Internal Temperatures of Relay Windings

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The steady state temperature distribution of a relay winding depends upon the power supplied and upon the rates of heat removal at the inner and outer surfaces. This is analyzed in terms of a more general form of the temperature distribution relation discussed by Emmerich (*Journal of Applied Physics*)[1]. This analysis is used to determine empirical constants for the rates of heat removal at the surfaces. Illustrative data are given for a stepping magnet.

Introduction

EMMERICH[1] has developed an expression for the steady state temperature distribution in a magnet coil when the heat flow is wholly radial, and the temperatures of the inner and outer surfaces are the same. A more general problem arises in the case of relays and other electromagnets used in telephone switching apparatus. In these cases, the coil is mounted on an iron core. Heat is withdrawn from the coil partly by conduction through this metal path, and partly by radiation from the outer surface. In consequence of this, the temperatures of the inner and outer surfaces are in general different.

In the relay and switching magnet problem, primary interest attaches to the rate at which heat is withdrawn through these two paths, as their combined effect determines the maximum temperature attained within the coil. The analysis outlined below has been employed to determine the division of heat between these two paths, and for the evaluation of empirical constants of heat removal. These constants are used in estimating the relation between the temperature of the winding and the power supplied to it.

Theory

As in Reference (1), it is assumed that there is no heat loss through the ends of the coil, so that the temperature gradient is wholly radial, and that the actually heterogeneous coil structure can be treated as homogeneous. Then if Q is the heat supplied per unit volume per unit time, and K is the thermal conductivity, the radial distribution of temperature is the solution to Poisson's equation:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{Q}{K} = 0, \tag{1}$$

[1] C. L. Emmerich, Steady-State Internal Temperature Rise in Magnet Coil Windings, *Journal of Applied Physics*, 21, 75, 1950.

where r is the co-ordinate of a surface of temperature T. The general solution to equation (1) is given by the equation:

$$T = A + B \log r - \frac{Qr^2}{4K}, \tag{2}$$

where A and B are constants determined by the boundary conditions. The temperature has a maximum value T' at some radius r', at which the temperature gradient $dT/dr = 0$. Substituting the expression for T given by equation (2) in this condition, it is found that:

$$r'^2 = \frac{2KB}{Q}. \tag{3}$$

If the expression for B given by equation (3) is substituted in equation (2), and if A is taken as given by the resulting expression for T' when $r = r'$, equation (2) may be written in the form:

$$T = T' + \frac{Qr'^2}{4K}\left(1 + 2 \log \frac{r}{r'} - \left(\frac{r}{r'}\right)^2\right). \tag{4}$$

This equation gives the general expression for the temperature distribution in terms of the radius r' at which the temperature has its maximum value T'. In the special case in which the temperature T_1 at the inner radius r_1 is the same as that at the outer radius r_2, substitution in equation (4) of $r = r_1$ and $r = r_2$ gives two expressions for T_1. From these there can be obtained the same expression for the radius r' of maximum temperature as is given in Reference (1) for this special case. In the notation used here this expression is:

$$r'^2 = \frac{r_2^2 - r_1^2}{2 \log \frac{r_2}{r_1}}. \tag{5}$$

Substitution of this expression for r' in equation (4) gives an expression for $T - T_1$ which is identical with that given by equation (18) of Reference (1).

Using the expression for T given by equation (4), integration of $2\pi rTdr$ over the interval r_1 to r_2, and division of this integral by $\pi(r_2^2 - r_1^2)$, the coil volume per unit length, gives the following expresson for the mean coil temperature $\overline{T}$:

$$\overline{T} = T' + \frac{Qr'^2}{2K}\left(\frac{r_2^2 \log \frac{r_2}{r'} - r_1^2 \log \frac{r_1}{r'}}{r_2^2 - r_1^2} - \frac{r_1^2 + r_2^2}{4r'^2}\right). \tag{6}$$

Experimental

By means of equation (4), coil temperature measurements may be analyzed to determine the thermal conductivity and the rates of heat removal from the inner and outer surfaces of the coil. If the heat flow is wholly radial, all the heat supplied must pass through one or the other of these surfaces. The division of the heat between the two paths is determined by the radius r' of maximum temperature. The rate of heat flow to the core is therefore the rate of heat supply per unit volume Q, multiplied by the volume of the coil inside the radius r', or $\pi\ (r'^2 - r_1^2)$ per unit length of coil. Similarly, the rate of heat flow through the outer surface per unit length of coil is $Q.\pi\ (r_2^2 - r'^2)$.

It is therefore formally possible to determine the heat division by measuring the temperature distribution, and reading the radius of maximum temperature directly from it. When this is done it is found that the temperature gradient is comparatively flat in the vicinity of the maximum, and that it is therefore difficult to measure r' directly. An indirect determination of the radius r' may be made, however, by determining the maximum temperature T' and the temperatures T_1 and T_2 at radii r_1 and r_2 respectively. Expressions for T_1 and T_2 are obtained from equation (4) by letting $r = r_1$ in the one case and r_2 in the other. Then from these two expressions:

$$\frac{T_1 - T'}{T_2 - T'} = \frac{1 + 2\log\frac{r_1}{r'} - \left(\frac{r_1}{r'}\right)^2}{1 + 2\log\frac{r_2}{r'} - \left(\frac{r_2}{r'}\right)^2}. \tag{7}$$

Knowing T_1, T_2, T', and the radii r_1 and r_2, r' may be evaluated by numerical or graphical solution of equation (7). This solution is facilitated by the use of a table or plot of the function $Y\ (X) = 1 + \log X^2 - X^2$. The numerator of the right hand side of equation (7) is $Y\ (r_1/r')$, and the denominator is $Y\ (r_2/r')$.

A convenient procedure for determining the temperature distribution within a coil is to measure the resistance changes in different layers, tapping the coil between these layers. If there is more than one layer between taps, the resistance change measures the mean temperature of the layers included.

For an accurate determination of the gradient, it is convenient to make the resistance measurements on a comparative basis, as by use of the bridge circuit shown in Fig. 1. Here the terminals marked 0 and 6 are the inner and outer ends of the winding, while terminals 1 to 5 denote taps at intermediate layers. With the key K in Position 2, the bridge circuit may be balanced to establish the resistance between terminals 0 and 3, for example, relative to that between terminals 3 and 6. The resistance of the whole winding may be determined with sufficient accuracy (about one per cent) by voltmeter-ammeter readings made with K in Position 1. With this known, the resistance of the layers between taps, and hence the temperature differences, can be computed from the bridge readings.

The temperature corresponding to the resistance between taps may be taken as the temperature at the mean radius of the layers included. The

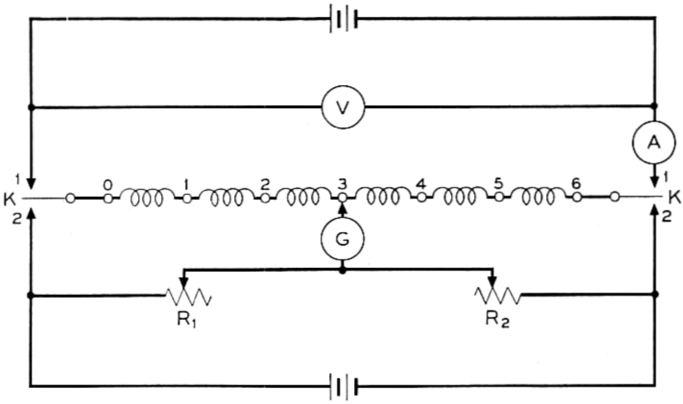

FIG. 1—Circuit for measuring temperature differences in tapped coils.

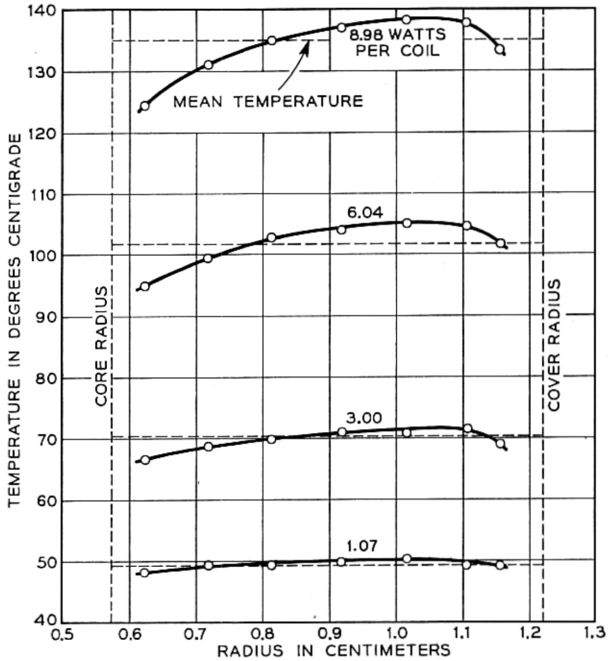

FIG. 2—Temperature distribution in coil of 197 switch magnet.

results of such measurements may be plotted as shown in Fig. 2. The results given in this figure were obtained with a sample of the Vertical Magnet of the Western Electric Company's 197 (Step-by-Step) switch.

The plotted points show the mean temperature of the layers between taps plotted against their mean radius; the dashed boundary lines show the inner and outer radii of the coil. The different curves correspond to the different levels of steady state power input indicated.

It is apparent in Fig. 2 that the central part of each curve is comparatively flat, and that it would therefore be difficult to determine the radius of maximum temperature directly. It may be noted in passing that the maximum temperature is not greatly in excess of the mean temperature. This justifies the usual engineering practice of taking the mean temperature as a criterion of whether the coil is overheated or not.

TABLE I
HEAT DISTRIBUTION IN 197 SWITCH COILS

Total Heat Input (Watts)	1.07	3.00	6.04	8.98
Q (watts per Cm.^{3})	0.096	0.268	0.540	0.802
Max. Temperature T' (°C)	50	72	105	139
Temperature T_1 (°C)	48	67	95	124
Temperature T_2 (°C)	49	69	102	132
K (Calories/°C/sec./cm.)	0.53×10^{-3}	0.69×10^{-3}	0.79×10^{-3}	0.82×10^{-3}
r' (Cm.)	0.92	0.93	0.95	0.94
Heat to Core (Watts)	0.48	1.39	2.99	4.37
Heat to Cover (Watts)	0.59	1.61	3.05	4.61
(Per Cent.)	55	54	51	51
Inner Surface Temperature (°C)	47	65	91	119
Outer Surface Temperature(°C)	49	68	99	130

From each curve in Fig. 2 there was read the maximum temperature T' and the temperatures T_1 and T_2 at $r_1 = 0.622$ cm. and $r_2 = 1.156$ cm. respectively, corresponding to the inner and outer points plotted. These temperatures are listed in Table I together with the corresponding values of Q, the power input divided by the volume of the coil (11.2 cm^3). From these data, the radius r' of maximum temperature has been computed by the procedure described above. With r' known, the quantity $\frac{Q}{K}$ was computed from equation (4) for the case $r = r_1$, $T = T_1$. The resulting values of r' and K are included in Table I. Using the value of r' thus determined, the division of the heat between that going to the core and that going to the cover was computed with the results shown in the table. The values of r_1 and r_2 used in the above computation are, as indicated in Fig. 1, internal to the coil. Taking new values of r_1 and r_2 corresponding to the core and cover radii respectively, the temperatures at these surfaces were computed from equation (4), using the values already found for r', T and $\frac{Q}{K}$. These core and cover temperatures are included in Table I.

It is of some interest to examine the observed values of apparent conductivity K. As the temperature differences for the two lower input measurements are small, the results for the other two cases are more accurate. The latter give a value for K of approximately 0.8×10^{-3} calories per °C per sec. per cm. In this form wound coil using No. 29 wire, the volume occupied by the insulation is approximately 36 per cent. As the conductivity of the copper is very large compared with that of the insulation, the conductivity of the latter may be estimated as of the order of K multiplied by the fraction of the volume occupied by the insulation. This gives an indicated conductivity for the insulation of the order of 3×10^{-4} calories per °C per sec. per cm. which is about the same as that of dry paper.

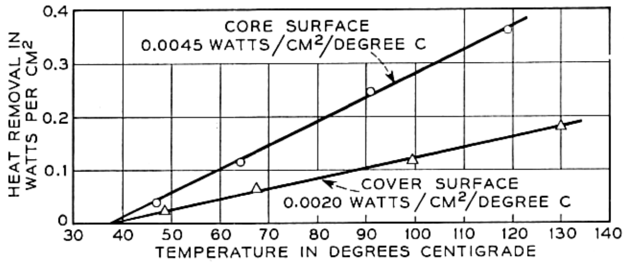

FIG. 3—Rates of heat removal from coil of 197 switch magnet.

For engineering purposes the results of major interest are those for the quantities of heat leaving the core and the cover. The results in Table I show that the heat flow is about equally divided between these two paths. As the core radius is about half the cover radius, the rate of heat flow to the core per unit area of surface is about twice that leaving unit area of the cover surface.

The rates of heat removal per unit area through the inner and cover surfaces are shown plotted against the corresponding temperatures in Fig. 3. It will be seen that the relation between the rate of heat removal per unit area and temperature is approximately linear, and intersects the temperature axis at 38°C (100°F), which was the ambient temperature in these tests.

It follows that for engineering purposes the rate of heat removal per unit area through either the inner or cover surface may be taken as proportional to the difference between the surface and ambient temperatures. A similar linear approximation has been found to apply for other relay and switch coils when mounted under conditions representative of telephone apparatus. Because of the multiplicity of mounting conditions and the complexity of conducting and radiating paths by which heat is removed, it is difficult to establish a relationship of the type shown in Fig. 3 by analysis of the paths of heat removal. For given mounting conditions, however, this relationship can be determined empirically by the procedure outlined above and used to estimate the heat removal from a given coil mounted under conditions for which such measurements have been made.

The rate of heat removal is thus measured by the slopes of the heat flow vs. temperature curves, which may be designated k_1 and k_2. Thus k_1 is the time rate of heat flow to the core per square centimeter of surface per °C difference between the inner coil surface and the ambient temperatures, while k_2 is the corresponding coefficient for the cover surface. For the case shown in Fig. 3, $k_1 = 0.0045$ watts per cm^2 per °C, and $k_2 = 0.0020$ watts per cm^2 per °C. This observed value of k_2 is characteristic of the cover surfaces of coils mounted as in telephone apparatus, where the heat removal is primarily by radiation to surfaces at or near the ambient temperature. While the value of k_1 observed for this case is representative of that applying to inner coil surfaces, the values of k_1 for such surfaces vary widely, and are particularly sensitive to variations in the clearance between the metallic core and the interior surface of the coil.

Prediction of Coil Temperatures

If values for the heat removal coefficients are known, the distribution of temperature within the coil for a given steady state power input may be determined from equation (4). The power input and the coil volume determine the rate of heat supply per unit volume Q. The rate of heat flow to the core per unit length of coil is therefore $\pi\,(r'^2 - r_1^2)\,Q$. The core area per unit length through which this heat passes is $2\,\pi r_1$. So from the empirical linear relation between the heat removed and the surface and ambient temperatures:

$$T_1 - T_0 = \frac{(r'^2 - r_1^2)Q}{2k_1 r_1}, \tag{8a}$$

where T_0 is the known ambient temperature. Similarly, for the cover surface:

$$T_2 - T_0 = \frac{(r_2^2 - r'^2)Q}{2k_2 r_2}. \tag{8b}$$

By substituting these expressions for T_1 and T_2 for T in equation (4), with r taken as r_1 in one case and r_2 in the other, there is obtained an expression for r' which reduces to the following equation:

$$r'^2 = \frac{\dfrac{r_1}{k_1} + \dfrac{r_2}{k_2} + \dfrac{r_2^2 - r_1^2}{2\mathrm{K}}}{\dfrac{1}{k_1 r_1} + \dfrac{1}{k_2 r_2} + \dfrac{1}{K}\log\dfrac{r_2}{r_1}}. \tag{9}$$

If r' is thus determined, and K is known from measurements of similar coils, T' can be determined by means of equation (4) from the values of T_1 or T_2 given by equations (8). If desired, the mean temperature $\bar{T}$ can then be determined by equation (6).

Conclusions

In most relay and switch magnet coils, of the type used in telephone apparatus, the heat flow under steady state conditions can be considered as wholly radial, and the temperature distribution conforms approximately to equation (4) above. In this expression, T' is the maximum temperature, and r' the radius at which this occurs, so that heat generated inside the surface of radius r' passes to the core, and that generated outside this radius passes to the cover. The rate of heat removal per unit area at either of these surfaces is found experimentally to be approximately proportional to the difference between the surface and ambient temperatures (for the temperature range of normal operation). The proportionality constant is the heat removal coefficient (k_1 or k_2 of equations (8)). Under conditions typical of telephone apparatus, this coefficient is of the order of 0.002 watts per cm^2 per °C for a cover surface, and 0.005 watts per cm^2 per °C for an inside surface in close proximity to the metal core. The heat removal coefficient for an inner surface is much more variable than that for a cover surface.

The coil temperature distribution [equation (4)] depends upon the rate of heat supply per unit volume Q, and upon the effective heat conductivity K. Q may be taken as equal to the total steady state power input divided by the coil volume. Correction might be made for the radial variation in Q resulting from the variation in copper resistivity with temperature. The relatively small temperature range observed in practice, as illustrated by the results of Fig. 2, makes this an unnecessary refinement. The heat conductivity constant K is an effective average value, applying to the coil as though it were a homogeneous structure. It is conveniently evaluated by measurements of actual coils, and is approximately a constant for a given wire size and type of insulation.

By measurement of the resistance changes in tapped coils, the internal temperature distribution can be determined. From this, values of the effective heat conductivity K and of the heat removal coefficients k_1 and k_2 can be determined by the use of equation (4), as described above. Conversely, if K and the heat removal coefficients are known, equation (4) may be used to estimate the internal temperature distribution, and thus the mean and maximum coil temperatures.