Cross-Modulation Requirements on Multichannel Amplifiers Below Overload

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Interchannel interference caused by non-linearity of multichannel amplifier characteristics is analyzed in terms of second and third order sum and difference products of the bands of energy comprising the various channels. Methods of relating the resulting disturbance to discrete frequency measurements are described and means for arriving at modulation requirements on individual amplifiers thus established.

1. Introduction

WHEN a repeater is used to amplify a number of carrier channels simultaneously, departure from linearity in the response as a function of input amplitude tends in general to produce interference between the channels. The non-linear component of the amplifier characteristic in effect acts as a modulator, changing the frequencies in the input wave and producing components which fall in bands assigned to channels other than the original ones. This phenomenon has been called "interchannel modulation" or "non-linear crosstalk." In formulating the requirements which are imposed on a repeater to insure that the resulting interference between channels will not be excessive it is convenient to treat separately two aspects of the problem namely—the condition when the total load on the amplifier is within the range for which the amplifier is designed and the severely overloaded condition. Actually a transition region between these two cases must also exist but when a considerable amount of negative feedback is used the break in the curve of response vs. input is quite sharp so that for practical purposes the input may be said to be either below or above the overload value.

Below overload, the amplifier characteristic may in most cases be represented with sufficient accuracy by the first few terms of a power series and the interchannel modulation analyzed in terms of the resulting combination tones of the frequencies present in the different channels. The total interference resulting from the combination tones falling in individual channels must be kept below prescribed limits. Above overload on the other hand the resulting disturbance in all channels becomes quite large and requirements are based on making such occurrences sufficiently infrequent.

The load capacity requirement has been discussed in a paper by B. D. Holbrook and J. T. Dixon.[1] In the present paper we assume that the principles there developed have been used to fix the maximum load which the amplifier must deliver and proceed with the second phase of the problem—the determination of the modulation requirements which the amplifier should meet below overload. A secondary objective is to set up simple testing procedures by which the performance of the system may be assessed without incurring the complications attendant upon loading the system with talkers. One such procedure involves the measurement of distortion products by a current analyzer when sinusoidal waves are impressed upon the system. Another involves the measurement of noise in a narrow frequency band when a band of noise uniformly distributed over the transmission range is impressed upon the system.

The path followed in reaching these objectives starts in Section (2) with a demonstration of the way in which non-linearity leads to interference in specific cases. Section (3) then considers the magnitudes of typical modulation products and arrives at volume distributions for them, by which the fluctuating character of speech may be taken into account. The basis for treating interchannel modulation as noise is given in (4). Since one of the most convenient testing methods involves the use of sine waves the relationship of distortion measured with sine waves to the distortion observed with speech on the system must be set up as in (5). The number of products falling in any single channel is considered in (6) and the average noise in any channel may then be evaluated in (7) with the aid of the results of preceding sections. The effects introduced by multiple centers of distortion in the amplifiers of a transmission link are considered in (8). The paper concludes with a discussion of test methods presented in the ninth section.

2. Interchannel Modulation as a Source of Crosstalk

We shall consider specifically a single sideband suppressed-carrier multichannel speech transmission system of the four-wire type although much of the treatment is also applicable to other kinds of carrier systems. The carrier frequencies will be assumed to be adjacent harmonics of a common base frequency greater than the highest signal frequency. The amplifier characteristic will be assumed to be expressible with sufficient accuracy by means of linear square and cube law terms. That is if i_p represents the output current and e_g the input voltage we write

$$i_p = a_1 e_g + a_2 e_g^2 + a_3 e_g^3. \tag{2.1}$$

[1] *Bell System Technical Journal,* Oct. 1939, Vol. 18, pp. 624–644.

Multivalued characteristics such as associated with ferromagnetic materials and reactive characteristics in which the coefficients vary with frequency are not included. The mechanism by means of which the characteristic (2.1) gives rise to interchannel interference may be illustrated by assuming that a sinusoidal signal of frequency q radians per second is impressed on the voice frequency channel associated with the carrier frequency mp, where p is the base frequency in radians per second and m is an integer. The resulting wave impressed on the amplifier is of the form

$$e_g = Q \cos (mp + q)t, \tag{2.2}$$

if upper sidebands are transmitted; the plus sign would be replaced by a minus sign in a system using lower sidebands. Substituting the value of e_g given by (2.2) in the characteristic (2.1), we find:

$$i_p = a_1 Q \cos (mp + q)t + \tfrac{1}{2}a_2 Q^2 + \tfrac{1}{2}a_2 Q^2 \cos (2mp + 2q)t \\ + \tfrac{3}{4}a_3 Q^3 \cos (mp + q)t + \tfrac{1}{4}a_3 Q^3 \cos (3mp + 3q)t. \tag{2.3}$$

Considering the terms which appear in the response (2.3), we note that the first term is the desired amplified signal. The second term is a direct current of trivial importance; if the system contains a transformer, this component is not transmitted. If q does not exceed $p/2$, the third term will be received in the channel associated with the carrier frequency $2mp$ and will there produce a detected frequency twice as great as the original signal frequency. In such a case, it represents interference produced in the $2mp$-channel when the mp-channel is actuated. If q exceeds $p/2$, the interference falls in the $(2m + 1)p$-channel. The fourth term of (2.3) is received in the mp-channel and represents a non-linearity in intrachannel transmission since its frequency is the same as that of the applied signal but its amplitude is proportional to the cube of the impressed signal amplitude. This term is of trivial interest in the study of transmission quality of individual channels of a well-designed multichannel system and is of no interest in the interference problem with which we are here concerned because it is received only in the originating channel. Finally, if q is less than $p/3$, the fifth term represents interference of frequency $3q$ received in the channel associated with the carrier frequency three times that of the originating channel; if q is greater than $p/3$, the interference falls in a higher channel.

Next suppose that a number of carrier channels are simultaneously transmitting signals. By substituting an expression representing the resulting carrier wave, which is a sum of several terms such as (2.2) with different values of Q, m and q, in the amplifier characteristic (2.1), we find that in addition to interference in channels having twice and three times the fundamental carrier frequencies, there are modulation terms appearing in channels having carriers which are various combinations of sums and differences of the original carrier frequencies. The second order term gives rise to crosstalk products with carriers $(m + n)p$ and $(m - n)p$ as well as $2mp$. The third order term causes products with carriers $(2m + n)p$, $(2m - n)p$, $(l + m + n)p$, $(l + m - n)p$, and $(l - m - n)p$ as well as $3mp$. In the above l, m and n are integers. For convenience we represent the tones associated with the carriers lp, mp, np by A, B and C and designate the various types of products as $A + B$, $A - B$, $2A + B$, $2A - B$, $A + B + C$, etc. It will be noted that the resultant modulation falling in a particular channel at any instant depends on the particular loading conditions prevailing on other channels at the same instant, and that a wide variety of amplitudes, numbers, and types of products are possible. Detailed study of these possibilities is necessary for the solution of our problem.

3. Nature of Multichannel Speech Load and Resulting Modulation Products

Considering any individual channel of the system, we note that (1) it may be active or inactive and (2) if active, the signal power being transmitted may vary throughout a considerable range of values. With regard to (1), we may estimate from traffic data a probability τ that a channel is active.[2] With regard to (2), data are available on the distribution of volumes corresponding to different calls at the toll switchboard. By "volume" is meant the reading of a volume indicator of a standard type. The distribution is approximately normal and hence may be expressed in terms of the average value V_0 and standard deviation σ. In mathematical language, the probability that the volume from any subscriber is in the interval dV at V is given by

$$p(V)dV = \frac{1}{\sigma\sqrt{2\pi}} e^{-(V-V_0)^2/2\sigma^2} dV. \tag{3.1}$$

The value of V_0 is about 16 db below reference volume, or about $-$ 8 vu when measured on the new volume indicator recently standardized in the Bell System. The value of σ is about 6 db. It is to be noted that volume is proportional to the logarithm of average speech power and hence V_0 is not the volume corresponding to the mean of the average speech powers of different talkers. The latter quantity, which will

[2] A channel is said to be active whenever continuous speech is being introduced into it. See Reference (1).

be designated by V_{0p}, may be calculated by averaging the distribution according to power, thus

$$\begin{aligned} V_{0p} &= 10 \log_{10} \overline{\text{antilog}_{10} V/10} \\ &= V_0 + \frac{\sigma^2}{20} \log_e 10 \qquad (3.2) \\ &= V_0 + .115\, \sigma^2, \end{aligned}$$

when V_0 and σ are expressed in db. The method of obtaining this result is indicated in Appendix A.

It will be convenient to extend the term "volume" to apply to modulation products by designating the modulation product produced by 0-vu talkers as a "zero volume modulation product" of its particular type. This is not its absolute volume as read by a volume indicator, but a reference value to which modulation products of the same type produced by talkers of other volumes may be referred. We assume on the basis of a power law of modulation that the volume of a product will increase one db for each one db increase in volume of a fundamental appearing once in the product, two db for each db increase in volume of a fundamental appearing twice, etc. Thus a $(2A - B)$-product should increase two db for one db increase in the volume of the A-component, and one db for one db increase in the volume of the B-component. If the fundamental talker volumes producing a particular product are normally distributed on a db scale, it follows from established relations concerning the distributions of sums [3] of normally distributed quantities that the volume of the product is also normally distributed. The relations between average and standard deviation for the modulation product and the corresponding quantities V_0 and σ for the fundamental are shown in Table I.

TABLE I

Modulation Product	Average in db Referred to Product from 0-vu Talkers	Standard Deviation in db
$2A$	$2V_0$	2σ
$A \pm B$	$2V_0$	$\sqrt{2}\sigma$
$3A$	$3V_0$	3σ
$2A \pm B,\ B - 2A$	$3V_0$	$\sqrt{5}\sigma$
$A + B \pm C,\ A - B - C$	$3V_0$	$\sqrt{3}\sigma$

That is, if the fundamental talker volumes are normally distributed with average value − 8 vu, and standard deviation 6 db, the

[3] Multiplying amplitudes of fundamental components is equivalent to adding logarithms of amplitudes; hence the volumes of the fundamental components add to determine product volumes. For a derivation of the distribution function of the sum of two independent normally distributed quantities, see Cramer, Random Variables and Probability Distribution, Cambridge Tract No. 36, 1937, p. 50.

$(A + B - C)$-type products, for example, are also normally distributed in product volume with average value 24 db less than the product produced by three 0-vu talkers and standard deviation $6\sqrt{3}$ or 10.4 db. To obtain the product volume corresponding to the average power of the product distribution, $.115(10.4)^2$ or 12.4 db must be added. In general if V_{0p} of an x-type product is desired, it may be expressed as $\eta_x V_0 + .115\lambda_x\sigma^2$ where η_x is the order of the x-type product, and the value of λ_x is given by the square of the coefficient of σ in the third column of Table I.[4] We observe that $\eta_x V_0 + .115\lambda_x\sigma^2 = \eta_x V_{0p} + .115(\lambda_x - \eta_x)\sigma^2$, and that $\lambda_x = \eta_x$ for $x = A \pm B$ and $A \pm B \pm C$.

The frequencies present in a typical commercial speech channel extend over a range of approximately 3000 cycles. The spacing of carrier frequencies must be made somewhat greater than this to allow for filter cut-offs. Figures 1 and 2 illustrate the spectra of the various second and third order modulation products resulting from two and three fundamental channel spectra respectively which are flat from 10 per cent to 80 per cent of the carrier spacing. Actual speech channels would have peaked spectra but the results would be roughly similar. Each second order band of products occupies twice the frequency range of one original speech band, and a third order band of products spreads over three times the fundamental range. Portions of one product band may thus be received in different channels, but with one part usually much larger than the others. It is to be noted that a $2A$-type product band does not consist merely of the second harmonics of all tones in the band A, but includes all possible sums of the tones in the fundamental band. The spectrum of the $2A$-type product is similar in shape to that of an $(A + B)$-type product but has half as much total power because only half as many sum products can be formed from a single band as from two equal bands. The interfering effect of a $2A$-type product from a speech channel may of course be quite different in character from that of an $(A + B)$-type product since in the latter case the result depends on two independent talkers.

4. The Noise Resulting from Modulation Products

It will be noted that the interference produced as described above may be classified as unintelligible, since in products involving one channel, the wave form is distorted, and in products involving more than one channel, sums and differences of independent signal frequencies are heard. It may be said therefore that interchannel modulation

[4] In general for a $(m_1A \pm m_2B \pm m_3C \pm \cdots)$-product, $\lambda_x^2 = m_1^2 + m_2^2 + m_3^2 + \cdots$.

may be treated as noise, and the usual noise requirements apply. If a great many products are superimposed, the noise heard will be fairly steady, and the average weighted noise power is a sufficient indication of the interfering effect. In systems with a small number of channels, large variations in the noise may occur, and it may be necessary to consider the infrequent large bursts of modulation from exceptionally loud talkers as a limiting factor. The allowance to be made can be estimated by determining the complete distribution curve of modulation noise. Computation of the required distribution function may be carried out by methods similar to those described in the paper by Holbrook and Dixon.[1] The fact that the products are not independent introduces a difficulty which complicates the calculation. For systems with a large number of channels, the requirements may be based on average values with a considerable resulting simplification.

If in addition to the sidebands due to speech, "carrier leaks" (partially suppressed carrier waves) are present, modulation products are produced which are sums and differences of carrier frequencies and speech sidebands. Products of this sort may cause intelligible crosstalk. For example the carrier frequency mp modulating with the channel frequency $np + q$ causes an $(A + B)$-product of frequency $(m + n)p + q$, which is received in the channel with carrier frequency $(m + n)p$ as the original signal frequency q. Requirements on intelligible crosstalk are in general more severe than on unintelligible; hence it is important that the carrier leaks be suppressed well below the level of the speech channels. The intelligibility tends to disappear as the number of channels is increased, since the number of superimposed products becomes larger thereby producing masking effects. Carrier leak modulation is however more serious than modulation from speech channels having the same power since carrier leaks are present all the time, while speech sidebands occur only in active channels. Similar considerations apply to pilot and control tones.

Quantitatively, the various frequency components in modulation noise must be weighted in terms of their interfering effect on reception of speech. In practice the weighting is done by a noise meter designed for that purpose. The noise meter readings are expressed in terms of db above reference noise. A reading of zero, or reference noise, is produced by a 1000-cycle sinusoidal wave with mean power equal to one micromicrowatt. The weighting incorporated in the noise meter is determined by judgment tests of the relative interfering effects of single frequencies and other reproducible noises.

5. Relation Between Speech and Sine Wave Modulation

A goal of our investigations is to express the requirements finally in terms of measurements which can be made on amplifiers with sinusoidal testing waves. A means of relating modulation products produced by speech channels to those occurring when discrete frequencies are applied is therefore needed. For our purposes here we shall express the needed relation [5] in terms of a "Speech-Tone Modulation Factor," which we shall abbreviate as S.T.M.F. and define in terms of the following procedure: Apply the fundamental single-frequency test currents necessary to produce the product in question, which we shall designate as an x-type product. Adjust each fundamental to give mean power of one mw. at the zero level point of the system. Measure the resulting x-type product at the point of zero transmission level of the system. Suppose the product is H_x db below one fundamental. Next load the system with speech from the combination of fundamental talkers required to form the talker product of x-type. Each talker must produce speech volume of 0 vu at the transmitting toll switchboard or point of zero transmission level. The product is then received from the appropriate channel and a comparison is made between it and the speech from one talker with both talker and product at the same transmission level point in the system. The comparison should be made on the basis of relative interfering effect. Suppose it is determined that an x-type product is L_x db below one 0-vu talker. Then s_x, the S.T.M.F. for an x-type product, is defined by

$$s_x = L_x - H_x. \tag{5.1}$$

The sign of the S.T.M.F. has been assigned here to be positive when the difference in db between effect of talker and talker product is greater than the difference between sine wave fundamental power and sine wave product power.

It is to be noted that not only does each type or product possess its own S.T.M.F., but also that the several portions of a product appearing in different channels have different S.T.M.F.'s. This may be clearly seen from Figs. 1 and 2. We note also that the S.T.M.F. is a property of the system on which the measurements are made, since it varies with the band width of the channels, the spacing of carrier frequencies, and the extent of departures from the simple square and cube law representation of the amplifier characteristic. It also varies with the type of transmitting and receiving instruments used. Theo-

[5] The quantity here defined is related to what has been called "staggering advantage" of modulation products. Since the term "staggering advantage" has been applied to various kinds of interference including linear crosstalk, its use here might lead to confusion and is avoided.

retically it should be possible to compute the S.T.M.F. for any particular product if sufficient information concerning the properties of speech, the transmitting and receiving instruments, the carrier system

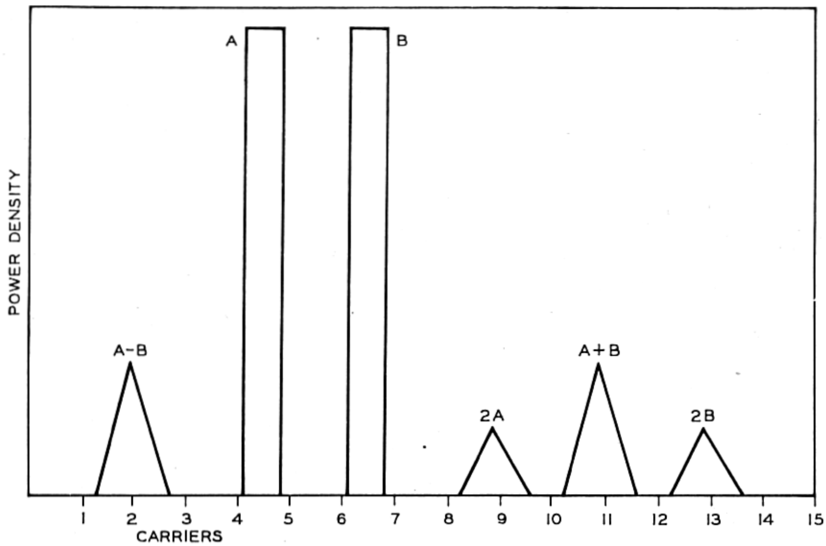

Fig. 1—Spectra of second order modulation products from two fundamental channels.

itself, and the ear were known, but in practice it is found best to use experimental determinations. In the case of new systems for which experimental data are not available, estimates based on known systems of similar type would be used.

6. Number of Products Falling in Individual Channels

In the appendix it is explained how the total number of possible products of each type falling in individual channels may be counted. Table II shows the result of counting all second and third order type products.[6] Results for products falling both within and without the fundamental band are given. In certain of the $(A + B - C)$- and $(A - B - C)$-type products, the channel in which the product occurs also is the source of one of the fundamentals. Since this would give a type of interference heard only when the disturbed channel is also carrying signal, it is not in general as serious a form of crosstalk as the cases of independent fundamental and product frequencies. Therefore the number of these special kinds of products has also been evaluated

[6] Mr. J. G. Kreer collaborated in the derivation of these formulæ.

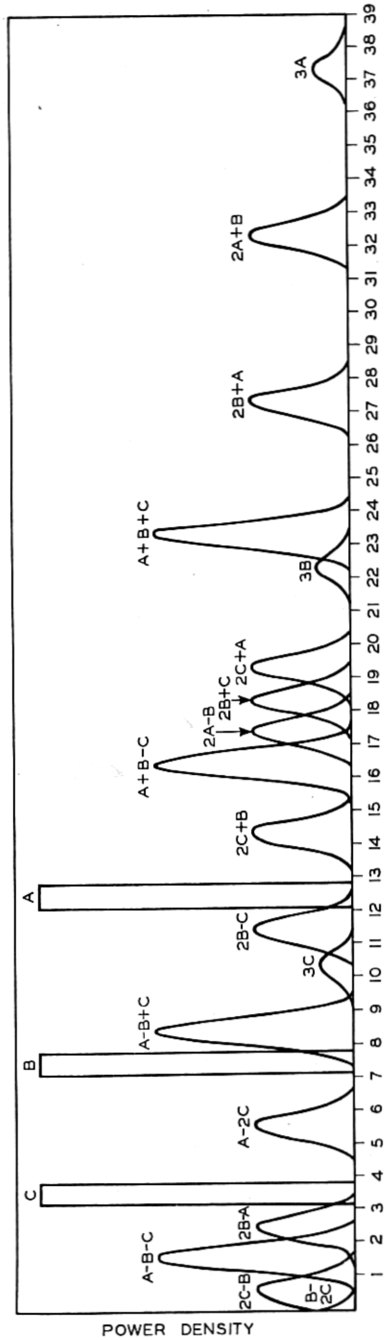

Fig. 2—Spectra of third order modulation products from three fundamental channels.

TABLE II

Number of Products Falling in kth Channel of Multichannel Carrier System with Harmonic Carrier Frequencies n_1p, $(n_1+1)p$, ... $(n_1+N-1)p$

n_1p = Lowest Carrier Frequency.
$n_2p=(n_1+N-1)p$ = Highest Carrier Frequency.
kp = Carr. Freq. Associated with Mod. Product.

N = Number of Channels.
$I(x)$ = "Largest Integer $\leq x$."
No. of Products is 0 Outside Ranges Indicated.

Number of Products

Type

Second Order

2A .... $1,\ n_1\leq\frac{k}{2}\leq n_2$ if $\frac{k}{2}$ is an integer

A+B

$$\begin{cases} I\left(\frac{k+1}{2}\right)-n_1, & 2n_1-1\leq k\leq n_1+n_2 \\ n_2-I\left(\frac{k}{2}\right), & n_1+n_2\leq k\leq 2n_2 \end{cases}$$

A−B ..... $N-k,\ 0<k<N$

Third Order

3A .... $1,\ n_1\leq\frac{k}{3}\leq n_2$ if $\frac{k}{3}$ is an integer

2A+B ....

$$\begin{cases} I\left(\frac{k-n_1}{2}\right)-I\left(\frac{k}{3}\right)+I\left(\frac{k-1}{3}\right)-n_1+1, & 3n_1\leq k\leq 2n_1+2n_2 \\ I\left(\frac{k-n_1}{2}\right)-I\left(\frac{k}{3}\right)+I\left(\frac{k-1}{3}\right)-I\left(\frac{k-n_2+1}{2}\right)+1, & 2n_1+n_2\leq k\leq n_1+2n_2 \\ n_2-I\left(\frac{k}{3}\right)+I\left(\frac{k-1}{3}\right)-I\left(\frac{k-n_2+1}{2}\right)+1, & n_1+2n_2\leq k\leq 3n_2 \end{cases}$$

2A−B ....

$$\begin{cases} I\left(\frac{n_2+k}{2}\right)-n_1+1, & 2n_1-n_2\leq k\leq n_1-1,\ k\geq 0 \\ I\left(\frac{n_2-k}{2}\right)-I\left(\frac{n_1+k+1}{2}\right), & n_1\leq k\leq n_2 \\ n_2+1-I\left(\frac{n_1+k+1}{2}\right), & n_2+1\leq k\leq 2n_2-n_1 \end{cases}$$

A−2B .... $I\left(\frac{n_2-k}{2}\right)-n_1+1,\ 0<k\leq n_2-2n_1,\ n_2\geq 2n_1$

A+B+C ..

$$\begin{cases} I\left(\frac{k-3n_1+3}{6}\right)+I\left[\frac{(k-3n_1-1)^2}{12}\right], & 3n_1+3\leq k\leq 2n_1+n_2+1 \\ (N-1)\left[k-2n_1+1-\frac{N}{2}\right]+I\left(\frac{3n_2-k+3}{6}\right)+I\left[\frac{(3n_2-k-1)^2}{12}\right]-I\left[\frac{(k-3n_1)^2}{4}\right] & \\ \quad -\frac{1}{2}I\left(\frac{k-3n_1+2}{3}\right)I\left(\frac{k+3n_1+5}{3}\right)-\frac{1}{2}I\left(\frac{3n_2-k}{3}\right)I\left(\frac{3n_2+k+5}{3}\right), & 2n_1+n_2+2\leq k\leq n_1+2n_2-2 \\ I\left(\frac{3n_2-k+3}{6}\right)+I\left[\frac{(3n_2-k-1)^2}{12}\right], & n_1+2n_2-1\leq k\leq 3n_2-3 \end{cases}$$

A+B−C ..

$$\begin{cases} I\left(\frac{N-n_1+k}{2}\right)I\left(\frac{N-n_1+k+1}{2}\right), & 2n_1-n_2+1\leq k\leq n_1-1 \\ I\left(\frac{k-n_1}{2}\right)I\left(\frac{k-n_1-1}{2}\right)+(k-n_1)(n_2-k)+I\left(\frac{n_2-k}{2}\right)I\left(\frac{n_2-k-1}{2}\right), & n_1\leq k\leq n_2 \\ \left[n_2-I\left(\frac{k+n_1}{2}\right)\right]\left[I\left(\frac{k+n_1}{2}\right)-k+N\right], & n_2<k\leq 2n_2-n_1-1 \end{cases}$$

Note: Number of products included in which k-channel signal is one fund. component $=\begin{cases} k-n_1, & n_1\leq k\leq n_1+\frac{N-1}{2} \\ n_2-k, & n_1+\frac{N-1}{2}<k\leq n_2 \end{cases}$

A−B−C .. $I\left(\frac{N-k-n_1}{2}\right)I\left(\frac{N-k-n_1-1}{2}\right),\ 0<k\leq n_2-2n_1-1,\ n_2\geq 2n_1+1$

Note: Number of products included in which k-channel signal is one fund. component $=\begin{cases} N-2k-1, & n_1\leq k\leq\frac{n_2-1}{3} \\ k-n_1, & \frac{n_2-1}{3}\leq k\leq\frac{n_2+1}{3} \\ N-2k, & \frac{n_2+1}{3}\leq k\leq\frac{N}{2} \end{cases}$

and shown in the table. The number of these is to be subtracted from the total of the $(A + B - C)$- or $(A - B - C)$-types to obtain the number of products not involving the listening channel. It should be pointed out also that kp is the derived carrier frequency throughout and that the products may extend over into adjacent channels. The principal component of the product usually falls in the channel with carrier frequency kp, but in some cases the amount of energy falling in adjacent channels may be quite considerable, as may be seen from Figs. 1 and 2.

The average number of products falling simultaneously in one channel is found by multiplying the total possible number by τ^2 for two-frequency products and τ^3 for three-frequency products, these factors being the probability that any particular product is present. The average number present is not affected by the dependence of the products arising from the fact that one talker may participate in the formation of more than one of the products falling in a channel. For convenience in making use of the results of Table II in evaluating the amplifier requirements, we shall represent the number of x-type products falling in channel number k when it is idle and all other channels are active by the symbol ν_{xk}. We shall also let $\mu(x)$ represent the number of distinct fundamental components required to produce an x-type product, e.g., $\mu(A + B) = 2$, $\mu(2A + B) = 2$, $\mu(A + B - C) = 3$, etc. It follows that the probability that any particular product is present is $\tau^{\mu(x)}$, since τ is the probability that any one required component is present. The average number of x-type products present in the k-channel is therefore $\nu_{xk}\tau^{\mu(x)}$, and may be considered as determined since ν_{xk} is the quantity tabulated in Table II.

7. Modulation Requirement in Terms of Average Total Noise Permissible in a Channel

From Section 3 we have a result for the volume of one product of arbitrary type, averaged on a power basis for a distribution of fundamental talker volumes, referred to the product of the same type produced by zero volume talkers. From Section 6 we have the average number of products of each type appearing in a channel. Combining these two results should give the average total modulation of each type present in a channel. A difficulty occurs however inasmuch as it is not certain how the interfering effect of superimposed modulation adds. The noise caused by one modulation product is of an irregular nature and it is probable that its most disturbing effect is associated with infrequent peak values. When two products are superimposed their individual peaks are not apt to coincide and hence the resultant disturbance may not be much greater than that of one alone. We shall introduce here the concept of "plural S.T.M.F." Suppose ν products of x-type are superimposed and comparison of the resulting noise with one fundamental talker shows that the difference is $L_{x\nu}$ db. If interfering effects add as mean power we should expect $L_{x\nu}$ to be equal to $L_x - 10 \log_{10} \nu$. Hence it seems logical to write

$$s_{x\nu} = L_{x\nu} + 10 \log_{10} \nu - H_x, \tag{7.1}$$

where $s_{x\nu}$ is the "plural S.T.M.F." to be used when ν products are superimposed in order that power addition of products may be valid. Combining (5.1) and (7.1),

$$L_{x\nu} - L_x = s_{x\nu} - s_x - 10 \log_{10} \nu, \tag{7.2}$$

which shows that the correction to be subtracted from power addition is

$$\rho_{x\nu} = s_{x\nu} - s_x. \tag{7.3}$$

The value of $\rho_{x\nu}$ is best determined by experiment. Superposition of a large number of products without using an excessive number of talkers can be accomplished by making phonograph records of individual products and combining their outputs in subsequent recordings.

The average total modulation of x-type in a channel is found by multiplying the average value for one product by the average number of products, and subtracting the quantity $\rho_{x\nu}$, which may be called the "plural S.T.M.F. correction," thus

$$V_x = \eta_x V_{0p} + .115(\lambda_x - \eta_x)\sigma^2 + 10 \log_{10} \nu_{xk}\tau^{\mu(x)} - \rho_{x\nu}. \tag{7.4}$$

where V_x is the volume averaged on a power basis of the x-type modulation in the k-channel referred to the volume of one x-type product from 0-vu talkers. We next wish to express V_x in terms of db above reference noise.

Let T_a represent the "noise" produced by a 0-vu talker in db above reference noise. This is an experimentally determinable quantity and is about 82 db. Let T_x represent the noise from an x-type product from 0-vu talkers. Then L_x, the quantity appearing in (5.1), is given by

$$L_x = T_a - T_x. \tag{7.5}$$

The average total noise produced by all x-type products in db above reference noise is given by

$$W_x = V_x + T_x = V_x + T_a - s_x - H_x. \tag{7.6}$$

If we assume that the total modulation noise allowable for an x-type product is X db above reference noise at zero level, we may equate X to W_x in (7.6) and solve for H_x, giving

$$H_x = V_x + T_a - s_x - X. \tag{7.7}$$

Substituting the value of V_x from (7.4) in (7.7), we get for the system requirement in terms of allowable ratio of fundamental to product when there is one mw. of each fundamental at the point of zero transmission level:

$$H_x = T_a + \eta_x V_{0p} - s_x + .115(\lambda_x - \eta_x)\sigma^2 + 10 \log_{10} \nu_{xk} + 10\mu(x) \log_{10} \tau - \rho_{x\nu} - X. \tag{7.8}$$

For the convenience of the reader, the following recapitulation of significance of the symbols used in (7.8) is given:

H_x = Ratio in db of power of each single frequency fundamental to power of resulting x-type product when each fundamental has power of one mw. at point of zero transmission level.

T_a = Reading of 0-vu talker on noise meter in db above reference noise.

η_x = Order of x-type product.

V_{0p} = Volume in vu corresponding to the average power of the talker volume distribution at the point of zero transmission level $= V_0 + .115\sigma^2$ where V_0 is average talker volume in vu.

σ = standard deviation in db of talker volume distribution.

s_x = Speech-Tone Modulation Factor of x-type product as defined in Section 5.

$\lambda_x = \sqrt{m_1^2 + m_2^2 + \cdots}$ for $(m_1A \pm m_2B \pm \cdots)$-type product.

ν_{xk} = total number of x-type products which can fall in channel with carrier frequency kp. See Table II.

$\mu(x)$ = number of distinct fundamentals required to produce x-type product.

τ = fraction of busiest hour that a channel is active.

$\rho_{x\nu}$ = correction to be applied to S.T.M.F. when ν products are superimposed. Defined in (7.1)–(7.3).

X = Allowable modulation noise in channel in db above reference noise at zero transmission level point.

The allowable noise may be divided equally between second and third order purely on a power basis by setting the requirement for each 3 db more severe than the total value allowed, or it may turn out that the noise from one order is much more difficult to reduce than that of the other in which the full allowance may be given to the more difficult one, and the other made to contribute a negligible amount. Usually one type of modulation product will predominate for a given order and the allowable noise for the order may be assigned to this type. If such is not the case division of the noise between the various types may be estimated.

8. Addition of Products in a Multirepeater Line

When a number of amplifiers are used in a carrier system, contributions to interchannel interference occur at each repeater. The considerations previously discussed have a bearing on the modulation requirements on the system as a whole, but it is evident that the individual amplifiers may have to meet much more severe requirements. The relation between total system modulation and single amplifier modulation depends to a considerable extent on the phase angles between products originating in the various repeater sections.

A discussion of the general problem of addition of modulation products from multiple sources is to be given in a forthcoming paper by J. G. Kreer. A point of particular interest in connection with broad band systems is the effect of a linear phase characteristic on the phase shift between modulation products originating in different repeater sections. The curve of phase shift vs. frequency throughout the frequency range occupied by a considerable number of adjacent channels will in general depart but little from a straight line, but the intercept of this straight line if produced to zero frequency is in general not zero or a multiple of 2π. The intercept of such a linear phase curve is effective in producing phase difference between contributions to modulation from successive repeater sections of all the second order products and of some of the third order products, namely the types $3A$, $2A + B$, $B - 2A$, $A + B + C$, and $A - B - C$. The phases of third order products of types $2A - B$ and $A + B - C$ however are unaffected by the value of the intercept and the contributions from the different repeaters of these types of modulation will add in phase to give the maximum possible sum whenever the phase curve is linear throughout the channels involved. Third order modulation requirements on individual repeaters of a system may therefore have to be based on the very severe condition of in-phase or voltage addition of separate contributions.

Experimental verification of in-phase addition of third order modulation products from the repeaters of a 12-channel cable carrier system are included in the paper by Kreer previously mentioned. Corroborating data obtained on an experimental system capable of handling 480 channels are shown in Fig. 3. The measurements there shown were made [7] on a loop approximately 50 miles in length with repeaters spaced 5 miles apart. The band transmitted extended from 60–2060 kc. Fundamental frequencies of 920 and 840 kc. were supplied from

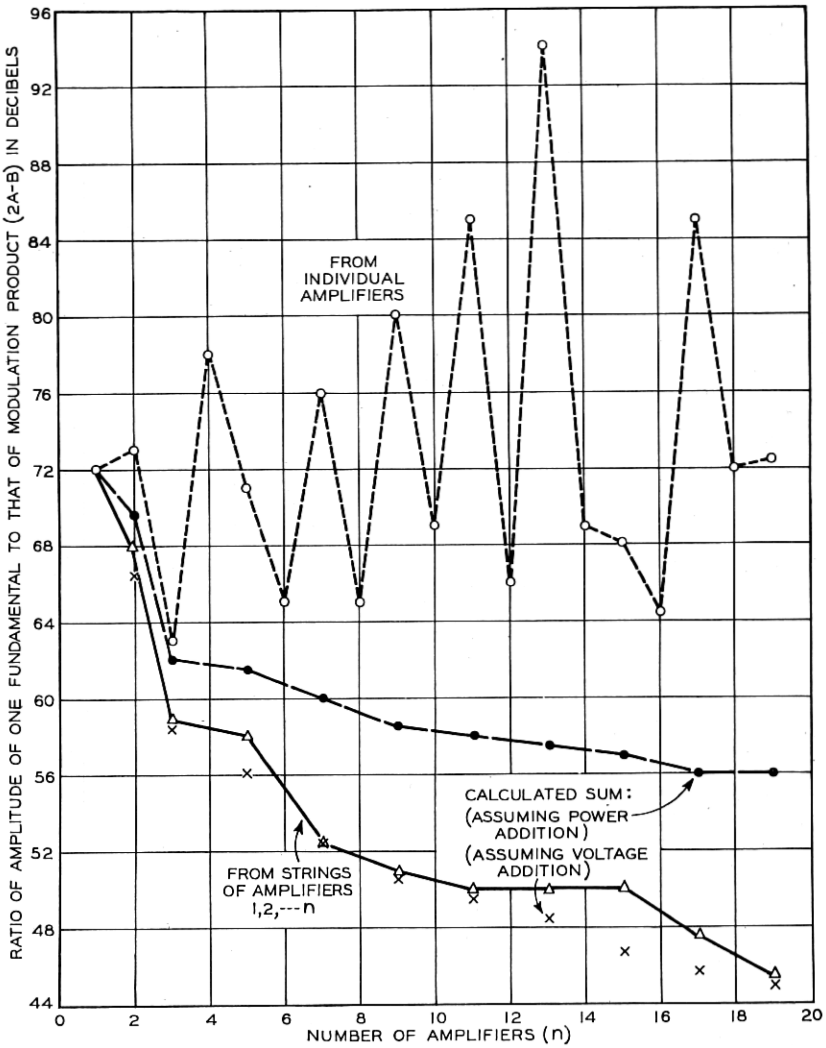

Fig. 3—Experimental data on addition of third order modulation from a multi-repeater line. Fundamental test tones $A = 920$ kc, $B = 840$ kc. Modulation product $2A-B = 1000$ kc.

two oscillators at the sending end, and measurements with a portable current analyzer were made at each repeater to determine the ratio of

[7] Messrs. M. E. Campbell and W. H. Tidd collaborated in these measurements.

amplitude of fundamental to that of the $(2A - B)$-product falling at 1000 kc. A band elimination filter having more than 100 db loss at 1000 kc. and suppressing a band approximately from 940 to 1070 kc. was inserted in the line at the repeater station to remove all contributions to the modulation product originating ahead of the station at which measurements were made. In this way, the modulation contributed by each amplifier and by various combinations of amplifiers could be measured without disturbing the operating levels throughout the system.

The data shown on Fig. 3 include measured modulation from individual amplifiers, and from tandem amplifiers with intervening cable sections. The summation of amplifiers proceeds in the same order as the plotted individual amplifier values. The crosses show the calculated sums of the individual contributions assuming in-phase addition. Agreement between these values and the measured sums is well within the accuracy of the measurements, considering the difficulties involved and the length of time required to complete the run. The dots show the resultant modulation which would be obtained by adding the power in the individual components instead of the voltages, which would be the expected result for a large number of components with random phase angles. The modulation thus calculated is much smaller than the measured values indicating that a hypothesis of random phasing is untenable for this product.

In actual systems both the magnitude and phase shift of modulation products in the different repeater sections exhibit variations because of non-uniform output levels, differences in tubes and other amplifier parts, and unequal repeater spacings. The addition factor for converting system requirements to single amplifier requirements should therefore contain a marginal allowance for these irregularities in performance.

Summarizing our conclusions on addition of modulation from multiple sources, we may state that the third order requirement invokes the most severe condition—that of in-phase addition. Second order products on the other hand will have enough phase shift, either inherent or from simple reversals of terminals at alternate sections, to make the addition no more rapid than on a power basis. In fact if there is a high degree of similarity with respect to both amplitude and phase increment of products from successive amplifiers throughout the system, the total second order modulation may be much less than calculated from addition of power. In setting the requirements which each amplifier must meet, marginal allowances should be made for differences in lineup throughout the system and aging effects which may take place after the amplifier is put in service.

Let A_x represent the ratio expressed in db between the total x-type modulation received from the system and the contribution of x-type from one amplifier, assuming the amplifiers contribute equally. For example, if there are K amplifiers in the system and if the contributions to the product add in phase, $A_x = 20 \log_{10} K$. If power addition occurs, $A_x = 10 \log_{10} K$. A favorable set of phase angles may reduce this factor by an amount depending on the uniformity of the repeaters. If the system is divided into K_1 links having K_2 amplifiers in each link, with phase shifts and changes of frequency allocations of individual channels present at the link junctions such that amplitude addition occurs within links and power addition from link to link, $A_x = 10 \log_{10} K_1 + 20 \log_{10} K_2$. We shall also introduce a lineup factor F_x defined as the number of db by which the x-type product requirement must be increased to allow for irregularities in lineup operating levels of the amplifiers. These may be due to initial differences in repeaters or cable sections and to subsequent changes which may occur because of aging effects. If H_{x0} represents the requirement on the ratio of fundamental to x-type product in the output of a single amplifier when one mw. of test tone power is delivered at zero level,

$$H_{x0} = H_x + A_x + F_x, \tag{8.1}$$

where H_x is the system requirement given by (7.8).

9. Testing Methods

It is difficult to test a broad band carrier system under conditions simulating normal operation because of the large number of independent conversations required to load the channels. We have seen that much information applicable to speech load can be deduced from current analyzer measurements of modulation products when discrete frequencies are applied. Since rather extensive calculations are required to evaluate the performance of the system from single frequency data, an overall test under conditions comparable to actual operation has considerable value as a check. A convenient method of simulating the speech load in the high frequency medium by means of a uniformly distributed spectrum of energy such as thermal noise or the output of a gas tube applied through a narrow band elimination filter has been developed [8] for this purpose. The band elimination filter suppresses the energy which would fall in several adjacent channels; hence anything received in these channels at the receiving end of the line is introduced by the system. We can thus measure interchannel modulation if it exceeds the background of noise from other sources. A

[8] E. Peterson, *Bell Laboratories Record*, Nov. 1939, Vol. 18, No. 3, pp. 81–83.

summation is obtained over all types of products but the predominant order may be distinguished by the power law followed. Effects not simulated are the frequency distribution of speech energy within individual channels and the idle periods which occur in various channels during normal operation, but these effects are minor in a system with a large number of channels. The noise loading method is particularly valuable on a multirepeater line, in which modulation measurements made with discrete frequencies may show large variations with frequency because of phasing between the various sources. Loading with a flat band of energy secures an average of these variations over the frequency range used.

Acknowledgments

Contributions to the solution of the various problems discussed here have been made by many members of the Bell Telephone Laboratories. The author wishes to take this opportunity to acknowledge his indebtedness to those of colleagues who have participated in the development of the ideas here discussed.

Other References

Following is a list of published papers (excluding those to which reference has already been made in the text) relating to various aspects of the general problem discussed here:

F. Strecker, "Nichtlineares Nebensprechen bei der gemeinsamen Übertragung mehrerer modulierten Trägerwellen," *Hoch. u. Elek.*, **49** (1937), 5, pp. 165–171.
H. Jacoby and G. Günther, "Über die Wahrscheinlichkeit der in Trägerfrequenz Vielfachsystemen auftretenden linearen und nichtlinearen Spannungen," *Hoch. u. Elek.*, **52** (1938), 6, pp. 201–209.
E. Hölzler, "Das Nichtlineare Nebensprechen in Mehrfach-Systemen mit Übertragenen Trägern," *Hoch. u. Elek.*, **52** (1938), 4, pp. 137–142.
H. F. Mayer and D. Thierbach, "Über den Einfluss von Nichtlinearität und Wärmerauschen auf die Reichweite von Trägerfrequenz-Vielfach-Systemen," *E. F. D.*, **48** (1938), pp. 6–12.
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Yonezawa and Hirayama, "Relation Between the Non-Linear Distortion and the Crosstalk on Multiplex Transmission," *Nippon Elec. Comm. Eng.*, June, 1938.
Yonezawa, "Non-Linear Distortion and Crosstalk Frequency Groups in Multiplex Carrier Transmission," *Nippon Elect. Comm. Eng.*, September, 1938.
Matzumae and Yonezawa, "Equipments for Multiplex Carrier Telephony on Ultra Short Wave," *Nippon Elect. Comm. Eng.*, Feb. 1939.

APPENDIX A

Evaluation of Volume Corresponding to Mean Power When Volume Distribution is Normal

The successive steps in the evaluation of (3.2) are as follows:

$$\overline{\text{antilog}_{10} V/10} = \overline{10^{V/10}} = \overline{\exp\left(\frac{V \log_e 10}{10}\right)}$$

$$= \int_{-\infty}^{\infty} \exp\left(\frac{V \log_e 10}{10}\right) p(V) dV$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{V}{10}\log_e 10 - \left(\frac{V-V_0}{2\sigma^2}\right)^2} dV$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{\log_e 10}{10}\left(V_0 + \frac{\sigma^2}{20}\log_e 10\right)} \int_{-\infty}^{\infty} e^{-\frac{\left(V - V_0 - \frac{\sigma^2}{10}\log_e 10\right)^2}{2\sigma^2}} dV$$

$$= e^{\frac{\log_e 10}{10}\left(V_0 + \frac{\sigma^2}{20}\log_e 10\right)} = 10^{\frac{1}{10}\left(V_0 + \frac{\sigma^2}{20}\log_e 10\right)}.$$

Hence

$$V_{0p} = 10 \log_{10} \overline{\text{antilog}_{10} \frac{V}{10}} = V_0 + \frac{\sigma^2}{20} \log_e 10.$$

APPENDIX B

Counting of Modulation Products

Consider an N-channel system with carrier frequencies $n_1 p$, $(n_1 + 1)p$, $\cdots$, $n_2 p$, where $n_2 = n_1 + N - 1$. Let q_n represent the frequency of the signal impressed on the voice frequency channel associated with the carrier frequency np. The wave to be amplified is then

$$e_g = \sum_{n=n_1}^{n_2} Q_n \cos\left[(np + q_n)t + \theta_n\right]. \tag{1}$$

The phase angles are of no consequence if the signal frequencies are incommensurable; hence we shall simplify the notation by setting $\theta_n = 0$. The square of the single series in n may be written as a double series in m and n, thus:

$$a_2 e_g^2 = \frac{a_2}{2} \sum_{n=n_1}^{n_2} Q_n^2 + \frac{a_2}{2} \sum_{n=n_1}^{n_2} Q_n^2 \cos(2np + 2q_n)t$$

$$+ a_2 \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m Q_n \cos[(m+n)p + q_m + q_n]t$$

$$+ a_2 \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m Q_n \cos[(m-n)p + q_m - q_n]t. \quad (2)$$

Similarly the term $a_3 e_g^3$ may be written as a triple series in l, m, n as follows:

$$a_3 e_g^3 = \frac{3a_3}{4} \sum_{n=n_1}^{n_2} Q_n^3 \cos(np + q_n)t$$

$$+ \frac{a_3}{4} \sum_{n=n_1}^{n_2} Q_n^3 \cos(3np + 3q_n)t$$

$$+ \frac{3a_3}{2} \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m^2 Q_n \cos(np + q_n)t$$

$$+ \frac{3a_3}{2} \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m Q_n^2 \cos(mp + q_m)t$$

$$+ \frac{3a_3}{4} \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m^2 Q_n \cos[(2m+n)p + 2q_m + q_n]t$$

$$+ \frac{3a_3}{4} \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m^2 Q_n \cos[(2m-n)p + 2q_m - q_n]t$$

$$+ \frac{3a_3}{4} \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m Q_n^2 \cos[(m+2n)p + q_m + 2q_n]t$$

$$+ \frac{3a_3}{4} \sum_{m=n_1+1}^{n_2} \sum_{n=n_1}^{m-1} Q_m Q_n^2 \cos[(m-2n)p + q_m - 2q_n]t$$

$$+ \frac{3a_3}{2} \sum_{l=n_1+2}^{n_2} \sum_{m=n_1+1}^{l-1} \sum_{n=n_1}^{m-1} P_l P_m P_n$$

$$\times \{\cos[(l+m+n)p + q_l + q_m + q_n]t$$
$$+ \cos[(l+m-n)p + q_l + q_m - q_n]t$$
$$+ \cos[(l-m+n)p + q_l - q_m + q_n]t$$
$$+ \cos[(l-m-n)p + q_l - q_m - q_n]t\}. \quad (3)$$

It is now a straightforward, though somewhat tedious process, to count the total number of possible products of each type falling in individual channels. The arrangement of terms above is such that $l > m > n$; this forms a convenient way of insuring that no product is counted twice. We shall illustrate by taking a simple case—the second order sum product, or $(A + B)$-type. Let kp represent the carrier frequency of the channel in which we wish to determine the number of possible $(A + B)$-type products. This means that in (2) we wish to find the number of terms in the third summation in which $m = n = k$. The resulting sum becomes:

$$\sum_{m=n_1+1}^{n_2} (n_1 \leq k - m \leq m - 1). \tag{4}$$

That is, there are as many terms as there are integer values of n satisfying the simultaneous inequalities,

$$\left[\begin{array}{c} n_1 + 1 \leq m \leq n_2 \\ \dfrac{k+1}{2} \leq m \leq k - n_1 \end{array}\right] \tag{5}$$

The number of terms is zero if $k > 2n_2 - 1$, because the lower limit of the second inequality exceeds the upper limit of the first. If $n_1 + n_2 \leq k \leq 2n_2 - 1$, the upper limit of the first inequality and the lower limit of the second inequality are governing, and the number of terms is $n_2 - I(k/2)$, where $I(x)$ is a symbolic representation for the largest integer $\leq x$. If $2n_1 \leq k \leq n_1 + n_2$, the second inequality is governing and the number of terms is $I\left(\dfrac{k+1}{2}\right) - n_1$. If $k \leq 2n_1$, the number of terms is zero.

In a similar manner the more complicated sums representing the number of third order products can be evaluated. It is to be noted that contributions to a particular type can come from more than one of the sums listed. For example, the $(A + B - C)$-type is made up of the summations from the $l + m - n$, $l - m + n$, and $l - m - n$ terms. In fact all these are of $(A + B - C)$-type except those in which $l - m - n$ is negative. The latter, since only positive values of frequency are significant, are of $(A - B - C)$-type. An $(A - B - C)$-type product differs from $(A + B - C)$-type not only in S.T.M.F., but also in manner of addition of contributions from a multi-repeater line as discussed in Section 8.

As an alternative to an actual count of the products falling in a channel, it is possible to approximate the sum by an integration process when the number of channels is large. This is an especially valuable simplification for products of high order for which the counting becomes very tedious. Suppose there are D components in unit band width in the range a to b. Let $x_1, x_2, \cdots x_n$ be n frequencies such that $a \leq x_1 < x_2 < \cdots < x_n \leq b$. The number of products of form $m_1x_1 + m_2x_2 + \cdots + m_nx_n$ which can be formed by selecting fundamentals from the n bands of width dx_1 at x_1, dx_2 at x_2, $\cdots dx_n$ at x_n is $D^n dx_1 dx_2 \cdots dx_n$. To count the total number of such products which can be formed in the band a to be such that the resultant frequency lies in the band x_0 to x, we form the integral

$$g(x, x_0) = D^n \int_a^b dx_n \int_a^{x_n} dx_{n-1} \cdots \int_a^{x_2} \varphi(x, x_0, x_1, x_2 \cdots x_n) dx_1, \quad (6)$$

where

$$\varphi(x, x_0, x_1, x_2 \cdots x_n) = \begin{pmatrix} 1, x_0 \leq m_1x_1 + m_2x_2 + \cdots + m_nx_n \leq x \\ 0, \text{ otherwise} \end{pmatrix}. \quad (7)$$

A suitable representation of φ is furnished by

$$\varphi(x, x_0, x_1, x_2 \cdots x_n) = \frac{1}{2\pi i} \int_C \left(e^{-ix_0 z} - e^{ixz} \right) e^{iz \sum_{r=1}^{n} m_r x_r} \frac{dz}{z}, \quad (8)$$

where C is a contour going from $z = -\infty$ to $z = +\infty$ and coinciding with the real axis except for a downward indentation at the origin. To obtain the number of products $\nu(x)dx$ falling in band of width dx at x, we write

$$\nu(x) = \operatorname*{Lim}_{\Delta x = 0} \frac{g(x, x - \Delta x)}{\Delta x}$$

$$= \frac{D^n}{2\pi} \int_C e^{-ixz}\, dz \int_a^b dx_n \int_a^{x_n} dx_{n-1} \cdots \int_a^{x_2} e^{iz \sum_{r=1}^{n} m_r x_r} dx_1. \quad (9)$$

If the spacing between carrier frequencies is used as the unit band width, we may set $D = 1$, $a = n_1$, $b = n_2$, $x = k$, in (9) and obtain the limiting forms of the results given in Table II as the number of channels is made large. Evaluation of (9) is easily carried out by means of the relation:

$$\int_C \frac{e^{ixz} dz}{z^m} = \begin{bmatrix} \dfrac{2\pi i^m x^{m-1}}{(m-1)!}, & x \geq 0 \\ 0, & x < 0 \end{bmatrix}. \quad (10)$$

As an example, suppose it is desired to calculate the approximate number of $(2A + B)$-type products falling in channel number k when the number of channels is large. If $x_2 > x_1$, this type of product may be either of form $2x_1 + x_2$ or $2x_2 + x_1$. Both are included by the expression

$$
\begin{aligned}
\nu_{2A+B}(k) &= \frac{1}{2\pi} \int_C e^{-kz} dz \int_{n_1}^{n_2} dx_2 \int_{n_1}^{x_2} \left[e^{iz(2x_1+x_2)} + e^{iz(x_1+2x_2)} \right] dx_1 \\
&= \frac{D^n}{4\pi} \int_C \frac{e^{-ikz}}{i^2 z^2} \left[e^{2in_2 z} - e^{i(2n_1+n_2)z} - e^{i(n_1+2n_2)z} + e^{3in_1 z} \right] dz \\
&= \begin{cases} 0, \; k < 3n_1 \\ \frac{1}{2}(k - 3n_1), \quad 3n_1 < k < 2n_1 + n_2 \\ \frac{1}{2}(n_2 - n_1), \quad 2n_1 + n_2 < k < n_1 + 2n_2 \\ \frac{1}{3}(3n_2 - k), \quad n_1 + 2n_2 < k < 3n_2 \\ 0, \quad k > 3n_2. \end{cases}
\end{aligned}
\tag{11}
$$

The method may be generalized to include the calculation of the modulation spectra produced by fundamentals having specified arbitrary spectra by inserting the appropriate function of the power in unit band width at $x_1, x_2, \cdots x_n$ in the integrand of (6).