An Aspect of the Dialing Behavior of Subscribers and Its Effect on the Trunk Plant

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Introduction

DURING the war it became necessary for the Bell System Companies to lower many service standards. Among these was the standard for the provision of trunks for handling subscriber-dialed calls. In the interest of economy the number of trunks for a given volume of traffic was lowered. It is evident that for any given case there is a lower limit to the number of trunks that should be provided for handling subscriber-dialed calls. Below this limit congestion of calls gets beyond control. The control of congestion is important. In the case of operator-handled calls it is possible to control congestion by filing tickets and placing calls in an orderly fashion. In the case of subscriber-dialed calls the subscriber may with impunity make many, indeed very many, successive dialing attempts to complete a call that is blocked due to a shortage of trunks. If, in a particular office enough subscribers do this simultaneously, a sender shortage may develop with its resulting reaction on the whole office.

From the foregoing it is evident that the standard of service for providing trunks in trunk groups handling subscriber-dialed calls is of importance. During the war years, the New York Telephone Company undertook a study to determine the limits below which it would be undesirable to degrade the service. This study was designed to test the reasonableness of the reduction in the inter-office trunk standard from the pre-war basis of providing enough trunks to delay only one out of a hundred calls in the busy hour to a wartime basis of providing enough trunks to delay two calls in every hundred during the busy hour. The conclusion from this study was that it was safe to use wartime standards.

The study reported herein is an analysis of the effect of repeated attempts when subscriber-dialed calls are blocked due to trunk shortages. The data upon which the results are based indicate that dial subscribers after encountering a *busy* condition make new attempts sooner and much more often than has been generally believed. The results indicate that one can reconstruct what happens when trunk groups carrying subscriber-dialed calls encounter serious overloads and that trunk capacity tables for such situations can be developed.

The study is based on extensive service observations taken at the New

York City Service Observing Bureaus during the winter of 1943–44. These observations dealt with the behavior of subscribers who encounter a *busy* on a dialed call. This behavior is assumed to apply to the situation when subscribers encounter an *all-trunks-busy* condition.

Inadequacy of the Poisson and Erlang B Formulae To Express the Situation when Shortages Occur in Trunk Groups Handling Subscriber Dialed Calls

In connection with the provision of trunks in the exchange plant, two sets of trunk-call-carrying-capacity tables are currently in use. One set of these tables is computed from the Poisson Formula and the other from the Erlang B Formula. The Poisson tables are used for trunk groups carrying non-alternate route traffic, whereas the Erlang B tables are used for trunk groups carrying traffic subject to alternate routing.

The assumption underlying the Poisson Formula, when a shortage of trunks occurs, is that of a partial delay. A call which encounters *all trunks busy* waits but not longer than a holding time interval for a trunk to become available.

The corresponding assumption underlying the Erlang B Formula is that of no delay. A call which encounters *all trunks busy* is cleared out. The call may be abandoned by the subscriber or advanced to an alternate route.

With respect to non-alternate route trunk groups handling subscriber dialed calls neither of the above two assumptions is realized in practice. When all trunks are busy, the dial equipment is arranged to return an *all-trunks-busy* signal to the subscriber rather than hold the call pending the outcome of a subsequent test for an idle trunk. The subscriber upon encountering an *all-trunks-busy* signal does not necessarily abandon the call. In most cases he redials the call.

The degree by which the assumptions are not realized depends upon the relative number of trunks that are provided for a given volume of traffic. For instance if, during an hour, 150 calls having an average holding time of 100 seconds are submitted to ten trunks and an equivalent volume of traffic is submitted to five trunks, the following theoretical results follow from the Poisson and Erlang B Formulae:—

Table I

Theoretical Results from Poisson and Erlang B Formulae

150 Calls of 100 Seconds Average Holding Time Submitted during an hour to	Number of Calls that Are Delayed on the Basis of the Poisson Formula	Number of Calls that Are Cleared Out on the Basis of the Erlang B Formula
10 trunks	1.6	1.0
5 trunks	60.6	32.0

The values in Table I indicate that, when a liberal number of trunks, i.e., ten trunks, is provided, the numerical difference between the results of the two formulae is small and the results of either formula can be used as an approximation of the number of calls affected by an *all-trunks-busy* condition. There are undoubtedly repetitious attempts, but because the number is small their effect can be neglected.

When, however, there is a serious shortage of trunks, as when only five trunks are provided, the numerical difference between the theoretical results of the two formulae is large. In addition, the repetitious attempts will be too numerous to ignore. Some of the repetitious attempts will encounter *all trunks busy* again and again. Other repetitious attempts will seize idle trunks thereby causing new calls to encounter *all trunks busy*. The effect is cumulative. Neither the Poisson nor the Erlang B Formula indicates to what extent the repetitious attempts take place nor their effect. A preliminary glimpse at the results of this study indicates that 150 calls of 100 seconds average holding time when submitted during an hour to five trunks become inflated by 99 repetitious attempts and appear as 249 calls being submitted to the trunks. Of these 249 calls, 108 encounter *all trunks busy*. Of the 108 calls, 99 become the aforementioned repetitious attempts and nine are abandoned. It is evident that neither formula presents this picture. For studies considering the effect of overloads due to trunk shortages, this is the type of information needed. A new approach is required to obtain such data. To do this, it is desirable to examine the habits of dial subscribers who have encountered *busies*.

The Dialing Behavior of Subscribers upon Encountering a *Busy*

In order to investigate the grade of service given to dial subscribers when trunk shortages occur it is desirable to know something about their behavior when they encounter *all-trunks-busy* signals. Specifically there are four items that need investigation; these are:—

1. How soon after encountering an *all-trunks-busy* signal does the subscriber redial his call?
2. What percentage of the subscribers make subsequent attempts?
3. How do the time intervals between successive subsequent attempts compare with each other; that is, are they about the same or do they differ widely?
4. What differences, if any, exist between classes of subscribers?

The first three items are answered from the results of service observations. The fourth item is answered indirectly.

The service observations consisted of 1,107 cases where line *busies* were observed (except for 35 cases of *all-trunks-busy* signals). Observations on line *busies* were used instead of *all-trunks-busy* signals because it would have taken too long to obtain sufficient observations, because it is undesirable to artificially degrade the service in order to obtain sufficient observations and because it is assumed that the average subscriber does not recognize the difference between a *busy* and overflow signal. It is considered that the data, while collected for *busy* signals, accurately represent the situation with regard to overflow signals.

Beginning on December 22, 1943 and ending on February 29, 1944, a special record of 1,107 subscriber-dialed calls, where line *busies* were observed, was taken at the three New York City service observation bureaus. Up to a point, regular service observation practices were followed and the regular service observing data concerning the calls were entered on the service-observing records. The data concerning the line *busies* were entered on a special form. This form is shown below. Instructions for the observers accompanied these forms; these instructions follow the form.

Form S.O. 171

SPECIAL RECORDS—*BUSY* CALLS

Calling No. . . . . Date . . . .

Enter in space under attempt number, the cumulative seconds from the start of the original attempt to the start of the attempt indicated. In addition for the last attempt show disposition.

Attempt Number

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Disposition of the call . . . .

Data for attempts over 20 should be entered on the reverse side.

Special service observing form used to collect data concerning the dialing behavior of subscribers upon encountering a *busy*.

Instructions Applying to the Use of Form S.O. 171

These instructions apply to the use of Form S.O. 171 which has been developed in connection with a study of the behavior of customers upon encountering a *busy* signal.

This study will not include observations originating on P.B.X. trunks or on coin lines. On all other calls encountering a *busy* signal or an overflow signal the observer will hold the line in the observing position until one of the following conditions occurs:

(1) Call is disposed of by reaching the desired number.—Code OK

(2) 10 minutes have elapsed since the last attempt for the desired number.—Code AB
(3) Call is disposed of by being given to the operator.—Code PR
(4) Call is disposed of by receiving a "Don't answer" on an attempt to reach the desired number—Code DA

All attempts made during the period that an observation is ordinarily held will be entered on the service observing detail sheets in the regular way. In addition, these entries and entries showing any other attemps to reach the desired number together with the proper code listed above to show the final disposition of the call will be recorded on Form S.O. 171.

In order to minimize the number of cases not completed at the end of an observer's trick, no cases will be recorded on the special record on which the original *busy* signal is received after $\frac{1}{2}$ hour prior to the finish of any trick.

From the instructions it may be noted that observations originating on P.B.X. trunks or on coin lines were not included. The reason for this is, when a *busy* is observed on a call originating on a P.B.X. trunk the subsequent attempt might be made on one of the other P.B.X. trunks, thus the subsequent attempt would be missed. Also, at a P.B.X. two extensions may place calls, within a few seconds of each other, to the same *busy* line. The service observations on any one trunk might therefore be a mixture of attempts involving two or more calls. When a *busy* is observed on a call made from a coin line, the calling party will in many instances vacate the coin box in favor of someone else, and the subsequent attempt may then be made from another coin line. For these reasons the observations were restricted to business and residential individual lines and to two-party lines (12 observations were on two-party lines).

It may also be noted that the observers were instructed to hold the line in the observing position until ten minutes have elapsed since the last attempt for the desired number. This was a departure from regular service observing practices when a line is held until 1 minute has elapsed.

Table II is a tabulation of the data observed at the Manhattan Service Observing Bureau on Manhattan dial subscriber lines. The observations are arranged in the order of increasing magnitude of the time intervals between the start of the first attempt and the start of the second attempt. Of interest is observation number 197 where a subscriber made 25 attempts in about an hour.

Data similar to that observed on Manhattan dial subscriber lines were likewise observed on Bronx-Westchester and on Brooklyn-Queens dial subscriber lines.

Figure 1(a) shows graphically the data listed in Table II. This graph shows, by dots, the cumulative percentage of the 451 Manhattan observa-

TABLE II

RESULTS OF OBSERVATIONS ON 451 DIAL SUBSCRIBERS IN MANHATTAN

Seconds elapsing between start of previous attempt and start of attempt listed below:

Observation No.	Attempt No. 1	2	3	4	5	6	7	8	9	10	Total Seconds	Disposition of the Call
1	0	13	24	13	11	20					81	O.K.
2	0*	16*	10*								26	AB.
3	0	16	48	54	82	108					308	O.K.
4	0	18									18	D.A.
5	0	19									19	PR.
6	0	19									19	O.K.
7	0*	20									20	O.K.
8	0	20									20	O.K.
9	0	20	64	189							273	PR.
10	0	21									21	AB.
11	0*	21									21	O.K.
12	0	21									21	PR.
13	0	21	208								229	O.K.
14	0	22	30	28							80	O.K.
15	0	22	22								44	O.K.
16	0	22	26	189	18	25					280	PR.
17	0	23									23	O.K.
18	0*	25									25	O.K.
19	0	25	28	33							86	O.K.
20	0	25	341	44							410	AB.
21	0	25									25	AB.
22	0	25	28	22	69	63	82	271			560	AB.
23	0	26	188								214	O.K.
24	0	27	35	35	29	38	36	42	43	53	338	O.K.
25	0	27									27	AB.
26	0	27									27	PR.
27	0	28									28	O.K.
28	0	28									28	AB.
29	0	29	110	22							161	PR.
30	0	30									30	AB.
31	0	30	31	23	20	20	86	20	54	26		
	51										361	AB.
32	0	30	440								470	O.K.
33	0	30									30	AB.
34	0	31	52	31	591						705	O.K.
35	0	31	45								76	AB.
36	0*	31									31	O.K.
37	0	31	105	66							202	O.K.
38	0	31									31	O.K.
39	0	31	98								129	O.K.
40	0	32									32	D.A.
41	0	32									32	AB.
42	0	32	32								64	O.K.
43	0	32									32	O.K.
44	0	33*									33	AB.
45	0	33	35	41	43	57					209	O.K.
46	0	35									35	O.K.
47	0	35	358								393	O.K.
48	0	36	88								124	O.K.
49	0	37	53	40							130	O.K.
50	0	39									39	O.K.
51	0	39									39	O.K.

*Overflow signal.

TABLE II (*Cont'd*)

Observation No.	Attempt No. 1	2	3	4	5	6	7	8	9	10	Total Seconds	Disposition of the Call
52	0	40	20	574							634	O.K.
53	0	40*	16	200	432						688	O.K.
54	0	40	409								449	O.K.
55	0	40									40	O.K.
56	0	41	45	55	52	27	25				245	AB.
57	0	41	45	84							170	O.K.
58	0	42	122	68							232	AB.
59	0	43	40	52	38	259					432	AB.
60	0	44									44	O.K.
61	0	46									46	O.K.
62	0	47	32	47	34	40	69				269	O.K.
63	0	47									47	O.K.
64	0	47	179	251							477	O.K.
65	0	48	64								112	AB.
66	0	49									49	AB.
67	0	49	51	57	62	71	60				350	O.K.
68	0	49	96	191							336	O.K.
69	0	50									50	O.K.
70	0	50									50	O.K.
71	0	50	85	151							286	O.K.
72	0	50									50	AB.
73	0	50									50	AB.
74	0	51									51	O.K.
75	0	51									51	AB.
76	0	52									52	O.K.
77	0	52	85	209							346	O.K.
78	0	53	195								248	O.K.
79	0	53									53	AB.
80	0	53									53	O.K.
81	0	55	43								98	AB.
82	0	55	43	27	170*						295	AB.
83	0	56	20	61	36	103					276	AB.
84	0	56	117	57							230	O.K.
85	0	56									56	O.K.
86	0	56	84								140	O.K.
87	0	57	74	81							212	O.K.
88	0	58									58	O.K.
89	0	58	139	84	163	62	127				633	O.K.
90	0	60									60	O.K.
91	0	60	139								199	O.K.
92	0	60									60	O.K.
93	0	60									60	AB.
94	0	61									61	O.K.
95	0	61									61	AB.
96	0	63									63	AB.
97	0	63	31	95	28	20					237	AB.
98	0	64	126	470	85	167					912	O.K.
99	0	64	61	67	84	67					343	O.K.
100	0	64	45	63	63	161					396	O.K.
101	0	65	482								547	O.K.
102	0	66	173	172							411	O.K.
103	0	66†	66	72							204	O.K.
104	0	66									66	AB.

*Overflow signal.
† Don't answer.

TABLE II (*Cont'd*)

Observation No.	Attempt No. 1	2	3	4	5	6	7	8	9	10	Total Seconds	Disposition of the Call
105	0	68	66								134	O.K.
106	0	68	330	380							778	O.K.
107	0	69									69	AB.
108	0	70									70	O.K.
109	0	71									71	AB.
110	0	71	95								166	O.K.
111	0	72	112								184	AB.
112	0	72									72	O.K.
113	0	74									74	O.K.
114	0	74	184	93							351	AB.
115	0	75									75	O.K.
116	0	75									75	O.K.
117	0	75	67	203							345	O.K.
118	0	76									76	O.K.
119	0	76									76	AB.
120	0*	77									77	O.K.
121	0	78									78	O.K.
122	0	78									78	O.K.
123	0	78	253	107	38						476	AB.
124	0	79	53								132	O.K.
125	0	80									80	O.K.
126	0	80	50								130	O.K.
127	0	80									80	AB.
128	0	80	117								197	O.K.
129	0	81									81	AB.
130	0	81									81	O.K.
131	0	83									83	O.K.
132	0	84									84	O.K.
133	0	85									85	O.K.
134	0	85	33	294	115						527	O.K.
135	0	88									88	AB.
136	0	88									88	O.K.
137	0*	89									89	O.K.
138	0	90	50	120							260	AB.
139	0	90									90	O.K.
140	0	90									90	AB.
141	0	90	51	39	46						226	AB.
142	0	91	78								169	O.K.
143	0	91									91	O.K.
144	0	91*	48								139	O.K.
145	0	91	116								207	O.K.
146	0*	91									91	O.K.
147	0*	92									92	O.K.
148	0*	92									92	O.K.
149	0	93									93	AB.
150	0	93	34	228	117						472	O.K.
151	0	94	94	75	91						354	AB.
152	0	95									95	AB.
153	0	95									95	O.K.
154	0	97	86	175							358	O.K.
155	0	97	143								240	O.K.
156	0	100									100	O.K.
157	0	100									100	O.K.
158	0	100									100	AB.
159	0	100									100	O.K.
160	0	102	115	198							415	O.K.

*Overflow signal.

TABLE II (*Cont'd*)

Observation No.	Attempt No. 1	2	3	4	5	6	7	8	9	10	Total Seconds	Disposition of the Call
161	0	102	80	96	152						430	O.K.
162	0	103									103	O.K.
163	0	104	17								121	O.K.
164	0	105									105	O.K.
165	0	105									105	O.K.
166	0	106	340								446	O.K.
167	0	108	98	140							346	O.K.
168	0	111									111	O.K.
169	0	111									111	AB.
170	0	111	94	125							330	O.K.
171	0	113									113	O.K.
172	0	114									114	O.K.
173	0	116									116	O.K.
174	0	116									116	O.K.
175	0	117									117	O.K.
176	0	120									120	O.K.
177	0	122									122	O.K.
178	0	124	131	209							464	O.K.
179	0	124									124	O.K.
180	0	125	354								479	O.K.
181	0	130									130	O.K.
182	0	130									130	O.K.
183	0	130	125								255	O.K.
184	0	130	56	101							287	O.K.
185	0	131	309								440	O.K.
186	0	134									134	O.K.
187	0	137	147	134	146						564	O.K.
188	0	139	125								264	AB.
189	0	139									139	AB.
190	0	139									139	O.K.
191	0	140	172	60							372	O.K.
192	0	140	400								540	A.B.
193	0	141									141	O.K.
194	0	143									143	O.K.
195	0	143	157								300	O.K.
196	0	144									144	O.K.
197	0	144	187	194	308	115	310	104	165	45		
	69	90	69	88	87	59	239	277	69	94		
	90	159	193	71	237						3,463	AB.
198	0	146									146	O.K.
199	0	146									146	O.K.
200	0	146	184	217							547	AB.
201	0	148									148	O.K.
202	0	149									149	O.K.
203	0	149	28	38	42	46					303	O.K.
204	0	149	121	84							354	A.B.
205	0	150									150	A.B.
206	0	150	26	142	119						437	A.B.
207	0	151	272								423	O.K.
208	0	152	90	95	89	79					505	O.K.
209	0*	155									155	O.K.
210	0	156									156	O.K.
211	0	156									156	O.K.
212	0	156	47	52	217						472	A.B.
213	0	160									160	A.B.
214	0	160									160	O.K.

*Overflow signal.

TABLE II (*Cont'd*)

Observation No.	Attempt No. 1	2	3	4	5	6	7	8	9	10	Total Seconds	Disposition of the Call
215	0	160									160	O.K.
216	0	160									160	O.K.
217	0	161									161	A.B.
218	0	164									164	O.K.
219	0	164									164	O.K.
220	0	165									165	A.B.
221	0	168									168	O.K.
222	0	169									169	O.K.
223	0	170									170	O.K.
224	0*	170									170	O.K.
225	0	171									171	O.K.
226	0	175									175	O.K.
227	0	179									179	O.K.
228	0	180									180	O.K.
229	0	181									181	O.K.
230	0	181	360								541	A.B.
231	0	182									182	O.K.
232	0	183									183	O.K.
233	0	183	312	33							528	P.R.
234	0	185									185	O.K.
235	0	186	251								437	A.B.
236	0	192	238								430	A.B.
237	0	195	477								672	O.K.
238	0	198									198	A.B.
239	0	202									202	O.K.
240	0	205	80								285	O.K.
241	0	208									208	O.K.
242	0	209									209	O.K.
243	0	209									209	O.K.
244	0	210									210	O.K.
245	0	214	50	33	29	34	79				439	O.K.
246	0	215	520								735	O.K.
247	0	215									215	A.B.
248	0	217									217	O.K.
249	0*	219									219	D.A.
250	0	219									219	O.K.
251	0	220	163	263	186	123	99	59	105		1,218	AB.
252	0	220	162								382	O.K.
253	0	222									222	O.K.
254	0	226									226	O.K.
255	0	228									228	O.K.
256	0	230									230	AB.
257	0	231	27								258	P.R.
258	0	232									232	O.K.
259	0	235									235	O.K.
260	0*	235									235	O.K.
261	0	238									238	O.K.
262	0	242									242	O.K.
263	0	245									245	O.K.
264	0	246									246	O.K.
265	0	252									252	AB.
266	0	252									252	AB.
267	0	258									258	O.K.
268	0	260	333								593	O.K.
269	0	267	193								460	AB.
270	0	272	219	88*							579	AB.
271	0	278									278	O.K.

*Overflow signal.

TABLE II (*Cont'd*)

Observation No.	Attempt No. 1	2	3	4	5	6	7	8	9	10	Total Seconds	Disposition of the Call
272	0	281									281	O.K.
273	0	287									287	O.K.
274	0	288									288	O.K.
275	0	289	256								545	O.K.
276	0	290									290	O.K.
277	0	296									296	O.K.
278	0	306									306	O.K.
279	0	319									319	O.K.
280	0	320									320	O.K.
281	0	320									320	AB.
282	0*	322									322	O.K.
283	0	331									331	O.K.
284	0	332									332	DA.
285	0	338									338	AB.
286	0	339									339	AB.
287	0	347									347	O.K.
288	0	351	454								805	O.K.
289	0	351									351	O.K.
290	0	363									363	O.K.
291	0	365									365	O.K.
292	0	369									369	DA.
293	0	376									376	O.K.
294	0	378									378	O.K.
295	0	382									382	O.K.
296	0	395									395	O.K.
297	0	398									398	O.K.
298	0	398									398	AB.
299	0	400									400	O.K.
300	0	402									402	O.K.
301	0	409									409	O.K.
302	0	416									416	O.K.
303	0	448									448	O.K.
304	0	449									449	O.K.
305	0	455									455	O.K.
306	0	473									473	O.K.
307	0	484									484	O.K.
308	0	484									484	A.B.
309	0	498									498	O.K.
310	0	505									505	O.K.
311	0	509									509	O.K.
312	0	510									510	A.B.
313	0	513									513	O.K.
314	0	526									526	O.K.
315	0	535	456	541							1,532	O.K.
316	0	543									543	O.K.
317	0	556	249								805	O.K.
318	0	561	389								950	O.K.
319	0	568									568	O.K.
320	0	569									569	O.K.
321	0	570									570	O.K.
322	0	586									586	O.K.
323	0	605	(over 600 seconds)								605	A.B.
324	0	624	(over 600 seconds)								624	A.B.
325	0	(At 30 seconds received on incoming call from the party desired)										A.B.
326–334	0*	(9 observations)										A.B.
335–451	0	(117 observations)										A.B.

*Overflow signal.

tions that equalled or exceeded particular time intervals between the starts of the first and second attempts. Figure 1(b) shows similar graphical data for 211 Bronx-Westchester observations and Fig. 1(c) shows similar graphical data for 445 Brooklyn-Queens observations. Each of these three graphs is compared with a composite curve for 1107 observations. This composite curve is developed from the data on Fig. 2(a).

Figure 2(a) shows, by dots, the cumulative percentage for 1107 observations, which are comprised of the 451 Manhattan, 211 Bronx-Westchester and 445 Brooklyn-Queens observations, that equalled or exceeded particular time intervals between the starts of the first and second attempts. A smooth curve was drawn through these plotted data. This curve is also shown on other figures, for the purpose of visual comparison of the various plots of data with the overall results.

Figure 2(b) shows a graph concerning 465 observations of the total 1107 observations. These are the cases where a *busy* was observed on a second attempt. (Of the 1107 total observations, 817 resulted in a second attempt within ten minutes and 290 were classified as abandoned. Of the 817 second attempts, 327 cases were able to complete their calls, 16 resulted in a don't answer, 9 were referred to an operator and 465 encountered a *busy*.) Figure 2(b) shows, by dots, the cumulative percentage of the 465 second attempts that equalled or exceeded particular time intervals between the starts of the second and third attempts. The graph of Fig. 2(b) does not differ significantly from the composite curve for 1107 observations. This feature indicates that, when observations concerning subscriber *busies* are made, it is not necessary to have the first observed attempts coincide with the first actual attempts. The observations can begin with any attempt.

Figures 3 and 4 are graphs similar to that shown on Fig. 2(a), the difference being in the graphical ordinates used in order to present additional pictorial representations of the data and to project the curve beyond the observed limits.

The percentage of subscribers who dial their calls again after encountering *busies* is estimated from Figs. 3 and 4 to be 90%. The data on Fig. 3 are projected to a time interval of 1,500 seconds (25 minutes). Judging by eye, beyond this point, it appears that the curve is asymptotic to the 10% horizontal line. This means that 10% of the subscribers abandon their calls and 90% try again. The part of the curve on Fig. 4 that projects beyond the limit of the observed data crosses the 10% line at 6,400 seconds, an interval of $1\frac{3}{4}$ hours. This seems to be a very long time for a subscriber to wait before redialing his call. It is unlikely that many attempts are made beyond this period.

Table III was prepared to determine the disposition of the calls on second attempts and to see if a correlation exists between certain time intervals,

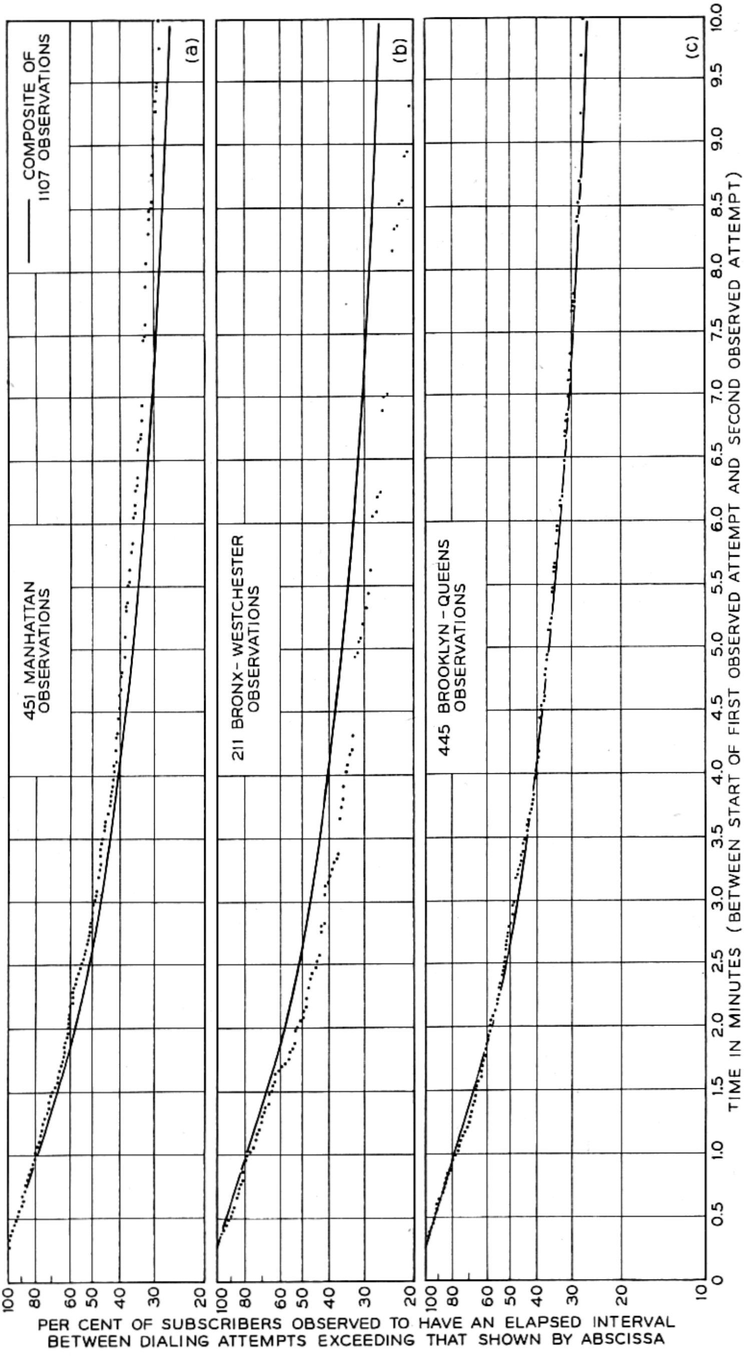

Fig. 1—Results of observations taken at three New York City service observing bureaus concerning the dialing behavior of subscribers upon encountering a *busy*.

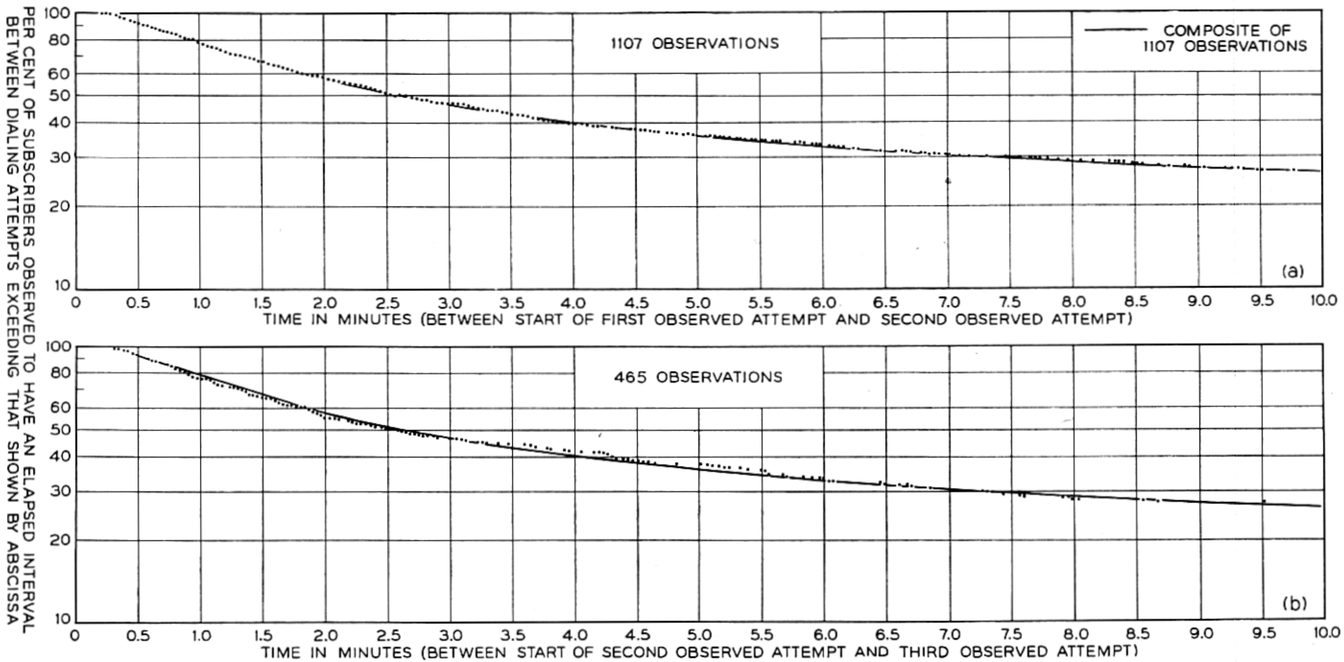

Fig. 2—Composite results of 1107 observations concerning the dialing behavior of subscribers upon encountering a *busy* on a first attempt and results of 465 observations concerning the dialing behavior of subscribers upon encountering a *busy* on a second attempt.

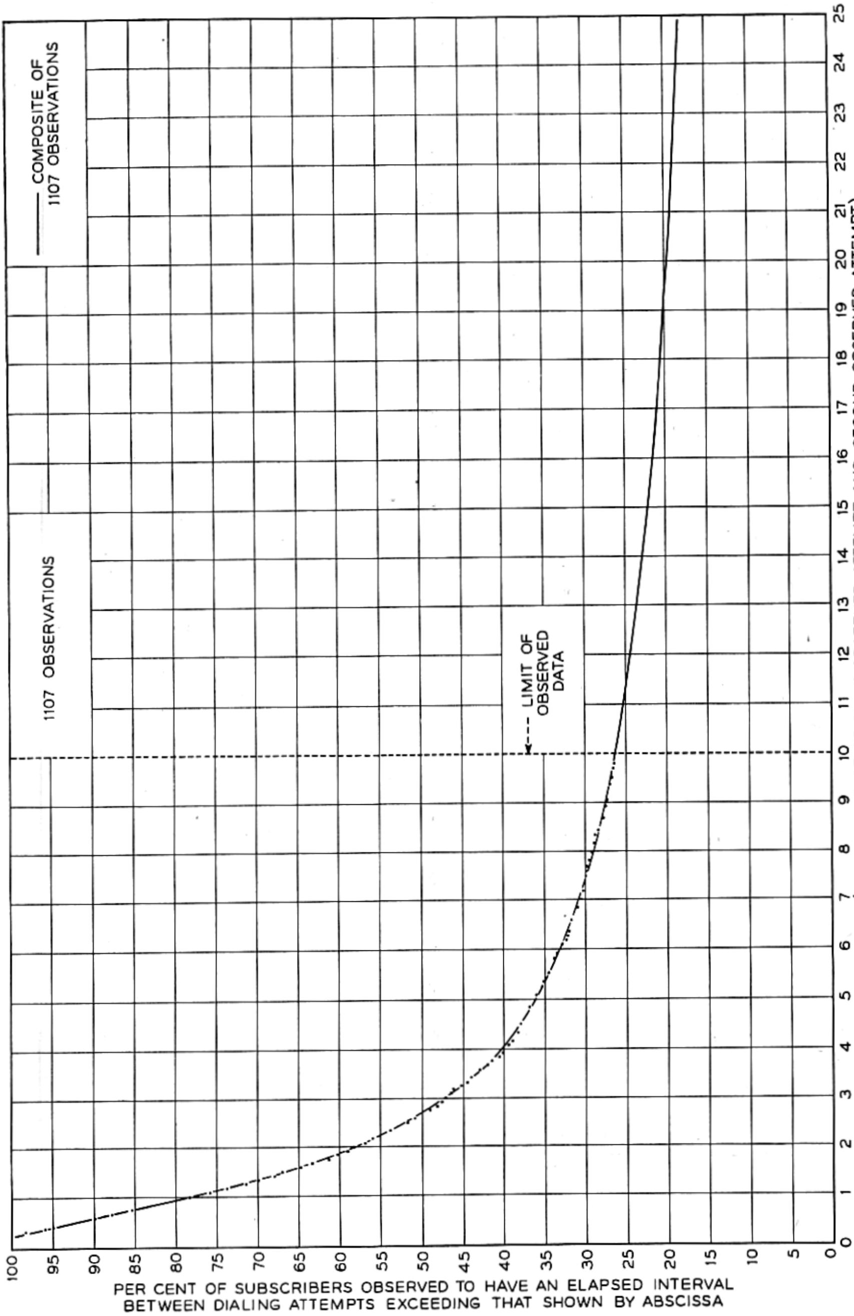

Fig. 3—Composite results of 1107 observations concerning the dialing behavior of subscribers upon encountering a *busy*.

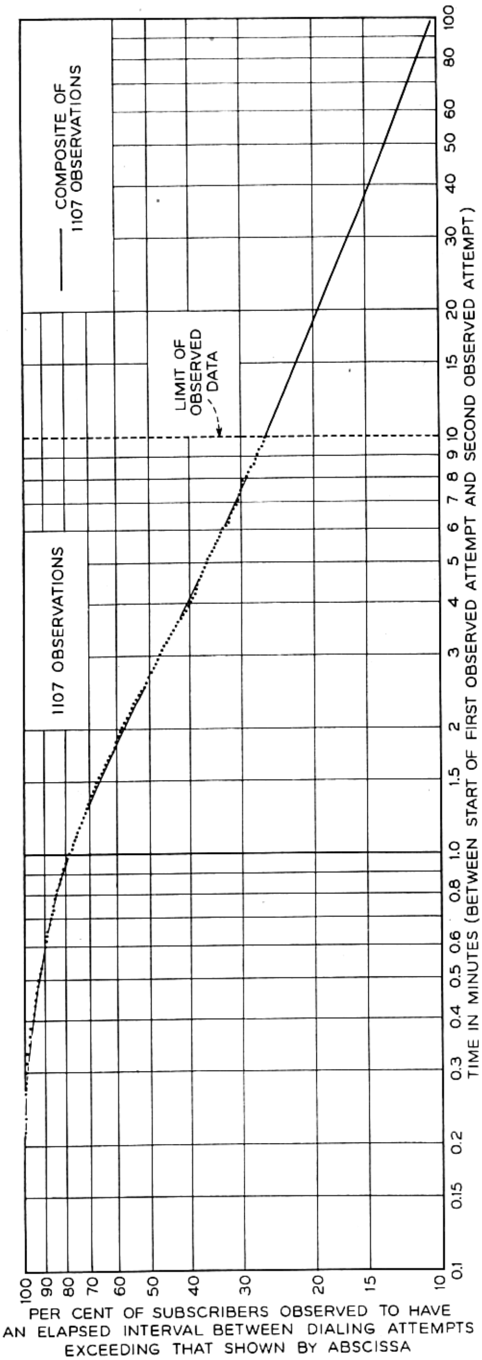

Fig. 4—Composite results of 1107 observations concerning the dialing behavior of subscribers upon encountering a *busy*.

namely, between the first and second attempts and between the second and third attempts. This table was developed by allocating the 817 observations where a second attempt occurred into 5 ranges of time intervals between the first and second attempts of about 163 observations each. For each range of time interval the number of calls that were respectively O.K., DA, PR and AB is listed. Where a third attempt occurred, the numbers of calls are tabulated by ranges of time intervals between the second and third attempts. The ranges of time intervals are the same as

TABLE III

DISPOSITION OF SECOND ATTEMPTS AND CORRELATION OF TIME INTERVALS BETWEEN DATA CONCERNING 817 OBSERVATIONS HAVING A SECOND ATTEMPT

Range of Time Intervals in Seconds Between the First and Second Attempts	Total Number of Observed Second Attempts	Disposition of the Second Attempt: Number of Second Attempts that				Correlation of Time Intervals Between Attempts: Number of Second Attempts Each of Which Resulted in a *Busy* and which was Followed by a Third Attempt Within the Range of Seconds Listed in the Column Headings Below				
		Were OK	Were DA	Were PR	Were AB	0–45	46–78	79–130	131–226	227–600
0– 45	164	36	7	5	26	43*	17	12	8	10
46– 78	164	44	1	2	24	14	26*	28	13	12
79–130	164	71	2	2	21	6	14	24*	11	13
131–226	162	83	2	0	27	5	9	5	15*	16
227–600	163	93	4	0	28	6	0	4	5	23*
	817	327	16	9	126	74	66	73	52	74

* The asterisk marks the items that had the same range of time intervals between the first and second attempts and between the second and third attempts.

those used between the first and second attempts in order to see if a correlation exists. The significant facts concerning these data are:—

1. The degree of success in obtaining an O.K. call was better for those subscribers who waited longer before making a subsequent attempt. Only 22% of the subscribers who waited from 0 to 45 seconds were successful as against 57% who waited from 227 to 600 seconds.
2. The number of calls referred to the operator or where don't answers occurred are not significant to the problem in hand.
3. The incidence of abandoned calls appears to be uniform for the five ranges of time intervals. This means that the 90% figure estimated from Fig. 3 can be considered to apply with equal effect to all subscribers without regard to the previous time interval between dialing attempts.
4. The correlation data indicate a tendency for subscribers to establish a tempo or pace which they follow when redialing their calls. If this tempo did not exist the items on Table III that are marked with asterisks would not be larger than the surrounding items.

It was previously indicated that no observations were taken on P.B.X. and coin lines. An earlier attempt to collect data concerning the behavior of subscribers when encountering *busies* produced data that showed fewer subsequent attempts than was believed to be the case. The differences between the earlier data, which included a high proportion of observations on P.B.X. and coin lines, and the data developed herein are believed to be fully accounted for and it is believed that the P.B.X. and coin lines have the same basic characteristics regarding dialing behavior upon encountering *busies* as have the subscribers who were observed. No significant differences between the results for residential and business offices were noted. From these indirect facts, it is concluded that no significant differences exist between classes of subscribers.

Effect on the Trunk Plant

As explained earlier, neither the Poisson nor the Erlang B formula gives an accurate picture of the facts when trunk shortages occur on trunk groups handling subscriber-dialed calls. In both formulae it is assumed that only one attempt is made per call. In the case of the Poisson formula, the call is assumed to be held by the dial equipment until a trunk becomes available or until the subscriber hangs up, and in the case of the Erlang B formula, the call is assumed to clear out. The data developed from the service observations, concerning the dialing behavior of subscribers when encountering *busies*, indicate that subscribers usually make many subsequent attempts when a *busy* is encountered. Also the dial equipment with which we are familiar clears out the calls by giving an *all-trunks-busy* signal. In order to determine what a trunk capacity table might be like that takes into account the habits of subscribers and the limitations of the dial equipment a study based on simulated traffic was made. This study consisted of 150 CCS (hundred call seconds per hour) of traffic offered to a trunk group varying from 5 to 12 trunks. This study utilized the data developed from the service observations.

A study based on simulated traffic is a method used to study the capacities of trunking arrangements where a formula is not available. This type of study is based on the idea that calls are placed at random, that holding times of the calls follow an exponential law, and that these characteristics can be simulated by random numbers drawn from an appropriate source.

The study of 150 CCS of simulated traffic was based on 1,000 calls offered to a trunk group during a ten-hour period. The average holding time per call was 150 seconds, with the total holding time being 150,000 seconds or 41.66667 hours. Sub-divisions of an hour were expressed in decimal terms, the smallest division being a hundred-thousandth part. Three sets of random numbers were used for the following purposes:

1. To determine at what time in the ten-hour period a particular call is offered to the trunk group.
2. To furnish the holding time of a particular call.
3. To define for each call the pattern of resubmission of the call to the trunk group should an *all-trunks-busy* be encountered by the call.

In each instance the numbers were taken from the tail-end portions of successive entries of 19 significant figures of e^x (Tables of the Exponential Function—WPA—1939). The numbers drawn and their functions in the study are as follows:

A set of 1,000 six-digit numbers was taken from the last six digits of entries of e^x from $x = 0.4000$ to $x = 0.4999$. These 1,000 six-digit numbers were arranged in numerical order to give the placing time of 1,000 simulated calls. The first digit in every number was used to represent the hour and the last five digits the hundred-thousands part of the hour when a particular call was placed. The randomness of this particular draw was checked by determining the differences between successive placement times and then arranging the differences in numerical order. The results were plotted on a cumulative basis on Fig. 5, where a visual comparison can be made with theoretical results.

A set of 1,000 seven-digit random numbers between 0,000,000 to 4,166,667 inclusive were taken from the last seven digits of entries of e^x from $x = 0.5000$ to $x = 0.7344$. Numbers above 4,166,667 were disregarded. These seven-digit numbers when arranged in numerical order accounted for the total holding time of all the calls. The difference between successive numbers arranged in numerical order, furnished 1,000 individual holding times.

A third set of 1,000 random numbers were taken from two sources in the e^x tables. These 1,000 numbers contained a variable number of digits. These numbers were for use when calls encountered *all trunks busies* in order to determine which calls were to be resubmitted and to determine the time interval for resubmitting a call. Previously, it was estimated from Fig. 3, that 90% of the subscribers after encountering a *busy* redial their call. This estimate was used by assigning to the numerals 1 to 9 in the third set of random numbers the characteristic that a call may make a subsequent attempt if it encounters an *all trunks busy* and by assigning to the numeral 0 the characteristic that the call drops out if it encounters an *all trunks busy*. About 10% of the 1,000 numbers show a numeral 0 in the first place and hence no further digits are needed because the call drops out. The remaining 90% of the numbers show numerals from 1 to 9 in the first place and hence may make a second attempt. If an *all trunks busy* is encountered on the second attempt, a numeral from 1 to 9 in the second place determines that a third attempt may be made while the numeral 0 determines that the call drops out. This process is repeated for each place of each number in the third set of 1,000 random numbers until the numeral 0 appears. The number of consecutive places showing only numerals from 1 to 9, indicates the total number of attempts that a particular call might make before it drops out. Thus for a particular number the numerals might be 4720. In this case, three subsequent attempts can be made. Another number might be 834650. In this case, five subsequent attempts can be made.

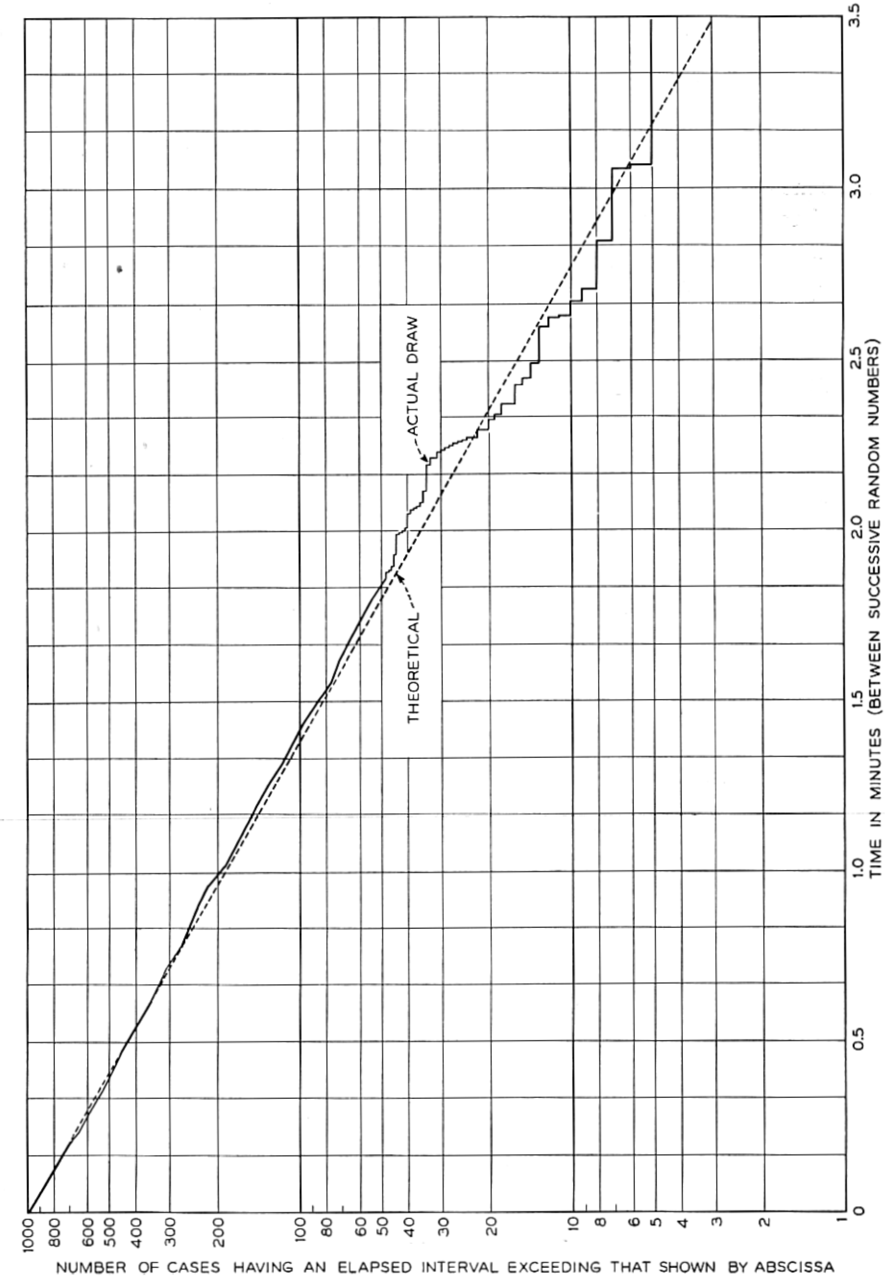

Fig. 5—Check of the randomness of numbers drawn that represent the time of placement of calls.

The effect of using numerals in this way is that 90% of the calls encountering *all trunks busies* appear as subsequent attempts.

The numeral in the first place of each of the third set of random numbers was used to establish the time interval for resubmitting each call. The time intervals were developed from the data on Fig. 4 by dividing the vertical scale into 10% bands. The time interval corresponding to the midpoint of each band was used as applicable to the 10% of the calls that fell within that band. The midpoint values, the corresponding time intervals, and the random numerals used are as follows:

TABLE No. IV

Midpoint Values of the 10% Bands of figure 4	Corresponding Time Intervals in Seconds	Equivalent Hundred-Thousandth Part of an Hour	Assignment of Random Numerals
a	*b*	$c = b \div .036$	*d*
95	25	700	9
85	46	1,300	8
75	67	1,900	7
65	93	2,600	6
55	132	3,700	5
45	195	5,400	4
35	320	8,900	3
25	665	18,500	2
15	2,250	62,500	1
5	Infinite	Call drops out	0

Based on the results indicated by Table III, that subscribers tend to make repetitious attempts at a uniform pace or tempo, the time interval determined by the numeral in the first place of a particular number of the third set of random numbers was repeated each time that a particular call was resubmitted.

The results of the study of simulated traffic are as follows:

TABLE V

Trunks Provided	Attempts (Calls Offered Plus All Subsequent Attempts)	Overflows (Calls Encountering *All Trunks Busies*)	Ratios of Overflows to Attempts	Calls Handled	Calls Abandoned	Approx. CCS Handled
a	*b*	*c*	$d = c \div b$	$e = b - c$	$f = 1000 - e$	$g = .150xe$
5	1,658	720	.4343	938	62	141
6	1,287	319	.2479	968	32	145
7	1,147	155	.1351	992	8	149
8	1,071	75	.0700	996	4	149
9	1,027	28	.0273	999	1	150
10	1,011	12	.0119	999	1	150
11	1,005	5	.0050	1,000	0	150
12	1,000	0	.0000	1,000	0	150

The ratios of overflows to attempts compared with theoretical results for the Poisson and Erlang B formulae for 150 CCS of offered traffic are as follows:

TABLE VI

Trunks Provided	Study of Simulated Traffic: Ratios of Overflows to Attempts	Theoretical Results	
		Erlang B: Ratio of Calls Lost to Calls Offered	Poisson: Ratio of Calls Delayed to Calls Offered
5	.4343	.2139	.4037
6	.2479	.1293	.2414
7	.1351	.0715	.1288
8	.0700	.0359	.0617
9	.0273	.0163	.0268
10	.0119	.0068	.0106
11	.0050	.0026	.0038
12	.0000	.0009	.0013

The ratios of overflows to attempts are apparently very close to the Poisson results. No further conclusion should be drawn from this, at this time, without further study.

SUMMARY

Data concerning the dialing behavior of subscribers who encounter *busies* have been obtained for New York City subscribers. These data indicate quantitatively: (1) how soon after obtaining a *busy*, a subscriber redials his call; (2) what percentage of subscribers make subsequent attempts; and (3) the pattern of time intervals between successive subsequent attempts. These data appear to have direct application in the development of trunk capacity tables for trunks handling subscriber-dialed traffic when trunk shortages occur.

ACKNOWLEDGMENTS

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