High Accuracy Heterodyne Oscillators

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The accuracy of a heterodyne oscillator after the low frequency check is made is of the same order of magnitude as that of an ordinary type of oscillator in which circuit elements of the same stability are used. It depends on the constants of the variable frequency oscillator only. This accuracy can be improved by a ratio of 10 to 1 by adding another and higher check frequency. The temperature coefficient of the circuit elements can be kept down to less than 6 parts per million. Scale errors can be reduced to a value comparable with the oscillator accuracy by spreading the scale. A precision oscillator having a frequency range up to 150 kc. and an accuracy of ± 25 cycles including a scale mechanism whereby a large scale spread is obtained on a direct reading scale is described.

Introduction

THE output frequency of a heterodyne oscillator is obtained by modulating the outputs of two oscillators of appreciably higher frequency, one of the oscillators having a fixed frequency, the other being continuously variable over a band width equal to the required output frequency range.

The circuit consists essentially of the two so-called local oscillators, the modulator, where the difference frequency is generated, and an amplifier where the modulator output is raised to the desired level.

The earliest designs of heterodyne oscillator were confined to the audio frequency range, but recently carrier-frequency applications have become more numerous. As the frequency range of the oscillators has increased, their per cent accuracy requirement has increased also. The required frequency accuracy of the oscillator is determined by the maximum slope of the frequency characteristic of the apparatus being measured. If this slope is great, as in the case of a sharply tuned circuit a relatively small displacement of the frequency will result in a large error in the value to be measured. In carrier-frequency systems where the signal is displaced upwards in the frequency scale by modulation, each channel has to meet same crosstalk and transmission requirements independent of its location in the carrier band. Therefore, the maximum slope of the characteristics is independent of the frequency and an oscillator used for measuring purposes has to meet a constant frequency error requirement. In addition the accuracy required when expressed in cycles is comparable with that of audio-frequency oscillators so that the percentage accuracy must be much higher.

The advantages of the heterodyne oscillator have made it desirable to study its sources of error to determine whether such an oscillator can be designed to have sufficient accuracy for these applications.

Oscillators With a Single Frequency Check

The frequency of a heterodyne oscillator is given by the expression:

$$F = f' - f, \tag{1}$$

where we will assume f' to be constant and f to be variable and less than f' whence the frequency of the variable frequency oscillator is lowered as the output frequency of the heterodyne oscillator is raised.

The value of F is usually much smaller than either f' or f and relatively small frequency shifts in the local oscillators produced by aging and temperature effects upon the elements of their resonant circuits and changes in vacuum tubes and in the stray capacitances of the circuits produce large relative variations in the output frequency. Usually the stability required of F and the ratio f'/F are so high that it is impracticable to design local oscillators of sufficient stability to meet requirements. Instead, in all heterodyne oscillators an adjustment in the form of a padding condenser in the circuit of the fixed frequency oscillator is used, whereby its frequency is adjusted shortly before the measurement until the oscillator reads correctly at the bottom of its frequency range. The adjustment is made by the zero beat method or by comparison with a low-frequency standard such as a vibrating reed or the 60-cycle power supply.

At the time of the adjustment the frequency of the oscillator is

$$F_o = f' - f_o, \tag{2}$$

where f_o is the value of f at the check frequency F_o. Eliminating f' between (1) and (2) we obtain

$$F = F_o + (f_o - f). \tag{3}$$

The frequency of the variable oscillator may be expressed as

$$f = 1/(2\pi\sqrt{L(C_o + C_a)}), \tag{4}$$

where C_a is the change in the variable air condenser capacitance from the value it has at f_o, and C_o is essentially the value of the fixed condenser, usually a good mica unit. L is the inductance of the resonant circuit.

Combining (3) and (4) we get

$$F = F_o + 1/(2\pi\sqrt{LC_o}) - 1/(2\pi\sqrt{L(C_o + C_a)}). \tag{5}$$

The accuracy of the oscillator will depend on the variations in the values of F_o, L, C_o and C_a and is independent of the constants of the fixed frequency oscillator.

By giving increments ΔF_o, ΔC_o, ΔC_a and ΔL to the constants F_o, C_o, C_a and L we obtain after simplifying the expressions

$$\Delta F_{Fo} = \Delta F_o, \tag{6}$$

$$\Delta F_{Co} = -\frac{\Delta C_o}{2C_o} f_o \left[1 - \left(1 + \frac{F_o}{f_o} - \frac{F}{f_o} \right)^3 \right], \tag{7}$$

$$\Delta F_{Ca} = \frac{\Delta C_a}{2C_a} f_o \left[1 + \frac{F_o}{f_o} - \frac{F}{f_o} \right] \left[1 - \left(1 + \frac{F_o}{f_o} - \frac{F}{f_o} \right)^2 \right], \tag{8}$$

$$\Delta F_L = -\frac{\Delta F}{2L} F \left(1 - \frac{F_o}{F} \right) = -\frac{\Delta L}{2L} (F - F_o), \tag{9}$$

giving the corresponding frequency errors ΔF where $f_o = 1/(2\pi\sqrt{LC_o})$ is the variable oscillator frequency at the check frequency F_o.

A variation in F_o will produce an error constant over the whole frequency range. On Fig. 1 the other errors are found plotted in parametric form. To find the error ΔF corresponding to a frequency F the ordinate y corresponding to the value of $x = (F - F_o)/(f_o)$ should be found. Then

$$\Delta F_{Ca} = y_{Ca} f_o \Delta C_a / C_a; \qquad \Delta F_{Co} = y_{Co} \Delta C_o / C_o; \qquad \Delta F_L = y_L f_o \Delta L / L.$$

It is found that F_o can be neglected in all practical cases. The ratio of the ordinate to the abscissa gives the percentage error in frequency caused by a one per cent variation in the element involved.

An examination of the curves shows that they differ only slightly from straight lines which can be interpreted as meaning that the errors are fairly independent of the choice of f_o. This constant should be chosen therefore sufficiently low to require infrequent adjustment at the low-frequency end of the scale. For low values of f_o such that $x > .3$ difficulties in shaping of the air condenser plates and in designing the modulator filter begin to appear. If the errors in an ordinary type of oscillator due to capacitance and inductance variations were plotted on the same set of coordinates the curves would coincide with the line

y_L. This means that if elements of the same accuracy were used, the heterodyne oscillator would be somewhat more accurate. Its total error would be represented by $y_{ca} + y_{co} + y_L$. Since $\Delta C_a/C_a$ and $\Delta C_o/C_o$ will be both positive and of about the same order of magnitude partial compensation will obtain and the error will be of the order of

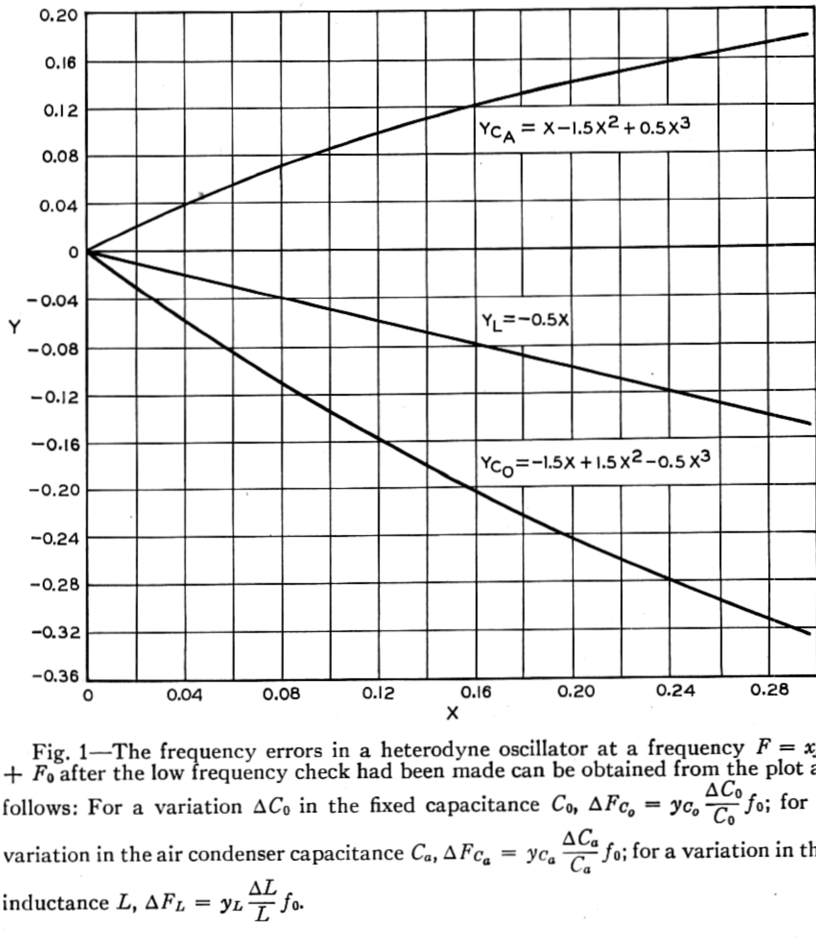

Fig. 1—The frequency errors in a heterodyne oscillator at a frequency $F = xf_0 + F_0$ after the low frequency check had been made can be obtained from the plot as follows: For a variation ΔC_0 in the fixed capacitance C_0, $\Delta F_{C_o} = y_{C_o} \frac{\Delta C_0}{C_0} f_0$; for a variation in the air condenser capacitance C_a, $\Delta F_{C_a} = y_{C_a} \frac{\Delta C_a}{C_a} f_0$; for a variation in the inductance L, $\Delta F_L = y_L \frac{\Delta L}{L} f_0$.

magnitude of ΔF_L. In the case of the ordinary type of oscillator the errors due to the capacitance and inductance variations will be equal and of the same sign so that the error will be of the order of magnitude of $2\Delta F_L$. For audio frequency applications this accuracy has been found to be adequate given sufficient care in the construction of the circuit elements.

Oscillators With a Double Frequency Check

For carrier frequency applications the tolerable error takes a constant value over the entire frequency range and it is found that if a single frequency check is used it is not possible to obtain sufficiently stable elements to maintain the required accuracy at points on the scale removed from the check frequency.

An increase in the accuracy of heterodyne oscillators has been obtained, however, by adding an adjustable condenser to C_o and checking the oscillator at two frequencies, the low frequency F_o and at another, higher, frequency F_s. Adjustment of this condenser by ΔC_o introduces a frequency change—$y_{Co} f_o \Delta C_o / 2C_o$ adjustable in sign and magnitude and this can be made to cancel the error $\Delta F_{Ca} + \Delta F_L$ for at least one frequency, the check frequency F_s. Obviously if the adjustment is made to correct for variations in C_o no residual error remains. The residual errors which remain after correcting for ΔF_{Ca} and ΔF_L are shown on Fig. 2. The residuals of ΔF_{Ca} and ΔF_L differ from each other

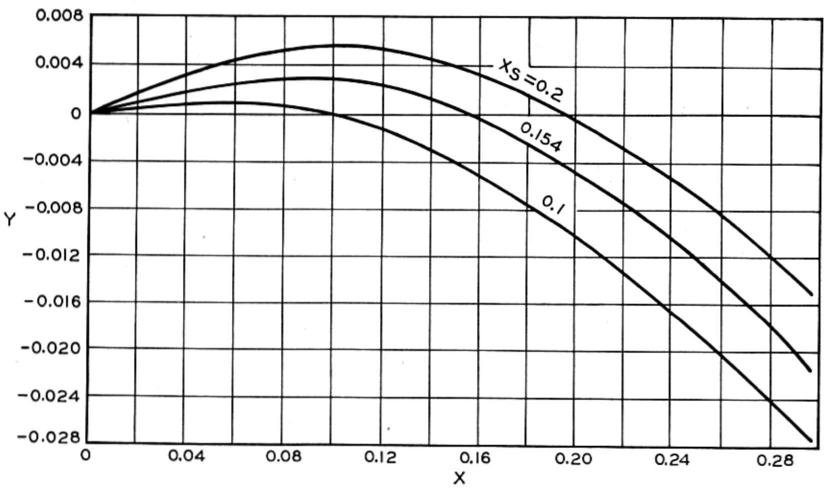

Fig. 2—The frequency errors in a heterodyne oscillator at a frequency $F = xf_0 + F_0$ after the low and high frequency checks had been made can be obtained from the plot as follows: For a variation in the air condenser capacitance C_a, $\Delta F_{C_a} = y \dfrac{\Delta C_a}{C_a} f_0$; for a variation in the inductance L, $\Delta F_L = y \dfrac{\Delta L}{L} f_0$.

so little that only one set of curves was drawn. The values of y were obtained by forming the sum $y = K y_{C_o} + y_{C_a}$ and choosing K so that $y = o$ for $x_s = (F_s - F_o)/f_o$.

For $x_s = .1$ better compensation is obtained at the lower end than at the higher. For a very wide frequency range up to $x = .25$ the best check frequency would be $x_s = .2$. A good practical limit to x is at 1.9 and here a value of x_s around .15 is best. A further improvement of about 50 per cent could be obtained by choosing a higher value of F_o. When comparing Fig. 2 with Fig. 1 it should be borne in mind that the scale spread for y on Fig. 2 is ten times that of the Fig. 1 which shows that an improvement in accuracy of at least ten to one is obtained by the adjustment. This means, that given two frequency standards F_o and F_s of sufficient accuracy a heterodyne oscillator can be built having a much higher accuracy than an ordinary oscillator having the same frequency range and same quality of circuit elements. This is somewhat contrary to what we are accustomed to think.

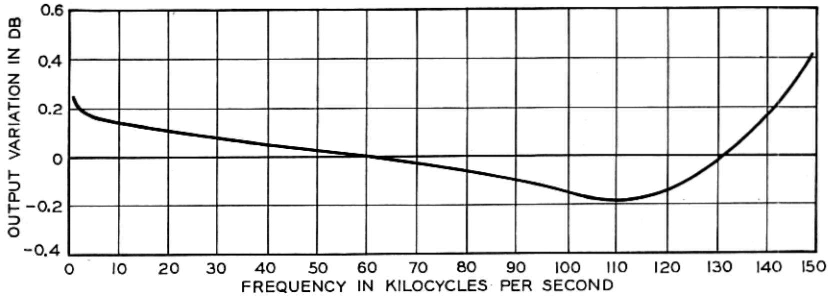

Fig. 3—Frequency-output characteristic.

One detail involved in the procedure of checking the oscillator which permits this high degree of accuracy to be obtained needs elaboration. As C_o is varied during the adjustment by the amount ΔC_o the value of f_o is changed and this destroys the low frequency adjustment at F_o. It is possible to obtain the adjustment by a process of successive approximations but the procedure is tedious. The difficulty can be overcome by the use of a mechanical device, however, as follows. The condenser ΔC_o is ganged to another condenser in the resonant circuit of the fixed oscillator, and the two condensers are so proportioned that the change in the fixed oscillator frequency is equal to the change in f_o as the condenser is adjusted. This makes the low frequency adjustment independent of the high frequency one. The oscillator is just set to the required reading at F_s and ΔC_o is adjusted until the frequency value is correct. Theoretically instead of two condensers two coupled inductometers could have been used to adjust the inductances in the resonant circuits. The net result obtained would have been the same and the mathematical treatment would be like the one given above. Condensers lend themselves better to such construction, however.

Stability of the Constants

Having determined the oscillator errors from the variations in its constants it will be of interest to inquire how large these may be.

When the zero beat method is used the error ΔF_{F_0} will depend on the value of the lowest beat frequency at which the local oscillators can operate. With a reasonable amount of shielding and some precautions in order to avoid mutual inductance in wiring loops it is quite practicable to keep this error below one cycle with local oscillators as high as 200 kc. The beat frequency may be observed on an ammeter placed in the plate circuit of the modulator. When alternating current from the power mains is used as a standard the accuracy is better than one cycle.

There are now available external frequency standards against which the high frequency check could be made which have such high accuracy that the resulting error in the heterodyne oscillator can be entirely neglected. It is desirable, however, to make the oscillator independent of external sources for its adjustment. A convenient checking circuit consists of a quartz crystal which is thrown in with a key across the grids of the output amplifier. At the series resonance frequency of the crystal the loss introduced reduces the output so sharply that the minimum output can be observed within 3 cycles at 100 kc. At any other frequency the error is therefore 30 ppm (parts per million). By using properly cut crystals the temperature variation error is made negligible.

The variations in L are chiefly due to temperature variations. Ordinary potted coils having a large number of layers have temperature coefficients up to 20 parts per million per degree Fahrenheit. The variation is chiefly due to the expansion of the wire.

This error is tolerable in audio frequency oscillators for most purposes. For carrier frequency oscillators unpotted coils having a single layer bank winding wound on a phenol plastic form may be used. Here the lengthwise expansion of the form, which tends to decrease the inductance partly compensates for the expansion of the winding which tends to increase the inductance. Coefficients from 0 to + 6 ppm per °F. are obtained.

The capacitance C_a in commercial air condensers has temperature coefficients of up to 25 ppm per °F. This, again, gives sufficient accuracy for audio frequency oscillators but is not satisfactory for carrier applications. The variations in capacitance with temperature are produced by increase in area of the plates with their expansion which increases by an amount equal to twice the linear coefficient of expansion of the material used. This change is partly compensated by the lengthening of the air-gaps. When, as usual, several materials are used in the construction, bending of the stator plates due to strains introduced by unequal expansions of the members produce unpredictable changes in capacitance. This is particularly true in the most common construction where the stators are held in place by rods of insulating material. The insulator having a different temperature coefficient of expansion than the plates, the difference in the expansion causes the plates to buckle.

Better stability can be obtained in a condenser built as follows. All parts determining the length of the condenser, including the stator supports and the stator plates are of aluminum. The ends of the stator supports are held in place by insulating bushings of sufficiently small dimensions to make the difference in expansion negligible. The bushings are made of Alsimag, a ceramic material which has a small dielectric constant and coefficient of dielectric constant.

With such a construction, the temperature coefficient of the condenser is equal to twice the temperature coefficient of expansion of the material of which the rotor plates are made minus the temperature coefficient of linear expansion of aluminum determining the length of the air-gaps. One half of the rotor plates are made of invar and one half of aluminum. The average expansion of the area of the rotor plates equals then the temperature coefficient of linear expansion of aluminum and the temperature coefficient of capacitance of the condenser should be equal to the temperature coefficient of air dielectric constant which is about 1 ppm per °F. negative. Measurements show that the temperature coefficient of the condenser varies from -3 to $+4$ ppm per °F., a quite acceptable value. The capacitance change due to a variation in the atmospheric pressure of one inch, a large variation, is 20 ppm.

Temperature coefficients, of paraffined mica condensers, can be adjusted by special manufacturing methods to 10 ppm negative. For the sake of increasing the instantaneous stability the two condensers used in each oscillator are paired within 3 ppm. As mentioned before, no residual error due to ΔC_o remains after the frequency check is made. The low temperature coefficients are desirable only to improve the stability of the oscillator.

By using high Q circuits and suitable corrective reactances, the variations in the frequency due to power line variations may be readily kept smaller than any one of the other errors discussed above.

Scale Errors

A heterodyne oscillator cannot be classified as a purely electrical circuit for it is used to translate a mechanical coordinate, the scale setting, into an electric coordinate, the output frequency. In planning the oscillator design, therefore, it is necessary to give as much attention to the construction of the scale as to the construction of the circuit elements.

For maximum scale length economy the scale should be so subdivided that a frequency interval equal to the tolerable frequency error ΔF could be read. The scale interval Δl corresponding to this frequency interval, will vary with the measuring conditions. For well illuminated scales on panel mounted equipment to be read conveniently at arm's length an interval Δl of at least .05″ is needed. For portable apparatus, intervals as small as .02″ have been used. With the aid of a vernier it can be brought down to .001″. Scale spreads such that a frequency interval much smaller than ΔF can be read are not only uneconomical but are also objectionable because they encourage the use of the instrument beyond its accuracy limits.

Having chosen Δl and the frequency error ΔF at all points of the scale, the scale shape $l = f(F)$ can be determined by the approximation

$$l = \int_0^F \frac{\Delta l}{\Delta F} dF.$$

As an example, in audio frequency applications the most common form of frequency accuracy desired is that having a constant percentage value $\Delta F/F = \rho$ at the upper part of the scale. At lower frequencies this accuracy is higher than necessary and the requirement is changed to a constant ΔF_o. A smooth shape is obtained by making the transition point F_T at such a frequency that $\Delta F_o/F_T = \rho$. The scale shape is then approximately

$$l = \int_0^F \frac{\Delta l}{\Delta F_o} dF = \frac{\Delta l}{\Delta F_o} F \qquad \text{for} \qquad F < F_T$$

and

$$l = \int_0^{F_T} \frac{\Delta l}{\Delta F_o} dF + \int_{F_T}^F \frac{\Delta l}{\rho F} dF = \frac{\Delta l}{\Delta F_o} F_T + \frac{\Delta l}{\rho} \log_e \frac{F}{F_T}, \quad \text{for} \quad F > F_T$$

The scale of common type of audio frequency oscillator can be spread over a ten inch dial giving a satisfactory accuracy.

For carrier applications, where the spread of any voice band is independent of its position in the frequency range the error function takes the form of a constant and the scale should be linear. Usually the scale lengths involved are much larger than in audio oscillators. To obtain sufficient scale length a precision worm and gear mechanism has to be used to drive the tuning condenser of the heterodyne oscillator. It gives a scale length of 300 inches, the equivalent of a 5-foot dial and can duplicate settings to better than one part in 10,000.

One detail of construction of such long scales deserves mention. Commercial worm driven air condensers carry on the worm shaft a drum or a dial on which fractions of a revolution of the worm shaft are recorded, while the number of revolutions is recorded on a main dial fixed on the rotor shaft. The effective scale length is then equal to the total displacement of the periphery of the small dial or drum. Using such a construction the oscillator has to be set by consulting a calibration chart where the position of the main dial and of the worm shaft is recorded against the oscillator frequency. Thus one of the most valuable properties of the short scale audio frequency oscillator, its direct reading, is lost.

To remedy this situation a special scale mechanism has been developed for carrier frequency oscillators which combines great scale length with good spread and compactness. It consists of a long motion picture film strip engaged by a two inch film sprocket mounted in place of the conventional drum on the worm shaft. The rotation of the shaft determines the displacement of the film against an index which reads the frequency directly in kilocycles. The loose ends of the strip are wound up on two spools interconnected by a spring mechanism which takes up the slack. The whole mechanism is confined in a space about 4″ by 5″ by 5″ accommodating a scale length up to 450 inches with a scale spread corresponding to $\Delta l = .05''$.

Specific Application

An example of application of these methods in the design of a heterodyne oscillator is furnished by an oscillator built for use in connection with the installation and maintenance of broad band transmission systems. It is shown on Fig. 4.

It has a frequency range of from 1 to 150 kc. Its variable frequency oscillator covers a range of from 500 to 650 kc. This was chosen as low as possible to obtain good instantaneous stability, but high enough not to introduce difficulties in designing the filter following the modulator. The capacitance of the air condenser is about 800 $\mu\mu$f. From the circuit design standpoint a larger capacitance would be desirable, but for the stability required the overall size of the condenser sets an upper limit to the capacitance. The frequency and air condenser capacitance values determine the value of the fixed condenser at 1000 mmf and the coil inductance at 50 microhenries. The fixed oscillator is similar to the variable oscillator except for the omission of the variable air condenser.

The frequency setting is recorded on a 300-inch film scale such as described above. This gives a spread of two inches per kilocycle. With the 50-cycle divisions marked directly the mechanism can be readily set to an accuracy better than 25 cycles. The visibility of the scale is greatly enhanced by a pilot lamp placed in back of the scale

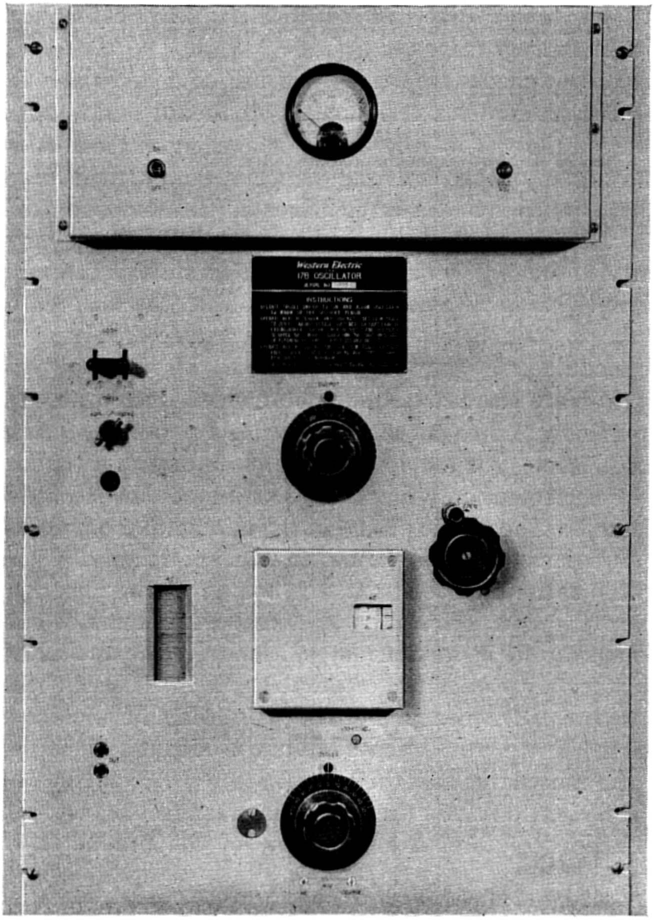

Fig. 4—Front view of the oscillator.

window with an intervening opal glass. A crank on the front of the panel is used to set the oscillator, the range being covered in 47 revolutions. When changing the frequency setting even at a moderate rate the speed with which the film moves prevents the operator from observing the frequency setting. To make the adjustment more convenient, a coarse scale is recorded on a dial which can be read easily to one kilocycle while the mechanism is in motion. It can be seen under the hood in the center of the panel.

Below the coarse frequency dial is seen a small dial connected to a variable condenser which permits the operator to vary the frequency of the oscillator up to $\pm$ 50 cycles from the frequency to which it is set and to read the frequency change with an accuracy of about 3 cycles. This feature is found to be useful in locating peaks of frequency characteristics of sharply resonant circuits.

The frequency checks are made by operating a key which throws the oscillator output across a telephone switchboard lamp and a 100 kc crystal across the grid of the output stage. For the low frequency check another key superposes the 60 cycles power main frequency on the oscillator output and a screwdriver adjustment operating a condenser in the fixed oscillator adjusts the oscillator frequency to synchronism with the scale set at 60 cycles. For the high frequency check a minimum signal is obtained on the lamp with the scale set to 100 kc by adjusting a padding condenser in the variable oscillator.

In the modulator a pentode type vacuum tube is used, which has a control grid-plate current characteristic which over nearly the entire region from zero bias to cut-off approaches a parabola so closely, that where modulation products lower than 40 db down on the useful output can be neglected, only first and second modulation products need be considered. The bias is placed in the middle of the parabolic range and the two input signals are adjusted to equality and to a value covering the entire parabolic range. This gives the maximum useful modulation output necessitating the smallest amount of gain in the output stage at little sacrifice in efficiency. The modulator being parabolic, the only products of modulation other than the useful output are the two high frequency input signals, their harmonics and sum frequencies. These are eliminated from the output by inserting a filter between the modulator and the output stage. Advantage is taken of phase discrimination since the circuit is arranged in push-pull to decrease the filter requirements for some of the products, which are generated in phase.

The plate supply is obtained from a rectifier operating on the 60-cycle main supply. It is provided with a vacuum tube regulator circuit which keeps the plate and screen voltages constant over a $\pm$ 5 volt variation of the power line voltage. The output control is obtained by means of a potentiometer in the output amplifier input. Two output impedances, 600 and 135 ohms, may be selected by operating a key.

The apparatus is mounted on a standard 19-inch panel 28 inches high. The bottom, the coolest part, is occupied by the oscillators; the middle by the modulator and amplifier; and the top by the power pack. Perforations in the oscillator cover provide ventilation to reduce warming-up effects. A close-up giving the details of the scale mechanism and the shielding is shown on Fig. 5.

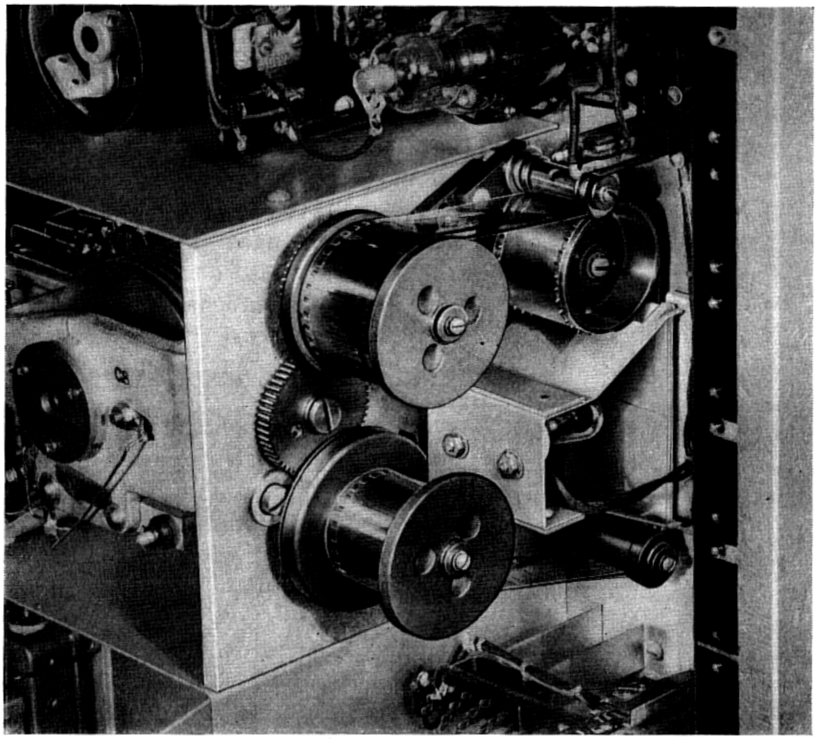

Fig. 5—Details of the scale mechanism.

Tests on the oscillator show that the overall frequency accuracy throughout its range can be maintained to ± 25 cycles. The harmonics are down 40 db from the fundamental at 100-milliwatt output. With the full output of one watt the harmonics are 30 db down. The total output variation with frequency is shown on Fig. 3.

This oscillator has found a wide range of applications as an accurate source of frequency in the communications field.

List of Symbols

F output frequency of the heterodyne oscillator

F_o standard frequency used to check the oscillator at the low end of the scale

F_s	standard frequency used to check the oscillator at the high end of the scale
F_T	frequency at which the scale changes from linear to logarithmic
l	length of the scale interval from 0 to F
f	frequency of the variable oscillator
f'	frequency of the fixed oscillator
f_o	frequency of the variable oscillator at the setting $F = F_o$
L	inductance in the resonant circuit of variable oscillator
C	total capacitance in the resonant circuit of variable oscillator
C_o	total capacitance in the resonant circuit of variable oscillator when set to $F = F_o$
$C_a = C - C_o$	capacitance change in the air condenser
ΔF_o	variation in the standard frequency F_o
ΔC_a	variation in C_a
ΔC_o	variation in C_o
ΔL	variation in L
Δl	smallest readable scale interval
ρ	relative frequency error $\Delta F/F$
ΔF	error in F
ΔF_{Fo}	error in F caused by ΔF_o
ΔF_{Co}	error in F caused by ΔC_o
ΔF_{Ca}	error in F caused by ΔC_a

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