 — Frequency dependence for a 10-kmc resonance isolator.

vary in the frequency range from 9.5 to 10.5 kmc for the isolator described in the same figure. This isolator was a laboratory model which was lined up at 10 kmc, and no attempt was made to optimize the design for a broad band. The curves shown are what resulted from the adjustment procedure of the type described above. As may be seen, a reverse-to-forward loss ratio of better than 20 to 1 in db is maintained over at least a 5 per cent band.

The field of usefulness of the resonance isolator is probably limited to frequencies below about 20 kmc. Above this frequency the magnetic field required to produce resonance becomes large and inconvenient. Below this frequency, however, fields of resonant strength are easily obtained from Alnico magnets of convenient size, and the extreme simplicity and compactness of the device makes it attractive. Other isolators (to be described) have similar attractive characteristics.

5-6. *Field-Displacement Isolators and Circulators*

We turn now to devices which utilize the non-reciprocal field displacement effects which occur in a ferrite-loaded guide:

(*a*) *Resistance-Sheet Isolator*

Fig. 42 illustrates a field displacement isolator which has already been found attractive in practice. In this device the predominance of one type of circularly polarized magnetic intensity at each of the ferrite sections results in a shift of the energy density and the associated electric field to one side of the center line for propagation in one direction and to the other side of the center line for propagation in the opposite direction, as qualitatively sketched in Fig. 42. These two distributions of electric field would result in appreciably different attenuations for forward and reverse directions of propagation, due to the addition of the resistance sheet shown adjacent to the left hand ferrite element in Fig. 42; such a difference in attenuation would of course result in isolator behavior.

Fig. 43 shows the observed performance of a resistance-sheet isolator (see Fig. 44) made for operation in the 24,000 mc region using 0.170″ x 0.420″ I.D. waveguide and 0.043″ x 0.162″ x 1.5″ sections of Ferramic J, quoted by General Ceramics Company as having a saturation magnetization of 2,900 gauss. Let us first note that ratios of reverse loss to forward loss as high as 30 were observed in this model. Several additional details of the observed performance merit recording in order to provide guidance for theoretical analysis. As would be expected, optimum performance was obtained when a maximum amount of ferrite was employed, the upper limit being determined by the onset of mode conversion. However, it was also observed that the resistance sheet isolator could employ more ferrite than could the same rectangular guide in the absence of the resistance sheet. For example, the 0.043″ thickness of ferrite employed in the model of Fig. 44 was adequate to cause mode conversion in the absence of the resistance sheet, such mode conversion

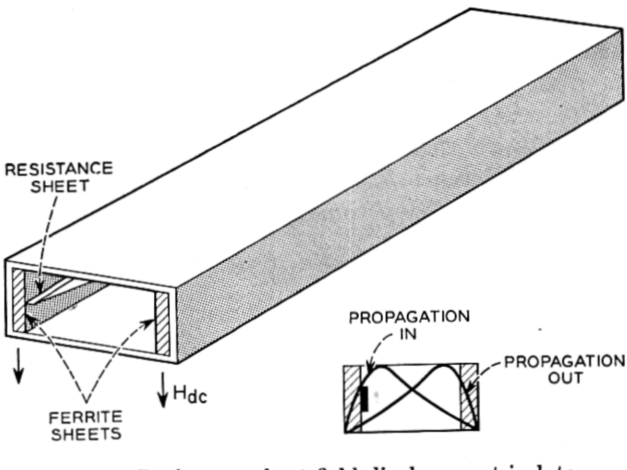

Fig. 42 — Resistance-sheet field-displacement isolator.

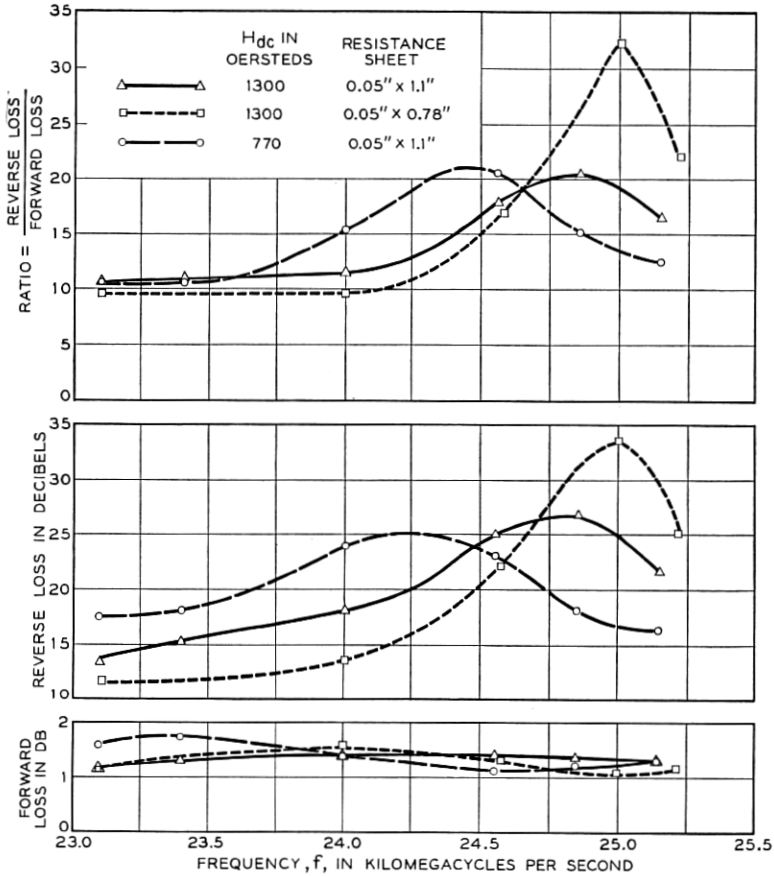

Fig. 43 — Observed performance of a 24,000-mc resistance-sheet isolator.

being characterized by fluctuation between zero loss and loss in excess of 5 db for varying magnetic field of either polarity. Evidently the presence of the resistance sheet damps out the higher-order modes. An increase in ferrite thickness to 0.053″ was adequate to cause mode conversion in the presence of the resistance sheet.

It is noteworthy also in connection with the data of Fig. 43 that the forward loss is approximately independent of frequency whereas the reverse loss is rather frequency sensitive.

The magnetizing field in the model of Fig. 44 was adequate to bring the ferrite near saturation, and in this region variations in field strength produced only small changes in observed performance. For example, curves I and III apply to the identical isolator structure at applied fields of 1,300 and 770 oersteds, respectively.

One of the subtleties in the observed performance of the resistance sheet isolators is illustrated by comparison of curves I and II of Fig. 43. These two sets of data apply to the same configuration with the exception of the length of the resistance sheet. Both resistance sheets gave essentially the same forward loss, but the shorter sheet gave a more pronounced peak in the reverse loss, resulting in a higher reverse-loss to forward-loss ratio.

Another model of the resistance-sheet isolator was made in the 6-kmc region employing the parts shown in Fig. 45 comprising a 0.622″ x 1.372″ I.D. waveguide, a 0.003″ x 0.2″ x 2.48″ resistance sheet, a 0.225″ x 0.570″ x 2.97″ and a 0.196″ x 0.570″ x 2.12″ sections of magnesium aluminum ferrite having a saturation magnetization of about 1,800 gauss. (The latter ferrite pieces have odd dimensions because they were fabricated from pieces remaining from other experiments). The observed performance of this model is recorded in Fig. 46. For the data shown, an applied field of about 470 oersteds was employed, but a reduction of the applied field to 270 oersteds had only a minor effect on the observed data — namely, the peak ratio of reverse-loss to forward-loss remained

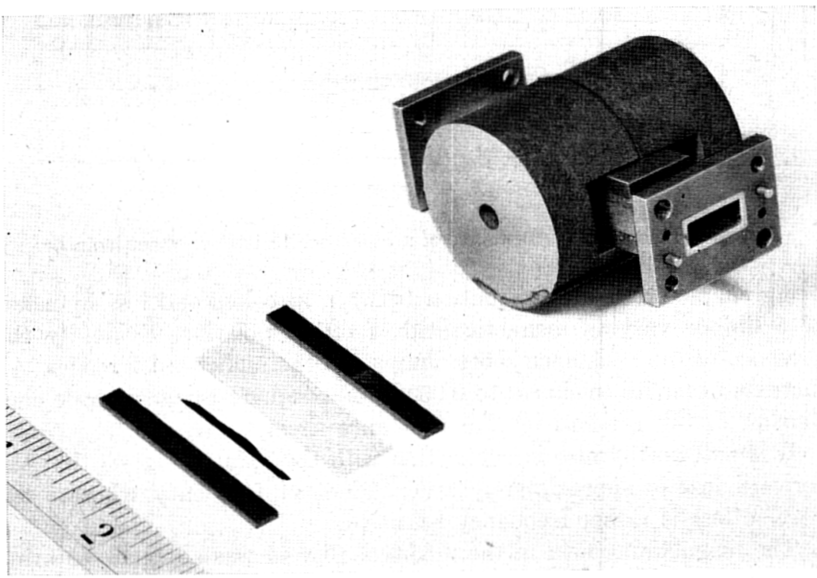

Fig. 44 — The resistance-sheet isolator used to obtain the data of Fig. 43.

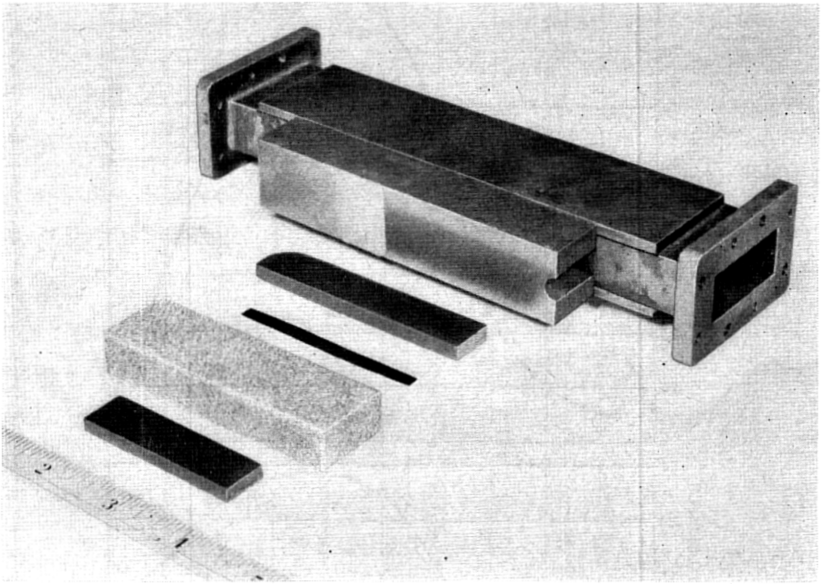

Fig. 45 — The resistance-sheet isolator used to obtain the data of Fig. 46.

80 to 1, but moved down to 6.1 kmc. Likewise the reduction in applied field translated the observed reverse-loss and forward-loss curves toward slightly lower frequencies.*

The return loss for the input and output terminals of the above isolators was in the region 26 to 29 db when the ferrite ends were untapered. The 24,000-mc model showed return losses in the range 33 to 37 db over the 23- to 25-kmc region using $\frac{3}{8}''$ long linear tapers at the ferrite ends.

The 6-kmc model was produced with small effort compared with that devoted to the 24,000-mc model of Figs. 43 and 44. It is likely, therefore, that there is an inherent reason why the resistance-sheet isolator behaves better in the lower frequency region.

When looking at the 0.3 db observed forward loss of the 6 kmc model in contrast to the reverse loss of greater than 20 db, one sees immediately that the electric field at the resistance sheet must be very small in the forward direction as compared to the electric field at the resistance sheet in the reverse direction. (There is no assurance, however, that the elec-

* Subsequent work by S. Weisbaum of Bell Telephone Laboratories has produced a resistance-sheet isolator having a reverse-loss to forward-loss ratio greater than 150 over a 500-mc band near 6,000 mc. A single ferrite element, spaced from the side wall, was used in these models.

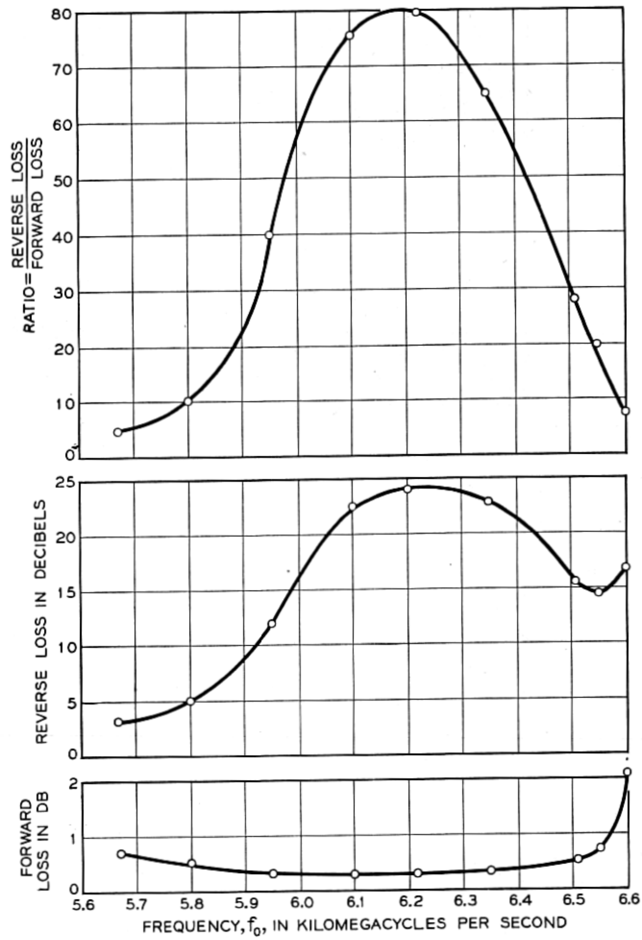

Fig. 46 — Observed performance of a 6 kmc resistance-sheet isolator.

tric field causing the reverse loss is a transverse component). Work done by E. H. Turner of Bell Telephone Laboratories subsequent to that described above has led him to conclude that a complete electric-field null at the edge of the ferrite element may occur provided that the ferrite's tensor permeability component k^2 is equal to or greater than μ^2. The two isolators described above are operated at magnetizing fields such that k^2 is small compared to μ^2 and Turner's condition for the null is not met. However, the ferrite's k/μ for the 6-kmc model was larger than that for the 24-kmc model, and since the 6-kmc model performed better with less effort it is likely that the larger k/μ is desirable in field-displacement devices. The requirement of appreciable k/μ suggests the need for ferrites of higher saturation magnetization for field-displacement devices to be used at 24 kmc or higher frequencies.

(b) *Slotted-Wall Field-Displacement Isolator*

Another form of isolator depending on non-reciprocal field displacement is sketched in Fig. 47. This circuit depends upon the fact that a longitudinal slot may be cut in the wide face of a dominant-mode rectangular waveguide along a line of zero longitudinal magnetic intensity without appreciably altering its transmission characteristics. This is a familiar property which is utilized in standing wave detectors. The addition of magnetized ferrite in the manner sketched in Fig. 47 has the effect of displacing the null for the longitudinal intensity to one side of the center line for one direction of transmission and to the opposite side of the center line for the reverse direction of transmission. The slot of Fig. 47 is located so as to coincide with the null for the longitudinal magnetic intensity for one direction of propagation, and as a consequence there will be negligible attenuation for that direction of propagation. However, for propagation in the reverse direction a very appreciable value of longitudinal magnetic intensity is present at the slot with the result that radiation up into the lossy dielectric occurs.

Exploratory experiments were carried out at 24,000 mc using the structure of Fig. 47 modified to have ferrite only on the side of the waveguide toward which the longitudinal slot was displaced. Using a ferrite section 0.050″ x 0.162″ x 1.5″ and a somewhat shorter slot, a forward

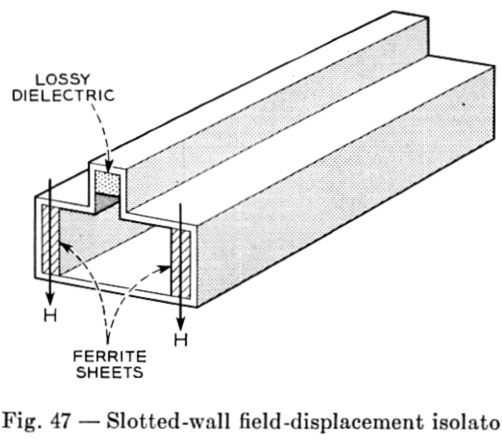

Fig. 47 — Slotted-wall field-displacement isolator.

loss of 0.6 db and a reverse loss of 20.5 db were observed at one frequency. A frequency characteristic was not taken. It is noteworthy, however, that the forward loss in the presence of the magnetizing field was relatively independent of the strength of the magnetizing field, whereas the reverse loss was critically dependent on the strength of the magnetizing field. It is possible that the longitudinal dimension of the slot was near a resonant value for the reverse waves under the condition of observation. In general, it is somewhat difficult to abstract power from a waveguide in a short length interval through a non-resonant slot even though appreciable magnetic intensity is present to excite the slot. A preferred form of this type of isolator might therefore be to replace the continuous longitudinal slot with a series of slots having lengths which are resonant in the frequency band of interest.

(c) A Field-Displacement Four-Terminal Circulator

A circulator utilizing the non-reciprocal field displacement idea is sketched in Fig. 48. Due to the orientation of waveguide 2-3 relative to waveguide 1-4, the only field component of waves propagating in guide 1-4 which will excite a wave in guide 2-3 is the longitudinal rf magnetic intensity. The ferrite loaded guide 1-4 behaves in a manner very similar to the description for the waveguide of Fig. 47. For propagation in the direction of 4 to 1, the longitudinal h is zero to the left of the center line of waveguide 1-4, and a series of coupling apertures between guides 2-3 and 1-4 are positioned along this null. As a result no power is coupled from either guide into the other for waves propagating in this direction.

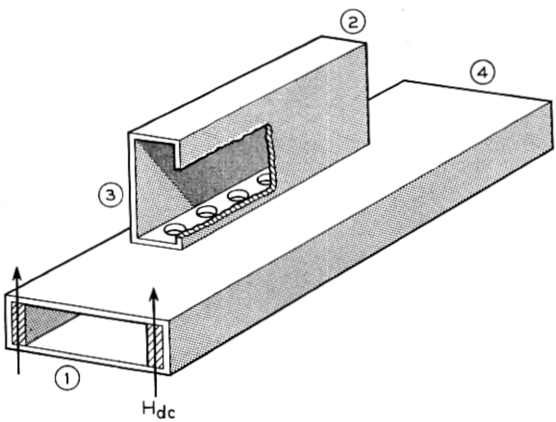

Fig. 48 — A field-displacement four-terminal circulator.

For propagation from 1 to 4, longitudinal h will appear at the apertures, and the number of coupling apertures and the coupling per individual aperture is predetermined,* so as to cause complete transfer of power from terminal 1 to terminal 2, and from terminal 3 to terminal 4. The direction of circulation is then $1 \to 2 \to 3 \to 4 \to 1$.

(d) A Field-Displacement Three-Terminal Circulator

Application of the field-displacement effect to form a three-terminal circulator is sketched in Fig. 49. Guide 1-2 is once more ferrite-loaded and at the coupling slot which is common to branch guide 3 the longitudinal magnetic intensity is zero for one direction of transmission and finite for the reverse direction of transmission in guide 2-1. The coupling slot is so positioned that a wave entering terminal 1 does not excite the slot and therefore passes without reflection to terminal 2. This situation is sketched diagrammatically in Fig. 49(b) as transmission from terminal 1 to terminal 2 only. Energy entering at terminal 2 will propagate with a finite value of longitudinal rf magnetic intensity at the coupling slot and transmit power to both terminals 3 and 1. The energy which sets up a dipole at the longitudinal slot cannot radiate a reflected wave back toward terminal 2 since a wave propagating in the direction 1-2 has no magnetic intensity vector at the coupling slot. This situation is sketched

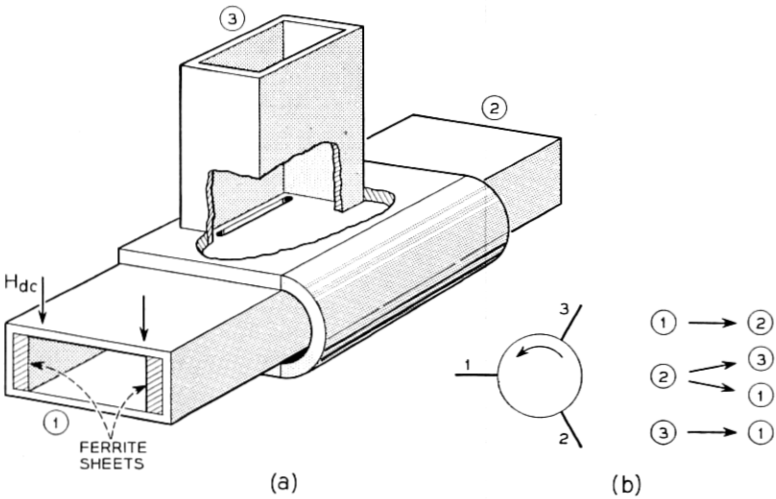

Fig. 49 — A field-displacement junction circulator.

* See S. E. Miller, Reference 8, pp. 694–695.

diagrammatically in Fig. 49(b) as transmission from terminal 2 to terminal 1 and to terminal 3. Energy which is introduced at terminal 3 will set up a magnetic intensity vector along the coupling slot which, for reasons already described, can radiate in the ferrite-loaded guide only in the direction of terminal 1. There may of course be a reflection component for a wave introduced at terminal 3 so that, in general, energy introduced at terminal 3 may propagate to terminal 1 or be reflected back upon itself.

If we now introduce in branch waveguide 3 a tuning reactance such as to cancel out the reflected wave for energy entering at terminal 3, we will transmit all of the power entering terminal 3 to terminal 1. This situation is sketched in Fig. 49(b) as transmission from terminal 3 to terminal 1 only.

The general set of conditions sketched in the power flow diagram of Fig. 49(b) can not exist when the external loads at terminals 1, 2 and 3 match the waveguide impedance. We may demonstrate this by observing that thermal noise power KTB flows from terminal 3 to terminal 1 but, as sketched in Fig. 49(b), there is also power flow from terminal 2 to terminal 1. This would seem to imply that more than KTB of energy is being transmitted to terminal 1, which is impossible. Actually, *under the conditions of matched input impedance at terminal* 3 there can be no transmission from terminal 2 to terminal 1. Instead the transmission is from 1 to 2, 2 to 3, and 3 to 1, and the junction is a circulator.

One may visualize what has happened to the component of transmission from 2 to 1 as follows: the matching mechanism which was introduced in branch 3 to prevent reflection for waves entering at terminal 3 also reflects some of the wave energy which flows from 2 toward 3. This reflected component from the matching mechanism travels back toward

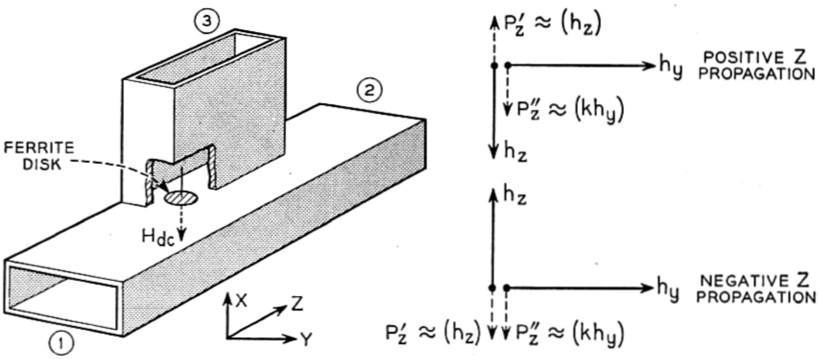

Fig. 50 — A junction circulator using a ferrite-loaded aperture.

the coupling slot and a portion proceeds in the direction of terminal 1 only. For the condition of matched input to terminal 3, the component of wave propagation $2 \rightarrow 3$ which is reflected from the matching mechanism will reappear in branch 1 in such an amplitude and phase as to cancel the direct transmission component of wave propagation $2 \rightarrow 1$.

Experimental models of the junction circulator of Fig. 49 have been built for operation at 6,000 mc and 24,000 mc, and the circulator type of transmission was observed. For the 6,000-mc model which used the same ferrite elements described above for the 6-kmc resistance-sheet isolator, the losses for the various transmission paths are tabulated below:

Path	*Insertion Loss* db
$1 \rightarrow 2$	0.7
$2 \rightarrow 3$	1.3
$3 \rightarrow 1$	1.3
$2 \rightarrow 1$	14.
$1 \rightarrow 3$	13.5

A rather lossy (sliding-screw) tuner was employed in branch 3 in this experiment, so these data do not represent the best that can be achieved. With a good piston in branch 3, the loss for path $2 \rightarrow 3 \rightarrow 1$ was observed to be about 0.1 db.

(e) A 3-Terminal Junction Circulator Utilizing a Ferrite-Loaded Aperture

E. H. Turner has pointed out that a ferrite-loaded aperture has nonreciprocal properties.* The present discussion indicates the way such an aperture can be utilized to form another kind of junction circulator. The structure is shown in Fig. 50. The coupling aperture, which is ferrite-loaded, is slightly to the left of the center-line of the guide 1-2, and filled with ferrite. The waveguides may otherwise be empty.

In order to explain the operation of this device, we note first that the branch guide 3 will be excited only by a Z-axis component of magnetic polarization in the aperture. Such a polarization will result from the component h_z in guide 1-2, and it will point in a direction opposite to h_z .† It is caused in part by the geometry of the aperture, and is further enhanced by the Z-axis component of ferrite magnetization due to h_z when ($\omega > \omega_0$). In addition there is another Z-axis component of polari-

* Unpublished work.

† For a picture of magnetic polarization of an unloaded coupling aperture see R. L. Kyhl, Reference 8, p. 860.

zation caused by h_y as a result of the precession of ferrite magnetization about H_{dc} as explained in connection with Fig. 5.

Fig. 50 shows the time relations between the rf vectors for propagation in both directions in guide 1-2. h_y is the largest vector (since the aperture is near the centerline) and is taken as reference. The Z-axis polarization (p_z'') caused by h_y is proportional to kh_y, and lags h_y by 90 degrees. The Z-axis polarization (p_z') caused by h_z will have a phase which depends upon the direction of wave propagation in guide 1-2. For propagation in the plus Z direction, h_z will lag h_y by 90 degrees, and consequently the polarization will lead h_y by 90 degrees. It will be seen that the Z-axis components of polarization p_z' and p_z'' point in opposite directions. By adjusting the dc magnetic field (which controls k) or the displacement of the aperture from the centerline of the waveguide, the relative magnitudes of p_z' and p_z'' can be adjusted so as to obtain zero net polarization for this direction of propagation. Thus no wave will be excited in branch guide 3. For propagation in the minus Z direction, however, h_z will lead h_y by 90 degrees and the vector relations are given by the lower diagram of Fig. 50. In this case p_z' and p_z'' point in the same direction, and there will be a net polarization of the aperture.

Thus, for propagation $1 \rightarrow 2$, no transmission to 3 occurs, but for propagation $2 \rightarrow 1$ there will be transmission to 3. A line of reasoning similar to that given for Fig. 49 indicates that circulator behavior can also be obtained in this structure. In a 24,000-mc model of the structure of Fig. 50, transmission $3 \rightarrow 1$ was 14–20 db greater than transmission $3 \rightarrow 2$. No attempt was made to match the input impedance at terminal 3.

There is another induced component due to the ferrite response to driving force h_z which is either in phase or out of phase with p_y' (due to h_y) depending on direction of propagation. The guide 3 may be rotated 90° to respond to components of p in the y-direction only, and the circulator may be built in this form as an alternative.

6. MAGNETICALLY CONTROLLED RECIPROCAL PHASE SHIFT IN FERRITE-LOADED STRUCTURES

Although most of the attention of this paper has been directed at the non-reciprocal microwave properties of ferrites, their reciprocal properties, which are under the control of a magnetic field, can also be of great importance. Some aspects of these reciprocal effects have been discussed in an earlier publication[23] where it was shown that, for a uniform plane wave propagating through a ferrite magnetized perpendicular to the rf magnetic vector, the rf permeability is given by:

$$\mu_\perp = \frac{\mu^2 - k^2}{\mu} = 2\,\frac{\mu_+\mu_-}{\mu_+ + \mu_-} = \left[\mu_0 + \frac{\gamma^2M^2 - \gamma M\omega_0\mu_0}{\mu_0(\omega_0^2 - \omega^2) - \gamma M\omega_0}\right] \quad (27)$$

independent of the direction of propagation. If the rf magnetic vector is parallel to the dc magnetization, the rf permeability no longer differs from μ_0. Thus, a transverse magnetic field causes the infinite ferrite medium to be reciprocally birefringent. This reciprocal effect can be large only in the general vicinity of resonance as may be seen by examining equation (27) and Fig. 6. Since μ_+ and μ_- have odd symmetry about $H_{dc} = 0$, the application of small magnetizing fields will cause nearly equal and opposite changes in the μ_+ and μ_-. To first order these effects cancel and as a result $\mu_\perp$ departs from μ_0 very slowly. For large values of field, the odd symmetry no longer holds and $\mu_\perp$ may be appreciably different from μ_0. Thus, for small values of applied magnetic field, (unsaturated region where M is proportional to H), $\mu_\perp - \mu_0$ varies as H_{dc}^2; for large frequencies $\mu_\perp - \mu_0$ varies as $1/\omega^2$ as seen from equation (27). In contrast, the birefringence for positive and negative circularly polarized waves due to an axial magnetic field is a first-order effect since it depends directly on μ_+ and μ_- whose departure from μ_0 is directly proportional to H_{dc}. Thus, $\mu_- - \mu_+$ is given by

$$\mu_- - \mu_+ = \frac{-2\omega\gamma M}{\omega^2 - \omega_0^2} \quad (28)$$

showing that this non-reciprocal effect is linear in M (rather than quadratic) and approximately proportional to $1/\omega$ (rather than $1/\omega^2$). Both $\mu_1 - \mu_0$ and $\mu_- - \mu_+$ are plotted in Fig. 51 as functions of ω/ω_0 for

$$-\frac{\gamma M}{\mu_0\omega_0} = 1.5 \qquad \text{and for} \qquad -\frac{\gamma M}{\mu_0\omega_0} = 0.5.$$

By reading the abscissa as a difference in "external μ's" (see end of Section 2) and the ordinates as the ratio of ω to the external resonant frequency, these curves become applicable to cases frequently encountered in practice. If we bias a given ferrite at a fixed H_{dc} sufficient to bring about saturation, then M and ω_0 are constant and we can use the plots of Fig. 51 to compare the reciprocal and the non-reciprocal effects at various signal frequencies, ω.

A comparison between reciprocal and non-reciprocal phase shifts for various positions of a longitudinally placed ferrite slab in a rectangular waveguide at 10,500 mc can be obtained from Fig. 52. The non-reciprocal phase shift plotted is the *difference* in phase for the two directions

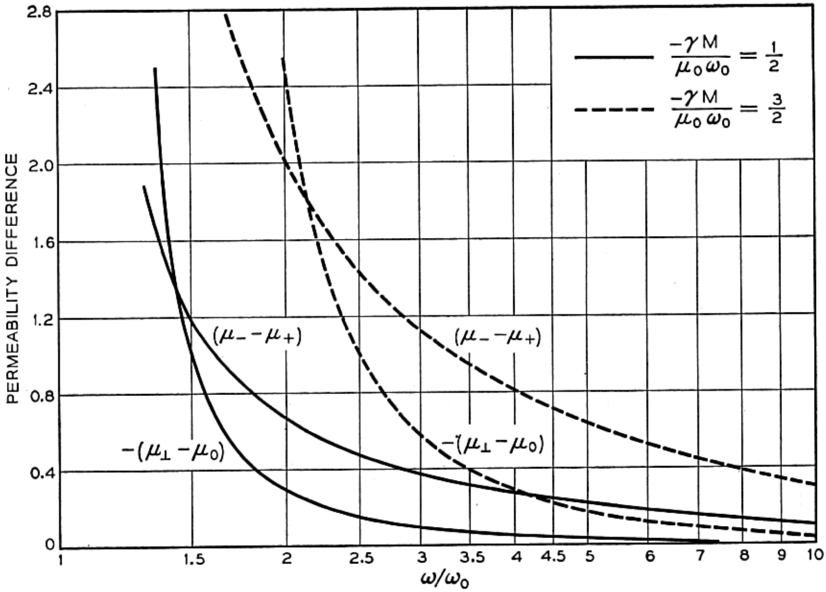

Fig. 51 — Calculated reciprocal $(\mu_\perp - \mu_0)$ and non-reciprocal $(\mu_- - \mu_+)$ ferrite permeability differences as a function of frequency.

of transmission for *fixed* magnetizing field, whereas the reciprocal phase plotted is one-half the sum of the phase changes for the two directions of propagation caused by going from zero magnetizing field to abscissa value of magnetizing field. With the ferrite at the wall of the waveguide, there is a small amount of reciprocal phase shift varying as H^2 with negligible non-reciprocal phase shift for fields below resonance. As the ferrite is moved away from the guide wall, the reciprocal phase shift increases because more energy is concentrated in the ferrite. The non-reciprocal phase shift increases very rapidly as discussed in Section 5-2. With the ferrite moved to the center of the guide, the non-reciprocal phase shift is reduced to zero with the reciprocal phase shift being at a maximum.

7. BIREFRINGENCE IN ROUND WAVEGUIDE

A round waveguide containing an axially symmetric ferrite element is inherently a two-mode guide. If the ferrite is unmagnetized, normal modes are degenerate in the sense that they have the same phase velocity, and we are at liberty to define them either as two orthogonal linearly polarized waves, or as (+) and (−) circularly polarized waves. With the application of any magnetic field, the degeneracy is removed. For longitudinal magnetization the circularly polarized waves remain the normal modes, while the linearly polarized waves become coupled modes. On the

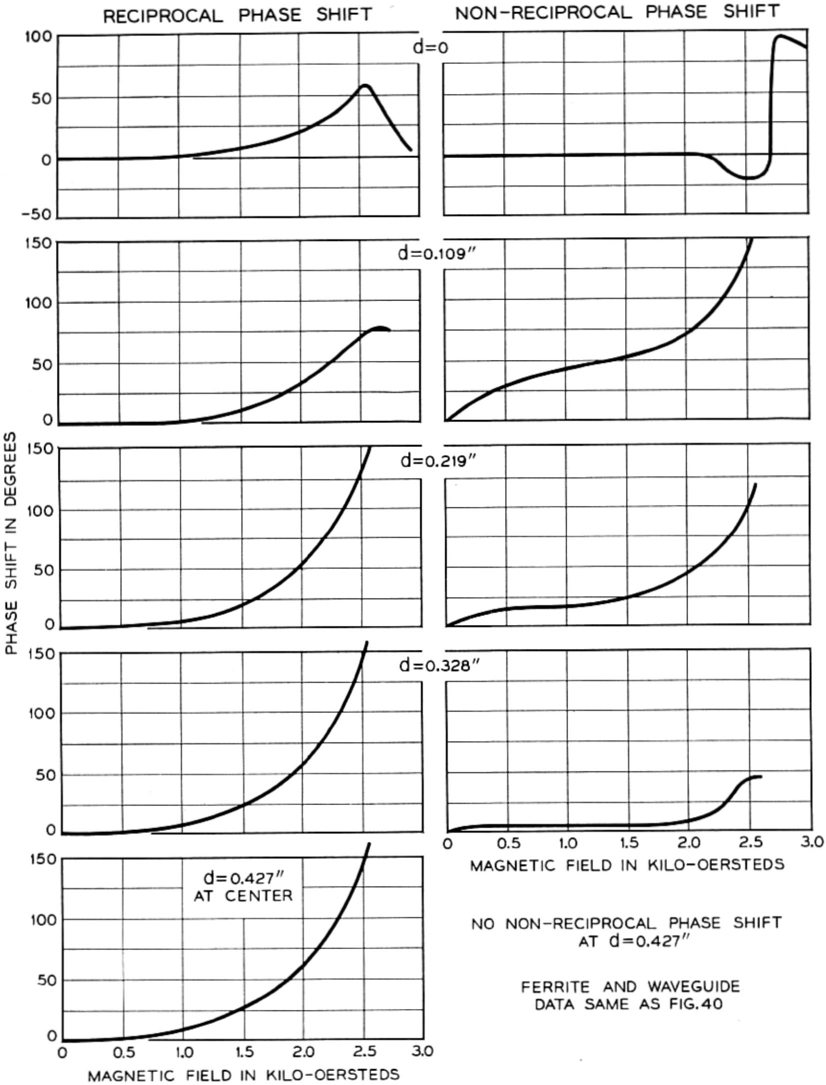

Fig. 52 — Measured reciprocal and non-reciprocal phase shifts for the ferrite-loaded rectangular guide of Fig. 40.

other hand, if the ferrite is magnetized in a transverse direction, the normal modes are two linearly polarized waves having different phase velocities, and the circularly polarized waves become the coupled modes. We may therefore note a form of duality: when longitudinally magnetized, a ferrite medium is birefringent for circularly polarized waves; and when transversely magnetized it is birefringent for linearly polarized waves. The former type of birefringence results in Faraday rotation. The latter type of birefringence has as yet received comparatively little attention but it is likely to prove important in a number of applications. It can be manifested as either a reciprocal or a non-reciprocal effect, and the relative importance of these two effects depends upon the frequency of the rf wave and upon the direction of the magnetization.

The reciprocal birefringence has been discussed for the plane-wave case in Section 6. We now consider this effect for a slender axial pencil in round waveguide, as shown in Fig. 53. With magnetization normal to the paper, a point-field examination indicates that excitation of the ferrite by the dominant wave is essentially transverse (dominant wave

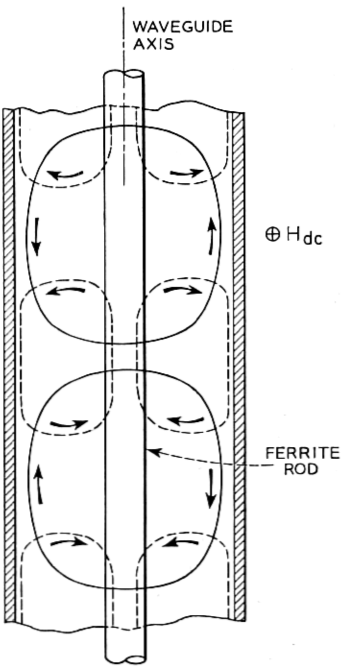

Fig. 53 — Longitudinal section of ferrite-loaded round waveguide with transverse magnetizing field.

magnetic loops shown solid). Ninety degrees later in time the precession of electron spins will produce an axial magnetization at every point where it was formerly transverse. This will tend to generate new b loops shown dashed. These correspond qualitatively to the magnetic field pattern of a TE_{01} wave. Thus, as a byproduct of our examination, we discover that this structure will tend to convert dominant waves into circular electric waves. However, if circular electric waves cannot be propagated, the dashed b loops cannot exist. Instead, an added reactance is presented to the driving wave similar to that which appeared in the infinite transversely-magnetized medium (Section 6). For the orthogonal TE_{11} wave (rf h parallel to the magnetizing field), μ equals μ_0 and the magnetizing field does not alter the phase constant. Thus, the medium is birefringent due to the transverse field, and this birefringence is small for frequencies far above the resonant frequency.

We may also look at the symmetrical ferrite in a somewhat different way. Assuming that the ferrite does not perturb the dominant mode field pattern, all ferrite lying to the right of the center line of Fig. 53 will be exicted by (+) elliptically polarized rf magnetic field vectors. All ferrite to the left of the center line will be excited by (−) eliptically polarized field vectors. As a result, ferrite to the right of the center line will exhibit a permeability less than μ_0, while ferrite to the left of the center line will exhibit a permeability greater than μ_0. To a first approximation, these two effects tend to cancel one another and the total effect on the phase velocity of the dominant wave will be small. We again conclude that the reciprocal birefringence which results from a symmetrical ferrite structure will be small unless the resonant frequency approaches the frequency of the applied wave.

As in the case of the rectangular waveguides considered earlier, this first order cancellation of (+) and (−) permeabilities can be eliminated

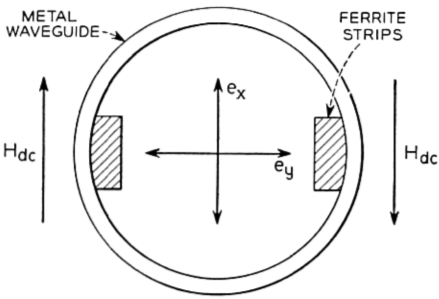

Fig. 54 — Cross section of round waveguide with ferrite next to the metal wall.

by reversing the dc magnetic field on opposite sides of the waveguide center line. Fig. 54 shows the cross section of a waveguide containing two strips of ferrite running along opposite walls and magnetized in opposite directions. If a dominant wave having electric polarization e_x propagates into the paper, the permeability of both strips of ferrite will appear to be less than μ_0, and there will be a first order effect on the phase velocity of this wave (aside from any dielectric effect). A dominant wave having polarization e_y should see a permeability of μ_0 since the rf magnetic vector is parallel with the dc magnetization, and there will be no effect on the phase velocity of this wave (aside from the di-

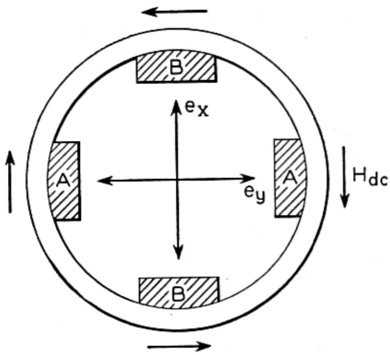

Fig. 55 — Further development of structure of Fig. 54 to eliminate dielectric birefringence and enhance the magnetic birefringence.

electric effect). Thus, there will be a magnetically induced birefringence for waves of these two polarizations.

In addition to the magnetically induced birefringence, the structure of Fig. 54 also introduces a dielectric birefringence due to the presence of the ferrite. This dielectric birefringence can be eliminated and the magnetic birefringence augmented through the use of the structure show in Fig. 55. For polarization e_x propagating into the paper, ferrite strips A-A produce a permeability less than μ_0, while ferrite strips B-B have no magnetic effect on this wave. For polarization e_y propagating into the paper, ferrite strips B-B have a permeability greater than μ_0, while strips A-A have no effect. Consequently, polarization e_x will have a higher phase velocity and polarization e_y will have a lower phase velocity than if the dc magnetic fields were not present. If the direction of propagation is reversed, and waves travel out of the paper, polarization e_x will have the lower phase velocity and polarization e_y will have the higher phase velocity. As will be shown presently, such a non-reciprocal birefringent medium can be made to perform all of the functions of a Faraday rotation medium, and with it circulators, isolators, and gyrators can be built.

The arrangement of Fig. 55 is not the only one which exhibits strong birefringence. Another arrangement is shown in Figure 56. A vertically polarized wave e_x will produce rf magnetic vectors next to the wall of the waveguide which are transverse at points 1 and 3 and longitudinal at points 2 and 4. In between 1 and 2, as for example at A, the magnetic vector will be elliptically polarized tangential to the wall. If a piece of ferrite is placed at A and a dc magnetic field is oriented normal to the wall as shown, a wave e_x propagating into the paper will see a permeability less than μ_0 in the ferrite, and a wave e_y will see a permeability greater than μ_0. In this manner E. H. Turner obtained the first non-reciprocal birefringent medium.[24] While the single ferrite strip has both magnetic and electric birefringence, the addition of the other three ferrite strips, B, C and D of Fig. 56, eliminates dielectric birefringence. With dc field applied as shown, all four strips have a permeability less than μ_0 for e_x propagating into the paper, and a permeability greater than μ_0 for e_y propagating into the paper. For the opposite direction of propagation, the phase velocities are interchanged, and this medium is also non-reciprocally birefringent.

It is possible to interlace the structures of Figs. 55 and 56 to obtain the structure of Fig. 57. With external magnetic poles shown, the magnetization of the ferrite ring will be such as to make all parts of the ferrite active in producing birefringence. The section shown in Fig. 58 was built to test this principle at 24 kmc. It was verified that while a simple

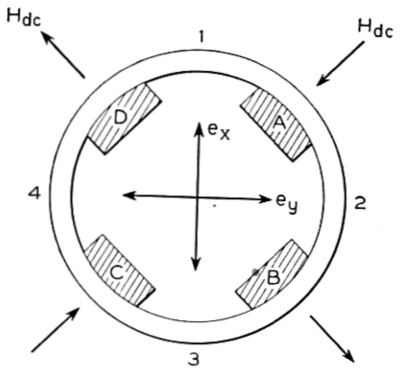

Fig. 56 — Alternate form (to that of Fig. 55) for a magnetically birefringent structure.

transverse magnetizing field produced no noticeable birefringence, a 4-pole field as shown in Fig. 57 produced a differential phase shift of 180°. Thus, as the 4-pole field was rotated a given amount, the output polarization was rotated by twice this amount. This technique may be useful in constructing continuously running phase changers, as will be shown later.

The shape and positioning of the ferrite in the above structure was a compromise selected for its symmetry and mechanical simplicity. Clearly, the ferrite is not used in the most effective places in the waveguide, since where the dc magnetization is tangential to the wall, the rf field is almost

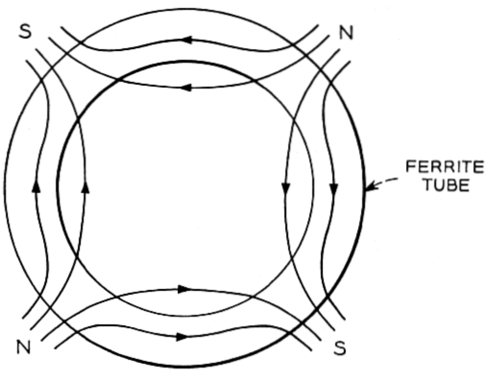

Fig. 57 — Composite of Figs. 55 and 56.

linearly polarized. We may then ask where the ferrite should be placed in order to be most effective.

In order to orient ourselves with regard to the field components in round guide we may note that the total h vector is circularly polarized at the loci drawn in Fig. 59, which also shows for comparison the loci of circularly polarized h in a rectangular guide having the same cut-off frequency. In the round guide it is important to note that the loci are normal to the planes of the polarization vectors at all points, and the dc magnetic field should be directed along the loci for maximum effectiveness.

As in rectangular waveguide, the position of pure circularly polarized h is *not* the optimum ferrite location for producing non-reciprocal phase shift. The latter locus is along points for which the product of transverse-h and longitudinal-b is a maximum, as discussed in Section 5-2. Fig. 60 illustrates such a locus for an unperturbed guide (i.e., for thin

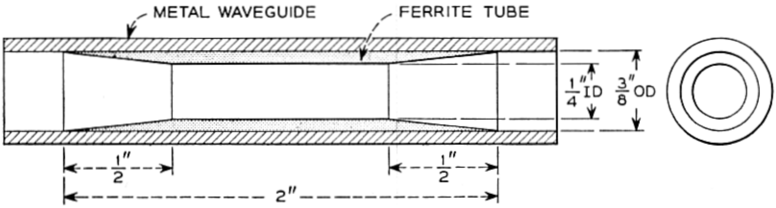

Fig. 58 — Longitudinal section showing the dimensions of a 24-kmc model of the structure of Fig. 57.

ferrite sheets added). Just as in rectangular guide, the most effective ferrite location along axis b–b is about ¼ the diameter away from the wall. The optimum ferrite location near the wall can be obtained by noting that h is trnsverse at points a and longitudinal at points b, with sinusoidal variation between. Therefore, the product of h transverse and h longitudinal will be maximum half way between points a and b, namely,

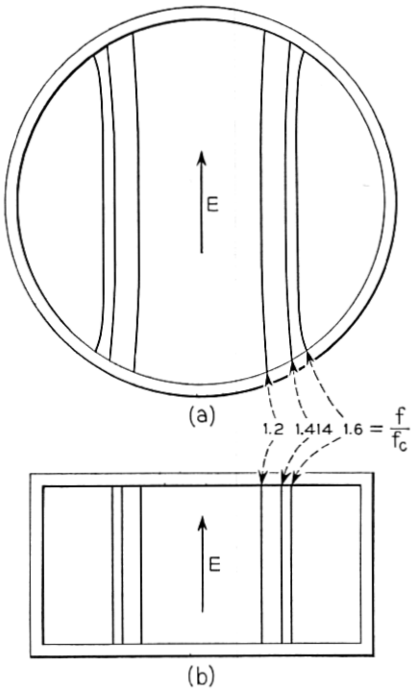

Fig. 59 — Locus of points of circularly polarized magnetic intensity for TE_{11}○ and TE_{10}□ in guides having the same cut-off frequency.

at points 2. Thus, there will be a locus of points shown by the dashed lines, along which a tiny amount of ferrite will be most effective in perturbing the phase velocity of a vertically polarized wave, and these loci are independent of frequency.

As in the rectangular waveguide case, when the amount of ferrite present is sufficient to perturb the wave, the position for maximum effectiveness moves toward the side wall, as illustrated in Fig. 61 for both TE_{11} waves. The thin-walled cylinder of Fig. 58 is a practical approximation to this optimum.

7-1. *Equivalence of Faraday Rotation Elements and Non-Reciprocal Birefringent Elements*

Linear polarization birefringent elements of the type described are quite different from Faraday rotation elements in the way they transform the polarization of transmitted waves. Yet both are basically two-mode elements, and either one can be made to duplicate the performance of the other. Thus, by combining two non-reciprocal birefringent elements with a reciprocal birefringent element, a Faraday rotation element can be simulated. This is indicated in Fig. 62.

Elements a and c are reciprocal $\Delta 90°$ birefringent elements having their axes of low phase velocity oriented vertical and horizontal as indicated by the solid lines. Element b is a non-reciprocal birefringent element having θ degrees differential phase delay with its axes of low phase velocity oriented at 45° to the vertical as shown by the solid and

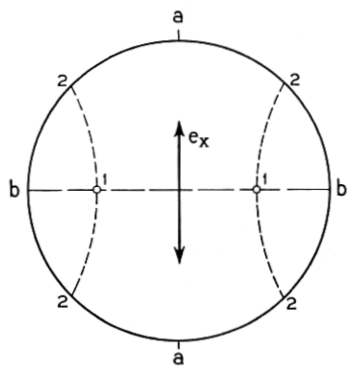

Fig. 60 — Calculated locus of optimum ferrite locations for producing non-reciprocal phase shift of the TE_{11}○ wave (applicable only for very thin ferrite sheets).

dashed lines for propagation from left to right and right to left respectively. Similarly, a Faraday rotation element combined with two reciprocal birefringent elements is the equivalent of a non-reciprocal birefringent element as shown in Fig. 63.

In practical applications some of the reciprocal elements may be superfluous, and simpler equivalent structures can be used. For example, a Faraday rotation type of circulator can be built using one non-reciprocal and one reciprocal birefringent element as shown in Fig. 64. Element *a* may contain a ferrite tube with a four-pole transverse magnetic field producing a differential phase shift of 90°. Its axis of high refractive index is at 45°, as shown by the solid line, for propagation from left to

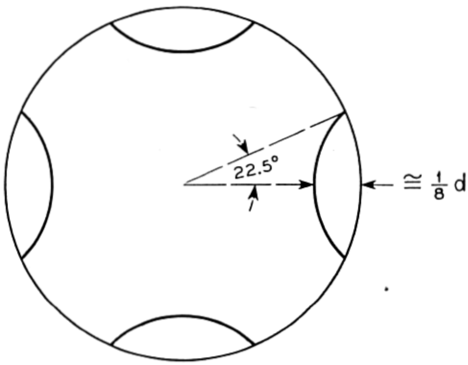

Fig. 61 — Estimated optimum positions of thick ferrite elements for producing non-reciprocal phase shift of the $TE_{11}{}^{\circ}$ wave.

right. Element *b* is a reciprocal Δ90° phase shift element with its axis of high refractive index at 45° as shown. For propagation from left to right the two sections constitute a Δ180° section and polarizations 1 and 3 are rotated into polarizations 2 and 4, respectively. For propagation from right to left the axis of high refractive index for section *a* is the dashed line. Thus, the two sections constitute a Δ0° element and the polarizations are not rotated; polarization 2 is transmitted to polarization 3, and polarization 4 is transmitted to polarization 1. Thus, these two elements comprise a circulator with commutation in the sequence of numbering. This combination is more flexible than the equivalent 45° Faraday rotation circulator in that the output polarizations may be arbitrarily oriented with respect to the input polarizations by orienting the axes of section *b* relative to the axes of section *a*.

7-2. *Other Transverse-Field Effects in Round Waveguide*

By using circumferentially magnetized ferrite, it is possible to obtain non-reciprocal phase shift, loss, or field displacement for any transverse electric mode. When a thin-walled cylinder of ferrite is mounted coaxially in a round waveguide and magnetized in a circumferential direction, the components of the rf magnetic field transverse to this magnetization are elliptically polarized in the same sense in all parts of the ferrite. The ferrite will produce a non-reciprocal phase delay for any transverse-electric wave, and because of the symmetry of the system, there will be no birefringence. By requiring that the radial h be equal to the longitudinal h, we may determine the locus of points for which the component of the magnetic vector perpendicular to the dc magnetization is circularly polarized. This locus is defined by the following equation for TE_{nm} waves:

$$\frac{J_n'\left(k_{nm}'\frac{\rho}{R}\right)}{J_n\left(k_{nm}'\frac{\rho}{R}\right)} = \frac{1}{\sqrt{\left(\frac{2\pi R}{\lambda_0 k_{nm}'}\right)^2 - 1}} \tag{29}$$

where R = radius of the waveguide
λ_0 = free space wavelength
ρ = radial distance coordinate inside waveguide
J_n and J_n' = Bessel function of n^{th} order and its derivative, respectively
k_{nm}' = m^{th} positive root of $J_n'(x) = 0$ which is 1.84 for TE_{11}

Equation (29) may be solved for the fractional radius ρ/R, and we find that the locus of circular polarization relative to a circumferential

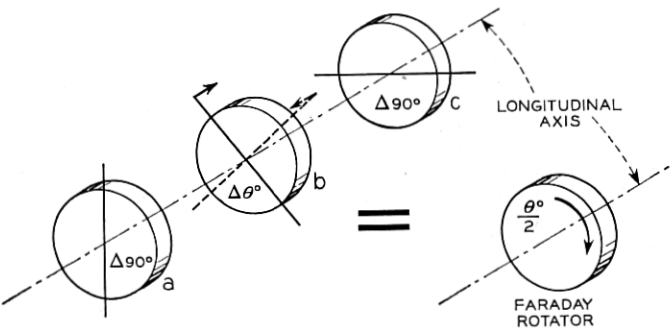

Fig. 62 — Diagram of a birefringent structure which is electrically equivalent to a Faraday rotator.

magnetization will be a circle whose radius depends upon the proximity of the operating frequency to cut-off frequency, as shown in Fig. 65. Again we note, however, that the locus of positions for maximum effectiveness of a given amount of ferrite in producing non-reciprocal phase shift will not be the same as the locus for pure circular polarization. This locus will also be a circle but will occur at a radius which produces a maximum value for the product of h longitudinal and h radial. For TE_{11} waves this occurs at a radius equal to 0.493 of the waveguide radius, and is independent of proximity to cut-off frequency.

It is noted above that the circumferentially-magnetized thin-walled cylinders of ferrite can be used to produce non-reciprocal phase delay

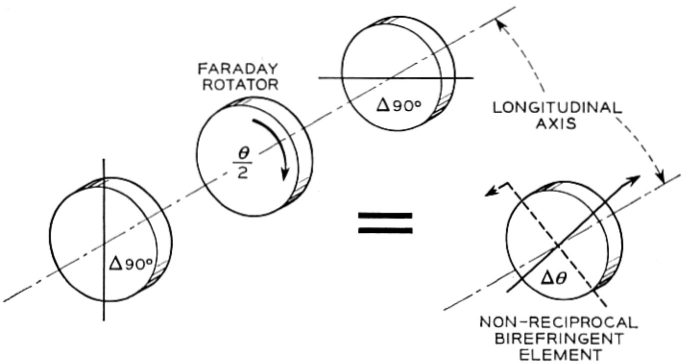

Fig. 63 — Alternate form showing equivalence between Faraday rotation and birefringent structures.

for any of the transverse electric modes of propagation. The optimum radius for the cylinder will, of course, depend upon which mode is used. For the circular electric mode (TE_{01}°), there will be two radii at which the magnetic field vector will have pure circular polarization. At one of these the circular polarization will have one sense, and at the other it will have the opposite sense. In this case it would be possible to employ two thin-walled cylinders of ferrite placed at the two most effective radii, and magnetized circumferentially in opposite directions.

Since there is no angular variation in the h components for the circular electric wave, circumferential ferrite biasing for TE_{01}° is the direct analogue of the transverse ferrite biasing for TE_{10} in rectangular guide. A field-displacement isolator analogous to Fig. 42 can be made for TE_{01}° using two circumferentially magnetized ferrite cylinders, one coated with a resistance film.

8. NON-RECIPROCAL DIELECTRIC WAVEGUIDE CIRCUITS

The behavior of ferrite-loaded hollow metallic waveguides, as previously discussed, depends on wave-electron interactions which can also be utilized in other types of microwave transmission lines. For example, a number of the non-reciprocal transmission elements may be built in dielectric waveguide form.* Fig. 66 shows a longitudinal section of a round dielectric waveguide containing ferrite loading for the purpose of producing Faraday rotation. The rf magnetic intensity vector at the center of the dielectric waveguide is transverse and linearly polarized for a linearly-polarized lowest-order wave. Therefore, with longitudinal

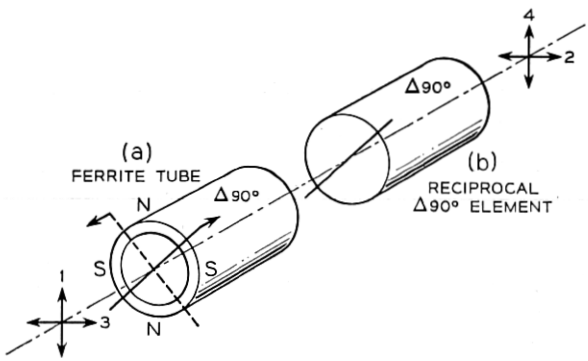

Fig. 64 — Birefringent elements capable of replacing a 45° Faraday rotator in a circulator.

magnetization, a linearly polarized input wave is coupled to the similar wave polarized at 90° to the first in a manner entirely analogous to the coupling between the two TE_{11}° waves in hollow metallic waveguide. Experiments were conducted at 48,000 mc using a structure of the form of Fig. 66. The ferrite diameter was ⅛″ and the outside diameter of the polystyrene in the ferrite-loaded region was ¼″. The ferrite consisted of powdered Ferramic J molded in a polystyrene binder. More than 90° rotation was observed with a biasing field of a few hundred gauss; the transmission loss was ½ db or less.

The Faraday rotator of Fig. 66 may be combined with dual polarization take-offs to form a circulator in dielectric waveguide as illustrated in Fig. 67. The dual-polarization take-offs have the function of providing

* The general principles involved in dielectric transmission lines are discussed in Reference 12, pp. 129–131. A. G. Fox gave a paper on dielectric-waveguide circuit elements at the March, 1952, I.R.E. National Convention.

separate terminals for the two polarizations of dominant wave at both ends of the Faraday rotator. One polarization of wave on the round dielectric rod passes by the region in which it is coupled to the rectangular dielectric guide with negligible loss, because the phase constants of the dielectric waveguide and rectangular waveguide are dissimilar.* The other polarization of wave on the round dielectric guide is arranged to have the same phase constant as has the parallel polarization on the rectangular guide, and therefore conditions may be so adjusted as to produce complete power transfer between the two guides. On a 48,000 mc experimental model of such a dual polarization take-off the transfer loss for the polarizations having the same phase constants was about ½ db, and the straight-through loss on the round rod for the polarization at 90° to the above waves was 0.1 db. In Fig. 67, a wave entering at 1 is transferred completely to the parallel polarization of wave on the round rod at the input to the Faraday rotator, and after rotation emerges in the same polarization as the take-off labeled 2. Transmission in at 2 again produces a wave of corresponding polarization on the round rod, which is then rotated an additional 45° and emerges on the near end of the Faraday rotator in the polarization labeled 3 which passes by the coupling region unaffected. In similar fashion, transmission occurs from terminal 3 to terminal 4 and from terminal 4 to terminal 1.

Non-reciprocal phase shifts in ferrite-loaded rectangular dielectric

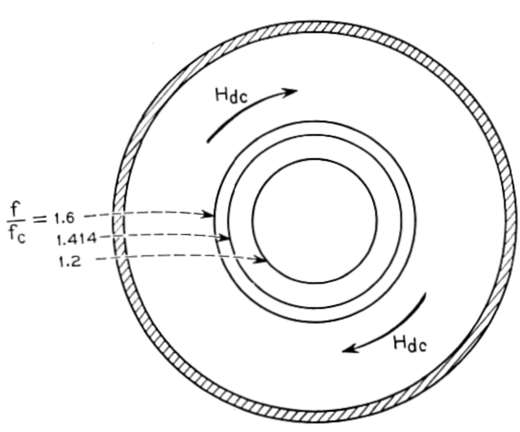

Fig. 65 — Calculated locus for which the component of h perpendicular to the circumferential dc magnetization is circularly polarized.

* This type of behavior is discussed in more detail in pp. 680–682 of S. E. Miller, Reference 8.

a second isolator having an input impedance match which is appreciably better than that of the antenna itself. For such systems applications of isolators, the requirements on the isolator for impedance match and discrimination against backward travelling waves are frequently rather stringent. Isolators can also be of very great value in experimental and measuring techniques. Here the requirements may be very much less stringent. Even a transmission ratio as low as 9 to 1 in decibels can be of great use. Thus, in most cases where a 10 db attenuator pad is used in a measuring setup, it could be replaced by an isolator having a forward loss of 2 db and a reverse loss of 18 db. The round trip attenuation will be the same in either case, but there will be an 8 db increase in the available forward traveling wave. Where the available signal power is low, and it is important to have a large measuring range, several such substitutions can increase the measuring range by 20 to 30 db.

9-2. *Circulators*

Circulators are considerably more complicated circuit elements, and the uses proposed for them are typically more sophisticated than those for isolators. An important distinction between the circulator and the isolator is that the circulator diverts or otherwise makes use of power which is reflected from the receiving end of the system rather than destroying it, as in the case of an isolator. One possible application of circulators is in channel frequency filtering systems. Such a system requires two vital components. It requires a filter which can pass certain frequencies and reflect others; and it requires some power dividing network which can separate out the reflected frequencies for further use. In other words, it is normally not satisfactory to allow the frequencies rejected by the filter to return toward the source. One method of constant impedance branching which can separate channels from one another without reflecting energy toward the source has been proposed by one of the authors, as shown in Fig. 69.[25] Incoming wave power containing all channel frequencies is divided at a first hybrid junction and passes to two identical transmission filters tuned for one of the channels. The channel which passes through these filters is recombined at a second hybrid and drops out at the point marked Channel 1. All other channels are reflected from the two pass filters but, upon returning to the first hybrid junction, differ in their round trip delay by 180° by virtue of a $\lambda/4$ difference in path length between the two filters and the hybrid junctions. As a result, all the rejected channels emerge from the remaining arm of the first hybrid junction and pass on down into the second such network. A dual of this scheme has been proposed by W. D. Lewis.[26] The structure of a single channel-dropping section is the same as that of Fig. 69, but in this case the filter pairs are of the band rejection rather than the band pass type. All frequencies *except* the desired channel pass through the filters and are recombined at the second hybrid to pass on to a second such dividing network. These examples of constant impedance branching filters are given to indicate the degree of complexity required and to show how circulators can simplify this complexity appreciably.

Fig. 70(a) is a diagram showing how circulators can be combined with pass filters to provide a constant impedance channel-branching system. All channel frequencies entering from the left pass around the first circulator and impinge on the pass filter for channel 1. Channel 1 is dropped out at this filter and all other channel frequencies are reflected back to the first circulator, where they pass on around to the channel 2 filter. This process continues indefinitely with as many circulators and filters added as are necessary to separate out the channels. Nowhere is power reflected back to the source. Any residual power at the end of the array may be absorbed in a resistive load. It can be seen that a pair of hybrid junctions and a pair of identical matched filters for each channel

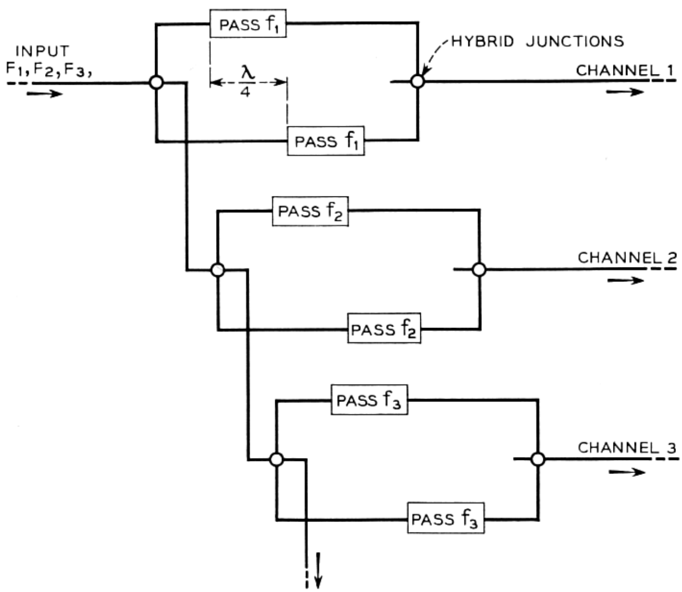

Fig. 69 — Channel-branching filter system using conventional hybrids.

in Fig. 69 have been replaced by a single transmission filter and one-half a circulator in the configuration of Fig. 70(a).

In Fig. 70(b) is shown an arrangement similar to the one of Fig. 70(a), except that rejection filters have been used. In this example only three arms are needed for the circulator. If four arms are available, the fourth one may be terminated in a characteristic impedance.

The three-arm circulator of Fig. 49 may be particularly well suited for use in filter applications, where channels are to be dropped off through pass filters. The tuning required to make such a circulator function can be provided as part of a transmission filter occupying the side arm of the device. As a result, a particularly convenient mechanical layout can be achieved where the main line proceeds straight through the structure, as shown in Fig. 71, with each of the channels dropping out of one of the side arms which project from the main line structure.

9-3. *Phase Changers*

A variable microwave phase changer described by one of the authors[27] has proven to be of considerable value in antenna scanning arrays[27] and in certain microwave measuring techniques.[28, 29] In these applications,

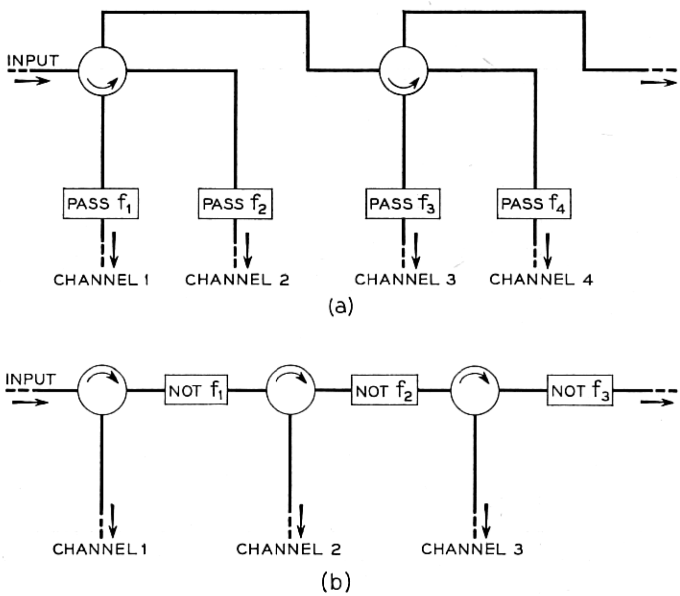

Fig. 70 — Channel branching with circulators.

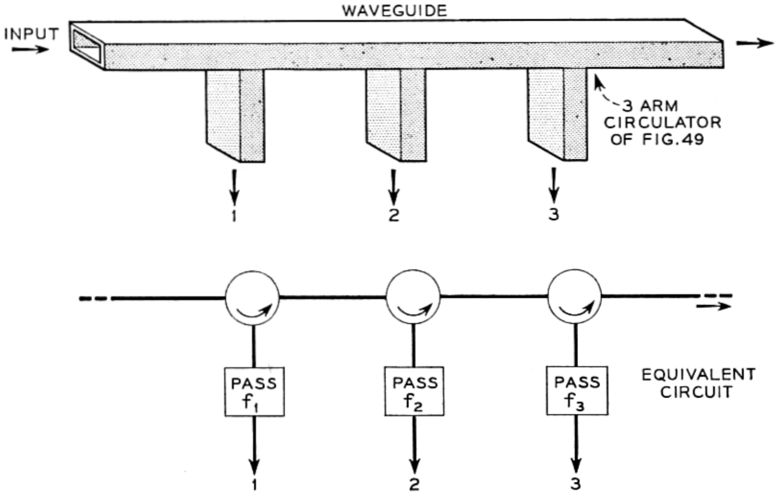

Fig. 71 — Waveguide embodiment of a channel branching filter using three-arm junction-circulators.

the 180° differential phase section in the middle of the device [Fig. 72(a)] is mechanically rotated in order to change overall phase shift. In some instances it would be very much more convenient if this rotation could be made to take place electrically. Since the function of this Δ180° section was to rotate the output polarization relative to the input polarization of this section, we can substitute a rotary medium comprising an axial pencil of ferrite in a round section of waveguide, so as to obtain the structure of Fig. 72(b). By varying the applied longitudinal magnetic field, the ferrite effectively rotates the output polarization relative to the input polarization of the middle section and achieves the same effect as physical rotation of the Δ180° section of Fig. 72(a). If a limited amount of phase shift (less than 1,000°, for example) is required, this Faraday-rotation type of phase changer can provide a convenient way of obtaining variable phase shift by means of electrical control. The limitation of the device is that the phase shift can be increased only up to the point where the ferrite is saturated. Beyond that point further increase of the applied field cannot change the output polarization, and as a result the total phase shift will be limited.

In some of the applications of the mechanical phase changer a continuously rotating Δ180° section was necessary in order to obtain continuous change in phase. In order to apply ferrites to such a device, it would be necessary to employ a birefringent medium whose axes of birefringence could be controlled magnetically. At frequencies of 10 kmc and below, the use of a round waveguide containing a rod of ferrite or a thin-walled tube of ferrite saturated by a simple transverse magnetic field, can produce enough birefringence to make such a Δ180° section possible. In this case the external magnet structure would be arranged like a rotating two-pole motor stator, and when properly excited, the axes of birefringence of the section could be made to rotate continuously. However, as pointed out earlier, this reciprocal birefringence may become so small at frequencies above 20 kmc as to be of little practical value. In this case, use may be made of the non-reciprocal birefringence obtainable from the four-pole field structures as described in Section 7.

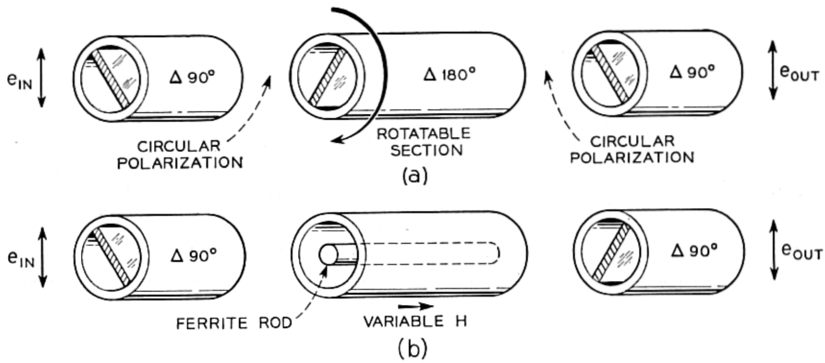

Fig. 72 — Phase changers — (a) using mechanical rotation, and (b) using variable magnetization of a ferrite element.

It is interesting to note that for pure non-reciprocal phase changers (which can be approximated using either the Faraday rotation element of Fig. 72(b) or the four-pole field birefringent element) no round-trip variation in phase delay is exhibited as the one-way phase delay is changed. In other words, the phase change for the waves traveling in one direction is equal and opposite to the phase change for waves traveling in the opposite direction. As a result, reflections arising on the load side of the phase changer will return to the source side with a phase (relative to the incident wave) which is independent of the phase changer setting. This is to be contrasted with the reciprocal type of phase changer for which the round trip phase delay is twice that of a one-way transmission.

10. SUMMARY AND CONCLUSIONS

The behavior of many configurations of ferrite-loaded waveguides has been deduced using a "point field" analysis. In this method it is assumed that the ferrite-loaded waveguide contains a field configuration bearing a strong resemblance to one or more of the modes which are characteristic of the empty waveguide. The interaction between the added ferrite and the presumed distribution of field components is determined qualitatively at each point in the cross section, and the total behavior deduced as the weighted sum of the effects throughout the cross section.

The particular characteristic of ferrite materials which is responsible for the non-reciprocal behavior of ferrite-loaded microwave circuits is a tensor permeability — i.e., a driving-wave magnetic intensity in one direction may cause a component of magnetic flux at right angles to this direction as well as a component in the same direction. The magnetization of all "magnetic" materials may respond in the above manner, but the coupling between the electrons responsible for this behavior and the microwave field components has been found useful only in ferrites, which have negligible conductivity and within which microwave energy can propagate without appreciable attenuation.

Section 3 explains the effects of ferrite-loading a two-mode transmission medium (such as a dominant-wave round guide) using two viewpoints: (1) the normal mode approach in which the input and output waves are resolved into two modes which are orthogonal in the *presence* of the ferrite, and (2) the coupled-mode approach in which the behavior of the ferrite-loaded region is described in terms of a coupling introduced by the ferrite between the two modes which are orthogonal in the *absence* of the biased ferrite. These concepts are applied to Faraday rotation in Section 3, and to birefringence in ferrite-loaded round waveguide in Section 7.

Section 4 contains a discussion of the principles of Faraday rotation elements. The optimum location for the ferrite element in Faraday rotators is shown to be on the waveguide axis, and it is pointed out that there is an optimum diameter of ferrite rod with respect to obtaining a maximum of Faraday rotation per unit transmission loss. The problems encountered in impedance matching at the ferrite ends are discussed. It is shown that mode-conversion difficulties can appear as impedance match or Faraday rotation irregularities. The reasons for the variation of Faraday rotation as a function of frequency are given, and the dependence of rotation and transmission loss on temperature and on the Curie point of the ferrite is also discussed.

An important use of ferrite-loaded waveguide circuits lies in building passive non-reciprocal devices. It is pointed out that there are numerous forms of non-reciprocal devices which accomplish the same functions, just as there are numerous forms of amplifiers and filters. The authors propose the names for the new elements be based on function and, in accordance with this point of view, definitions are given for the gyrator, circulator, and the isolator (Section 1).

Methods of employing Faraday rotation for the construction of gyrators, isolators, circulators and non-reciprocal power dividers are described, and a novel method of avoiding the deterioration of isolator performance due to frequency or temperature changes is described (Fig. 18).

A rectangular waveguide suitably loaded with ferrite can be non-reciprocal despite the existence of only one mode of propagation in each direction. Such guides may be non-reciprocal with respect to (1) phase constant (2) attenuation constant (3) magnetic and electric field configurations in the cross section. The circuits using these effects are fundamentally different from those depending on Faraday rotation, since the latter depend on the existence of two orthogonal waves in the ferrite-loaded region. The point-field concept is utilized to explain the non-reciprocal properties noted above and is used to indicate approximately the optimum ferrite position and direction of magnetization associated with obtaining these effects. When developing final models, one utilizes the maximum amount of ferrite which is tolerable in order to maximize these non-reciprocal effects, and it is indicated that the appearance of higher-order modes of propagation frequently limits the amount of ferrite which can be so utilized. Calculations are presented to serve as guidance for avoiding higher-order mode propagation, but the configuration of ferrite loading which is most attractive from the practical point of view is one which has not yet been adequately analyzed theoretically. Rectangular-waveguide isolators, circulators, and directional phase shifters are described, and experimental data on the performance of many of them are reported. In particular, the *resonance isolator* (Figs. 39–41, incl.) and the resistance-sheet type of *field-displacement isolator* (Figs. 42–46, incl.) have been explored in some detail.

Reciprocal phase shifts in ferrite-loaded structures may be important since they are under the control of a magnetizing field which may, under certain circumstances, be varied rapidly. In Section 6, the particular ferrite characteristics responsible for reciprocal phase shifts are discussed and the reciprocal phase shift obtained in practical ferrite-loaded structures is compared with the non-reciprocal phase shift in a similar structure. It is shown that the reciprocal phase shift requires ferrites of such saturation magnetization and with such magnetizing fields that the tensor permeability component k^2 is appreciable compared to tensor permeability component μ^2 (terminology defined in Section 2). This implies operating the ferrite in a region not too remote from ferromagnetic resonance, but despite this condition very usable reciprocal phase shifts can be obtained with negligible ferrite loss. Reciprocal phase changes vary as the square of the applied field, whereas non-reciprocal phase changes vary linearly with the applied field.

Section 7 discusses birefringence in round waveguide. It is pointed out that the orthogonal TE_{11} waves for round guide containing an axially symmetric ferrite element are no longer degenerate when a magnetizing field is applied in any direction. For transverse magnetization, the medium is birefringent for the linearly polarized TE_{11} waves. Such birefringence may appear as either a reciprocal or non-reciprocal effect, the relative importance being determined by the frequency of the rf wave and by the detailed shaping of the magnetization in the transverse plane.

Section 7-1 compares the operation of non-reciprocal birefringent elements with Faraday rotation elements, and shows how either type can be substituted for the other in building non-reciprocal circuits.

Section 7-2 shows the manner in which non-reciprocal phase shift, loss, or field displacement may be obtained for any of the transverse-electric modes in a round waveguide through the addition of transversely magnetized ferrite. The optimum conditions for obtaining non-reciprocal phase shift or non-reciprocal loss are discussed. Faraday rotation for any of the transverse electric modes in round or square waveguide can be obtained, and the optimum geometry of ferrite for producing such rotation is indicated.

The wave-electron interactions which have been utilized in ferrite-loaded hollow metallic waveguides can also be employed in other types of microwave transmission lines. To illustrate this possibility, Section 8 describes Faraday rotators, directional phase shifters, circulators, and isolators in dielectric waveguides.

In Section 9 are indicated some of the advantages which may be obtained by employing isolators in waveguide circuits. The application of circulators to channel frequency filtering systems is described in connection with Figs. 69–71, and several forms of phase changer utilizing Faraday rotation or birefringence in round waveguide are described.

Not mentioned explicitly in earlier portions of this paper are the numerous possibilities for using ferrite-loaded circuits with time-varying magnetizing fields. The various directional or reciprocal phase changers, the isolators and the circulators which have been described may each be employed in conjunction with a modulating current which varies the magnetization on the ferrite and which thereby switches the transmission properties of the ferrite-loaded circuit from one form to another. For example, an isolator may act as a matched-impedance On-Off switch simply by reversing the direction of the magnetizing field. Similarly, the direction of power circulation between the various terminals of a circulator may be reversed by reversing the direction of the magnetization. Continuous variation of the power division may in many of the circulators be achieved through variation of the magnetizing field.

It may be helpful to indicate the relative merits of several of the isolators which have previously been described. Both the Faraday rotation isolator and the resistance sheet isolator have produced comparable ratios of reverse-loss to forward-loss (about 150). The resonance isolator and the resistance-sheet isolator are simpler in form than Faraday rotation isolators and therefore may prove less expensive. The Faraday rotation isolator, and the resonance isolator, have provided reverse-loss to forward-loss ratios in the region 20–35 over frequency bands on the order of 10 per cent, whereas the resistance sheet isolator has provided ratios of 150 over the same band. However, the Faraday rotation isolator has not been perfected to the degree which appears possible with regard to broad-band operation. In comparing the resonance isolator with the resistance sheet isolator, the latter appears to have considerable advantage in requiring smaller magnetizing fields. This advantage of the resistance sheet isolator over the resonance isolator is more pronounced at the higher frequencies (such as 24,000 mc) where the magnet weight necessary to produce ferromagnetic resonance becomes objectionable.

ACKNOWLEDGMENT

The authors wish to acknowledge the stimulating influence of other Laboratories' workers in this general field, as specifically noted in the text. In addition, we are grateful for the help of L. G. Van Uitert, who made available to us many of the ferrites which were used in the experimental program.

Part of the work on this investigation was done under United States Navy Contract Nonr-687(00).

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Correction

J. L. Merrill, Jr., A. F. Rose, and J. O. Smethurst, authors of the paper, "Negative Impedance Telephone Repeaters", which appeared in the September Issue of the B.S.T.J. on pages 1055 to 1092 have brought the following correction to the attention of the editors.

In Fig. 15, page 1074, the value of β_1 and β_2 were shown as follows:

$$\beta_1 = \frac{c}{a + b + c}$$

$$\beta_2 = \frac{a}{a + b + c}$$

This should be corrected to read:

$$\beta_1 = \frac{a}{a + b + c}$$

$$\beta_2 = \frac{c}{a + b + c}$$