

Compressed Powdered Molybdenum Permalloy for High Quality Inductance Coils *

By V. E. LEGG and F. J. GIVEN

Molybdenum-Permalloy is now produced in the form of compressed powdered cores for inductance coils. Its high permeability and low losses make possible improved coil quality, or decreased size without sacrificing coil performance. Its low hysteresis loss reduces modulation enough to permit application where large air core coils would otherwise be required.

INTRODUCTION

THE introduction of loading coils in the telephone system at about the turn of the century brought special demands on magnetic and electrical properties of core materials, and set in motion investigations which have had wide influence on the theoretical and practical aspects of ferromagnetism. The first step in this development led to cores of iron wire, which sufficed for loading coils on circuits of moderate length.¹ With the development of telephone repeaters and the extension of circuits to transcontinental length some twenty-five years ago, there arose need not only for loading coils, but also for network coils, which would have high stability with time, temperature and accidental magnetization. Magnetic stability was at first secured² by employing iron wire cores provided with several air gaps. Later, commercial and technical considerations led to a core structure made from compressed insulated powdered material, first electrolytic iron³ and later permalloy powder.⁴ This type of core is mechanically stable; it introduces in an evenly distributed fashion the requisite air-gaps, while avoiding undesirable leakage fields; and it sub-divides the magnetic material so as to reduce eddy-current losses. Although other means have been suggested,^{5, 6} no way has yet been devised which provides these features so well and at so low a cost as the compressed powdered type of core.

Loading coil cores made from electrolytic iron powder generally satisfied the stability requirements for long lines, but on account of their low magnetic permeability they were large and costly. The search for materials with higher permeability and lower hysteresis loss

* Presented at Winter Convention of A.I.E.E., New York, N. Y., January 22-26, 1940.

led to permalloy⁷ which, by 1925, had been produced in powdered form and fabricated into cores. This development provided coils for voice frequency applications (loading coils and filter coils) which were cheaper and yet superior electrically to those made from electrolytic iron. These coils became available at a time when the telephone plant was undergoing a very large extension of loaded cables. As a result, large economies in cost and space were realized in the more than six million coils involved in this plant expansion.

Iron powder, and later permalloy powder, ground to a finer size and diluted to lower permeability than used in loading coils, also found application in coils for oscillators, filters and networks of multiplex carrier telephone and telegraph systems employing frequencies up to 30 kc.⁸ and in receivers for transoceanic radio telephone communication employing frequencies up to approximately 60 kc.⁹ Permalloy powder improved the electrical characteristics—particularly modulation—of coils for use in high frequency circuits, because of its low hysteresis losses.

Continued research for a powdered material having still better intrinsic properties has recently made available new compressed powder cores which permit further important gains in coils for voice-frequency circuits and in coils for high-frequency carrier system applications. The latter take on considerable significance at this time because they play an important part in making practical for commercial use the new broad-band carrier telephone systems intended for use on existing open-wire and cable lines and on new types of cable. Again, therefore, the advent of a new core material is well-timed to be of assistance in further growth of the telephone system.

The development of this core material was based on the discovery¹⁰ that the addition of a small percentage of molybdenum to permalloy increases its permeability and electrical resistivity, and decreases its eddy current and hysteresis losses. Decreased losses are necessary for improvements and economies for both voice and carrier frequency operation. The increased permeability of this alloy is essential for the improvement of voice-frequency coils. It is readily reduced to the proper values for high-frequency coils by diluting the powdered magnetic material with insulating material before compressing into core form. In this development many problems of alloy embrittlement, pulverization, insulation and heat treatment had to be solved both on a laboratory and factory scale. The alloy composition finally selected as giving the best combination of desirable properties, contains approximately 2 per cent molybdenum, 81 per cent nickel, and 17 per cent iron, and is designated as 2-81 molybdenum-permalloy. This new

alloy is manufactured commercially by the Western Electric Company for use in loading coils and filter coils.

PHYSICAL AND MAGNETIC CHARACTERISTICS

The raw materials and necessary embrittling agents¹¹ are melted together and cast into ingots which are rolled to develop the desired grain structure. The density of this alloy is 8.65 gm/cm³. The brittle material is pulverized to the desired fineness and finally annealed to soften the alloy particles before insulation and pressing into core form.

The distribution by weight of the particle sizes of a sample of 120-mesh powder is given in Fig. 1, showing a root mean square size of 50

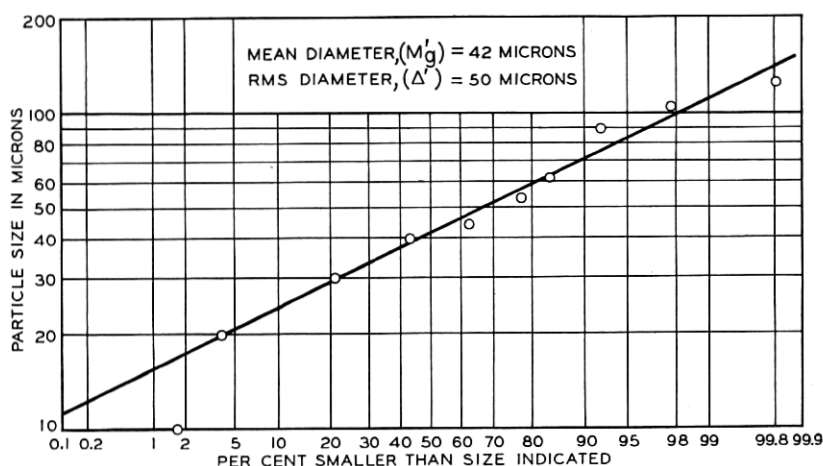


Fig. 1—Distribution of particle size of 120-mesh powder, by weight.

microns.¹² Since the effective resistance of a coil due to eddy-current losses in its core is proportional to the mean square particle diameter,¹³ it can be decreased when desired by the use of more finely pulverized material.

The problem of insulating 2-81 molybdenum-permalloy powder is to coat the particles with a minimum thickness of a material which will not break away during the pressing operation, which will not fuse and flux the magnetic particles together during the core heat treatment, which will prevent the flow of eddy currents between metallic particles, and which will be chemically inert throughout the lifetime of the magnetic core. The difficulty of the problem will be appreciated from the fact that the separation between adjacent particles of a core of 125 permeability is approximately equal to the wave-length of visible

light (0.5 micron). This thickness of insulating film may be shown to be approximately $\frac{rt}{300p}$, where r is the percentage of insulating material by volume, p is the packing factor of the magnetic material, and t is the r.m.s. particle diameter (assuming spherical particles).

A new type of ceramic insulation has been introduced with these cores which fulfills the above requirements and which is more inert than the previous type. This new insulating material is free from water soluble residue. It thus eliminates the final washing treatment which was required with the earlier type.

For applications where a low permeability is desired, non-magnetic powder is added to further dilute the magnetic material. The permeability of the finished core depends largely on the quantity, particle size, and thoroughness of admixture of non-magnetic powder. Various attempts to derive theoretical relations between core permeability and dilution have been made,^{14,15} but they generally fail in some detail. An empirical representation of this relationship is found to be

$$\mu = \mu_i^p \quad \text{or} \quad \log \mu = p \log \mu_i,$$

where μ_i is the intrinsic permeability of the magnetic material, and p is the packing factor, or fraction of the core volume occupied by magnetic material. This equation is found to be valid for a wide range of dilution, effected either by adding insulating material or by reducing the load during core compression. However, the intrinsic permeability must be determined experimentally for each type of particle, size distribution, method of admixture of non-magnetic powder, and annealing process. Figure 2 shows curves of permeability and percentage diluting material vs. metallic packing factor. A permeability of 125 has been selected for most loading coil cores, while permeabilities of 14 and 26 have been chosen for two important types of high-frequency filter coils.

A pressure of 100 tons/sq. in. is employed in forming molybdenum permalloy cores, to attain proper density and mechanical strength. The effect of pressure on core density and strength is shown in Fig. 3 for cores having 2.5 per cent dilution. The tensile strength of diluted cores is decreased somewhat; for example, cores with 25 per cent non-magnetic materials have a tensile strength of about 250 lbs./sq. in.

In annealing the compressed cores to remove stresses incident to pressing, it has been found that the insulating material remains intact at a considerably higher temperature if oxygen is excluded. An improved annealing treatment has therefore been introduced by which

the cores are heated in an atmosphere of hydrogen, with the attainment of high core permeability and low hysteresis loss. The ability of the insulating material to withstand such a heat treatment testifies

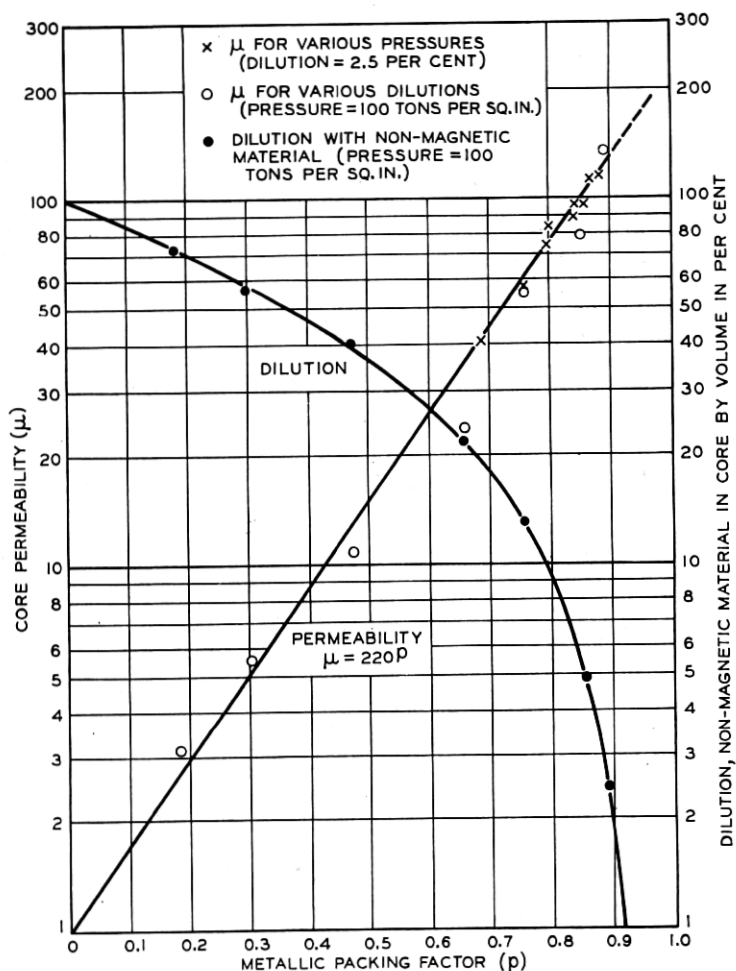


Fig. 2—Relation between metallic packing factor, permeability and percentage dilution.

to its extreme stability and recommends it in preference to organic materials.

An essential core requirement of precision inductance coils is that the permeability remain unaltered during the life of the coil. The greatest difficulty with cores having no air gaps is the large and more or

less permanent shift of permeability due to accidental strong magnetization. Such variations have recently been overcome to a degree in continuous cores made of hard rolled nickel-iron alloy sheet,⁶ but they have been found to be rather large immediately after strong magnetization, decreasing slowly to tolerable limits only after two or three days. With compressed powdered molybdenum-permalloy cores, permeability shift due to strong magnetization is remarkably small even within a fraction of a minute after the magnetization is released, and any

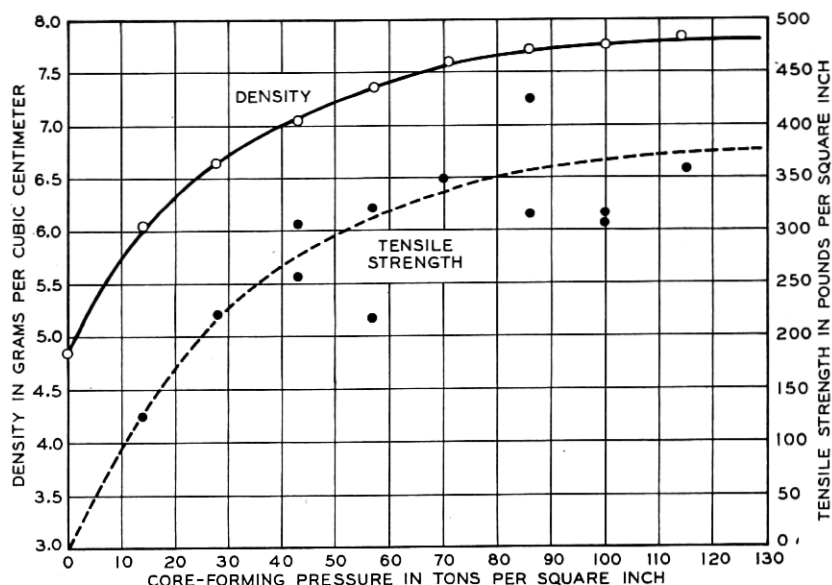


Fig. 3—Effect of core forming pressure on density and tensile strength.

further drift of permeability with time is negligible. In typical cores of the new material, the shift in permeability after strong magnetization is less than 0.2 per cent for cores of permeability 125, and less than 0.05 per cent for cores of permeability 14. Figure 4 shows the residual effect of the application and removal of various magnetizing forces on cores of both these permeabilities.

When a direct current is superposed on an alternating current in the windings of a coil, the inductance is altered because the magnetic field set up by the direct current modifies the core permeability. Figure 5 shows the effect of superposed d-c. fields on the permeability of 2-81 molybdenum-permalloy powder cores of various permeabilities.

A further important core property is the constancy of permeability with respect to flux density B . This is of particular importance in

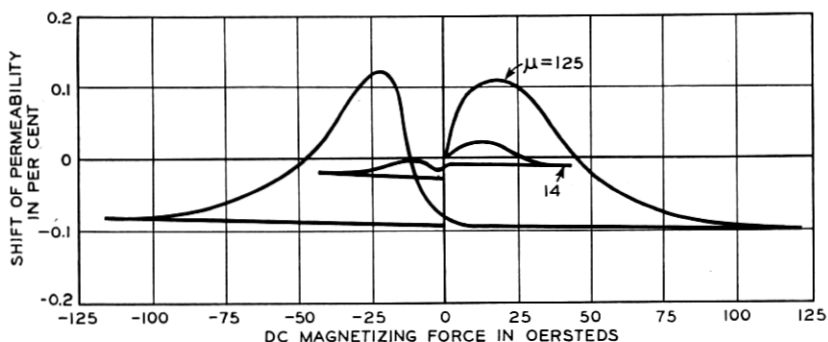


Fig. 4—Residual effect of d-c. magnetization on initial permeability—measured three minutes after release of direct current.

precision filters, to insure that changes in transmission level do not produce serious alterations in the frequency discrimination characteristics. Figure 6 shows the superiority of the new material over the earlier permalloy.

A new requirement for cores has been introduced by quartz crystal filters used in wide-band carrier systems. In order to secure the necessary precision in this type of filter, measures have to be taken to prevent departures from the initial frequency adjustment due to changes in core permeability ordinarily occurring with room temperature changes. Extremely small temperature coefficients of permeability have now been achieved by adding to the new 2-81 molybdenum-permalloy powder a very small percentage of special permalloy powder having a molybdenum content of about 12 per cent. Such an alloy has a non-magnetic or Curie point close to room temperature, and for a

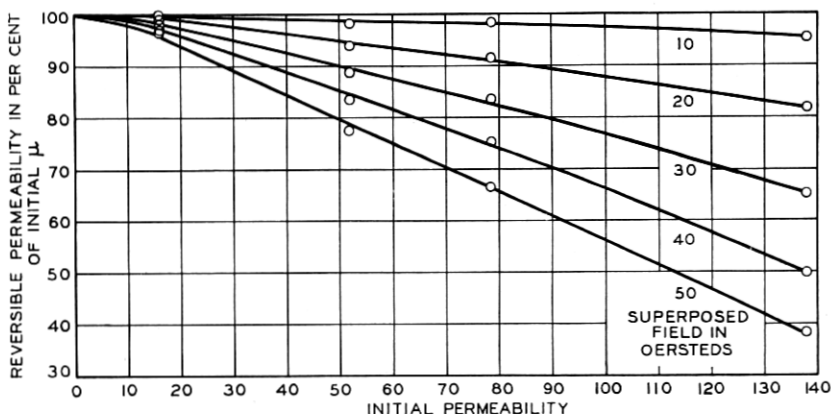


Fig. 5—Effect of superposed magnetization on permeability.

small temperature range just below its Curie point, it has a negative temperature coefficient several hundred times as large as the positive coefficient of 2-81 molybdenum-permalloy. By choosing suitable compositions and percentages of such compensating alloys, the net temperature coefficient of permeability of a core can be adjusted to any reasonable value, positive or negative, over a desired temperature range. Figure 7 shows a permeability vs. temperature curve for a

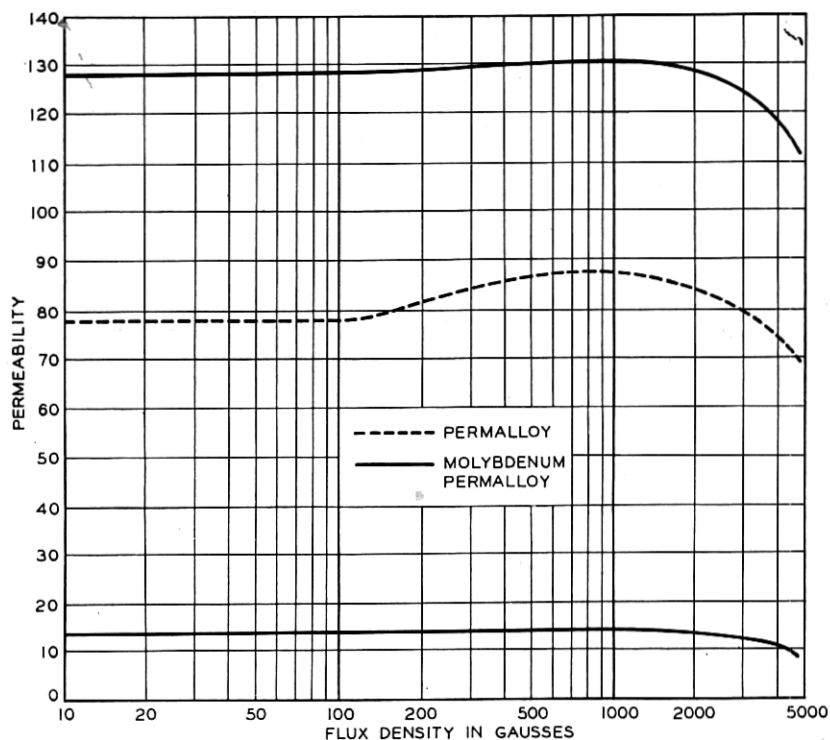


Fig. 6—Permeability-induction characteristics.

core stabilized to give a small negative coefficient, compared to a similar curve for a core not stabilized.

CORE LOSSES

The desirability of core materials increases in general as their loss characteristics decrease. Low total eddy-current and hysteresis losses give low contributions to attenuation. Hysteresis loss is frequently of especial importance because it appears fundamentally as a resistance which varies with coil current, and because it incidentally generates harmonic voltages. A low value of hysteresis loss thus

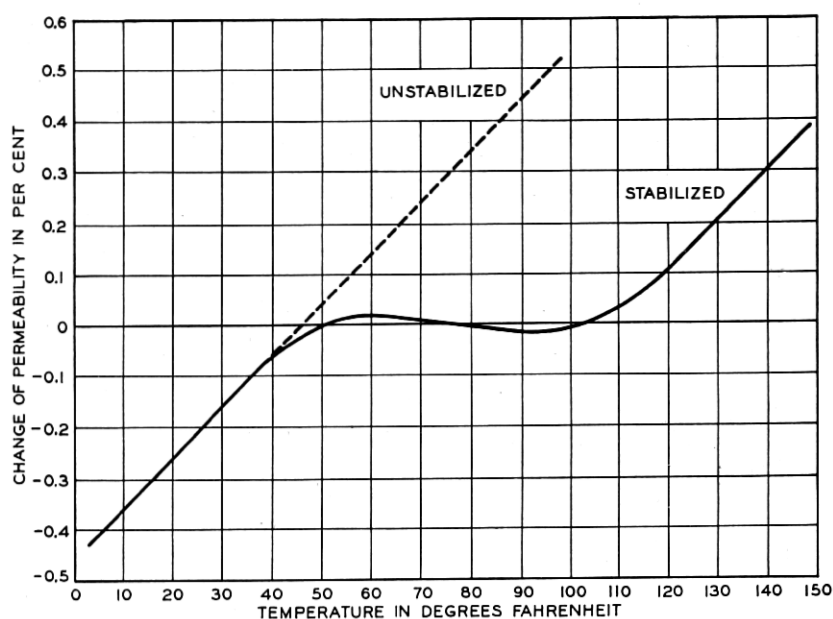


Fig. 7—Effect of temperature on permeability of stabilized and unstabilized cores.

simplifies circuit problems arising from resistances which vary with energy level, and it avoids troublesome modulation conditions.

The total resistance per unit inductance arising from eddy-current and hysteresis losses may be expressed as ¹⁶

$$R_m/L = \mu(aB_m + c)f + \mu e f^2,$$

where the symbols are as given in the Appendix.

Table I gives the loss coefficients for various core materials, and for

TABLE I
LOSS COEFFICIENTS OF POWDERED CORE MATERIALS

Material	μ	Hysteresis		Residual		Eddy Current	
		$a \times 10^6$	$\mu a \times 10^3$	$c \times 10^6$	$\mu c \times 10^3$	$e \times 10^9$	$\mu e \times 10^6$
Grade B Iron	35	49	1.7	109	3.8	88	3.1
Grade C Iron	26	81	2.1	139	3.6	31	0.8
81 Permalloy	75	5.5	0.41	37	2.8	51	3.8
	26	11.5	0.30	108	2.8	27	0.7
2-81 Molybdenum Permalloy	125	1.6	0.20	30	3.8	19	2.4
	26	6.9	0.18	96	2.5	7.7	0.2
	14	11.4	0.16	143	2.0	7.1	0.1

2-81 molybdenum-permalloy insulated to several permeabilities. The low loss coefficients of the new material as compared with the best previous materials are of importance from two standpoints. First, core permeabilities as much as 50 per cent greater can be now utilized in coils without increasing the total core loss resistance. Second, by utilizing the same permeabilities, core loss resistances about 60 per cent smaller can be obtained.

In many coil design problems, harmonic generation or modulation assume controlling importance. The modulation factor m which denotes the ratio of the generated third harmonic to the applied voltage may be expressed as follows ¹⁷

$$m = E_3/E_1 = 3\mu a B_m/10\pi.$$

The low values of a obtained with the new material yield values of m that are about 6 db and 20 db lower than possible with powdered permalloy and electrolytic iron cores, respectively.

The wide range of core permeability available with this new material permits a ready choice of the proper values of permeability and core size to suit any particular needs. In the usual design problem the following main requirements must be considered, in addition to providing the desired inductance.

1. D-C. Resistance, R_c .
2. Coil quality factor, $Q = \omega L/(R_c + R_m)$.
3. Modulation Factor, m .
4. Coil size (which depends directly on core size).

These requirements can not be satisfied independently however, as fixing any two of them automatically fixes the values of the others. In each case, a particular value of core permeability is required for the proper fulfillment of the conditions. Appendix I lists the formulae essential to the determination of these factors. Although these formulae imply an entire freedom of choice of core permeability and size, it becomes necessary for practical purposes to standardize on a limited number of values of permeability and a limited number of sizes of core. It is possible by the proper choice from these types to approach rather closely to an ideal solution for each problem.

IMPROVED DESIGNS OF LOADING COILS

A study of alternate ways of utilizing the advantages offered by the new material in coils for voice frequency loaded cables showed that the greatest immediate benefit to the telephone plant would accrue from making the new coils substantially duplicate performance characteristics of previous designs. The new designs chosen are in fact better

in most respects and have been made approximately 50 per cent smaller in volume by using a molybdenum-permalloy core with a nominal permeability of 125. Table II summarizes data comparing

TABLE II
COMPARATIVE SIZE AND WEIGHT DATA OF TYPICAL NEW AND SUPERSEDED COILS

Type of Coil	Type of Compressed Powdered Core	Inductance (Henrys)	Coil Volume (Cu. In.)	Coil Weight (Lbs.)
Small Exchange Area	Permalloy	0.088	2.5	0.4
"	Molybdenum Permalloy	0.088	1.5	0.2
Program Circuit	Permalloy	0.022	11.8	1.6
"	Molybdenum Permalloy	0.022	4.4	0.6
Toll-Side Circuit	Permalloy	0.088	13.5	1.7
"	Molybdenum Permalloy	0.088	5.1	0.7

the electrical characteristics, sizes, and weights of coils commonly used on exchange and toll cables. Figure 8 shows the improved

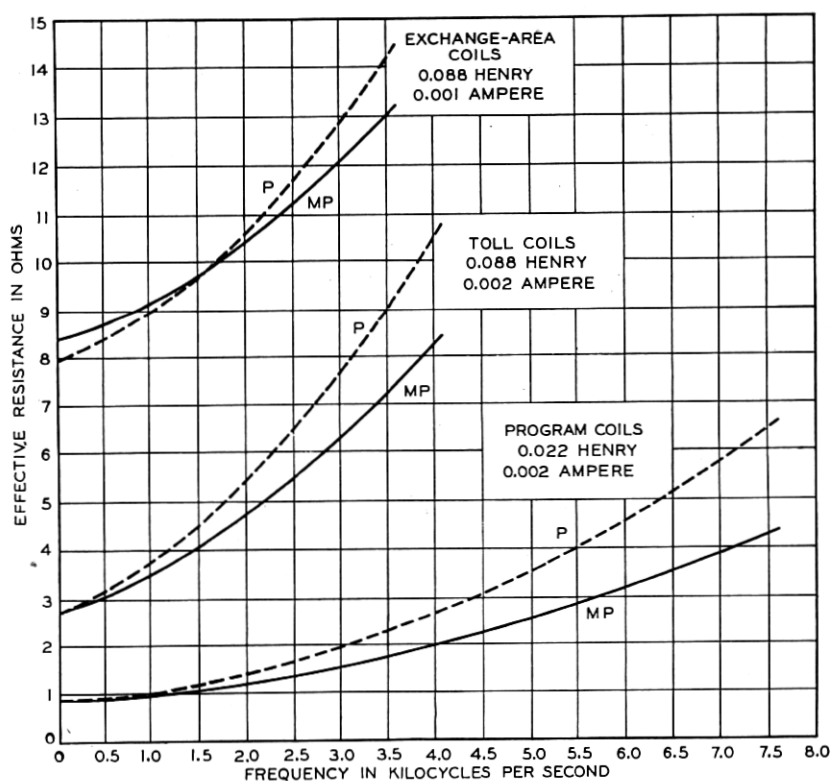


Fig. 8—Effective resistance-frequency characteristics of typical loading coils.

resistance-frequency characteristics for typical coils. Figure 9 shows the improved telegraph flutter¹⁸ characteristics of a commonly used toll type coil.

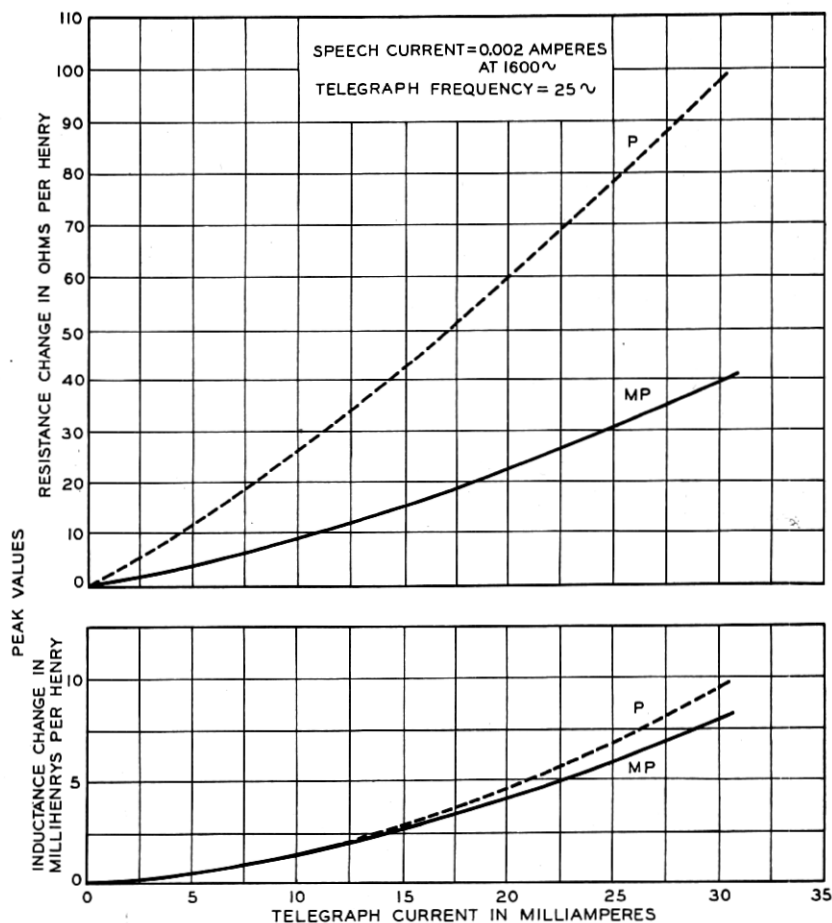


Fig. 9—Flutter characteristics of typical toll loading coil; P—with permalloy core, MP—with molybdenum-permalloy core.

Figure 10 pictures the reduction in size of cores and coils which are commonly used in toll and exchange area circuits. In the preparation for commercial manufacture of the smallest of the new coils, a difficult problem in the development of winding machinery was involved because of the small dimensions of the hole in the finished coil. This problem has been successfully solved by the Western Electric Company.

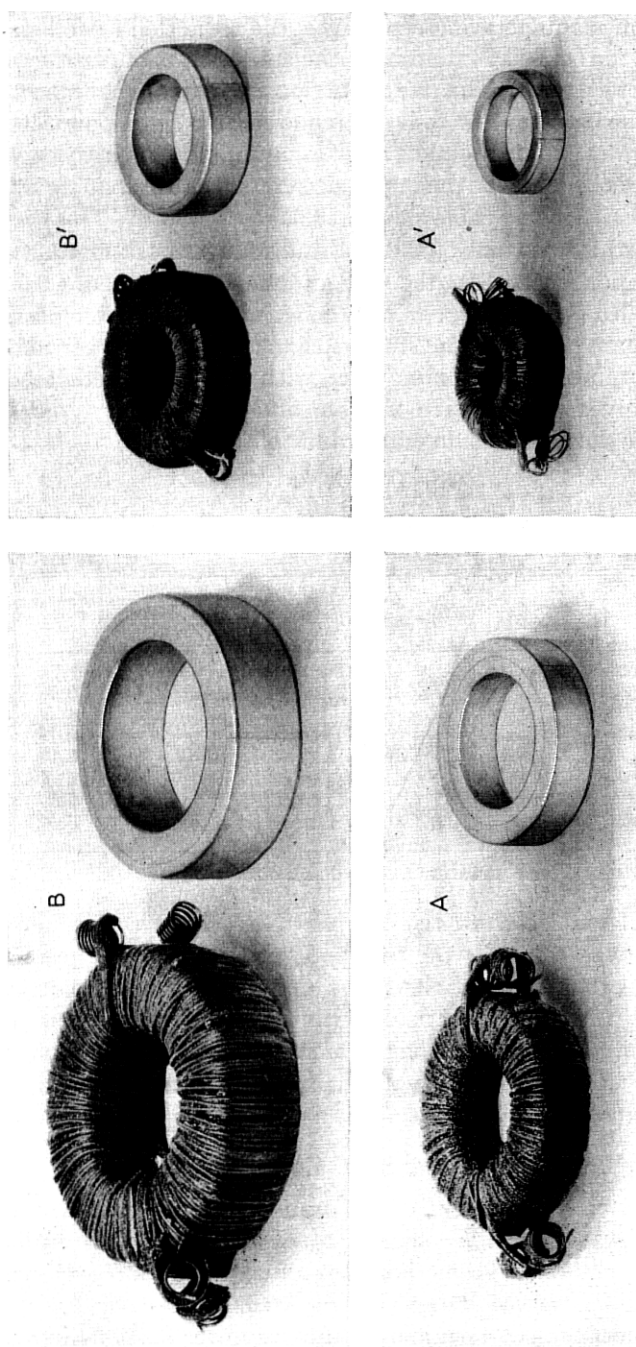


Fig. 10—Comparative sizes of the new molybdenum-permalloy (A, A') and the superseded permalloy (B, B') cores and coils:
left—program circuit loading coils; right—exchange area loading coils.

Aside from manufacturing economies, the reduction in coil size is of importance to the Bell System from the standpoints of plant construction and installation. The size reduction is particularly important in instances where only a few coils are required at a given point, as is the case in program circuit loading. It is now practicable to enclose as many as six coils, even of the larger size employed for loading program circuits, directly in the cable splice at a loading point. This dispenses with the need for conventional cases, and reduces both manufacturing and installation costs. In the field of small complements using the inexpensive lead sheath type of case construction, it is now possible to furnish as many as 100 of the small exchange area coils whereas 15 was the maximum number accommodated with the superseded coil design. Table III gives comparative weights and volumes of typical cases provided for potting exchange area and toll type coils.

TABLE III
COMPARATIVE DATA ON REPRESENTATIVE CASES FOR NEW
AND SUPERSEDED LOADING COILS

Type Cable	Size of Complement	Type Coil or Unit	Approx. Volume Cu. Ft.	Approx. Weight Lb.
Exchange Area	200	Permalloy	1.7	480
"	"	Molybdenum-Permalloy	0.8	350
Program-Toll	6	Permalloy	0.15	70
"	"	Molybdenum-Permalloy	0.11	50
Toll	50 units*	Permalloy	5.5	1350
"	"	Molybdenum-Permalloy	3.5	950

* A unit consists of one phantom and two side circuit coils.

Figures 11 and 12 show the comparative sizes of typical steel and lead sleeve type cases for the new and superseded coils. In Fig. 13, the midget proportions of the latest design of case for plotting 100 exchange area coils are contrasted with those of the case utilized up to 1922 containing only 98 exchange area coils. The reduction in size of cases for this type of coil is of particular importance in larger cities where underground vault space is at a premium.

IMPROVED INDUCTANCE COILS FOR FILTERS

The trend of development of toll transmission circuits is now very definitely toward multiple channel carrier systems utilizing a much wider frequency band than has heretofore been employed on wire circuits.^{19,20} These systems involve extensive use of selective or equalizing networks at terminals and at repeater stations in order to

obtain proper separation of the frequency bands of the various channels or to insure suitable transmission properties of the individual channels. While these networks involve coils, condensers and crystals, it is frequently the case that their size, cost and performance are determined chiefly by the quality factor Q of the inductance coils. This follows from the fact that Q values of coils are usually considerably

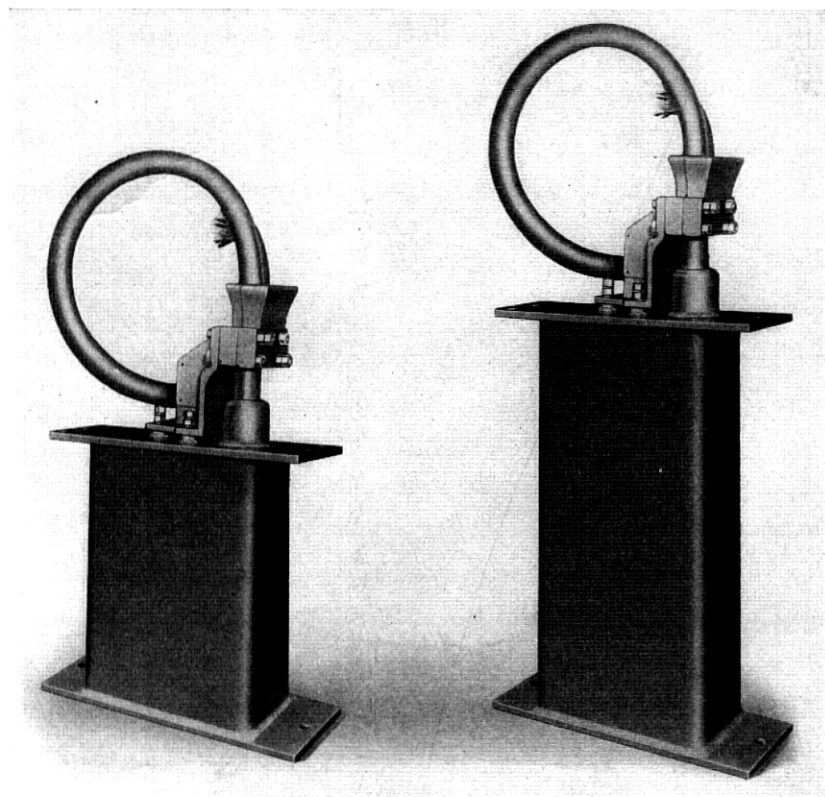


Fig. 11—New and superseded cases containing 200 exchange area coils.

lower than those obtainable readily in condensers and crystals. Accordingly, it is very desirable to have as high a value of Q as possible economically. In addition, such coils must have low hysteresis resistance to limit modulation, and a low temperature coefficient of inductance to secure stability of attenuation or impedance characteristics of the filters and networks.

Due to the improvements in these respects, molybdenum-permalloy core coils can be used quite extensively in new types of carrier tele-

phone systems. In such systems for existing lines and cables, as well as for projected new types of cables, those filters are of key importance which separate individual message channels in the frequency range from 3 to 108 kc. By using coils of powdered molybdenum-permalloy insulated to permeabilities of 14 or 26, valuable economies in space and cost of filters are realized.^{24,22} Figure 14 shows a typical coil employing a 14 permeability core designed for use in one of these channel filters having its transmitted band in the vicinity of 108 kc,

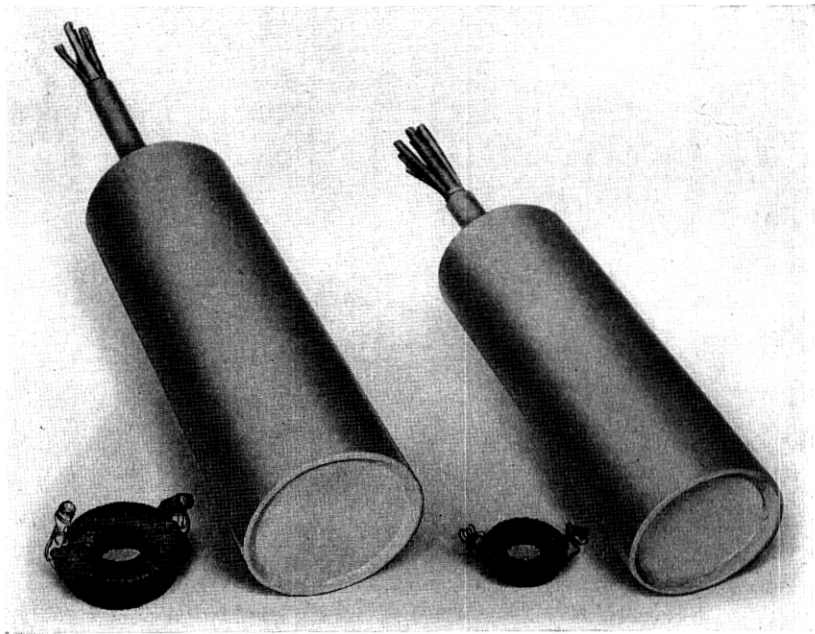


Fig. 12—New and superseded cases containing six program loading coils.

together with a shielded solenoidal air core coil which might be employed for the same purpose. The molybdenum-permalloy coil has a Q at 100 kc about twice that of the air core coil, yet it occupies approximately $1/10$ as much space. The third order modulation products are approximately 80 db below the level of the normal channel currents. This is considered to be tolerable from the standpoint of interchannel crosstalk on circuits used for one-way transmission. An inductance-temperature coefficient of about -20×10^{-8} per degree F. has been chosen to compensate for the positive capacity-temperature coefficient of associated condensers.

In Fig. 15 data are presented illustrating the Q -frequency characteristics that can be obtained on typical coil designs using the new material in the frequency range from 300 cycles to 200 kc. The characteristics shown apply to coils wound on three sizes of cores that

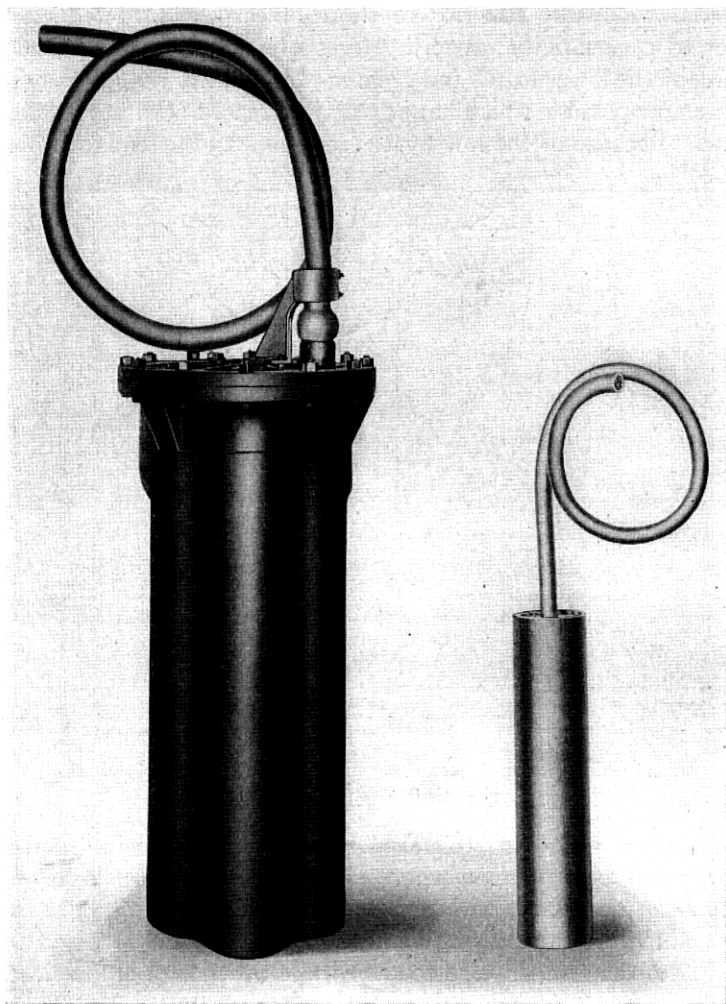


Fig. 13—Comparative size of equivalent 1939 and 1922 cases.

are suitable for use in this range. For comparison, similar characteristics are also included for coils using cores of equal size but made of permalloy and electrolytic iron powder. These data include all effects on Q resulting from winding capacities and losses, which have

been made tolerably small by suitable choice of insulating materials, stranding of conductor and configuration of winding.

CONCLUSION

Compressed powdered cores of 2-81 molybdenum-permalloy have properties which are superior to those of earlier powdered cores in respect to permeability range, hysteresis loss and eddy current loss. Because of the lower losses and greater permeability range, inductance coils are now possible which have greatly increased Q values for a given volume. Because of the low hysteresis losses and attendant lowering

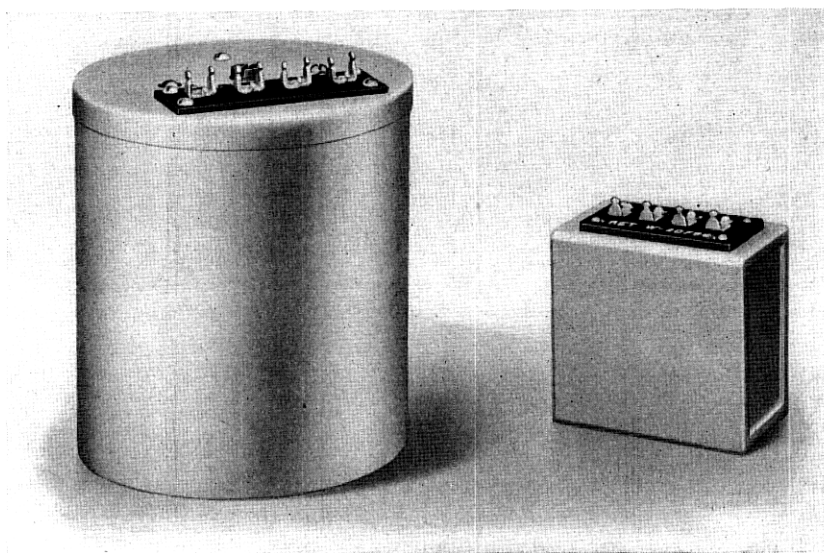
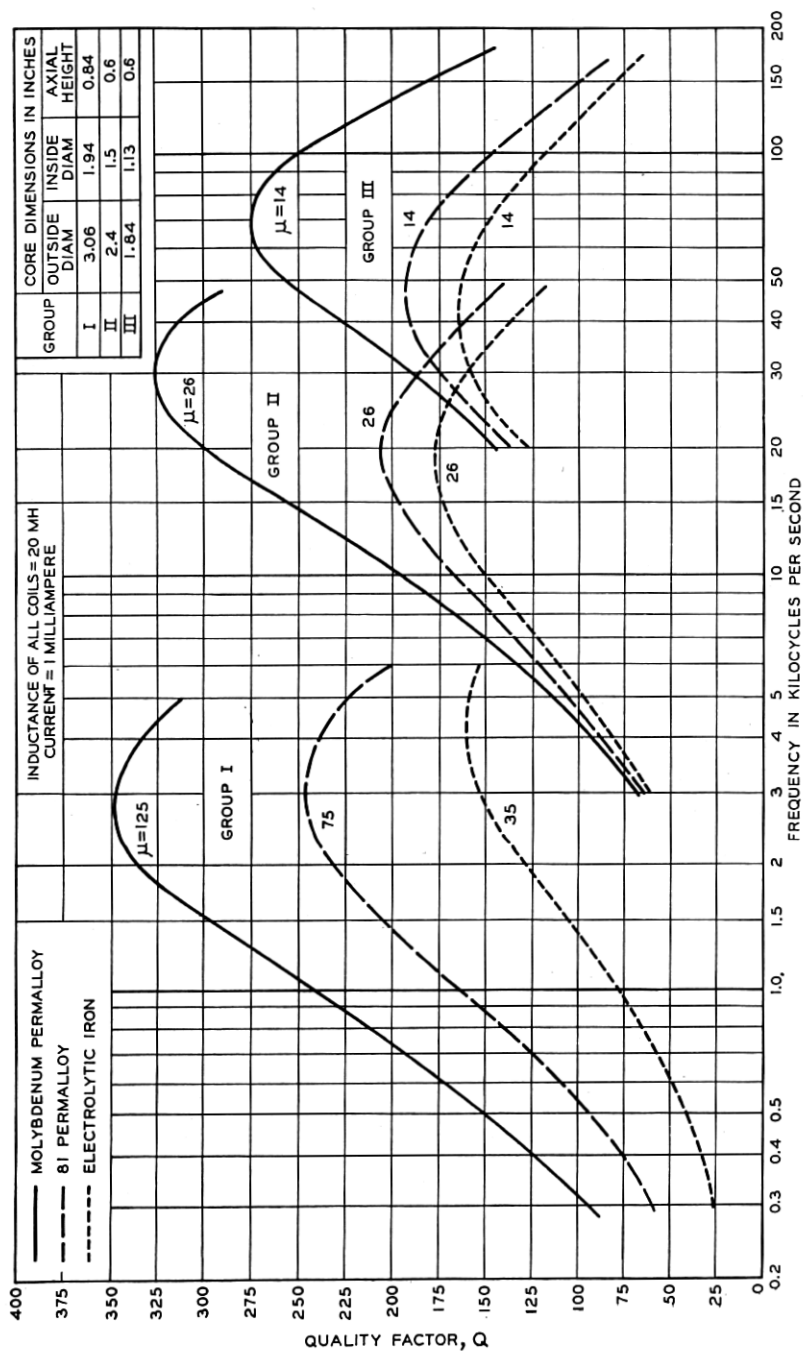


Fig. 14—Comparative size of non-magnetic core and molybdenum permalloy core filter coils.

of modulation effects to tolerable levels, magnetic core inductance coils can now be employed where non-magnetic core coils have previously been necessary, with a very great increase in Q values for a given volume. Temperature coefficients can now be obtained which are equal to the temperature coefficients of other high grade electrical elements such as mica condensers and quartz crystals. Moreover, the ability to make the temperature coefficient of coils negative or positive at will permits the attainment of remarkable stability in resonant combinations of coils and condensers. Two important applications have been made in the field of communication apparatus. For voice-frequency circuits, new loading coils of improved quality and reduced



size have been standardized. For broad band carrier systems, the compactness of channel separating filters is largely due to the volume economies introduced by molybdenum-permalloy coils.

APPENDIX

The following formulae illustrate the essential steps in selecting the size and permeability of an annular core best suited for a coil having a desired set of characteristics. For sake of simplicity, they assume throughout a coil of arbitrarily chosen proportions, in which all dimensions bear fixed ratios to the mean core diameter, and approximating those that are practicable from a manufacturing standpoint. The core is assumed to be rectangular in section. It is assumed that wire of any diameter can be used and that the winding efficiency is independent of the wire diameter. Any inductance due to air space outside of the core is neglected. Because of these simplifications, the expressions are of somewhat restricted applicability. However, they yield solutions for optimum permeability and corresponding values of Q and core size which are sufficiently accurate for most practical purposes.

The inductance in henrys due to a core of permeability μ , and mean diameter d cm., wound with N turns of wire is

$$(1) \quad L = \frac{3}{8} N^2 \mu d \times 10^{-9}.$$

If the coil is wound with wire of resistivity ρ_c ohm-cm., with winding efficiency s (i.e., the ratio of copper area to total available winding area), the direct current resistance in ohms will be

$$(2) \quad R_c = \frac{19 \rho_c L \times 10^9}{s \mu d^2}.$$

The maximum flux density due to sine wave measuring current of effective value I amperes is

$$(3) \quad B_m = \frac{8I}{5} \sqrt{\frac{\mu L \times 10^9}{3d^3}}.$$

In terms of the hysteresis loss coefficient $k_2 = \mu a$, the modulation factor thus becomes¹⁷

$$(4) \quad m = \frac{E_3}{E_1} = \frac{3k_2 B_m}{10\pi} = \frac{4k_2 I}{25\pi} \sqrt{\frac{3\mu L \times 10^9}{d^3}}$$

or

$$(5) \quad d^3 = 3\mu L \times 10^9 \left(\frac{4k_2 I}{25\pi m} \right)^2.$$

When the core permeability μ is reduced by dilution of a given material, the hysteresis and residual loss coefficients a and c vary so as to make the products $\mu c = k_1$ and $\mu a = k_2$ approximately constant, as may be seen by reference to Table I. The core loss resistance in ohms at frequency f cycles per second may therefore be expressed with reasonable accuracy as

$$(6) \quad R_m = Lf(k_1 + k_2 B_m + \mu ef) = Lf \left(k_1 + \frac{10m\pi}{3} + \mu ef \right),$$

where the eddy current coefficient e depends upon the particle diameter t , and the alloy resistivity ρ being proportional¹³ to t^2/ρ .

The coil quality factor is thus

$$(7) \quad Q = \frac{2\pi fL}{R_c + R_m} = \frac{2\pi f}{\frac{19\rho_c \times 10^9}{s\mu d^2} + f \left(k_1 + \frac{10m\pi}{3} + \mu ef \right)}.$$

Case I: If the value of m is fixed and d and R_c can be freely chosen, it is desirable to know the value of μ which will yield the highest possible value of Q . By substituting in (7) the value of d^2 obtained from (5) and setting the derivative with respect to μ equal to zero, the following is obtained for the optimum permeability:

$$(8) \quad (\mu')^8 = \frac{52.5 \times 10^{16} \rho_c^3 m^4}{e^3 s^3 f^6 k_2^4 I^4 L^2}.$$

The corresponding values of d and R_c can be obtained from equations (5) and (2). The corresponding value of Q , which is the greatest obtainable under these conditions is

$$(9) \quad Q'_{\max} = \frac{\pi}{\frac{5m\pi}{3} + \frac{k_1}{2} + \frac{4}{5} \mu' ef}.$$

If a smaller value of Q than that obtained from (9) is acceptable, equations (7) and (5) can be solved simultaneously for d and μ . A smaller value of μ than that obtained from (8) and a correspondingly smaller value of d will result.

Case II: If modulation is unimportant and the hysteresis loss resistance is negligible in comparison with other component losses, then d and μ can be selected without regard to modulation. Equation (7) can be differentiated directly and solved for the permeability required to yield the maximum value of Q . This optimum permeability is

$$(10) \quad (\mu'')^2 = \frac{19\rho_c \times 10^9}{s e d^2 f^2}.$$

The corresponding value of Q is

$$(11) \quad Q''_{\max} = \frac{2\pi}{k_1 + 2\mu''_{ef}}.$$

The values of Q''_{\max} and μ'' depend on the value of d chosen. For a desired value of Q , (11) can be solved for μ'' , and (10) for the corresponding diameter.

In any case, the ideal wire size has a cross-sectional area of conductor equal to

$$0.15sd^2 \sqrt{\frac{\mu d \times 10^{-9}}{L}} \text{ cm}^2.$$

It is usually desirable to subdivide the wire into insulated strands to minimize eddy current losses in the coil winding.

REFERENCES

1. "Commercial Loading of Telephone Circuits in the Bell System," B. Gherardi, *Trans. A.I.E.E.*, v. 30, 1911, p. 1743.
2. "Development and Application of Loading for Telephone Circuits," T. Shaw and W. Fondiller, *Jour. A.I.E.E.*, v. 45, 1926, p. 253; *B.S.T.J.*, v. 5, 1926, p. 221.
3. "Magnetic Properties of Compressed Powdered Iron," B. Speed and G. W. Elmen, *Trans. A.I.E.E.*, v. 40, 1921, p. 596.
4. "Compressed Powdered Permalloy Manufacture and Magnetic Properties," W. J. Shackleton and I. G. Barber, *Trans. A.I.E.E.*, v. 47, 1928, p. 429.
5. A. F. Bandur, U. S. Patent 1,673,790, June 19, 1928.
6. "Pupinspulen mit Kernen aus Isoperm-Blech oder -Band," H. Jordan, T. Volk and R. Goldschmidt, *Europaischer Fernsprechdienst* v. 31, 1933, p. 8.
7. "Permalloy, an Alloy of Remarkable Magnetic Properties," H. D. Arnold and G. W. Elmen, *Jour. Frank. Inst.*, v. 195, 1923, p. 621.
8. "Carrier Current Telephony and Telegraphy," E. H. Colpitts and O. B. Blackwell, *Trans. A.I.E.E.*, v. 40, 1921, p. 205.
9. "Radio Extension of the Telephone System to Ships at Sea," H. W. Nichols and L. Espenschied, *Proc. I.R.E.*, v. 11, 1923, p. 193; *B.S.T.J.*, v. 2, 1923, p. 141.
10. "Magnetic Alloys of Iron, Nickel, and Cobalt," G. W. Elmen, *Jour. Frank. Inst.*, v. 207, 1929, p. 583; *B.S.T.J.*, v. 8, 1929, p. 435.
11. "A Survey of Magnetic Materials in Relation to Structure," W. C. Ellis and E. E. Schumacher, *B.S.T.J.*, v. 14, 1935, p. 8.
12. "Statistical Description of the Size Properties of Non-uniform Particulate Substances," T. Hatch and S. Choate, *Jour. Frank. Inst.*, v. 207, 1929, p. 369.
13. "On the Self-induction of Wires," O. Heaviside, *Phil. Mag.*, v. 23, 1887, p. 173.
14. "Magnetostatik der Massekerne," F. Ollendorf, *Arch. f. Elektrotechn.*, v. 25, 1931, p. 436.
15. "Iron Powder Compound Cores for Coils," G. W. O. Howe, *Wireless Engineer*, v. 10, 1933, p. 1.
16. "Magnetic Measurements at Low Flux Densities Using the A-C Bridge," V. E. Legg, *B.S.T.J.*, v. 15, 1936, p. 39.
17. "Harmonic Production in Ferromagnetic Materials at Low Frequencies and Low Flux Densities," E. Peterson, *B.S.T.J.*, v. 7, 1928, p. 762.
18. "Hysteresis Effects with Varying Superposed Magnetizing Forces," W. Fondiller and W. H. Martin, *Trans. A.I.E.E.*, v. 40, 1921, p. 553.
19. "Communication by Carrier in Cable," B. W. Kendall and A. B. Clark, *Elec. Engg.*, v. 52, 1933, p. 477; *B.S.T.J.*, v. 12, 1933, p. 251.
20. "Systems for Wide Band Transmission over Coaxial Lines," L. Espenschied and M. E. Strieby, *Elec. Engg.*, v. 53, 1934, p. 1371; *B.S.T.J.*, v. 13, 1934, p. 654.
21. "The Evolution of the Crystal Wave Filter," O. E. Buckley, *Jour. Appl. Phys.*, v. 8, 1937, p. 40; *Bell Tel. Quarterly*, v. 16, 1937, p. 25.
22. "An Improved Three-Channel Carrier Telephone System," J. T. O'Leary, E. C. Blessing and J. W. Beyer, *B.S.T.J.*, v. 18, 1939, p. 49.