# Duality as a Guide in Transistor Circuit Design

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Because of a relationship which exists between the properties of a vacuum tube triode and those of a transistor, it is possible to start with a known vacuum tube circuit and to transform it into a completely different circuit suitable for use with transistors. The nature of this transformation is discussed and a number of examples are given.

## INTRODUCTION

SINCE the invention of the transistor there has been a natural tendency to compare its properties with those of a vacuum tube triode. This comparison indicates that the two devices are different in many important respects. For example, the grounded cathode vacuum tube is essentially a voltage amplifying device with a high input impedance and a relatively low output impedance, while the grounded base transistor is essentially a current amplifying device with a low input impedance and a relatively high output impedance. Furthermore, high gain vacuum tubes tend to be unstable with open circuit terminations, while high gain transistors tend, on the other hand, to be unstable with short circuit terminations.

The properties of the two devices are, in fact, so radically different that the development of the transistor has posed an entirely new set of circuit design problems. If the vacuum tubes in a known circuit are simply replaced by transistors (and appropriate changes are made in the supply voltages), it is usually found that the transistor is not used to best advantage and the circuit performance is not satisfactory. For this reason, circuit designers heretofore have exercised considerable ingenuity in devising new circuits which take into account the peculiarities of the transistor and use them to best advantage. It turns out that some of these circuits bear little resemblance to vacuum tube circuits designed to perform the same function.

Although there is a great difference between the electrical properties of transistors and vacuum tubes, there is a very simple approximate relationship between them. It is the purpose of this paper to show how it is possible, taking this relationship into account, to start with a known vacuum tube circuit and transform it into a completely different circuit suitable for use with transistors. Circuits derived in this way tend to take advantage of the peculiarities of the transistor, and in a number of cases have shown exceptionally good performance.

## The Relation between Vacuum Tube and Transistor Properties

It is the purpose of this section to show that the properties of a transistor are related to those of a vacuum tube triode through an interchange of current and voltage, and that transistor currents behave like vacuum tube voltages and vice versa. The discussion is aimed particularly at the large-signal properties of the two devices and is restricted to the frequency range in which static characteristics are sufficient to determine circuit performance.

Consider first the grid-cathode input terminals of a triode as compared to the emitter-base input terminals of a transistor. With respect to these terminals each device behaves as a diode rectifier the properties of which are relatively unaffected by biases applied to the third electrode (plate or collector). The grid conducts when biased in the forward direction and fails to conduct when biased in the reverse direction. A similar statement can be made about the emitter. Furthermore, either device behaves as a low impedance when biased in the forward direction and as a relatively high impedance when biased in the reverse direction.

The difference between the emitter circuit and the grid circuit comes about in the following way: The vacuum tube is most effective as an amplifier when the grid is biased in the *reverse* direction, while the transistor is most effective when the emitter is biased in the *forward* direction. With respect to these input terminals, then, the essential difference between the two devices amounts to the difference between "forward" and "reverse". But this, in turn, amounts to an interchange of current and voltage.

Whatever qualitative statements can be made about emitter current and voltage can also be made about grid voltage and current, respectively. For example, the grid is normally given a moderate voltage bias at which the grid current is essentially zero, while the emitter is normally given a moderate current bias at which the emitter voltage is essentially zero. Furthermore, the principal non-linearity in the grid circuit occurs when the grid voltage is allowed to swing through zero with the result that grid current begins to rise, while the principal non-linearity in the emitter circuit occurs when the emitter current is allowed to swing through zero with the result that emitter voltage begins to increase.

The comparison between the plate-cathode output circuit of the triode and the collector-base output circuit of the transistor is somewhat complicated by the effects of grid and emitter biases. Consider first the situation in which zero bias is applied to the input circuits ($v_g = 0$ and $i_e = 0$). In this case, both the plate and the collector behave like diode rectifiers, conducting readily when biased in the forward direction and conducting relatively poorly when biased in the reverse direction. When input biases are applied, however, the principal difference between the two devices becomes apparent and turns out again to be associated with the difference between forward and reverse. This is because biases applied to the grid affect only the forward part of the plate circuit characteristic while biases applied to the emitter affect only the reverse part of the collector circuit characteristic.

Thus the grid and plate are normally biased in the reverse and forward directions, respectively, with the result that the vacuum tube input impedance is high and the output impedance is relatively low. The emitter and collector, on the other hand, are normally biased in the forward and reverse directions, respectively, with the result that the transistor input impedance is low and the output impedance is relatively high.

The comparison of vacuum tube and transistor properties can be carried further with the help of Fig. 1(a) which shows the plate circuit characteristics of a particular vacuum tube triode and Fig. 1(b) which shows the collector circuit characteristics of a particular transistor. The axes in these two figures have been chosen to facilitate comparison of transistor currents with vacuum tube voltages and vice versa. The result is that the two families look quite similar. It is seen that the quantities to be compared are

$v_p$ with $-i_c$,
$i_p$ with $-v_c$,
$-v_g$ with $i_e$, and, though not shown,
$-i_g$ with $v_e$.

The consistent difference in sign between vacuum tube and transistor quantities holds only when the transistor is made from an $N$-type semiconductor. If the transistor is made of $P$-type material[1] there is no difference in sign between corresponding transistor and vacuum tube quantities.

By referring to Fig. 1(a) it can be seen that to a first approximation the effect of applying a negative voltage bias to the grid is simply to shift the plate circuit characteristic to the right along the $v_p$ axis. The number of volts shift caused by a change of one volt on the grid is called the voltage amplification factor, $\mu$, of the triode. Similarly, it can be seen from Fig. 1(b) that the principal effect of applying a positive current bias to the emitter is simply to shift the collector circuit characteristic to the right along the $-i_c$ acis. The number of milliamperes shift caused by a change in emitter current of one milliampere is called the current amplification factor, $\alpha$, of the transistor. Thus, $\alpha$ of the transistor corresponds to $\mu$ of the vacuum tube.

[1] The p-Germanium Transistor, W. G. Pfann and J. H. Scaff, *Proc. I.R.E.*, 38, 1151.

It is interesting to note that the gross non-linearities in the vacuum tube plate circuit have their counterparts in the transistor collector circuit. For example, the counterpart of plate *current* cutoff is collector *voltage* cutoff.

The relationship between vacuum tubes and transistors is not only qualitative, but can be made quantitative as well provided a suitable vacuum tube is chosen for comparison with the transistor. The requirements are that the vacuum tube and transistor have similar dissipation ratings and that $\mu$ be equal to $\alpha$. These conditions are roughly satisfied by the vacuum tube and transistor of Fig. 1(a) and Fig. 1(b). By comparing the axes in these two figures it may be seen that one milliampere in the transistor corresponds

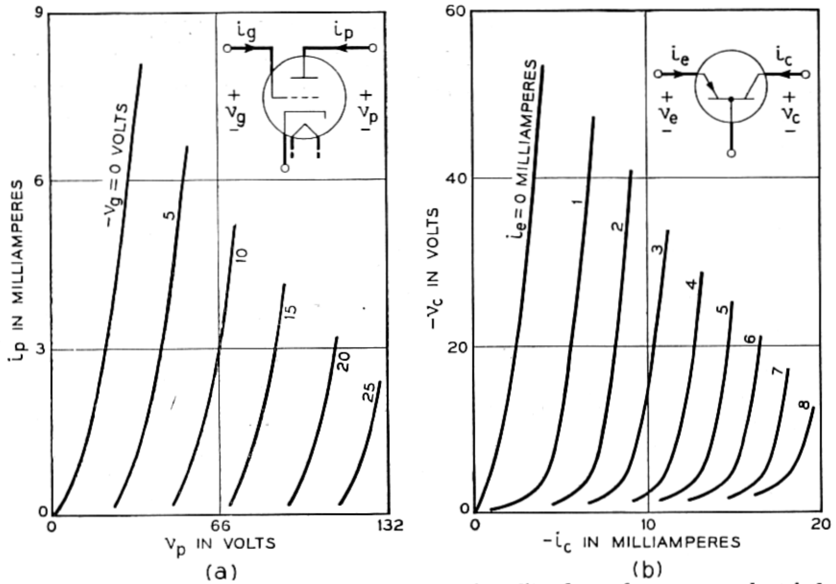



Fig. 1—The static characteristics of a transistor look like those of a vacuum tube triode provided transistor currents are compared with vacuum tube voltages and vice versa.

to 6.6 volts in the vacuum tube and vice versa. It follows that, in this case, transistor currents are related to vacuum tube voltages through a "transformation resistance," $r$, given by

$$r = \frac{6.6 \text{ volts}}{(10)^{-3} \text{ amps}} = 6{,}600 \text{ ohms.} \tag{1}$$

## Circuit Considerations

The internal structure of a vacuum tube imposes a particular set of relationships between the vacuum tube currents and potentials. At low frequencies these relationships are given by the static characteristics which show that over a fairly wide range of values the tube currents are roughly linear functions of the voltages. When the tube is connected into an external circuit, the circuit imposes a second set of algebraic relationships between vacuum tube currents and potentials and the performance of the circuit as a whole represents a simultaneous solution of these two sets of relationships. Now if the vacuum tube is replaced by a transistor and the external circuit is left unchanged, then the relationships internally imposed are markedly changed while the relationships imposed by the external circuit are left unaltered. Ordinarily this will lead to a completely different simultaneous solution for the two sets of conditions and hence to completely different circuit performance.

If circuit performance (with respect to the terminals of the tube or transistor) is to be maintained after substituting a transistor for a vacuum tube then the external circuit must be modified. One might suppose, for example, that it should be possible to find a new external circuit such that the collector voltage in the new circuit would behave exactly as did the plate voltage in the original circuit. To a certain extent this is possible, but this procedure meets with a serious difficulty. Although transistor voltages are fairly well behaved, roughly linear single-valued functions of transistor currents over fairly wide ranges of values, transistor currents are relatively more non-linear, often double valued, functions of the voltages. This means at once that if circuit performance is to be maintained for large signals, non-linear elements will be needed in the external circuit. This approach seems very much less promising than another to which we now come.

The new approach is to seek a transistor circuit in which every current behaves like a corresponding voltage in a known vacuum tube circuit and every voltage behaves like a corresponding current. This approach is relatively simple because, as has already been shown, half the problem is solved simply by exchanging transistor for vacuum tube. The remaining part of the problem is to find an external circuit which will impose the same relation between transistor potentials and currents as the original circuit imposed between vacuum tube currents and potentials. This amounts to saying that if the vacuum tube is to be replaced by a device in which the roles of currents and voltages are just interchanged then the external network should also be replaced by a new network which accomplishes this same interchange.

Networks in which this sort of interchange is accomplished are known as duals,[2] one of the other. It has been shown in the literature that it is possible to find and to realize physically the duals of most practical circuits. The total number of circuit elements in a network is ordinarily preserved when the

[2] Communication Networks, E. A. Guillemin, Vol. 2, pp. 246-254, John Wiley & Sons (1935); Network Analysis and Feedback Amplifier Design, H. W. Bode, p. 196, Van Nostrand, (1945). What Bode calls inverse we have called dual.

dual transformation is performed, each element being transformed into a new element which is its dual. The transformed elements are not, however, connected together in the same way as were the original ones. Elements in parallel are transformed into elements in series and vice versa. Nodes transform into loops and loops into nodes.

There are cases when finding the dual of a network is not as straightforward as the reader might infer from the above. Complications may arise when the network contains mutual inductance or non-linear elements, or if the network cannot be drawn on a flat surface without crossovers. Some of these questions are discussed by Bode.[2]

## Duality

Table 1 shows side by side a number of network elements and the duals of these elements related through the transformation resistance $r$. The table also shows the duals of a few simple networks. It is the purpose of this section to show by means of examples how these dual relationships are established.

One network element is the dual of another provided the role of current in one is played by voltage in the other, and vice versa. Consider what this means in the case of a capacitance in which current and voltage are related by the equation

$$e = \frac{1}{jC\omega}\, i. \tag{2}$$

Interchanging the roles of current and voltage means replacing $e$ in this equation by $i'r$ and replacing $i$ by $e'/r$. The value of $r$ determines how many volts across the condenser are to correspond to an ampere through its dual. Making the indicated substitutions gives

$$i' = \frac{1}{jr^2C\omega}\, e'. \tag{3}$$

This, however, is the kind of equation which relates the current through an inductance to the voltage across it. It is seen, therefore, that the dual of a capacitance $C$ is an inductance of value given by

$$L' = r^2C. \tag{4}$$

In the dual transformation of a network every capacitance in the original network will be transformed in this way into an inductance in the dual network. Also, if $e_C$ and $i_C$ represent the voltage across a capacitance and

[2] Bode, loc. cit.

the current through it in the Kirchoff equations of the original network, these quantities will be replaced by $i_{L'}$, and $e_{L'}$, respectively, in the Kirchoff equations of the dual network. The quantities $i_{L'}$, and $e_{L'}$, represent the current through an inductance of value $L'$ given by (4) and the voltage across it.

The argument just given can equally well be interpreted to mean that the dual of an inductance $L$ is a capacitance $C'$, the value of which is given by

$$C' = L/r^2, \tag{5}$$

so that every $e_L$ and $i_L$ in the Kirchoff equations of the original network are replaced by $i_{C'}$, and $e_{C'}$, respectively, in the Kirchoff equations of the dual network.

The dual of a resistance $R$ is found in the same way. The equation applicable to a resistance is

$$e = Ri, \tag{6}$$

which, with the substitution of $ri'$ for $e$ and $e'/r$ for $i$, becomes

$$i' = e'/(r^2/R). \tag{7}$$

Thus it is seen that a resistance $R$ transforms into a resistance $R'$ where

$$R' = r^2/R. \tag{8}$$

Also, $e_R$ and $i_R$ in the Kirchoff equations of the original network are replaced by $i_{R'}$, and $e_{R'}$, respectively, in the Kirchoff equations of the dual network.

The dual of a temperature sensitive resistance which changes value with changes in average signal level can be found by exchanging the labels on the axes of an $E$-$I$ plot of its characteristic. This shows that the dual of a resistance which has a positive temperature coefficient, and hence increases in resistance with increase in signal level, is a resistance with a negative temperature coefficient of resistance which decreases in resistance with increase in signal level. Similarly, the dual of a short-circuit-stable negative resistance is an open-circuit-stable negative resistance.

The equations applicable to an ideal transformer of impedance ratio $1{:}\alpha^2$ are

$$\begin{aligned} e_2 &= \alpha e_1 \text{, and} \\ i_2 &= i_1/\alpha. \end{aligned} \tag{9}$$

Making the substitutions previously indicated leads to

$$\begin{aligned} i_2 &= \alpha i_1 \text{, and} \\ e_2 &= e_1/\alpha. \end{aligned} \tag{10}$$

TABLE I

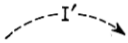
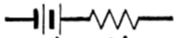
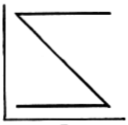
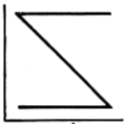
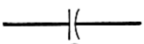
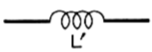
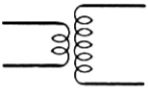
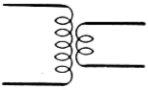
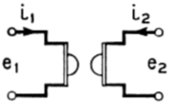
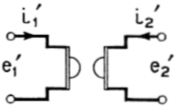
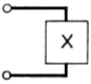
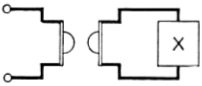
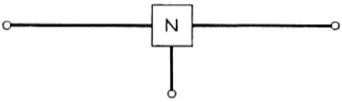
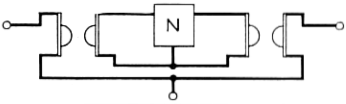
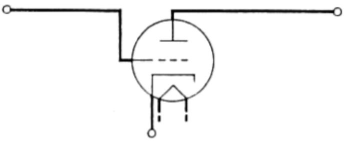
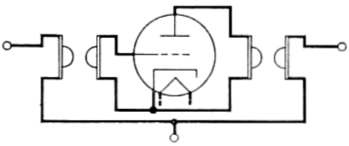
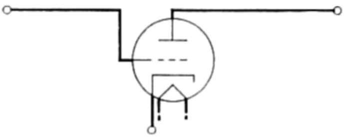
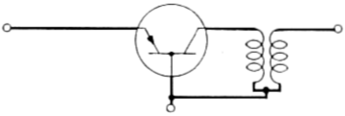
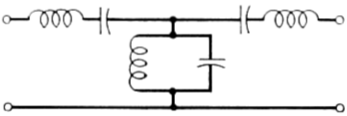
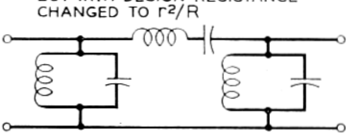
| | |
|---|---|
| (1a) CONSTANT VOLTAGE SUPPLY  E | (1b) CONSTANT CURRENT SUPPLY  I′ <br> $I' = E/r$ |
| (2a) SERIES BATTERY AND RESISTANCE  E R | (2b) CONSTANT CURRENT SUPPLY AND RESISTANCE IN PARALLEL  I′ R′ <br> $I' = E/r, \quad R' = r^2/R$ |
| (3a) SERIES BATTERY AND RESISTANCE  E R | (3b) SERIES BATTERY AND RESISTANCE  E′ R′ <br> $E' = (r/R)E, \quad R' = r^2/R$ <br> (EQUIVALENT TO (2b), BY THEVENIN'S THEOREM) |
| (4a) RESISTANCE  R | (4b) RESISTANCE  R′ <br> $R' = r^2/R$ |
| (5a) POWER-SENSITIVE RESISTANCE WITH POSITIVE TEMPERATURE COEFFICIENT  E, I | (5b) POWER-SENSITIVE RESISTANCE WITH NEGATIVE TEMPERATURE COEFFICIENT  I′, E′ <br> $I' = E/r, \quad E' = rI$ |
| (6a) SHORT-CIRCUIT-STABLE NEGATIVE RESISTANCE  E, I | (6b) OPEN-CIRCUIT-STABLE NEGATIVE RESISTANCE  I′, E′ <br> $I' = E/r, \quad E' = rI$ |
| (7a) CAPACITANCE  C | (7b) INDUCTANCE  L′ <br> $L' = r^2 C$ |

TABLE I—Continued

| | |
|---|---|
| (8a) IDEAL TRANSFORMER OF IMPEDANCE RATIO 1 : $a^2$<br> | (8b) IDEAL TRANSFORMER OF IMPEDANCE RATIO $a^2$ : 1<br> |
| (9a) IDEAL GYRATOR OF FORWARD TRANSFER IMPEDANCE R<br><br>$e_1 = -Ri_2$<br>$e_2 = Ri_1$ | (9b) IDEAL GYRATOR OF FORWARD TRANSFER IMPEDANCE $-r^2/R$<br><br>$e_1' = (r^2/R)\,i_2'$<br>$e_2' = -(r^2/R)\,i_1'$ |
| (10a) ANY TWO-TERMINAL NETWORK X<br> | (10b) THE SAME TWO-TERMINAL NETWORK, X, PLUS AN IDEAL GYRATOR OF TRANSFER IMPEDANCE r<br> |
| (11a) ANY THREE-TERMINAL NETWORK N<br> | (11b) THE SAME THREE-TERMINAL NETWORK, N, PLUS TWO IDEAL GYRATORS<br> |
| (12a) VACUUM TUBE TRIODE<br> | (12b) VACUUM TUBE TRIODE PLUS TWO GYRATORS<br> |
| (13a) SUITABLE VACUUM TUBE TRIODE<br> | (13b) TRANSISTOR PLUS IDEAL PHASE REVERSING TRANSFORMER<br><br>NOTE THAT (13b) AND (12b) ARE EQUIVALENT |
| (14a) ANY MID-SERIES TERMINATED CONSTANT-k FILTER SECTION OF DESIGN RESISTANCE R<br> | (14b) THE SAME CONSTANT-k FILTER SECTION MID-SHUNT TERMINATED BUT WITH DESIGN RESISTANCE CHANGED TO $r^2/R$<br> |

This indicates that the dual of an ideal transformer of impedance ratio $1:\alpha^2$ is another ideal transformer of impedance ratio $\alpha^2:1$. It follows that the dual of a 1:1 ideal transformer is a 1:1 ideal transformer.

The dual of a constant voltage supply $E$ is, of course, a current supply which maintains a constant current equal to

$$I' = E/r, \tag{11}$$

and the dual of a constant current supply $I$ is a supply of constant voltage equal to

$$E' = Ir. \tag{12}$$

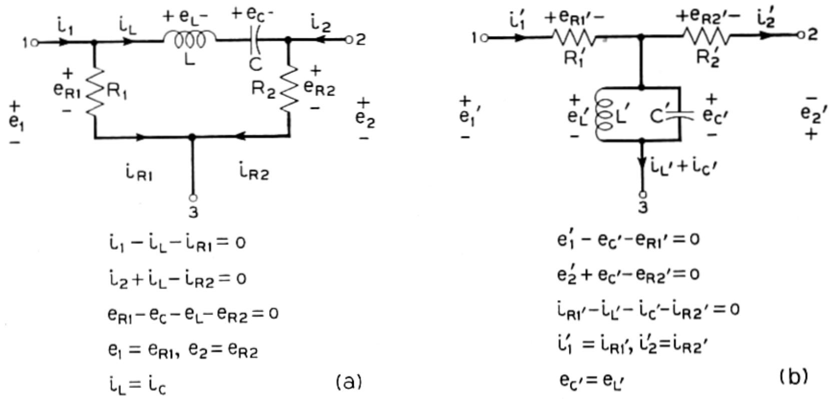


Fig. 2—The dual of a network is found by transforming the Kirchoff equations.

The procedure of substitution used in all the examples above can be used in a straightforward way to find the dual of a more complicated network, but, in view of what has already been said some labor can be saved by writing the Kirchoff equations in the abbreviated notation used in Fig. 2. The equations on the left corresponding to the original network are then transformed into the equations of the dual network by making the following substitutions:

$$\begin{aligned} e_1 &= i_{1'} \\ i_1 &= e_{1'} \\ e_{R_1} &= i_{R_1'} \\ i_{R_1} &= e_{R_1'} \\ e_L &= i_{C'} \\ i_L &= e_{C'}\text{ , etc.} \end{aligned}$$

From these transformed equations, shown on the right hand side of Fig. 2, the dual network shown above them can be drawn by inspection.

From the example given in this figure, it is seen that a ladder network is transformed into another ladder network with each series branch of the original network being transformed into a shunt branch in the dual network and vice versa. Note also that a series combination of $L$ and $C$ is transformed into a shunt combination of $C'$ and $L'$. The effect of a short circuit between terminals 1 and 2 in the original network (which makes $e_1 = e_2$) is an open circuit at terminal 3 in the dual network (which makes $i_1 = i_2$).

## The Dual of an Ideal Vacuum Tube Triode

In a previous section it was shown that transistor currents behave approximately like vacuum tube voltages and vice versa. In view of what has been said about duality it might be assumed that, as three-terminal networks, the transistor and the vacuum tube triode are approximate duals. It is the purpose of this section to examine the relationships between the two devices in detail and to show that they fail to be duals one of the other principally because of a sign. What it amounts to is that signals transmitted through the dual of a vacuum tube suffer a phase reversal while, on the other hand, signals are transmitted through a transistor without change of phase.

A convenient way of proceeding is to start with the 4-pole equations of an ideal vacuum tube triode and transform them, by the methods already presented, into the equations of the dual. These transformed equations will then be compared with the 4-pole equations of a transistor.

The small signal behavior of a vacuum tube triode is represented by the equations,

$$
\begin{aligned}
i_g &= (0)v_g + (0)v_p\,, \\
i_p &= g_m v_g + k_p v_p \\
&= k_p(v_p + \mu v_g),
\end{aligned}
\tag{13}
$$

where $\mu = g_m/k_p\,,$

and $k_p = 1/r_p\,.$

These equations apply when the positive directions of current and voltage are as indicated in Fig. 3. The equations corresponding to the dual of the ideal vacuum tube triode are found by substituting in equations (13),

$$
\begin{aligned}
i_g &= v_1/r, \\
i_p &= v_2/r, \\
v_g &= ri_1\,, \text{ and} \\
v_p &= ri_2\,.
\end{aligned}
$$

The quantities $i_1$ and $v_1$ will then represent the current and voltage at the input terminals of the dual device and $i_2$ and $v_2$ will represent the current and voltage at the output terminals. Making these substitutions leads to

$$
\begin{aligned}
v_1 &= (0)i_1 + (0)i_2 , \\
v_2 &= r^2 g_m i_1 + r^2 k_p i_2 \\
&= r^2 k_p (i_2 + \mu i_1).
\end{aligned} \tag{14}
$$

It remains to be seen how the directions of current and voltage must be assigned in Fig. 3. If the directions of $v_1$ and $i_1$ are arbitrarily assigned as indicated in the figure, then the directions of $v_2$ and $i_2$ can be found by an argument like that used in connection with the passive three-terminal network just discussed. The dual of making $v_p = v_g$ by placing a short circuit between plate and grid is making $i_1 = i_2$ by opening terminal 3 of the dual. This says that the positive direction of $i_2$ is as shown in Fig. 3. Similarly, the dual of making $i_g = -i_p$ by opening the cathode connection to the vacuum tube is making $v_1 = -v_2$ by placing a short circuit between terminals 1 and 2 of the dual. This requires that positive values of $v_1$ and $v_2$ have opposite signs when measured with respect to terminal 3 and so fixes the positive direction of $v_2$ as shown in Fig. 3.

The 4-pole equations for a transistor[3] are

$$
\begin{aligned}
v_e &= r_{11} i_e + r_{12} i_c , \\
v_c &= r_{21} i_e + r_{22} i_c \\
&= r_{22}(i_c + \alpha i_e),
\end{aligned} \tag{15}
$$

where $\alpha = r_{21}/r_{22}$ .

These equations are similar in form to equations (14) which correspond to the dual of an ideal vacuum tube triode. Comparing the two sets of equations shows that the following transistor and vacuum tube quantities correspond to each other:

$r^2 g_m$ and $r_{21}$ ,
$r^2 k_p$ and $r_{22}$ , and
$\mu$ and $\alpha$.

Comparing the first of equations (14) with the first of equations (15) shows that the transistor quantities $r_{11}$ and $r_{12}$ should be zero if the transistor is to be an accurate dual of the vacuum tube triode. These quantities are small in present day transistors and there is hope that they may be made still smaller in the future. In the transistor of Fig. 1(b), for example, $r_{11}$ is approximately 200 ohms. This corresponds to a grid-to-cathode leakage resistance in the triode which can be computed from

$$r_g = r^2/r_{11} .$$

[3] Some Circuit Aspects of the Transistor, R. M. Ryder and R. J. Kircher, *Bell Sys. Tech. Jl.*, *28*, 367 (July 1949).

Since $r = 6600$ ohms for the transistor and vacuum tube of Fig. 1, $r_g$ amounts to 218,000 ohms. This is large compared to $r_p$ and would not seriously impair the operation of the tube for many purposes.

What has been said indicates that transistor currents and voltages are fairly accurate duals of vacuum tube voltages and currents. As a three-terminal network, however, the transistor fails to be the dual of a vacuum tube because the values of $i_c$ and $v_c$ which behave as duals of $v_p$ and $i_p$ are measured with a convention of signs which is not consistent with Fig. 3. This can be seen by comparing the directions of $i_2$ and $v_2$ in the dual of a vacuum tube (Fig. 3) with the convention of signs for the transistor indicated in Fig. 1(b). A transistor like present day ones in all respects except for a reversal in sign of $i_c$ and $v_c$ would be a fairly good dual for a vacuum tube triode. This discrepancy in sign means, of course, that the grounded base transistor fails to give the phase reversal which would be given by the dual of a vacuum tube. This does not mean that the duals of vacuum tube circuits cannot be

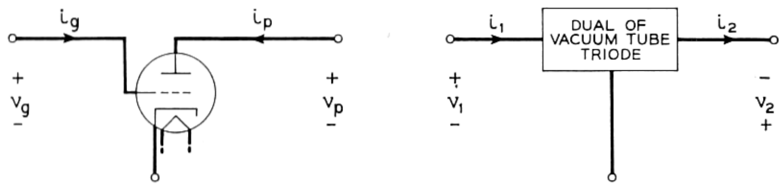



Fig. 3—The right-hand figure shows the convention of signs which must be used with the transformed equations (14).

found and used to advantage with transistors. It simply means that if the circuits are to be strictly dual an ideal transformer or some other means must be used to supply the phase reversal.

In finding the dual of a vacuum tube circuit there are several equally satisfactory ways of proceeding. Perhaps the simplest is to begin by treating the transistor as though it were a perfect dual of a vacuum tube triode. In this case, the transistor is substituted for the vacuum tube—emitter for grid, base for cathode, and collector for plate—and then the remaining part of the vacuum tube circuit is replaced by its dual. The resulting circuit fails to be a dual of the original only with respect to a phase reversal which can be corrected by inserting a phase reversing ideal transformer at the most convenient appropriate place.

Another procedure, which is perhaps more straightforward but which may also require more work, takes care of the phase reversal automatically. The first step in this case is to write down the Kirchoff equations for the vacuum tube circuit and then transform them into a new set of equations in the manner illustrated in a previous section (see Fig. 2). In doing this, replace

$$v_p \text{ by } -i_c\,,$$
$$i_p \text{ by } -v_c\,,$$
$$v_g \text{ by } -i_e\,, \text{ and}$$
$$i_g \text{ by } -v_e\,.$$

This gives a set of Kirchoff equations which apply to the dual circuit and it remains only to find, by inspection, a circuit which satisfies them. It will often be found that, in order to satisfy these equations, it is necessary to introduce a phase reversing transformer. Several examples of this method of procedure will be given in sections to follow. In the appendix a great many more examples of vacuum tube circuits and their transistor duals are shown.

## Gyrators and Duality

Tellegen[4] has shown that in principle it is possible to make a new kind of passive 4-pole circuit element to which he has given the name "ideal gyrator". This device is characterized by the 4-pole equations

$$e_1 = Ri_2 \text{ and}$$
$$e_2 = -Ri_1\,.$$

Though such a device is not known to have been realized in a practical physical form as yet, its properties are so closely related to duality as to be worth mentioning here.

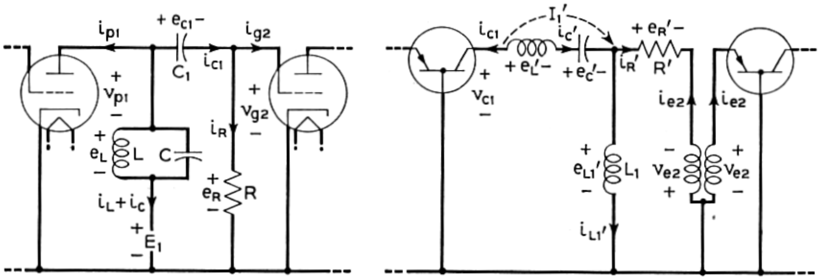
The following interesting properties can readily be deduced from the equations above. First, signals are transmitted through the device in one direction without phase reversal, while signals transmitted in the other direction are reversed in phase. Second, if an impedance $Z$ is connected across the output terminals of an ideal gyrator, the impedance seen at the input terminals is $R^2/Z$. This means that the ideal gyrator has the property of transforming any two-terminal network into its dual. Also a three-terminal network can be converted into its dual by connecting one gyrator to the input terminals of the network and another to the output terminals. These gyrators must be so poled that no phase reversal is produced in either direction by the action of the two together.

This means that the dual of a vacuum tube triode can be obtained by using a triode plus two gyrators as shown in Table I and, of course, the dual of a transistor can be obtained by using a transistor plus two gyrators. Also,

[4] The Gyrator, A New Electric Network Element, B.D.H. Tellegen, *Phillips Research Reports*, 1948, pp. 81–101.

since the gyrator gives a phase reversal or not, depending on the direction of transmission, a transistor plus two gyrators can be made the *equivalent* of a vacuum tube triode by poling the gyrators in such a way as to take care of the phase reversal.

Ideal gyrators are not yet available but passive circuit elements having very similar properties over a narrow frequency range are available and are used in certain vacuum tube circuits. A quarter-wave line or its lumped-constant equivalent (which amounts to a full section of low- or high-pass filter) has the property of impedance inversion at a single frequency. Instead of giving zero or 180° phase change as does the ideal gyrator discussed above, this single frequency gyrator can be designed to give either +90° or −90° phase change. In either case, the phase shift is independent of the




(a)

$$v_{p1} - e_L - E_1 = 0$$
$$E_1 + e_L - e_{c1} - e_R = 0$$
$$v_{g2} - e_R = 0$$
$$e_L = e_C$$
$$i_{p1} + i_{c1} + i_L + i_C = 0$$
$$i_{g2} + i_R - i_{c1} = 0$$

(b)

$$-i_{c1} - i_{c'} - I_1 = 0$$
$$I_1 + i_{c'} - i_{L1'} - i_{R'} = 0$$
$$-i_{e2} - i_R = 0$$
$$i_{c'} = i_{L'}$$
$$-v_{c1} + e_{L1'} + e_{c'} + e_{L'} = 0$$
$$-v_{e2} + e_{R'} - e_{L1'} = 0$$

Fig. 4—A tuned amplifier stage and the transistor dual.

direction of transmission. This leads to the possibility of exchanging a transistor for a vacuum tube plus two quarter wave lines. An application of this sort will be discussed in a later section.

## The Dual of an Amplifier with Shunt Tuned Interstage

Figure 4(a) shows a vacuum tube amplifier with the Kirchoff equations which apply to it. Figure 4(b) shows the transformed equations and a transistor circuit which satisfies them.

The ideal transformer in this circuit has no effect on any aspect of the circuit performance except the phase of the output and, if this is not important, the transformer may be left out. The curved arrows represent constant current supplies. All the element values in the transistor circuit may be computed from the values in the vacuum tube circuit by means of the relations of Table 1.

Figure 5, (a) and (b), shows a different arrangement of power supplies in the vacuum tube circuit and the resulting more convenient arrangement of power supplies in the transistor circuit. In this case, the ideal transformer has been omitted but in all other respects the circuits are duals.

The application of duality in this case has led to the use of a series tuned circuit in series with the load instead of the shunt tuned circuit in shunt with the load, which the vacuum tube circuit might otherwise have suggested. The series tuned circuit has the advantage when used with short-circuit unstable transistors of insuring stability outside the pass band.

## The Dual of a Push-pull Class B Amplifier

Figure 6(a) shows the circuit of a push-pull amplifier and the Kirchoff equations which apply to it. Figure 6(b) shows the transformed equations

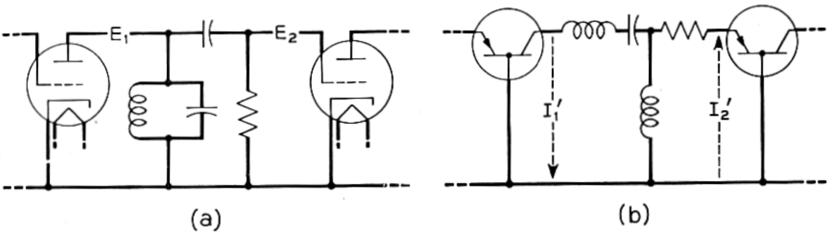
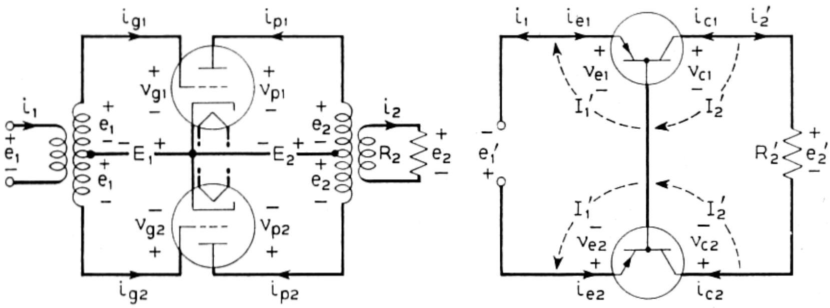



Fig. 5—An unconventional arrangement of power supplies in the vacuum tube circuit leads to a convenient arrangement for transistors. The phase reversing transformer which would make the transistor circuit strictly dual has been omitted.

and the dual transistor circuit. In this case, not only the circuit configuration but also the choice of operating point is important. For class B operation the two tubes are given a high negative grid bias, so that in the absence of an input signal the two plate currents are nearly zero while the plate voltages are quite high. In the transistor case, the dual situation is that the emitters are given a high positive emitter current bias so that in the absence of an input signal the two collector *voltages* are nearly zero while the collector *currents* are quite high. During a positive half cycle of input voltage the upper vacuum tube plate circuit begins to conduct and the plate current of the upper tube goes through a positive half cycle while the plate current in the lower tube remains essentially at zero. During this half cycle the plate current of the upper tube is coupled through the output transformer to the load while the lower tube contributes nothing, behaving simply as an open circuit element in shunt with the load and with the upper tube. The corresponding set of events in the transistor amplifier is that, in response to a positive half cycle of input current, the collector voltage of the upper transistor goes through a negative half cycle while the collector voltage of the lower transistor remains essentially zero. All the collector voltage swing of the upper transistor is impressed directly on the load because, during this half cycle, the lower transistor serves as a short circuit element in series with the load and with the upper transistor. The next half cycle is, of course, like the first except that the lower tube and the lower transistor assume the active roles.

It was this circuit which first showed the great advantage which can sometimes be achieved through the use of duality. Using two type A transistors in the circuit of Fig. 6(b), it has been possible to obtain 400 milliwatts of




$$v_{g1} - e_1 + E_1 = 0$$
$$v_{g2} + e_1 + E_1 = 0$$
$$v_{p1} - e_2 - E_2 = 0$$
$$v_{p2} + e_2 - E_2 = 0$$
$$i_{g1} - i_{g2} - i_1 = 0$$
$$i_{p1} - i_{p2} + i_2 = 0 \qquad \text{(a)}$$

$$-i_{e1} - i_1' + I_1' = 0$$
$$-i_{e2} + i_1' + I_1' = 0$$
$$-i_{c1} - i_2' - I_2' = 0$$
$$-i_{c2} + i_2' - I_2' = 0$$
$$-v_{e1} + v_{e2} - e_1' = 0$$
$$-v_{c1} + v_{c2} + e_2' = 0 \qquad \text{(b)}$$

Fig. 6—A class B amplifier and the transistor dual.

audio output with a collector circuit efficiency of 60%. The same two transistors which gave this result could not be made to deliver more than 25 milliwatts output when used as grounded base amplifiers in a conventional circuit like that of Fig. 6(a).

## The Dual of a Bridge Stabilized Oscillator

Figure 7(a) shows the circuit of a bridge stabilized oscillator due to Meacham,[5] in which amplitude stabilization can be achieved through the action of a temperature sensitive resistance $R_T$ which has a positive tem-

[5] The Bridge-Stabilized Oscillator, L. A. Meacham, *Proc. I.R.E.* 26, 1938, p. 1278; *Bell Sys. Tech. Jl.* 17, 1938, p. 574; *U. S. Pat.* 2,303,485.

perature coefficient of resistance. At the resonant frequency of the tuned circuit

$$\frac{v_g}{v_p} = \frac{\dfrac{R_T}{R_x} - 1}{\dfrac{R_T}{R_x} + 1}$$

where $R_x$ is the equivalent resistance of the tuned circuit at resonance. The circuit values are chosen so that $R_T$ is smaller than $R_x$ and therefore the feedback is positive. As the amplitude of oscillation builds up, the increasing signal level across $R_T$ increases its temperature thereby increasing its

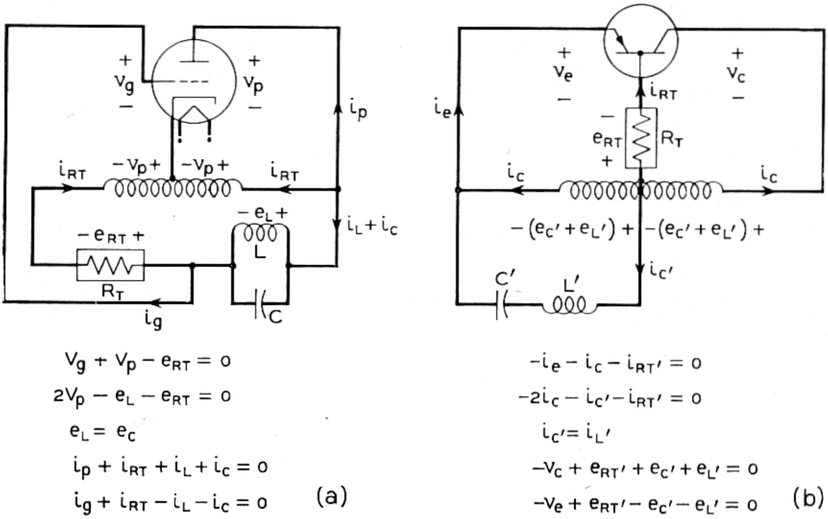



Fig. 7—A bridge-stabilized oscillator and the transistor dual.

resistance and bringing the bridge nearer to balance, so that the amount of positive feedback is reduced. This results in a stable amplitude of oscillation sufficient to make $R_T$ slightly smaller than $R_x$.

Figure 7(b) shows the transformed equations and the dual transistor oscillator. In this case, $R_{T'}$ is a thermistor with a negative temperature coefficient of resistance. At the resonant frequency of the series tuned circuit

$$\frac{i_e}{i_c} = \frac{1 - \dfrac{R_{T'}}{R_{x'}}}{1 + \dfrac{R_{T'}}{R_{x'}}},$$

where $R_{x'}$ is the effective series resistance of the tuned circuit at resonance. The circuit values are so chosen that $R_{T'}$ is greater than $R_{x'}$ and therefore the feedback is positive. As the amplitude of oscillation increases, $R_{T'}$ is heated so that its resistance decreases and brings the bridge more nearly into balance. This reduces the amount of positive feedback until a stable amplitude of oscillation is reached with $R_{T'}$ only a little greater than $R_{x'}$.

Meacham has shown that the stability of such an oscillator increases as the gain of the amplifier is increased. Since the vacuum tube amplifier of Fig. 7(a) can be made to give more gain than can be obtained from a single transistor, the transistor oscillator of Fig. 7(b) is not as stable as its vacuum tube dual. If increased stability is desired, it can be obtained by using a two-stage transistor amplifier instead of the single transistor shown.

## Circuits Using Vacuum Tubes and Transistors Together

Since the vacuum tube and the transistor are basically different kinds of circuit elements, it seems reasonable to suppose that there may be circuits in which both can be used together to advantage. Two examples of such circuits will be discussed. The first has to do with a very ingenious high efficiency linear amplifier designed by Mr. W. H. Doherty.[6] This amplifier is particularly suited for use with amplitude modulated radio frequency inputs.

Figure 8 shows the basic features of one form of the Doherty amplifier. The networks $N_1$ and $N_2$ are impedance inverting networks of the type already discussed and amount to ideal gyrators for frequencies near the carrier frequency. Tube $T_1$ is biased nearly to cutoff and works, for small rf inputs, as a linear class B amplifier; while $T_2$ is biased well below cutoff and is inactive except when the rf input is higher in level than the unmodulated carrier. Downward swings of modulation are amplified by $T_1$ alone, which sees an effective load impedance just twice the value into which it could deliver maximum power. Under these conditions the peak voltage swing of $T_1$ begins to approach the supply voltage just as the rf input reaches a value corresponding to the unmodulated carrier. For greater input signals $T_1$, if acting alone, would begin to distort. But as the input signal is increased above the value corresponding to the unmodulated carrier, $T_2$ comes into action and contributes in two different ways to increasing the output signal linearly. First, $T_2$ acts as a class C amplifier and delivers power to the load and second, through the action of the impedance inverting network $N_2$, $T_2$ acts in such a way as to lower the effective load impedance seen by $T_1$. This makes it possible for $T_1$ to deliver more power to the load without an increase in plate voltage swing. The result of all this, which is discussed in much greater detail in Doherty's papers, is a linear amplifier of unusually high efficiency.

[6] A New High-Efficiency Power Amplifier for Modulated Waves, W. H. Doherty, *Proc. I.R.E.*, *24*, 1163 (September, 1936).

The part of the circuit of Fig. 8 shown inside the dotted box amounts to a vacuum tube plus two gyrators, which is just the dual of a vacuum tube. Apart from a phase shift of 180° this is *equivalent* to a transistor. This part of the circuit can therefore be replaced by a transistor plus a phase reversing transformer to obtain the basic transistor-vacuum-tube circuit of Fig. 9. This results in a considerable simplification because the impedance inverting networks are no longer needed.

The operation of the circuit of Fig. 9 is exactly similar to that of Fig. 8 except that the transistor operates as the dual of $T_1$. This means that the transistor is given a large forward emitter bias so that collector voltage is

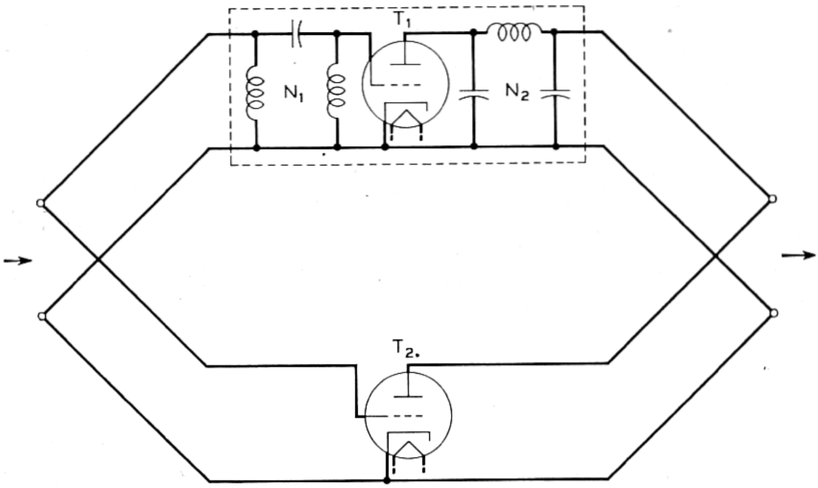



Fig. 8—The basic arrangement of a Doherty amplifier.

almost cut off. Under these circumstances, it is capable of operation as a linear amplifier. The load resistance is just half that into which the transistor could deliver maximum power. The transistor acts alone to amplify downward swings of modulation ($T_2$ being biased well below cutoff as before) but as the input signal exceeds that of the unmodulated carrier the collector current swing begins to approach the maximum value permitted by the (current) supply and $T_2$ begins to contribute to the output in just the ways it did in the circuit of Fig. 8. First, it acts as a class $C$ amplifier delivering power directly to the load and second, it behaves as a negative resistance bridged across the load and thereby increases the impedance into which the transistor works. This permits the transistor to deliver more power without increasing the collector current swing.

Just as the basic Doherty circuit of Fig. 8 needs tank circuits to suppress carrier harmonics, so also does the circuit of Fig. 9. When these are added the circuit of Fig. 10 is obtained.

Doherty shows that there are two basic circuit arrangements for obtaining the high efficiency linear amplifier action which he describes. One of them has been discussed above. By starting with Doherty's other arrangement,

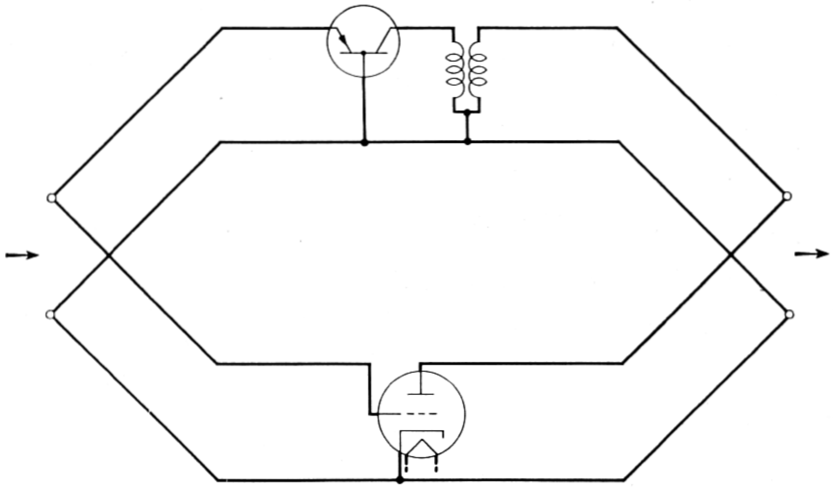


Fig. 9—The basic circuit of a Doherty-type amplifier using a transistor to replace a vacuum tube and two impedance inverting networks.

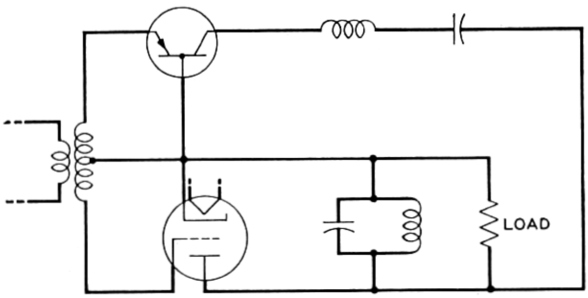



Fig. 10—A Doherty-type amplifier in which low level signals are amplified by the transistor alone.

one arrives at the circuit of Fig. 11. In this case, it is the vacuum tube which operates class $B$ and the transistor which helps to supply the peaks by class $C$ operation. At low input levels the transistor behaves as a short circuit and the vacuum tube works into an impedance just twice the value into which it can deliver maximum power. As the input signal increases above the carrier level the transistor begins to operate, contributing in two ways to increasing the power output. First, it delivers power directly to the load and, secondly, it behaves as a negative resistance in series with the load, thereby decreasing the impedance into which the vacuum tube works and permitting it to deliver more power without increasing its plate voltage swing.

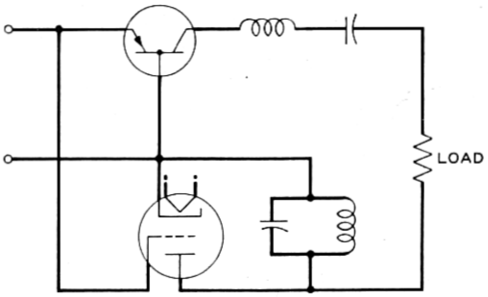



Fig. 11—A Doherty-type amplifier in which low level signals are amplified by the vacuum tube alone.

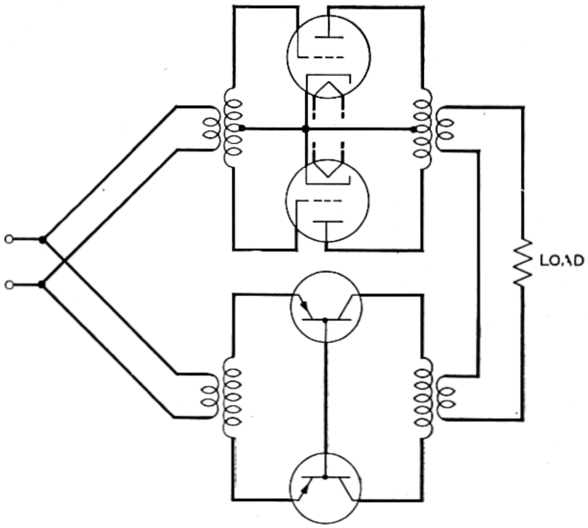



Fig. 12—A high efficiency untuned amplifier in which small signals are amplified by the vacuum tubes alone.

The Doherty amplifier is limited to narrow band operation only because the networks $N_1$ and $N_2$ will behave as gyrators only over a narrow range of frequencies. Apart from a phase change of 180°, however, the transistor behaves as a vacuum tube plus two *ideal* gyrators and is therefore capable of acting as the dual of a vacuum tube over a wide band of frequencies. This leads to the possibility of an entirely new, wide band, high efficiency amplifier which operates on the same principles as the Doherty circuit.

Both the transistor part and the vacuum tube part of the amplifier must be made push-pull in order that both halves of the input wave be amplified equally. In the circuit of Fig. 12 the vacuum tubes are biased for class *B* operation, while the transistors are given a large forward emitter current bias so that they are operated well below collector voltage cutoff. For small input signals the transistors are inactive, serving simply as short circuit elements in series with the load. As the input signal reaches half the peak

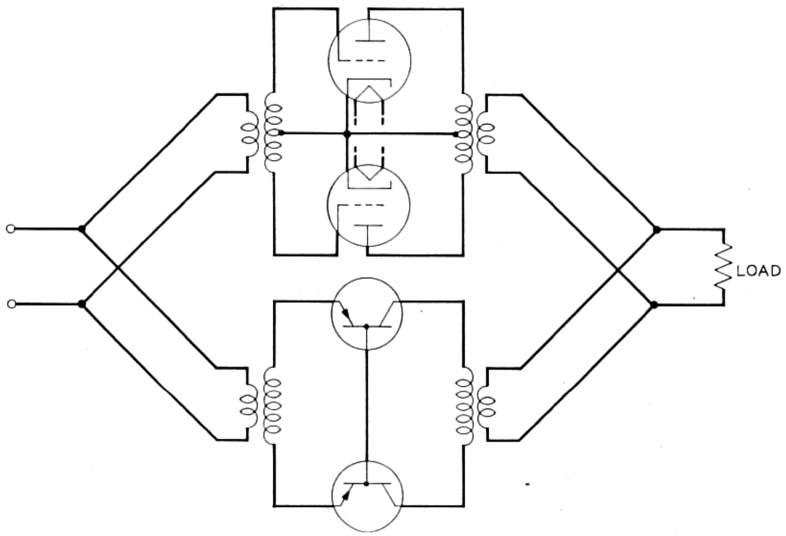



Fig. 13—A high efficiency untuned amplifier in which small signals are amplified by the transistors alone.

permissible value the vacuum tubes begin to distort because their voltage swings approach the supply voltage. At this point the transistors begin to operate in two separate ways, just as in the Doherty amplifier. First, they work as class *C* amplifiers delivering power directly to the load and second they behave as a negative resistance in series with the load thereby serving to reduce the impedance into which the vacuum tubes work. This permits the vacuum tubes to deliver more power without increasing their plate voltage swing.

Just as there are two forms of the Doherty amplifier, there are also two forms of this wide band arrangement. In the second form shown in Fig. 13 the transistors are biased for class *B* operation (near collector voltage cutoff) while the vacuum tubes are biased well below cutoff. For small signals the transistors act alone as class *B* amplifiers and the vacuum tubes act simply as open circuit elements in shunt with the load. As the instantaneous input signal reaches half the permissible peak value, the transistors begin to distort because the collector current swing begins to approach the value of the (current) supply. At this point the vacuum tubes begin to operate in two separate ways to increase the power output. First they act as class $C$ amplifiers delivering power directly to the load and second, they behave as negative resistance elements in shunt with the load and thereby increase the impedance into which the transistors work. This permits the transistors to deliver more power without increased collector current swing.

The circuits of Figs. 12 and 13 can both be adjusted to give reasonably linear performance. Perhaps the most interesting aspect of these circuits is that the theoretical maximum efficiency (for sinusoidal signals) is 93%. This should be a matter of importance in applications where the greatest possible power output is desired from transistors and tubes of limited dissipation rating.

It has been pointed out by Ryder and Kircher[3] that a transistor with $\alpha$ just equal to unity behaves like a vacuum tube triode when operated with the emitter grounded. If transistors can be made to operate satisfactorily in this way with large signal swings then all the vacuum tubes in the circuits discussed in this section can be replaced by grounded-emitter transistors.

## General Comments

It is obvious that not all useful transistor circuits can be found in the manner presented in this paper and, furthermore, not all of the circuits found through the application of duality are useful.

One limitation of the method is imposed by the fact that present day transistors correspond to rather low $\mu$ vacuum tubes. On this account, vacuum tube circuits which require high $\mu$ tubes for satisfactory performance will lead to inferior transistor circuits. If further development of the transistor produces higher values of $\alpha$, this limitation will be reduced.

Another limitation of the method comes from the failure of the transistor to produce a phase reversal. Although this is not important in many cases, and in other cases in which it is important a transformer provides a satisfactory solution, still the fact remains that transformers do not respond at d.c. and because of this fact some transistor dual circuits are useless.

In spite of these limitations, the methods presented in this paper have led to a number of useful transistor circuits and may be expected to yield still more in the future.

## Acknowledgment

The authors wish to express their gratitude for the keen interest in this work shown by Mr. W. E. Kock and Mr. R. K. Potter under whose direction

[3] Ryder and Kircher, loc. cit.

it has been carried out. We are indebted to Mr. B. McMillan and also to Messrs. Harold Barney, R. J. Kircher, L. A. Meacham, J. A. Morton, L. C. Peterson and R. M. Ryder for encouragement and helpful comments.

## APPENDIX

The appendix is a brief account of some of the circuits which have been investigated with the aid of the methods described in the main text. The circuits shown do not exhaust all possibilities, and the specific configurations shown are not to be regarded as optimal choices.

Figure 14 shows a one-stage resistance-capacitance coupled amplifier and its dual. In the input circuit of 14(a), $C$ is a series element which passes alternating currents and blocks direct currents. The dual element, $L$, in the input circuit of 14(b) is a shunt element which is a short circuit to direct voltages, but not to alternating voltages. The resistance $R_1$ is a shunt element which provides a path through the battery without creating a short circuit to ground for the alternating signal. Correspondingly, the resistance $R_1'$ provides a path around the current supply, which otherwise would be an open circuit for the alternating signal.

The passive elements in the input circuit are also capable of acting as a source of self-bias. Suppose, for example, that $R_1$ be connected directly to ground with no battery interposed and that the emitter current supply be removed. The resulting vacuum tube circuit is familiar. The usual explanation of its behavior is that when the grid is driven positive and draws grid current, the condenser $C$ becomes charged, and that subsequently the condenser discharges through the resistance $R_1$, supposed large enough to assure a long discharge time constant. In this way the condenser is kept charged so that grid current flows only a small portion of the time.

The behavior of the dual circuit is exactly analogous, but is much harder to explain simply because words and expressions dual to those used above do not exist or are not in current use. For example, we speak of a condenser as "charged" when there is a potential between the terminals. There is no corresponding term for an inductor with a current passing through it. The explanation, nevertheless, might be as follows: The emitter normally presents a low impedance to positive input currents, and a high impedance to negative input currents. When the input current is negative, the high impedance of the emitter blocks the current and a current is therefore drawn through the inductor $L$, in an upward direction as the figure is drawn. Subsequently, when the input current becomes positive, the emitter presents a low impedance, and the current in the inductor is free to pass through $R_1'$ and the emitter. It is supposed that the inductance is large enough so that the decay time constant of the inductor through $R_1'$ and the emitter is large. Then the current through the inductor will be approximately constant over a short period and will be a bias current. This current will regulate itself so that the resultant emitter current is positive most of the time, becoming negative only long enough to keep the inductor "charged."

The output circuit is easier to explain. The resistance $R_2$ provides a path through the battery which is not a short circuit. The resistance $R_2'$ provides a path around the collector current supply which does not have infinite impedance to the signal. The loads $Z_L$ and $Z_L'$ are the ultimate receivers of the amplified signal. The duality of the loads may be emphasized by pointing out that the condition corresponding to $Z_L = \infty$ is $Z_L' = 0$.

If the circuits are analyzed with the aid of the equivalent circuits discussed in the text, the voltage amplification of the vacuum tube circuit will be found to be

$$\frac{g_m}{\frac{1}{r_p} + \frac{1}{R_2} + \frac{1}{Z_L}},$$

while the current amplification of the transistor circuit is found to be

$$\frac{r_m}{r_c + R_2' + Z_L'}.$$

These expressions are obviously duals.

Transistor amplifiers like the one shown in Fig. 14 can be connected in cascade. Three examples are shown in Fig. 15 (b) and (c) and (d). Figure 15(b) is the most obvious connection, and 15(c) and 15(d) have provisions for correcting the relative phase inversion that occurs in the transistor circuit. If the circuit equations for the three examples are written out, it will be discovered that only 15(c) and 15(d) are duals (in the sense defined in the text) of the vacuum tube circuit 15(a). The remaining example, 15(b), is the dual of a peculiar looking circuit with one vacuum tube inverted.

In the range where operation is nearly linear, the three cascaded amplifiers behave much alike; and 15(c) and 15(d) can be regarded as pedantic attempts to make the signs come out "right." As soon as non-linear operation is encountered, however, the differences between the circuits become pronounced. This will be clearer when multivibrators are considered.

Figure 16 shows a variation of one of the circuits of Fig. 15 designed to operate on a single power supply. A circuit like this with four cascaded stages has been built and tested, and was found to work satisfactorily with selected matched transistors. The two extra resistors in each stage, $R_1$ and $R_2$, are voltage dropping resistors, chosen to balance the voltage drops in the emitter and collector circuits respectively.

Figure 17 shows a multivibrator, conveniently illustrated as a two-stage RC coupled amplifier with its output connected to its input. Below are shown three circuits, of which the first is almost a dual and the other two are duals of the vacuum tube circuit. The first transistor circuit, 17(b), fails to be a dual in that besides having positive feedback around two stages, it has positive feedback in each stage separately. This is avoided in 17(c) by an isolating

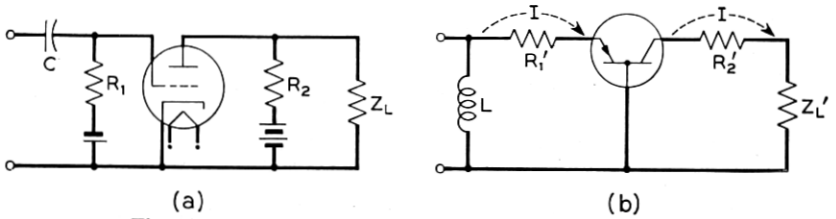



Fig. 14—Resistance—capacitance coupled amplifier and dual.

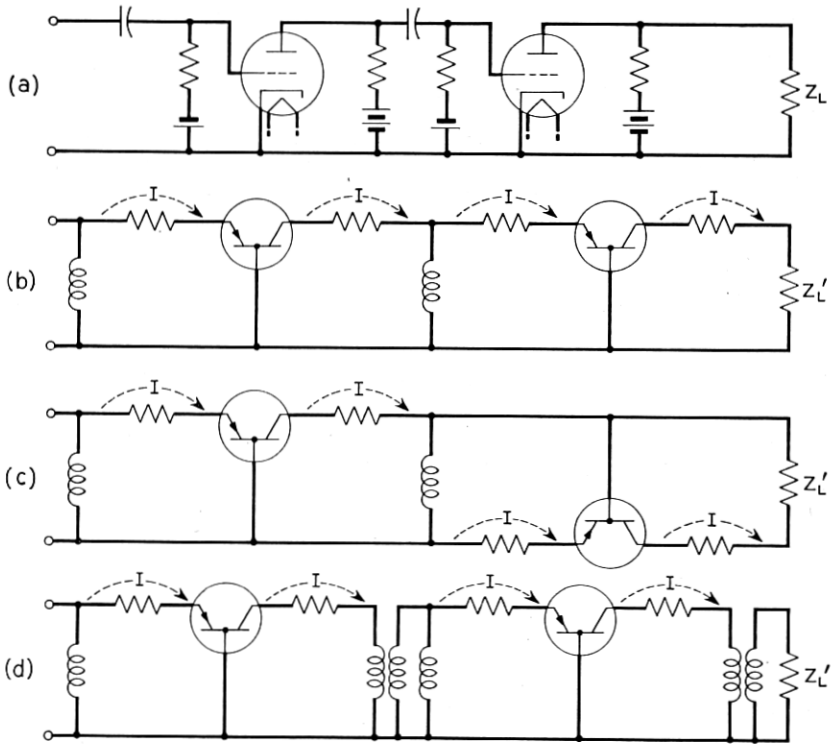



Fig. 15—Cascaded amplifier and duals.

transformer. It has been shown nevertheless by practical tests that 17(b) acts as a multivibrator, and is perhaps even a better circuit than the others. It has the interesting characteristic that the two inductors are in parallel, and hence may be replaced by a single inductor.

One explanation of the operation of a vacuum tube multivibrator is this.

Suppose one grid is at a large negative potential, cutting off that tube, and the other is at a positive potential or at zero potential. The potential of the

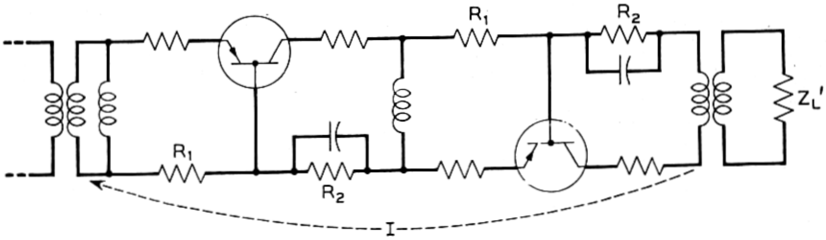



Fig. 16—Two-stage transistor amplifier using a single power supply.

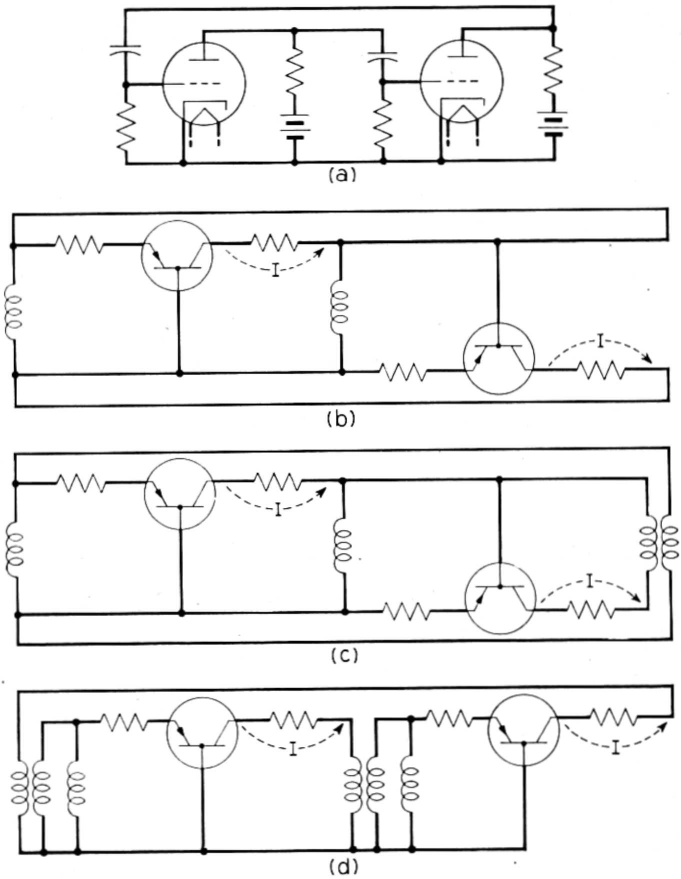
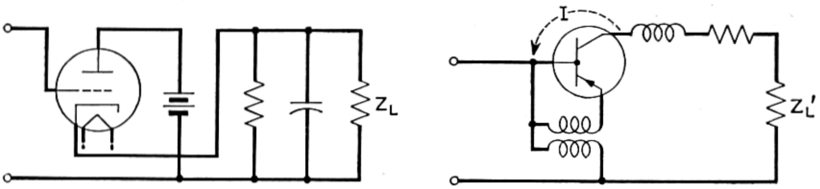



Fig. 17—Multivibrator and duals.

negative grid rises toward zero at a rate controlled by the grid resistor and the coupling condenser. When the grid potential rises above the cutoff voltage, the plate potential falls, and the positive feedback accelerates the process so the first grid rises to a positive potential and the other grid falls to a large negative potential. The process is then repeated.

The dual behavior of the transistor multivibrator is as follows: Suppose that the emitter current of one transistor is very large, and of the other about zero. The large emitter current passes through the emitter, an inductance, and a small resistor, and will decay at a rate controlled by the inductance and the effective resistance of the resistor and emitter. As it decays, no effect will result in the collector circuit until the emitter current falls below the collector voltage cutoff point, after which the collector current will decrease. The emitter current of the other transistor will *increase*, as a consequence of the phase inversion built into the circuit, and the collector current of the second transistor will increase. As a consequence of positive feedback, the whole process will accelerate suddenly and proceed until the



(a) (b)

Fig. 18—Cathode follower and dual.

emitter current of the second transistor is large and that of the first is zero or negative. The process will then repeat.

Figure 18 shows a simple cathode follower and its dual. It has been explained in the text that, in circuits where the cathode current or the grid-to-plate voltage play an important part, the dual circuit will usually require a transformer. Alternatively it may be said that, in any circuit in which feedback in a single stage plays an important role, a transformer may be a necessity. In fact, we have found it impossible to avoid the use of a transformer in the dual of the cathode follower.

The transistor circuit shown will need another power supply for emitter bias, and a blocking condenser to prevent the bias current from flowing through one winding of the transformer. The power supply is required because the transformer will not transmit d-c. signals, and the condenser is necessary because the d-c. impedance of a transformer winding is nearly zero. Extra blocking condensers will appear in association with transformers in many circuits, especially in cases where the transformer is being used as the dual of another transformer.

A vacuum tube cathode follower ordinarily has a high input impedance and a low output impedance, and has a voltage gain nearly equal to one. The dual circuit has a low input impedance and a high output impedance, and a current gain nearly equal to one. This comes about as follows: Suppose the collector circuit resistance and load are small. Then the collector is approximately at ground potential. Let the input current increase. This tends to increase the voltage drop from base to collector, and therefore the base potential tends to rise. This rise is transmitted to the emitter by the transformer, and therefore the emitter current rises. The rise in emitter current causes a drop in collector resistance, and counteracts the tendency for the collector-to-base voltage to rise. The result is that the input current passes through the collector circuit into the load without any corresponding rise in voltage between base and ground. This means that the input impedance is low. Of course, not all the input current passes to the load; some is passed to the emitter circuit. In a practical case tested, using a transistor whose

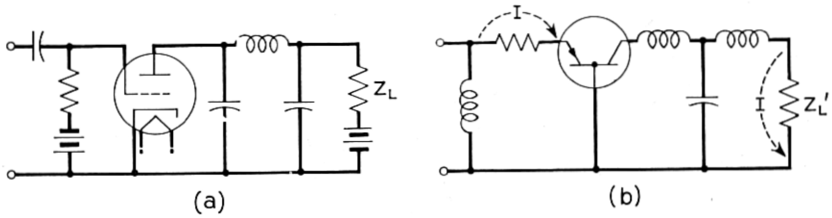



Fig. 19—Plate detector and dual.

base and emitter resistances were of the order of a few hundred ohms, with a load of 5000 ohms, the current gain was about .70 and the input impedance was about 40 ohms.

Figures 19, 20, and 21 show several amplitude modulation detectors and their duals. The purpose of all these circuits is to derive from an amplitude modulated wave a wave proportional to the envelope of the given wave.

Figure 19 shows a plate detector and its dual. The plate detector looks like a single-stage amplifier with a low-pass filter in its output circuit. It is biased approximately to plate current cutoff. As an amplifier it amplifies approximately the upper half of the input wave and does not pass the lower half. The filter smooths the succession of current pulses in the plate circuit and gives an output proportional to the average of the upper half of the input wave. If the input is a true amplitude modulated wave, this is also proportional to the envelope.

The dual circuit operates in the same way. The circuit looks like a one-stage amplifier with a low-pass filter in the collector circuit. It is biased to collector voltage cutoff. The negative part of the input signal is amplified, and the positive part is not. The collector voltage is a succession of pulses of varying amplitudes. These are smoothed by the filter; and the output, as before, is proportional to the envelope of the input.

It is important for the proper operation of these circuits that the filter in the plate detector have low input impedance outside of the pass band, and that the filter in the dual circuit have low input admittance outside of the pass band. The exact form of the filter is immaterial.

Figure 20 shows a grid leak detector and its dual. The operation of the grid leak detector depends on the same principles as that of the grid leak bias circuit described before. The time constant of the bias circuit is chosen, however, so that the bias will be able to vary fast enough to follow the en-

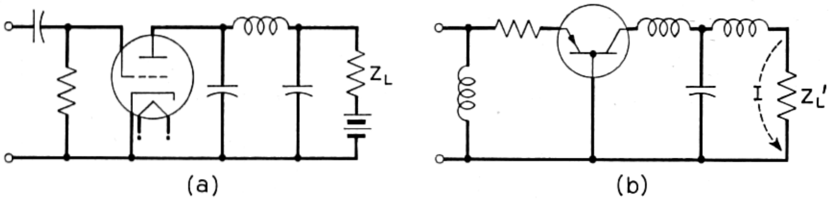



Fig. 20—Grid leak detector and dual.

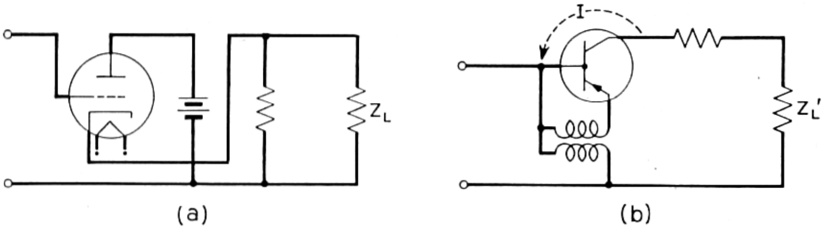



Fig. 21—Infinite impedance detector and dual.

velope of the input wave. The overall grid voltage or emitter current is then the input with a super-imposed wave proportional to the envelope of the input. These are amplified together, and the undesired high-frequency components are removed by a filter in the output circuit, leaving only the envelope wave. The filter must have approximately the same qualities as in the previous case.

Figure 21 shows an infinite impedance detector and its dual. This can be thought of as a cathode follower with a large capacitor across the cathode resistor. This capacitor charges through the comparatively low impedance of the tube when the signal is positive and reaches approximately the peak potential of the input wave. When the input voltage falls, the tube is cut off and the condenser must discharge through a comparatively high impedance. If the time constant of the discharge is properly chosen, the condenser will remain charged approximately to the instantaneous peak value of the input wave, which is the envelope of the input. The transistor circuit works in a similar way. When the input current is positive, the circuit behaves like a cathode follower, and a current is sent through the inductor $L$ which is approximately equal to the input current. When the input current falls, the emitter current rises, and the collector presents a low impedance.

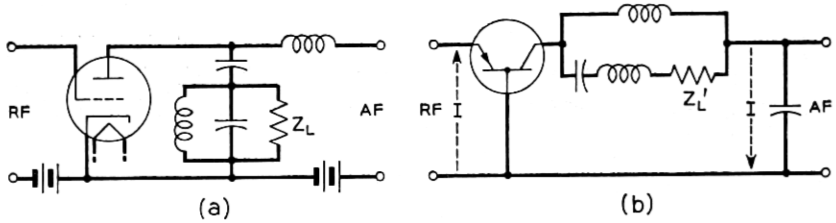



Fig. 22—Plate modulator and dual.

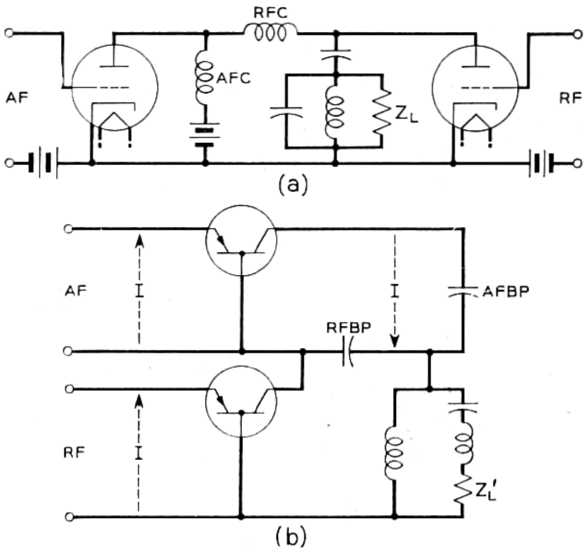



Fig. 23—Constant current modulator and dual.

The current through the inductor then decays slowly through the load and the low collector impedance. Each time the input signal has a positive peak, the effect is to draw a large current through the inductor, which persists during the rest of the cycle.

The dual of the infinite impedance detector is not a very attractive circuit, because the transformer must act for the carrier frequency as well as for the envelope frequencies.

Figures 22, 23, 24, 25, 26 and 27 show various amplitude modulator circuits and their duals. These have as their object the production of a carrier frequency wave with a given envelope.

Figure 22 shows a plate modulator and its dual. This circuit makes use of the fact that the output of a class *C* amplifier is proportional approximately to the plate supply voltage. In 22(a) the vacuum tube acts as a class *C* amplifier amplifying the carrier frequency wave, and the supply voltage is varied

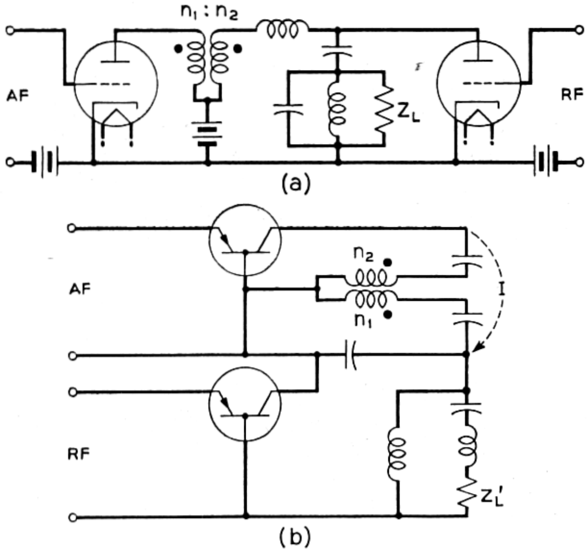



Fig. 24—Modified constant current modulator and dual.

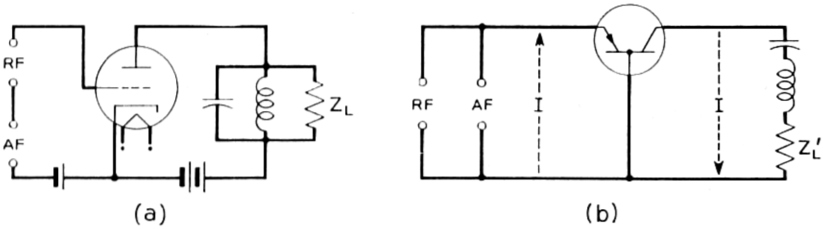



Fig. 25—Grid modulator and dual.

by adding to it the modulating voltage. The peak output voltage thus varies with the modulating voltage. The dual transistor circuit, 22(b), is also a class *C* amplifier, with its collector supply current varied by the addition of a modulating current. The output is proportional approximately to the total supply current, and hence varies with the modulating current.

Figure 23(a) shows a particular embodiment of the plate modulator which is called a constant current modulator, because the total supply current to the two tubes in the circuit is (approximately) constant. It is easier to explain this circuit by saying that the output of class $C$ amplifier is proportional to the supply current. Two tubes, one a sort of audio amplifier and one a class $C$ amplifier, are connected in parallel to a single constant current power supply consisting of a battery in series with a large inductor. The

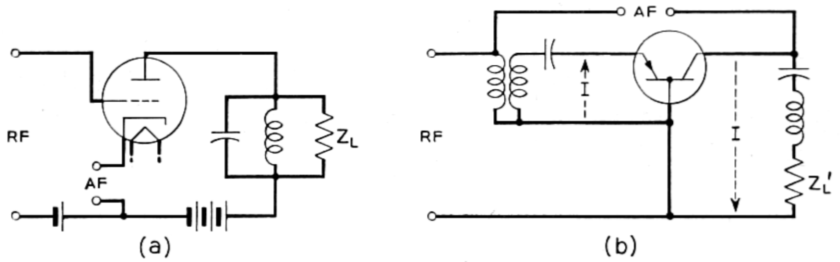



Fig. 26—Cathode modulator and dual.

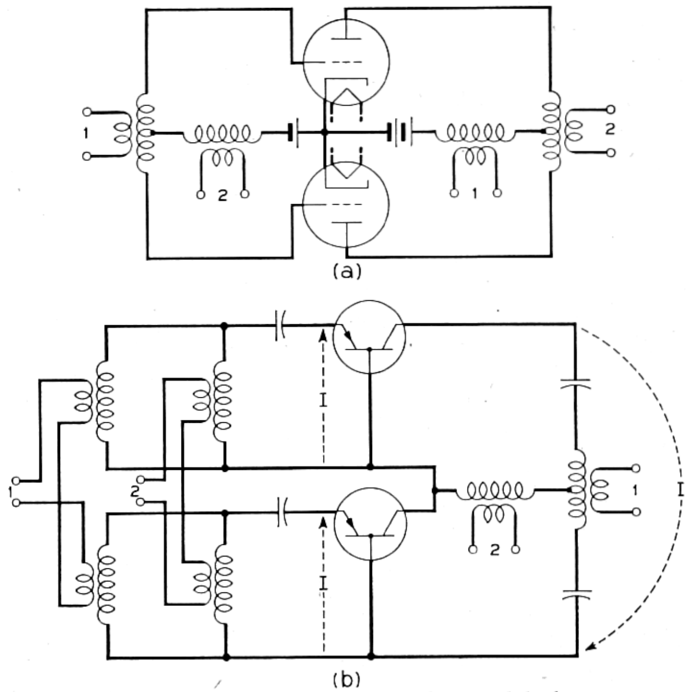



Fig. 27—General balanced modulator and dual.

class $C$ amplifier can use only the current not used by the modulator tube, and hence its output varies inversely with the plate current of, and hence inversely with the grid voltage of, the modulator tube. The dual, Fig. 23(b), consists of a class $C$ transistor amplifier in series with a class $A$ modulator transistor, connected to a source of constant voltage approximated here by a constant current supply in parallel with a large capacitor. The voltage available for the operation of the class *C* amplifier is the difference between the supply voltage and the collector voltage of the modulator transistor. Inasmuch as the output of the class *C* amplifier is proportional to its supply voltage, it is clear that the output will vary directly with the emitter current of the modulator transistor.

Both the vacuum tube and transistor circuit of Fig. 23 suffer because the modulator element can never take all of the available power supply voltage or current, and hence 100% modulation cannot be attained. The circuits of Fig. 24 correct this defect with a transformer, which amplifies slightly the variations in current or voltage in the modulator element. In both circuits $n_2 > n_1$, and the total supply current or voltage is no longer constant, but it is nearly so.

Figure 25 shows a grid modulator and its dual. Here the non-linearity of the transfer characteristics in the neighborhood of the cutoff point is made use of directly to produce modulation products. The tube is biased approximately to plate current cutoff, and the transistor approximately to collector voltage cutoff. The desired modulation products are selected by a tuned circuit.

Figure 26 shows a cathode modulator and its dual. These circuits combine some of the features of the grid modulator and of the plate modulator. Unfortunately the phase relationships are such in the transistor circuit that a transformer is required, and this transformer must be able to pass modulation frequencies as well as carrier frequencies.

The circuits of Fig. 25 and Fig. 26 can be operated as large-signal devices, using the gross non-linearities of the circuits to produce modulation products, or as small-signal devices, when they operate as 'square law' devices. They can, moreover, be combined to form various push-pull or balanced modulators. An example of such a circuit is shown in Fig. 27(a). This circuit has two inputs and two outputs. If it is operated as a square-law device the relations between the input and output frequencies will be as follows:

| *Input* | | *Output* | |
|---|---|---|---|
| 1 | 2 | 1 | 2 |
| a | — | 2a | a |
| — | a | a, 2a | — |
| a | b | b, 2b, 2a | a, a+b, a–b |
| a, b | — | 2a, 2b, a+b, a–b | a, b |
| — | a, b | a, b, 2a, 2b, a+b, a–b | — |

The same relations hold for the dual circuit, Fig. 27(b). The action of the dual circuit is analogous to that of the vacuum tube circuit. It is a two-transistor circuit operated at the same time as a push-pull circuit and as two transistors in series, in phase. At various points in the circuit certain components of the signal are zero because of the symmetries of the circuit. Notice that the dual of two vacuum tubes in parallel is two transistors in series.

Figure 28 shows a modulator which bears the same relation to the modulator of Reise and Skene (*U. S. Patent* 2,226,258) that the amplifier of Fig. 11 of the test bears to the Doherty amplifier. The carrier wave is fed into the tube and the transistor in the same way that the signal is fed into the amplifier, and the modulating signal is fed into the grid and the emitter through the transformers $AF_1$ and $AF_2$. The effect of the modulating signal is to vary the biases of the active elements. Inasmuch as both elements are used as class $B$ or $C$ amplifiers, their outputs are dependent on their biases

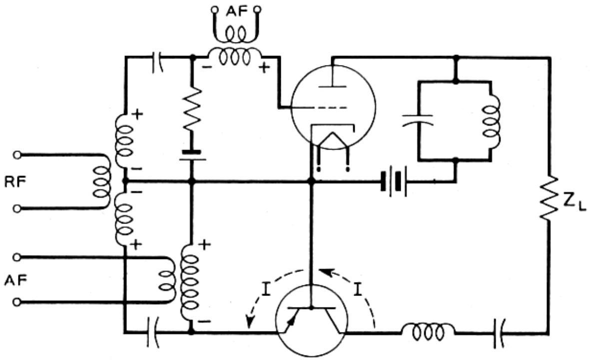



Fig. 28—High efficiency modulator.

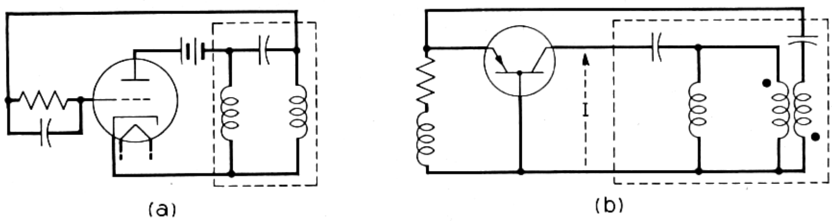



Fig. 29—Hartley oscillator and dual.

If the *RF* signal is large enough, and if the phases and turns ratios of the transformers are carefully chosen, the amplitude of the output will be nearly proportional to the modulating signal. A similar modulator can be based on the circuit of Fig. 10.

Figure 29 shows a Hartley oscillator and its dual. The configuration of elements in the Hartley oscillator may seem unfamiliar, but is chosen deliberately to emphasize the point of view that the Hartley oscillator is an amplifier with feedback through a coupling network. The part of the circuit enclosed in dotted lines is the coupling network. The dual circuit is also an amplifier with a coupling network, but because of the fact that the vacuum tube has an inherent phase reversal and the transistor has not, an extra transformer is needed in the coupling network. It is conveniently placed as shown, where it can be combined with the inductor. The input circuits to the amplifiers have self bias circuits which have already been described.

The Colpitts oscillator is similar to the Hartley oscillator. The difference lies in the coupling circuit. Figure 30 shows the coupling circuit for the Colpitts oscillator and its dual. The tuned-plate tuned-grid oscillator can be treated in exactly the same way.

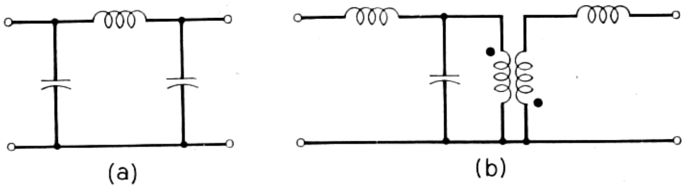



Fig. 30—Colpitts coupling network and dual.

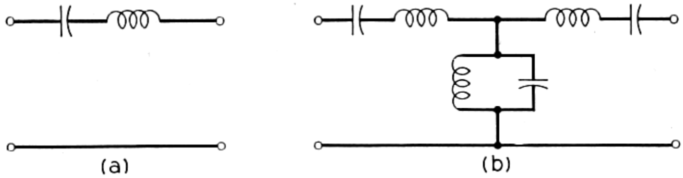



Fig. 31—Transistor oscillator coupling networks.

The coupling circuits used in successful vacuum tube oscillators are characterized by having a phase shift of 180°. This can be done with structures like low- or high-pass filter sections, by *R-C* networks, and in other ways. On the other hand, the coupling network required for a good transistor oscillator must have zero (or 360°) phase shift. It is therefore probably most easily arrived at by designing a 180° phase shifting network and adding a phase inverting transformer. Two simple coupling networks which have zero phase shift are shown in Fig. 31. These are both band-pass structures which lead to oscillation in the pass band. Of the two, the network of Fig. 31(b) gives a more rapid change of phase with frequency and hence leads to a more stable oscillator. In addition, this circuit has the potential advantage of providing means for matching impedances.