Crosstalk Between Coaxial Conductors in Cable

By R. P. BOOTH and T. M. ODARENKO

The available literature on crosstalk between coaxial conductors in contact makes it clear that the presence of any other conducting material in continuous or frequent contact with the coaxial outer conductors simply reduces the coupling per unit length without altering the law of crosstalk summation with length.

When the conducting material is insulated from the coaxials, as in the case of quads and sheath in coaxial cables, the situation is more complicated. Instead of simply reducing the coupling per unit length the quads and sheath, with the outer conductors for a return, provide a tertiary circuit in which interaction crosstalk can take place between elementary line sections. The summation with length for this type of crosstalk is quite different from that between two coaxials in contact and therefore the combined summation is obviously more involved.

Tests on sections of a five-mile length of coaxial cable were made at Princeton, New Jersey, in the latter part of 1937 and early in 1938 in order to obtain experimental verification of the manner in which the quads and sheath affect crosstalk summation with length. It is shown that the crosstalk component due to the presence of the sheath and quads opposes the component which is present between two coaxials in free space so that the resultant crosstalk is considerably lower than would be computed ignoring the tertiary effects.

Introduction

In spite of the geometrical and electrical symmetry of the coaxial circuit and the excellent shielding properties of the outer conductor, a part of the electromagnetic energy escapes from the circuit through the outer conductor and sets up an electromagnetic field in the space around it. Any circuit, be it even another coaxial placed in this field will absorb a part of the energy stored in the field and deliver it to the terminals of the circuit in the form of an unwanted or interfering current—the crosstalk current. The magnitude of this crosstalk current depends on a variety of factors, such as the physical characteristics of the conductors and of the intervening space, the frequency and the length of the circuit.

Expressions for two important cases of crosstalk between two coaxial circuits in *free space*, namely, the so-called "direct" crosstalk with the outer conductors in continuous contact and the "indirect" crosstalk with the outer conductors insulated from each other, were determined and discussed in a previously published paper.[1] It was shown there that the direct far-end crosstalk is directly proportional to l and the direct near-end crosstalk is proportional to

$$\frac{1 - \epsilon^{-2\gamma l}}{2\gamma},$$

where l is the length and γ is the propagation constant of either coaxial unit. The indirect crosstalk was shown to be a more complicated function of the length.

The present paper extends this earlier work to include the case where the coaxials are enclosed in a common sheath or, in the general case, paralleled by any conducting material symmetrically disposed.[2] When this conducting material is introduced in the neighborhood of two coaxials in contact the conditions for crosstalk production are naturally changed from those existing in free space. If the material is uniformly distributed along the coaxials and is in continuous or frequent contact with the outer conductors the summation of crosstalk with length is the same as before but the magnitude is reduced. This reduction is due to the fact that part of the current formerly flowing on the disturbed outer conductor now flows on the new conducting material instead, thus reducing the direct crosstalk coupling per unit length.

In most cables, the coaxial outer conductors are in contact but the other conducting material (sheath and quads) is insulated from the outer conductors. The quads must obviously be insulated for normal use and the sheath is kept insulated except at the ends of a repeater section in order to permit the use of insulating joints for electrolysis prevention where required. This material thus provides an extra transmission circuit, or tertiary circuit, in which tertiary currents can be propagated up and down the line. In such a case the resulting crosstalk in any length consists of both the direct crosstalk between the contacting coaxials and the indirect crosstalk via the outer conductor-sheath and quad tertiary circuit. The general formulas given in the Schelkunoff-Odarenko paper apply for these components. Since the two components follow different laws regarding summation with length the resultant summation is quite complicated except for very short or very long lengths.

The study of the tertiary effects on crosstalk summation is the main contribution of this paper to crosstalk theory. Emphasis will be placed on the development of a simple physical picture which will help one to

[1] Schelkunoff-Odarenko paper in *Bell Sys. Tech. Jour.*, April, 1937.

[2] In the interim between our tests and this publication a paper by H. Kaden concerning this general subject was published in the Europaischer Fernsprechdienst, no. 50, October, 1938, pp. 366–373.

visualize clearly the influence of the tertiary circuits in the summation process. To produce such a picture a certain amount of review of the general crosstalk problem will be necessary. This is undertaken in Part I of this paper.

Part II is devoted mainly to the presentation of test data taken in November and December, 1937, January and February, 1938 on sections of a five-mile length of a twin coaxial cable near Princeton. These data confirm and graphically illustrate certain relationships developed in Part I. In addition they provide information on the tendency of tertiary circuits to complicate the effectiveness of transpositions and show how interaction crosstalk takes place around repeaters via the tertiary circuits.

PART I—THEORY

In any series of crosstalk tests on short lengths of paired or quadded cable where the problem of combining a number of such lengths is concerned it has generally been the practice to terminate both the test circuits and important tertiary circuits in characteristic impedance. Under such a condition the normal influence of all circuits in the production of crosstalk within each short section is provided for and the summation process, including interaction between successive sections, can be studied under actual line conditions. This is a general method applicable to any type of coupling and was adopted for the Princeton investigation. The effect of discontinuities such as short-circuited tertiaries at the extreme ends of a repeater section can be readily handled mathematically as correction terms due to "end effect."

To simplify the presentation of the factors involved, the discussion in this section will be confined mainly to the case of far-end crosstalk. In a twin coaxial cable where the transmission in the two units is in opposite directions there actually exists no far-end crosstalk problem since only talker echo, a near-end crosstalk phenomenon, is involved.[3] In multi-unit cable, however, there will be far-end crosstalk between different systems. Since this type of crosstalk tends to increase directly with the number of repeater sections it is important to understand its nature thoroughly. Moreover, in a study of fundamentals it is possible to avoid certain complications not essential to an understanding of the problem by investigating far-end rather than near-end crosstalk.

To present a clear picture of the physical meaning of some of the forthcoming mathematical expressions their derivations will be ap-

[3] This statement may not hold if the repeater impedances fail to match the line impedance since in that case the far-end crosstalk can be reflected and appear as near-end crosstalk.

proached in as elementary a fashion as possible. In order to do this we shall start with the simple arrangement of two coaxial conductors in free space, a case already covered in previous papers. To the crosstalk equations covering this case will then be added terms to allow for the effects of quads and sheath. In all that follows in Part I the quads and sheath will be considered as one unit referred to as the "sheath." This is a good approximation as will be shown in Part II.

The conception of two independent crosstalk components—a direct or transverse component between coaxials in contact and an indirect or interaction component via the sheath tertiary circuit—is not necessary for the solution of the problem. It is preserved here, however, as offering a familiar and much simpler approach to a clear understanding of the processes involved in crosstalk summation with length.

Far-End Crosstalk

Consider first an elementary section, dl, of a long single coaxial in free space as indicated in Sketch (a) of Fig. 1. If the current at this point in the center conductor is I_1 the current in the outer conductor is practically $-I_1$ since there is no other return path (except through the air dielectric which offers a high impedance especially at the lower broad-band frequencies considered here). Using Schelkunoff's nomenclature we may state that an open-circuit voltage equal to $e_1 = I_1 Z_{\alpha\beta} dl$ is developed on the outer surface of the outer coaxial conductor. The term $Z_{\alpha\beta}$ represents the surface transfer impedance (mutual impedance) per unit length between the inner and outer surfaces of the outer coaxial conductor.

Now suppose that we place another long coaxial parallel to the first one and, for generality, insulated from it as shown by Sketch (b) of Fig. 1. The open-circuit voltage e_1 on length dl of the first coaxial outer conductor will now cause current to flow in the intermediate circuit composed of the two outer conductors. The parameters of this circuit are γ_3 and Z_3 as shown on the sketch. In returning on the second coaxial outer conductor this current causes crosstalk into the second coaxial circuit.

It is convenient at this point to replace the original impressed voltage e_1 by the set of emf's shown in Sketch (c) of Fig. 1. The insertion of equal and opposite voltages $e_1/2$ on the outer surface of the disturbed coaxial outer conductor does not change conditions but enables us to consider certain effects separately. The first effect to be considered is that due to the pair of equal and opposite voltages $e_1/2$ in the loop composed of the two coaxial outer conductors. These voltages combine to form a "balanced" voltage e_1 which tends to drive current around the balanced circuit composed of the two outer conductors. For the present we shall not consider the voltages $e_1/2$ which are in the same direction in the outer conductors.

The current in the "balanced" intermediate circuit of characteristic impedance Z_3 and propagation constant γ_3 due to the balanced voltage e_1 in the elementary length dl is $i_3 = e_1/2Z_3$. This current flowing along the outer coaxial conductor of the disturbed circuit produces a voltage $e_2 = i_3 Z_{\alpha\beta} dl$ on the inner surface of this outer conductor and this voltage in turn causes a current i_{2a} in the disturbed coaxial circuit

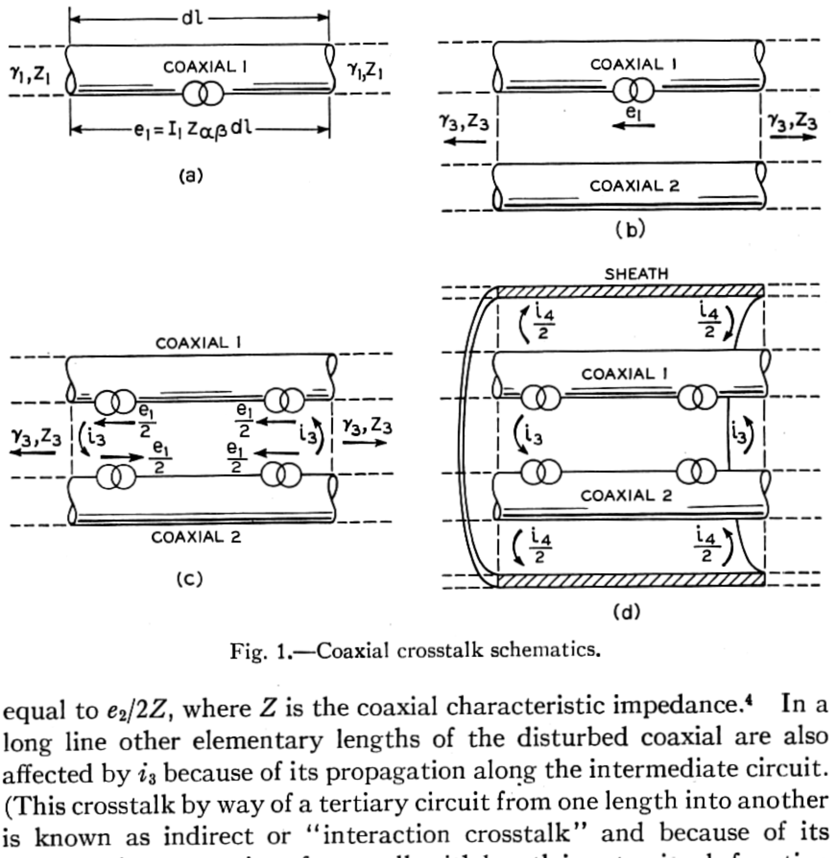

Fig. 1.—Coaxial crosstalk schematics.

equal to $e_2/2Z$, where Z is the coaxial characteristic impedance.[4] In a long line other elementary lengths of the disturbed coaxial are also affected by i_3 because of its propagation along the intermediate circuit. (This crosstalk by way of a tertiary circuit from one length into another is known as indirect or "interaction crosstalk" and because of its presence the summation of crosstalk with length is not a simple function of length even for systematic coupling such as occurs with coaxials.) This is a crosstalk case for which the general solution is already

[4] The subscript "a" in i_{2a} relates this current to the so-called "mode a" current used by Carson and Hoyt in their paper entitled "Propagation of Periodic Currents Over a System of Parallel Wires," *Bell Sys. Tech. Jour.*, July, 1927.

available. When the effects are integrated it is found that the far-end crosstalk is quite a complicated function of length and of the tertiary and coaxial propagation and impedance characteristics.[5] However, if the coaxial units are in actual contact as in the case of the coaxial cable to be considered here, the formula for the far-end crosstalk F_3 expressed as a current ratio is quite simple, namely,

$$F_3 = \frac{Z_{\alpha\beta}^2}{2ZZ_{33}} \cdot l, \tag{1}$$

where $Z_{33} = Z_3\gamma_3$ is the series impedance per unit length of the circuit composed of one coaxial outer conductor with return on the other. Thus, for this component, the far-end crosstalk is directly proportional to length. This simple relation results from the fact that the intermediate circuit, being continuously shorted, has such high attenuation that no interaction crosstalk between elementary lengths can exist.

We shall now consider the crosstalk contribution due to the longitudinal voltage $e_1/2$ acting along both coaxial outer conductors in parallel. Suppose that a sheath is placed symmetrically around the two coaxials but insulated from them as shown in Sketch (d) of Fig. 1. The longitudinal voltage sends a current around the circuit composed of the two parallel outer conductors with sheath return equal to $i_4 = e_1/(2)(2Z_4)$, where Z_4 is the characteristic impedance of this circuit. Half of this longitudinal current flows on the disturbed coaxial outer conductor *in opposition* to the balanced current i_3 flowing there.

Following previous procedure it can be shown that in the elementary length a crosstalk current $i_{2c} = i_4 Z_{\alpha\beta} dl/4Z$ will flow in the disturbed coaxial circuit.[6] Other elementary lengths are also affected by i_4 thus producing interaction crosstalk. When the effects are integrated over a length l the far-end crosstalk for this component is found to be as follows:

$$F_4 = -\frac{Z_{\alpha\beta}^2}{16ZZ_4}\left[\frac{2l}{\gamma_4}\cdot\frac{\gamma_4^2}{\gamma_4^2-\gamma^2} - \left(\frac{2(\gamma_4^2+\gamma^2)}{(\gamma_4^2-\gamma^2)^2} - \frac{\epsilon^{-(\gamma_4-\gamma)l}}{(\gamma_4-\gamma)^2} - \frac{\epsilon^{-(\gamma_4+\gamma)l}}{(\gamma_4+\gamma)^2}\right)\right], \tag{2}$$

where γ_4 is the propagation constant of the sheath-outer conductor circuit. If the sheath is in actual contact with the coaxial units the

[5] See equation (40) in the Schelkunoff-Odarenko paper in *Bell Sys. Tech. Jour.*, April, 1937.

[6] The subscript "c" in i_{2c} relates this current to the "mode c" current used by Carson and Hoyt in their paper of July, 1927.

formula reduces to the simple relation

$$F_4 = -\frac{Z_{\alpha\beta}^2}{16ZZ_4}\cdot\frac{2l}{\gamma_4} = -\frac{Z_{\alpha\beta}^2}{8ZZ_{44}}\cdot l, \tag{3}$$

where $Z_{44} = Z_4\gamma_4$ = series impedance per unit length of the circuit composed of the outer conductors with sheath return.[7] For a sheath in contact this component is thus directly proportional to length since interaction crosstalk from one elementary length into another has been eliminated.

Now, while we are actually concerned with the insulated sheath as covered by (2) it is of considerable interest to study equations (1) and (3) at this point. The total far-end crosstalk when the outer conductors and sheath are in contact is the sum of the crosstalk components F_3 and F_4 as given in equations (1) and (3), or

$$F_3 + F_4 = F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{l}{Z_{33}} - \frac{l}{4Z_{44}}\right]. \tag{4}$$

This simple addition follows from the fact that the circuits for the two modes of propagation covered by equations (1) and (3) are mutually non-inductive because of symmetry so that there is no reaction between them. The recognition of this fact does away with the necessity of complicated mathematics which would otherwise have to be used in the general solution.[8]

In formula (4) the second term in the bracket represents the contribution of the tertiary circuit involving the sheath and is seen to be opposite in sign to the first term which represents the crosstalk which would exist in the absence of the sheath. The equation illustrates mathematically the previous statement that conducting material in contact with the coaxials acts to reduce the crosstalk. Since both components are directly proportional to length, the total is also directly proportional to length.

It is apparent in formula (4) that the crosstalk would be zero if the values of Z_{33} and $4Z_{44}$ were equal. In cables where steel tapes are used on the outer surface of the coaxials this condition is approached. For example, if we neglect external inductance and proximity effects, Z_{33} would be equal to twice the surface self-impedance of a single outer

[7] It should be noted here that it is not really necessary to postulate a separate sheath return in order to obtain expression (3) for F_4 due to the longitudinal voltage $e_1/2$, since the return *in continuous contact* with the outer conductors will actually tend to lose its identity. The device of introducing sheath return insulated from the outer conductors and then shorting it to the conductors serves only to simplify the concepts of Z_{44}, Z_4, γ_4.

[8] This principle of symmetry can be extended to the case of four coaxial units whether insulated or in contact.

conductor while Z_{44} would equal one-half of the surface self-impedance of a single outer conductor (neglecting the self-impedance of the lead sheath in comparison with the iron outer conductors). Thus, neglecting differences in external inductance and in proximity effects $1/Z_{33}$ would equal $1/4Z_{44}$ and the crosstalk would vanish. Actually, the observed reduction in crosstalk due to these opposing terms is about 32 db at 50 kilocycles in a 145-foot section of twin coaxial with quads and sheath shorted to the coaxials at the ends only. Physically this means that the current due to the voltage on the outer conductor surface of the disturbing coaxial flows mainly in the sheath and quads rather than on the high impedance surface of the disturbed coaxial.

Now let us consider the case where the sheath is insulated from the coaxial outer conductors. For this case equations (1) and (2) may also be added directly to give

$$F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{l}{Z_{33}} - \frac{l}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2-\gamma^2}\right) + \frac{\gamma_4}{4Z_{44}}\left(\frac{\gamma_4^2+\gamma^2}{(\gamma_4^2-\gamma^2)^2}\right) - \frac{\gamma_4}{4Z_{44}}\left(\frac{\epsilon^{-(\gamma_4-\gamma)l}}{2(\gamma_4-\gamma)^2} + \frac{\epsilon^{-(\gamma_4+\gamma)l}}{2(\gamma_4+\gamma)^2}\right)\right]. \quad (5)$$

This equation appears quite formidable but it has been split purposely into three terms which will be examined individually. The first term is directly proportional to length, the second term is independent of length and the third term involves length exponentially. For lengths where the tertiary circuit is electrically long the third term vanishes and we have

$$F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{l}{Z_{33}} - \frac{l}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2-\gamma^2}\right) + \frac{\gamma_4}{4Z_{44}}\left(\frac{\gamma_4^2+\gamma^2}{(\gamma_4^2-\gamma^2)^2}\right)\right]. \quad (6)$$

In electrically short lengths we get

$$F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{l}{Z_{33}} - \frac{l}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2-\gamma^2}\right) + \frac{\gamma_4}{4Z_{44}}\left(\frac{\gamma_4 l}{\gamma_4^2-\gamma^2} - \frac{l^2}{2}\right)\right]$$
$$= \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{l}{Z_{33}} - \frac{\gamma_4}{4Z_{44}}\cdot\frac{l^2}{2}\right] \cong \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{l}{Z_{33}}\right], \quad (7)$$

in which it is seen that terms two and three of (5) combine to cancel the second half of term one.

From equations (5), (6) and (7) we are now ready to build a physical picture of what takes place as l is increased for cable sections where the sheath is insulated from the coaxial outer conductors but terminated to them at each end in characteristic impedance, Z_4. Starting with equation (7) we see that for very short lengths the term involving

l^2 becomes negligible, that is, the crosstalk is practically all due to the component which exists in the complete absence of a sheath (see equation (1)). In the range of lengths where this is true *the crosstalk increases directly with length.*

Quite a different state of affairs exists for a section electrically long enough for equation (6) to hold. The first bracketed term is still proportional to length but now consists of the difference of two components. The first of these represents the crosstalk between the coaxials *with no sheath present* while the second is a part of the crosstalk component introduced by the presence of the sheath. Except for the factor $\gamma_4^2/\gamma_4^2 - \gamma^2$ this first bracketed term in equation (6) is the same as equation (4) for a sheath in contact where, as we have already noted, the cancellation of the two components is quite effective when steel tapes are used on the outer conductors. Since γ_4^2 is necessarily considerably greater than γ^2 because of these steel outer conductors, it is reasonable to expect that the factor $\gamma_4^2/\gamma_4^2 - \gamma^2$ is nearly unity and that, therefore, the two components in the first bracketed term of equation (6) will also tend to cancel leaving a residual proportional to length but *much* lower in magnitude than either component *alone.*

The second bracketed term of equation (6) is entirely independent of length. This term has also been introduced by the presence of the tertiary circuit and its magnitude depends on the characteristics of this circuit.

Thus, even without knowing the relative magnitudes of the two components of the first bracketed term of equation (6) for a given length, it is apparent that as l is increased this term must eventually be controlling. The crosstalk will then again be proportional to length as it was for very short lengths but at a reduced level proportional to

$$\frac{\dfrac{1}{Z_{33}} - \dfrac{1}{4Z_{44}} \cdot \dfrac{\gamma_4^2}{\gamma_4^2 - \gamma^2}}{\dfrac{1}{Z_{33}}} = 1 - \frac{Z_{33}}{4Z_{44}} \cdot \frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}.$$

It is quite evident, too, that for a range of lengths where the tertiary circuits are electrically long but where the first term of equation (6) has not had a chance to build up sufficiently the crosstalk will be about constant at a level determined mainly by the second term.

The above analysis may well suffice as a background for an interpretation of the measurements to be given in Part II. However, another and perhaps in some ways a more illuminating approach from a physical standpoint is possible.

Suppose, for example, that far-end crosstalk measurements are made on two cable sections each of length l with tertiaries terminated as illustrated in Sketch (a) of Fig. 2. Let the total crosstalk in each section be equal to F_l as defined by equation (5) above. If these two sections are joined together the total crosstalk is $2F_l$ plus some other terms which represent the interaction crosstalk between the two sections as illustrated in Sketch (b) of Fig. 2. We shall call the component F_{nn}

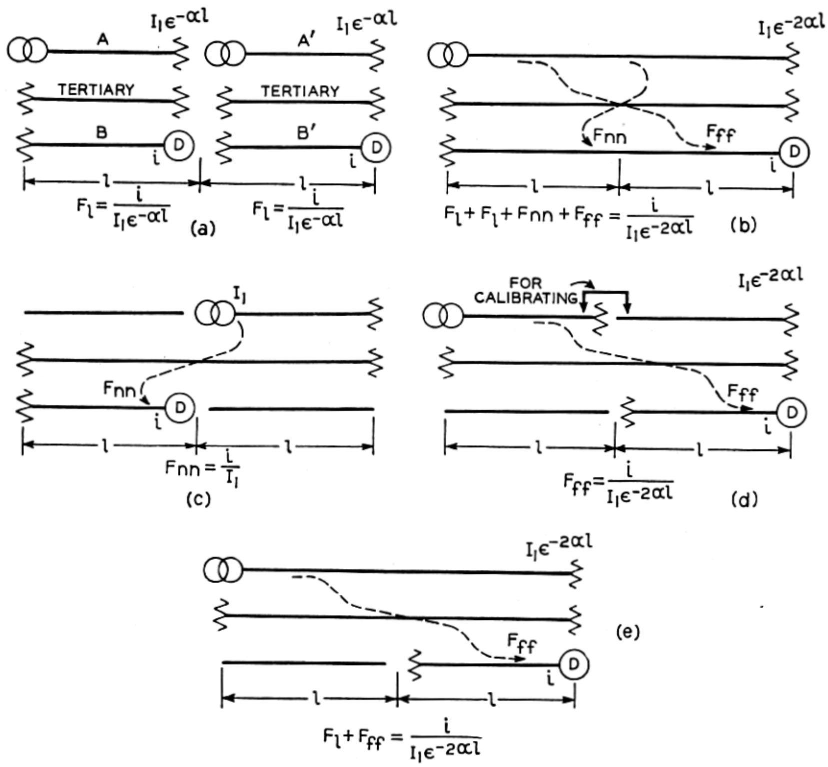

Fig. 2—Schematics illustrating far-end crosstalk summation.

near-end near-end and the component F_{ff} far-end far-end interaction crosstalk. Although inseparable under normal line conditions, these components are definite physical entities and can be isolated as shown schematically on Sketches (c) and (d) of Fig. 2. Thus, both F_{nn} and F_{ff} can be measured readily. In addition it is possible to measure directly $F_l + F_{ff}$ as shown on Sketch (e).

This interaction crosstalk between sections is due to crosstalk currents introduced into the outer conductor-sheath tertiary circuit in one section and propagated along this circuit into the next section and thence into the disturbed coaxial. Except for interaction crosstalk between sections the total crosstalk in $2l$ would simply be twice that in length l, that is, the crosstalk would be directly proportional to length.

Now, the expressions for far-end crosstalk due to such interactions between two sections each of length l are

$$F_{nn} = -\frac{Z_{\alpha\beta}^2}{4Z}\cdot\frac{\gamma_4}{4Z_{44}}\left[\frac{1-\epsilon^{-(\gamma_4+\gamma)l}}{\gamma_4+\gamma}\right]^2, \tag{8}$$

$$F_{ff} = -\frac{Z_{\alpha\beta}^2}{4Z}\cdot\frac{\gamma_4}{4Z_{44}}\left[\frac{1-\epsilon^{-(\gamma_4-\gamma)l}}{\gamma_4-\gamma}\right]^2. \tag{9}$$

Since the coefficients [9] outside of the brackets are the same for F_{nn} and F_{ff} the terms may be combined to give the total interaction crosstalk between the two sections, namely,

$$F_{nn}+F_{ff} = -\frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{\gamma_4}{4Z_{44}}\left(\frac{\gamma_4^2+\gamma^2}{(\gamma_4^2-\gamma^2)^2}\right) - \frac{\gamma_4}{4Z_{44}}\left(\frac{\epsilon^{-(\gamma_4-\gamma)l}}{(\gamma_4-\gamma)^2}+\frac{\epsilon^{-(\gamma_4+\gamma)l}}{(\gamma_4+\gamma)^2}\right) + \frac{\gamma_4}{4Z_{44}}\left(\frac{\epsilon^{-2(\gamma_4-\gamma)l}}{2(\gamma_4-\gamma)^2}+\frac{\epsilon^{-2(\gamma_4+\gamma)l}}{2(\gamma_4+\gamma)^2}\right)\right]. \tag{10}$$

As mentioned before, the crosstalk in length $2l$ *exclusive* of interactions *between* the two sections is equal to $2F_l$ or equation (5) multiplied by 2, namely,

$$2F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{2l}{Z_{33}}-\frac{2l}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2-\gamma^2}\right)+\frac{2\gamma_4}{4Z_{44}}\left(\frac{\gamma_4^2+\gamma^2}{(\gamma_4^2-\gamma^2)^2}\right) - \frac{2\gamma_4}{4Z_{44}}\left(\frac{\epsilon^{-(\gamma_4-\gamma)l}}{2(\gamma_4-\gamma)^2}+\frac{\epsilon^{-(\gamma_4+\gamma)l}}{2(\gamma_4+\gamma)^2}\right)\right]. \tag{11}$$

The total crosstalk in length $2l$ is then the sum of (10) and (11), namely,

$$F_{2l} = 2F_l + F_{nn} + F_{ff} = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{2l}{Z_{33}}-\frac{2l}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2-\gamma^2}\right) + \frac{\gamma_4}{4Z_{44}}\left(\frac{\gamma_4^2+\gamma^2}{(\gamma_4^2-\gamma^2)^2}\right) - \frac{\gamma_4}{4Z_{44}}\left(\frac{\epsilon^{-2(\gamma_4-\gamma)l}}{2(\gamma_4-\gamma)^2}+\frac{\epsilon^{-2(\gamma_4+\gamma)l}}{2(\gamma_4+\gamma)^2}\right)\right], \tag{12}$$

[9] These near-end near-end and far-end far-end coefficients are equal because the coupling through a coaxial is of a series voltage character. In open wire and non-shielded cables where there is also present coupling due to shunt admittances the coefficients for F_{nn} and F_{ff} are different in magnitude and their effects must be considered separately. See paper by A. G. Chapman in *Bell Sys. Tech. Jour.* for January and April, 1934.

wherein the second term in equation (10) is cancelled completely by the third term of equation (11). This equation (12) is exactly what we would get by substituting $2l$ for l in the general equation (5). The only reason for deriving it in terms of $2F_l$ plus interaction between the sections is to present a better physical picture of the mechanism of far-end crosstalk summation with length, that is, to show how the interaction crosstalk *between* two sections alters what otherwise would be a direct summation with length.

In lengths where the tertiary circuit is electrically long equation (12) for total crosstalk in length $2l$ becomes

$$F_{2l} = 2F_l + F_{nn} + F_{ff} = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{2l}{Z_{33}} - \frac{2l}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}\right) + \frac{\gamma_4}{4Z_{44}}\left(\frac{\gamma_4^2 + \gamma^2}{(\gamma_4^2 - \gamma^2)^2}\right)\right], \quad (13)$$

which differs from equation (6) for total crosstalk in length l only by the factor of 2 in the first bracketed term. Thus, as mentioned before, there is a range of lengths wherein the crosstalk will be constant at a level determined by the second term of (6) or (12) until the length becomes sufficient for the first term to become controlling.

In lengths where the tertiary circuit is electrically short equation (11) becomes

$$2F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{2l}{Z_{33}} - \frac{\gamma_4}{4Z_{44}}\cdot l^2\right], \quad (14)$$

which reduces simply to

$$2F_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{2l}{Z_{33}}\right] = \left[\frac{Z_{\alpha\beta}^2}{ZZ_{33}}\right] l \quad (15)$$

when the length is sufficiently short. The interaction crosstalk *between* two electrically short lengths becomes, from equation (10),

$$F_{nn} + F_{ff} = \frac{-Z_{\alpha\beta}^2}{2Z}\left[\frac{\gamma_4}{4Z_{44}}\cdot l^2\right] = \left[\frac{-Z_{\alpha\beta}^2}{8ZZ_4}\right] l^2, \quad (16)$$

one-half of which is due to component F_{nn} and the other half to component F_{ff}. The sum of (14) and (16) is

$$F_{2l} = 2F_l + F_{nn} + F_{ff} = \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{2l}{Z_{33}} - \frac{\gamma_4}{4Z_{44}}\cdot 2l^2\right], \quad (17)$$

which is exactly equal to equation (7) if $2l$ is substituted for l therein. From (15) and (16) it is apparent that for very short lengths the total crosstalk in length $2l$ will be simply twice that in length l since the interaction crosstalk between lengths l is proportional to l^2 and therefore is negligibly small.

The view of the mechanism of far-end crosstalk summation as developed above is illustrated by measurements to be presented in Part II. It may be pointed out here that the measurement of far-end and interaction crosstalk in phase and magnitude on short lengths where equations (15) and (16) hold gives the far-end and interaction crosstalk coefficients from which the crosstalk in any length of line may be computed provided the propagation constants and impedances of the coaxial and the tertiary circuits are known.

A practical difficulty may arise from the fact that the application of this method involves equations (12) or (5) where the first bracketed term consists of the difference of two quantities each of which is very large compared with this difference. Thus, a considerable error may be introduced in the computation of this term because of small errors in the measurement of its components. For some cases it is, therefore, better to use a method based on certain crosstalk measurements in a short length of cable with the tertiary circuits open and shorted.[10] There are cases, however, where the controlling crosstalk in a five-mile section is predominantly due to the second term of equation (5). One such case is for the crosstalk between diagonally opposite coaxials in a four-coaxial cable. In this case tests have shown that the cancellation of components in the first term is so complete that the second term is controlling in five miles. For such a case the more accurate method may be to determine the interaction coefficient from equation (16).

Near-End Crosstalk

It will be sufficient here to give simply the final equations for the two crosstalk components for any length l.

For the component which would exist for two contacting coaxials in free space we have

$$N_3 = \frac{Z_{\alpha\beta}^2}{2Z} \cdot \frac{1}{Z_{33}} \left(\frac{1 - \epsilon^{-2\gamma l}}{2\gamma} \right) \tag{18}$$

and for that component due to the presence of the sheath

$$N_4 = -\frac{Z_{\alpha\beta}^2}{2Z} \cdot \frac{1}{4Z_{44}} \left[\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2} \cdot \frac{1 - \epsilon^{-2\gamma l}}{2\gamma} - \frac{\gamma_4^2}{\gamma_4^2 - \gamma^2} \cdot \frac{1 - 2\epsilon^{-(\gamma_4 + \gamma) l} + \epsilon^{-2\gamma l}}{2\gamma_4} \right], \tag{19}$$

[10] The method described in a companion paper by K. E. Gould.

whence, for both components,

$$N_3 + N_4 = N_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{1}{Z_{33}} - \frac{1}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}\right)\frac{1 - \epsilon^{-2\gamma l}}{2\gamma} + \frac{1}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}\left(\frac{1 - 2\epsilon^{-(\gamma_4+\gamma)l} + \epsilon^{-2\gamma l}}{2\gamma_4}\right)\right]. \qquad (20)$$

In a section where the tertiary circuit is electrically long equation (20) reduces to

$$N_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\left(\frac{1}{Z_{33}} - \frac{1}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}\right)\frac{1 - \epsilon^{-2\gamma l}}{2\gamma} + \frac{1}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}\left(\frac{1 + \epsilon^{-2\gamma l}}{2\gamma_4}\right)\right] \qquad (21)$$

and when l is electrically short it reduces to

$$N_l = \frac{Z_{\alpha\beta}^2}{2Z}\left[\frac{l}{Z_{33}} - \frac{\gamma_4}{4Z_{44}}\cdot\frac{l^2}{2}\right], \qquad (22)$$

which is the same as for far-end crosstalk in very short lengths as given in equation (7).

As pointed out earlier the expression for near-end crosstalk even when the tertiary circuit is electrically long is more complicated in form than for far-end crosstalk because of the terms $1 - \epsilon^{-2\gamma l}$ and $1 + \epsilon^{-2\gamma l}$. This may be seen by comparing formulas (6) and (21).

Nevertheless it is possible to see from (21) that the presence of the tertiary circuit acts to reduce near-end crosstalk as it did in the case of far-end crosstalk. The first term of (21) is less than the near-end crosstalk without the sheath (equation (18)) by the factor

$$\frac{\dfrac{1}{Z_{33}} - \dfrac{1}{4Z_{44}}\cdot\dfrac{\gamma_4^2}{\gamma_4^2 - \gamma^2}}{\dfrac{1}{Z_{33}}} = 1 - \frac{Z_{33}}{4Z_{44}}\cdot\frac{\gamma_4^2}{\gamma_4^2 - \gamma^2}.$$

This is the same factor by which far-end crosstalk is reduced in very long lengths as brought out in the discussion of equation (6). However, the second term in equation (21) prevents this complete reduction from ever taking place in the case of near-end crosstalk.

PART II—EXPERIMENTAL RESULTS

The crosstalk measurements presented here were made on and between sections of twin coaxial cable of various lengths from 73 feet to about five miles. Primarily the tests were made to indicate the effect of sheath and quads upon the summation of crosstalk with length as a check on theoretical considerations and were the first extensive tests made on a coaxial cable with this end in view. The layup of the cable is shown in Fig. 3.

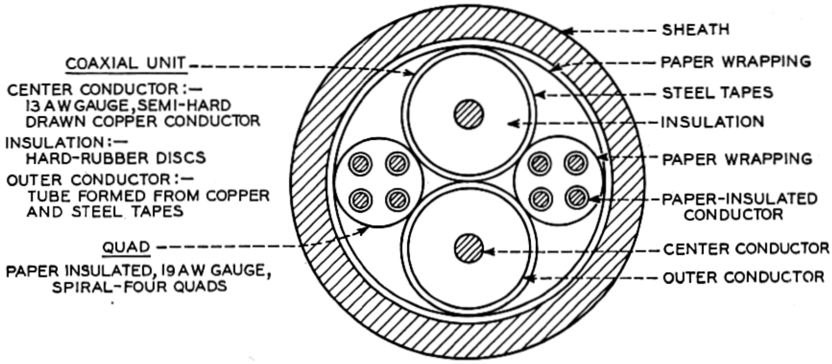

Fig. 3—Cross-section of twin coaxial cable.

As indicated in the latter portion of Part I the general procedure was to measure crosstalk in available sections of equal length, l, with the tertiary circuits terminated in approximately characteristic impedance. Interaction crosstalk between these sections was then measured and finally the two sections were combined to find the total crosstalk in length $2l$. This process was repeated until a total length of about five miles was built up.

Far-End Crosstalk Summation

The results of crosstalk tests on 73 and 146-foot lengths are shown in Fig. 4. The letters on the curves correspond to the crosstalk components discussed in Part I. Only far-end far-end interaction crosstalk was measured but for such short lengths the near-end near-end crosstalk would be nearly the same.

Remembering from the discussion in Part I that the total crosstalk F_{2l} in length $2l$ is equal to $2F_l + F_{nn} + F_{ff}$ it is evident that since in this case the measured components F_{nn} or F_{ff} are quite small the crosstalk in 146 feet should be approximately $2F_l$. That this is the case may be seen from the measured crosstalk in 146 feet which is about 6 db higher than for 73 feet. These lengths are apparently short enough for equations (15) and (16) to hold reasonably well at the lower frequencies.

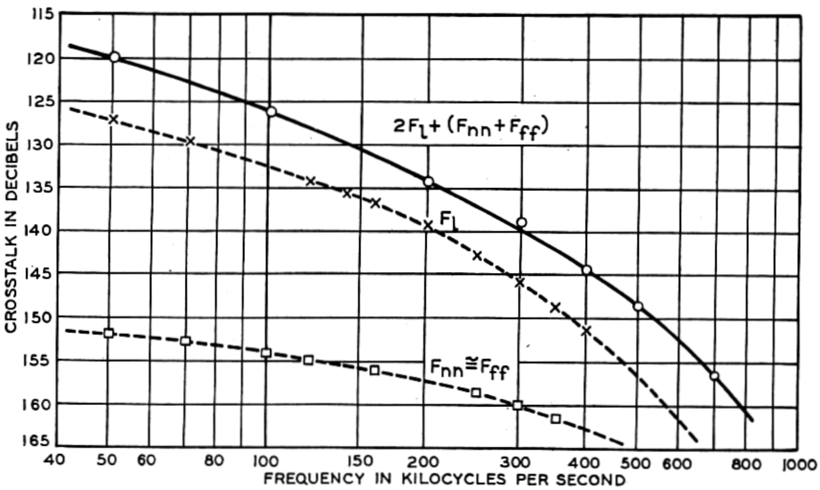

Fig. 4—Crosstalk components in 73-foot and 146-foot lengths.

Now suppose that we consider two lengths which are considerably longer so that equations (6) and (13) more nearly apply. Figure 5 shows the results of tests on and between two 1500-foot cable sections. Here, in contrast with the 73-foot measurements, components F_l and F_{ff} are nearly equal in magnitude while F_{nn} is quite small. Also, F_{ff} and F_l are in general phase opposition since their sum, $F_l + F_{ff}$, is considerably less than either component alone.

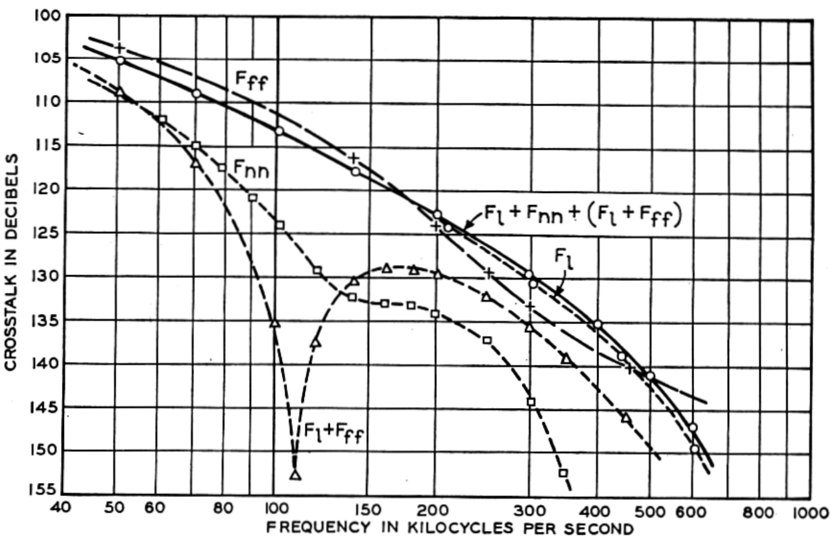

Fig. 5—Crosstalk components in 1500-foot and 3000-foot lengths.

Reference to equations (6) and (9) show that this tendency to cancel is to be expected provided the second term of (6) is the controlling term in F_l. Indeed, in lengths where the tertiary is electrically long, equations (8) plus (9) should exactly cancel the second term of (6). In other words, the total interaction crosstalk *between* two such sections should cancel a portion of the interaction crosstalk *within* a section. Since the portion which is cancelled is the controlling term the net result is that when two sections are combined the total crosstalk in length $2l$ is no more than was measured in length l, as evidenced by the measured curve $F_l + F_{nn} + (F_l + F_{ff})$ of Fig. 5.

This effect persists when two 3000-foot lengths are combined to form a 6000-foot section, as illustrated by the curves of Fig. 6. Here

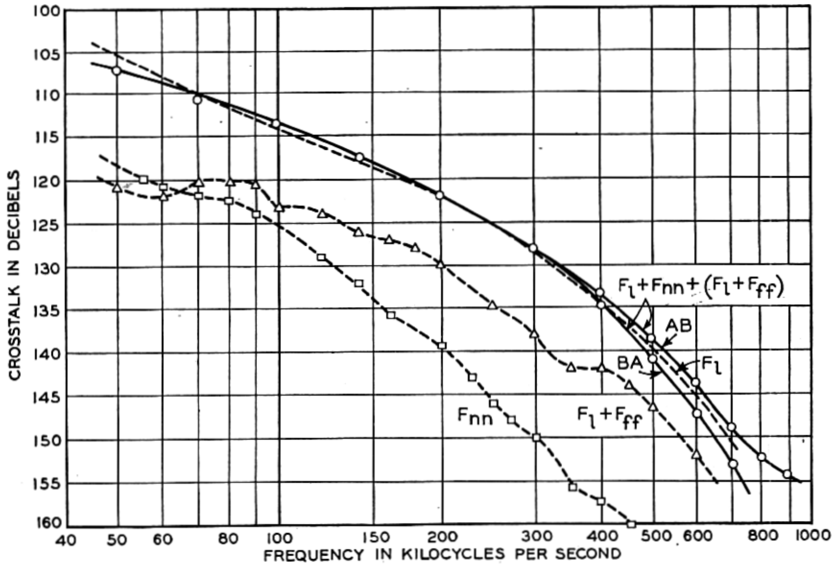

Fig. 6—Crosstalk components in 3000-foot and 6000-foot lengths.

again $(F_l + F_{ff})$ and F_{nn} are considerably smaller in magnitude than F_l so that the total crosstalk in 6000 feet cannot differ materially from the value F_l measured in 3000 feet.

The curves labelled AB and BA were made by using first coaxial A and then coaxial B as the disturbing circuit. The difference between the curves indicates that there is a certain amount of random unbalance within the section. For example, random deviations in the shielding of the two coaxials from a nominal value would result in different values of interaction crosstalk when the disturbed and disturbing circuits are interchanged. The direct crosstalk component would not exhibit this effect.

The results of tests on 6000 and 12,000-foot lengths are given in Fig. 7. Again, the trend is in the same direction as in Figs. 5 and 6 except that in this case $(F_l + F_{ff})$ is nearly equal to F_l and has an appreciable influence when the two components are combined to give far-end crosstalk in 12,000 feet. This indicates that the first term of F_l in equation (6) is becoming more important as l is increased as would be expected.

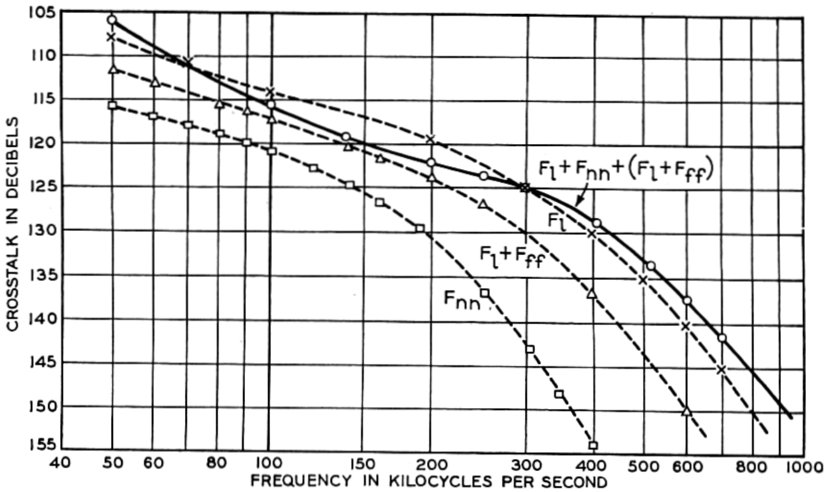

Fig. 7—Crosstalk components in 6000-foot and 12,000-foot lengths.

It may be noted here that curve F_l in Fig. 7 differs considerably from the *AB* and *BA* curves of $F_l + F_{nn} + (F_l + F_{ff})$ in Fig. 6 although all represent far-end crosstalk in 6000-foot sections. These differences in magnitude must be due to differences in the construction of the two cable sections. The difference between the curves varies from 3 to 8 db in the frequency range above 200 kilocyles. However, up to about 150 kilocycles the differences are not greater than 1 db. At the higher frequencies such differences naturally will introduce difficulties in any analysis since they superpose sizeable random effects on the major component of crosstalk which is systematic.

The curves in Fig. 8 present far-end crosstalk tests on 12,000 and 24,000-foot lengths. Here F_l and $(F_l + F_{ff})$ are of the same order of magnitude and combine in such a way that the crosstalk in 24,000 feet is from 3 to 6 db higher than that measured in 12,000 feet. Component F_{nn} is again negligible. This behavior indicates that the first term of equation (6) is controlling as the length is increased.

It appears from all these tests that the magnitude of the far-end crosstalk in this cable with tertiaries terminated does not vary materially from 1500-foot to 12,000-foot cable lengths, except for random effects. In other words, for this range of lengths the second term of (6) is controlling. For very short lengths the crosstalk varies directly with length due to the absence of interaction crosstalk of sufficient magnitude to exert any influence. Also, in going from 12,000 to 24,000 feet, there is a definite indication that the crosstalk is increasing with

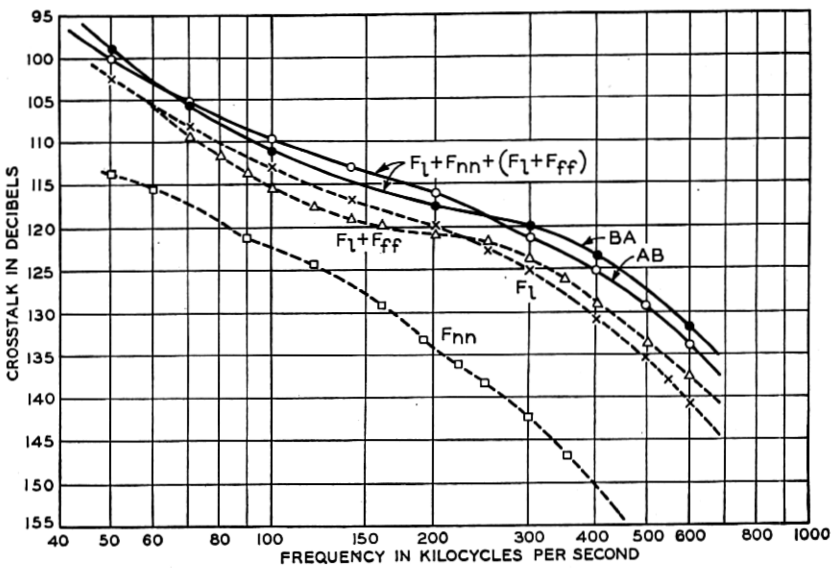

Fig. 8—Crosstalk components in 12,000-foot and 24,000-foot lengths.

length, so that for lengths over 24,000 feet the crosstalk would again tend to be proportional to length. We have shown in Part I that on the basis of theoretical considerations this law of crosstalk summation with length might be expected.

To illustrate this measured behavior the far-end crosstalk versus length for frequencies of 50, 100 and 200 kilocycles has been plotted on Fig. 9. For comparison are also plotted dashed curves based on the 73-foot tests and computed on the assumption that the crosstalk is directly proportional to length. The difference between corresponding curves shows the influence of the tertiary circuits. For a 24,000-foot length this difference amounts to 23, 26 and 27 db at 50, 100 and 200 kilocycles, respectively.

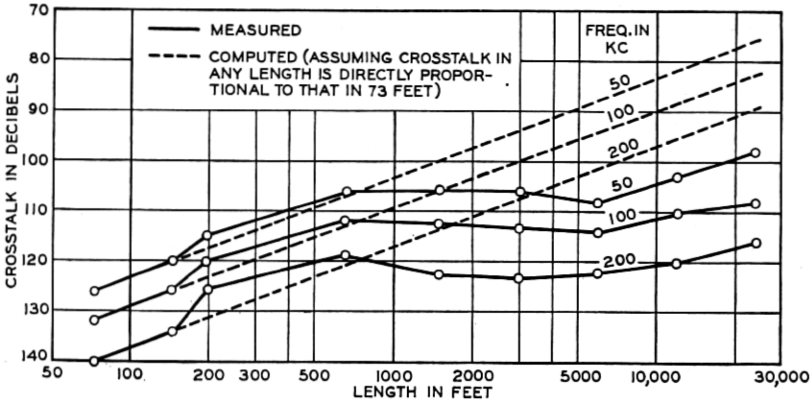

Fig. 9—Far-end crosstalk vs. length with tertiary terminated.

Near-End Crosstalk Summation

The curves on Fig. 10 show the amount of near-end crosstalk reduction due to the presence of the sheath and quads for a length of about five miles. The upper curve was computed from tests on a 73-foot length with tertiaries terminated by raising the values measured

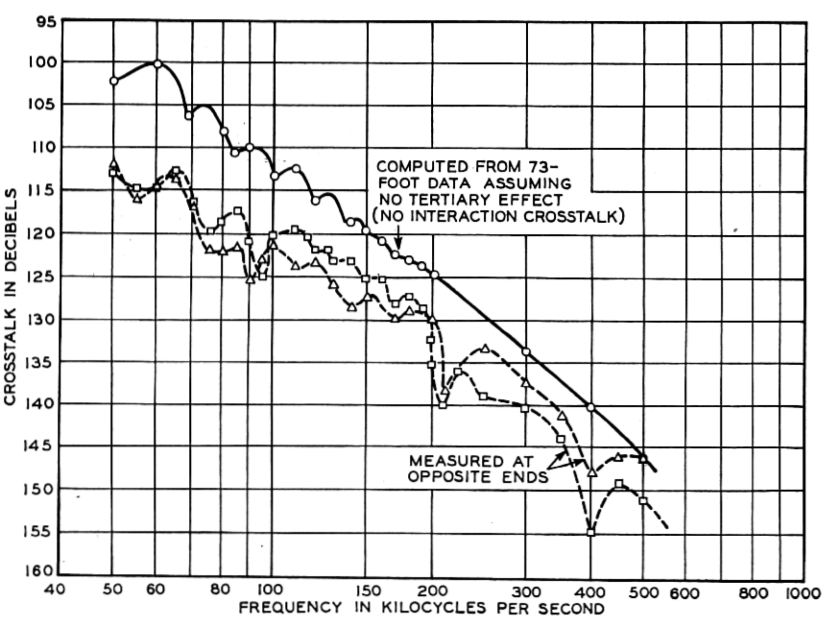

Fig. 10—Near-end crosstalk in a 5-mile length with tertiary terminated.

there by the factor

$$\frac{1 - \epsilon^{-2\gamma L}}{2\gamma l},$$

where $l = 73$ feet and $L = 5$ miles. This is the crosstalk which would exist in five miles *in the absence of a sheath and quads.*

The lower two curves were measured at opposite ends of the cable and the difference of about 10 db between these curves and the upper curve is due to the tertiary circuit effects. As might be expected from the discussion of equation (21), this reduction is considerably less than in the case of far-end crosstalk.

Interaction Crosstalk Between Sections

The methods of measuring the various types of interaction crosstalk between two sections have already been discussed in reference to Fig. 2. Besides showing the influence of interaction crosstalk in the summation of crosstalk *within* a repeater section the results presented below are indicative of the importance of interaction crosstalk which takes place *between* repeater sections, that is, around repeaters, when all or only a part of the tertiary is continuous at repeater points.

Values of near-end near-end interaction crosstalk, F_{nn}, were measured between various section lengths from 73 to 12,000 feet. It was found that the results are roughly independent of the section lengths above 1500 feet, and curve F_{nn} of Fig. 11 for the crosstalk measured between two 12,000 foot sections is typical. This independence of length is because of the high attenuation of the tertiary circuits which annihilates the effects of crosstalk in the more remote portions of the sections as may be seen from equation (8) if γ_4 is made large. The relatively unimportant contribution of this type of interaction crosstalk to the summation of far-end crosstalk *within* a repeater section has been discussed.

Similarly, measured values of far-end far-end and near-end far-end interaction crosstalk between various sections lengths were found to be practically independent of length above 1500 feet. Curves F_{ff} and N_{nf} of Fig. 11 for the crosstalk between 12,000-foot sections are typical. The far-end far-end component of interaction crosstalk has an important influence on the summation of far-end crosstalk within a repeater section as already mentioned in the section on far-end crosstalk summation. The influence of near-end far-end interaction crosstalk N_{nf}, on the summation of near-end crosstalk within a repeater section has not been very thoroughly investigated here but it is responsible for the results described in the discussion of near-end crosstalk in a five-mile length.[11]

The relative importance of various tertiary circuits in the production of interaction crosstalk between two sections was studied for the case of near-end near-end crosstalk between two 12,000-foot lengths. It was found that the outer conductor-quads and outer conductor-sheath circuits were about equally important and that crosstalk via the quad-sheath tertiary circuit was from 20 to 30 db less. These results are about as expected since the outer conductors are the source of the

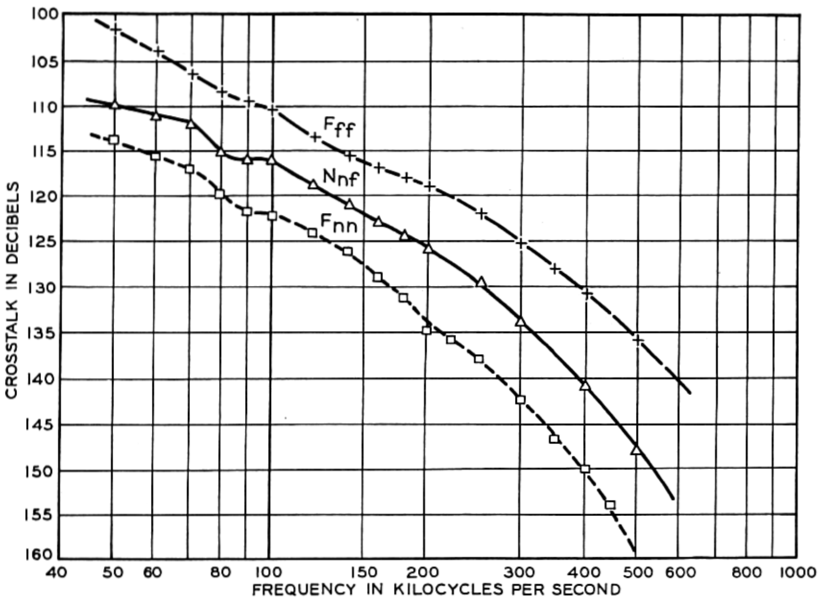

Fig. 11—Interaction crosstalk between two 12,000-foot lengths.

tertiary emf and thus the tertiary circuits involving the outer conductors should be the important ones. It is therefore permissible to consider sheath and quads as a single unit as was done in Part I.

Effectiveness of Transpositions on Far-End Crosstalk Reduction

In a long repeatered system the far-end crosstalk measured in successive individual sections inherently tends to sum up directly since all

[11] It should be noted that while Fig. 11 shows the measured values of the three types of interaction crosstalk between two 12,000-foot sections, the relative importance of the various types acting *between* repeater sections, that is, around repeaters, is not as shown there, since different correction factors have to be applied when estimating the total crosstalk at system terminals.

repeaters have practically the same phase shift and the propagation characteristics of the two coaxials are nearly identical. One way to prevent this direct addition is to transpose one section against another or one group of sections against another group along the line. In the case of unbalanced circuits these "transpositions" take the form of transformers or extra tube stages in one of the systems at repeaters, either of which will produce a 180-degree phase reversal.

If the far-end crosstalk in one transposition section is F_{l1} and that in another is F_{l2} the total in the two sections, *exclusive of interaction crosstalk between sections*, is inherently $F_{l1} + F_{l2}$. With a transposition in one coaxial at the junction the total becomes $F_{l1} - F_{l2}$. Hence, if $F_{l1} = F_{l2}$ it is possible to eliminate this crosstalk component entirely. However, due to irregularities in the cable and the practical impossibility of locating repeater points exactly, F_{l1} will not, in general, equal F_{l2} and even after transposing a small residual may remain.

This residual, however, may be negligible compared with the near-end near-end and far-end far-end interaction crosstalk components F_{nn} and F_{ff} between repeater sections (that is, around repeaters), unless transmission along the tertiary circuits from one repeater section into another is suppressed at repeater points. The interaction crosstalk tests already discussed may be used to compute this effect. However, in order to demonstrate the effectiveness of transpositions, far-end crosstalk tests were made in a 24,000-foot length with and without a transposition in one of the coaxials at the center and with various interaction crosstalk paths suppressed. The results are given in Figs. 12 to 14 and are discussed below.

To suppress entirely the interaction crosstalk between the transposed sections all tertiary circuits were shorted at the transposition point. In these measurements the tertiaries were also shorted at each end of the line in an effort to have both ends of each half of the line terminated as nearly alike as possible. The test results are given in Fig. 12.

For this condition the crosstalk measured in each half of the line is also shown. Curve AB represents the far-end crosstalk in one line section and $A'B'$ that in the other section. Curve $(AB + A'B')$ gives the results when the two sections are combined with no transposition. Curve $(AB - A'B')$ gives the results when a transformer is inserted in one coaxial at the center. (A similar set of curves are given for BA, $B'A'$, etc.)

Note that AB and $A'B'$ coincide very closely in magnitude. When combined with no transposition the crosstalk in two sections is nearly 6 db higher over the entire frequency range than in either individual section. When combined with a transposition the crosstalk in two sections is from 13 to 27 db below either individual section over the frequency range. *Such a reduction is possible only because AB and A′B′ are so nearly equal.*

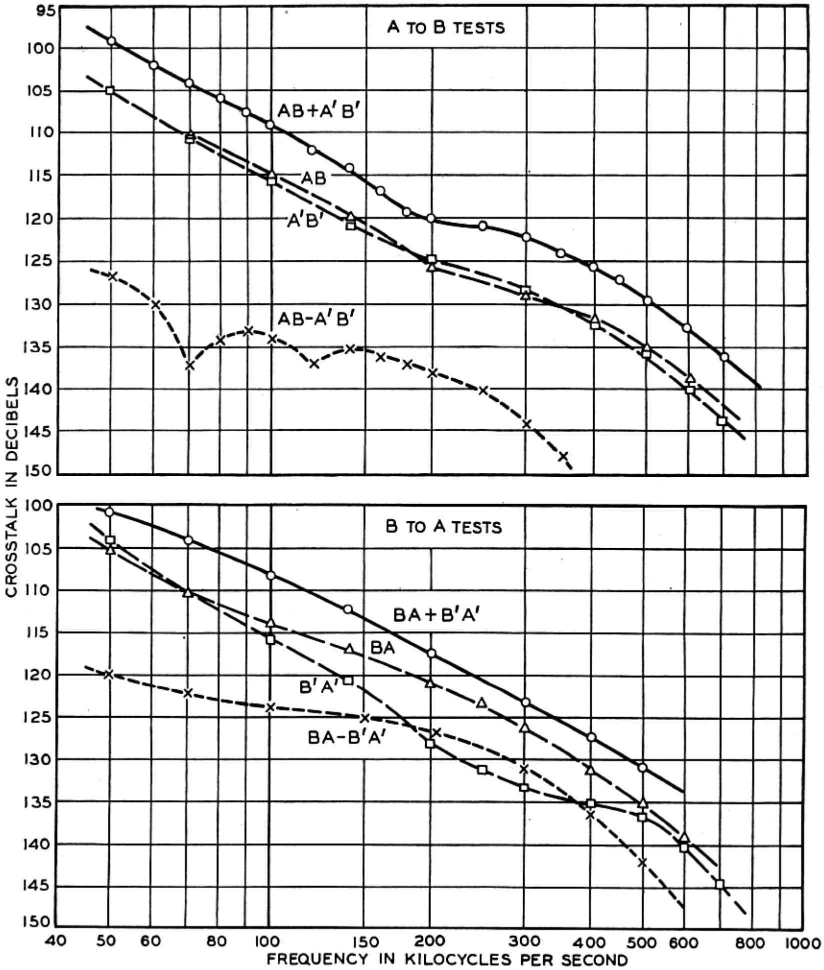

Fig. 12—Effect of a transposition on far-end crosstalk in a 24,000-foot length with all tertiary circuits suppressed at the transposition.

In contrast, *BA* and *B′A′* may be seen to differ considerably from each other at the higher frequencies. As a result, the transposition is not nearly so effective in that range. The improvement at the lower frequencies where it is needed most is still about 20 db.

In order to suppress only a portion of the interaction crosstalk between two sections, measurements were made with the coaxial outer conductor-sheath circuit shorted at the transposition point thus permitting continuity of the quad-outer conductor tertiary circuit. This tertiary circuit had been shown previously to be an important

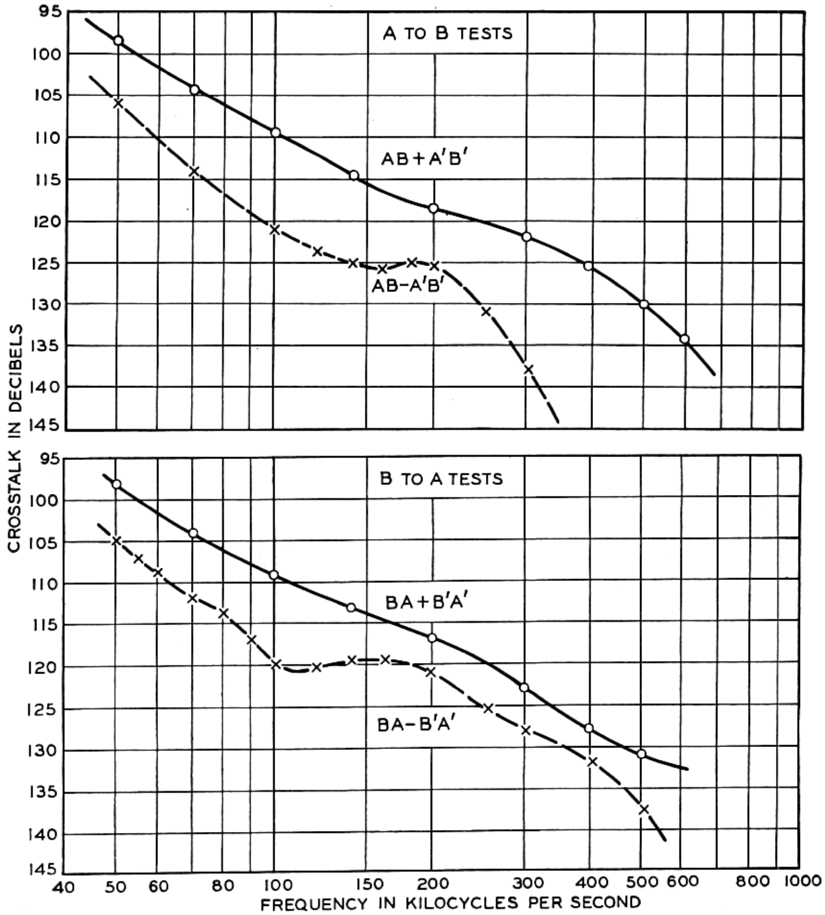

Fig. 13—Same as Fig. 12 except with quad-outer conductor tertiary circuit continuous past the transposition.

one in the production of interaction crosstalk. The measured far-end crosstalk results are given in Fig. 13.

It is at once apparent that the transposition is not so effective in this case. The crosstalk remaining after transposing is about what would be expected due to interaction crosstalk between sections via the quad-

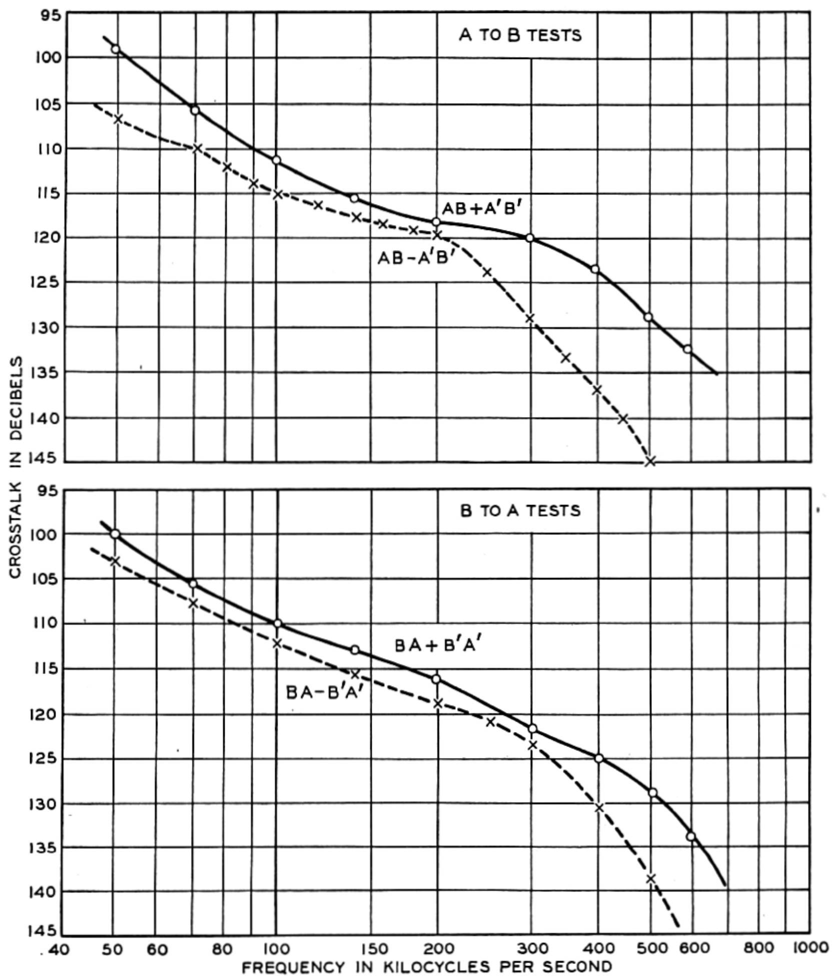

Fig. 14—Same as Fig. 12 except with all tertiaries continuous past the transposition.

outer conductor tertiary circuit.[12] However, a certain portion is also due to differences between AB and $A'B'$ (or BA and $B'A'$).

On Fig. 14 are plotted far-end crosstalk values when two 12,000-foot sections are combined with and without a transposition in one coaxial

[12] It should be noted here that these tests indicate directly the effect of a transposition *at the center* of a 24,000-foot section rather than at a *junction* between two repeater sections in a long repeatered system. If 12,000-foot repeater spacing is assumed with the transposition at the repeater point it is necessary to reduce the measured far-end far-end interaction crosstalk and increase the measured near-end near-end interaction crosstalk by an amount equal to the line loss in 12,000 feet. These corrections put interaction crosstalk between repeater sections on an output-to-output or equal level basis.

at the center and when all tertiary circuits are continuous at the transposition point and terminated at the ends. Curve $(AB + A'B')$ gives the results when the two sections are combined with no transposition. Curve $(AB - A'B')$ shows the result when a transformer is inserted in one coaxial at the junction. (A similar set of curves is given for BA, $B'A'$, etc.)

It is seen that in the 50–200 kc range there is an improvement in overall crosstalk of from 3 to 8 db due to the transposition. However, the overall crosstalk in the combined sections with a transposition is not appreciably less than that in an individual 12,000-foot section as shown by curve F_l on Fig. 8. Reference to Fig. 11 shows that this is due mainly to the far-end far-end interaction crosstalk between the two sections which is unaffected by the transposition.

The results shown in Fig. 12 give some indication of the extent to which far-end crosstalk may be reduced by means of a transposition, *provided interaction crosstalk between sections is entirely suppressed.* As illustrated in Figs. 13 and 14 a transposition at a repeater point is not nearly so effective if the interaction crosstalk is not suppressed.

Acknowledgment

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