

Nuclear Fission

By KARL K. DARROW

This article pertains to the most newly-discovered and most sensational mode of transmutation, in which the entry of a neutron into a massive atom-nucleus brings about an internal explosion in which the nucleus is "fissured" or divided into two fragments which share the total mass and charge between them in nearly equal proportions. (In all other modes of transmutation except those affecting the very lightest elements, the division is into fragments of very unequal mass and charge.) The conversion of rest-mass into kinetic energy, or (as is more commonly said) the release of energy, is unprecedented in scale. A multitude of radioactive bodies, many hitherto unknown, is formed; and there is spontaneous emission of fresh neutrons in great quantities, possibly sufficient to convert the process once initiated into a self-perpetuating one under realizable conditions.

EVERY now and then a physicist is liable to receive a letter from some yearbook or other, in which he is invited to write x thousand words on the "most important developments in physics during the year just ending." The only safe reply is of course that for ten years at least and perhaps for fifty it will be impossible to tell which is the most important development in physics during the year just ending. This year, however, it looks as though one need not be so cautious; for ever since the first few weeks of the year many have felt pretty sure that one particular discovery would long be recognized as the most important to be made, or at any rate to be revealed, in 1939. It came early—the first publication was on the sixth of January, and there was a rain or perhaps I should say a deluge of others before the end of February. Inasmuch as these others proceeded from laboratories sprinkled all of the way from Copenhagen to Berkeley, it is literally true for once that a discovery commanded immediate attention. Nor is attention even yet diverted, though the pace of publication has grown less.

The phenomena of fission are as yet confined to the last three elements of the periodic table: thorium, protactinium, uranium. I show their chemical symbols, their atomic numbers or nuclear charges, and the mass-numbers—to wit, the nearest integers to the actual values of the masses—of their several isotopes (charges expressed of course as multiples of e , masses as multiples of one-sixteenth the mass of the

commonest kind of oxygen atom); charge appears as a subscript before the symbol, mass as a superscript after it:



From this list I omit several very unstable isotopes of which we shall probably never be able to assemble enough to observe their fission. Protactinium 231 is itself so rare that only one man in the world (he is von Grosse, of Chicago) ever got enough of it together for this experiment. He brought his precious sample—less than 9 mg.—to Dunning at Columbia for the test, and the three of them found fission. I make this allusion at the start, because there will be little further occasion to refer to protactinium, and yet it should not be forgotten. There is danger of forgetting even thorium, since so disproportionately great an amount of study has been lavished on uranium. Neither thorium nor uranium is a very rare element, but more than 99 per cent of any sample of uranium consists of the isotope 238, so that the two other isotopes must also be classed as rare; yet it is believed at present that 235 is responsible for some of the most remarkable of the phenomena.

Now let me indicate two qualities shared by all five of these nucleus-types. First: all are radioactive, that is to say, they are unstable. I must not be too emphatic with this word; the average lifetime of nuclei of either Th^{232} or U^{238} is hundreds of millions of years, and there are not many organizations which would be considered unstable if they could bank on a probable lifetime of that scale. Still they are, in the physicist's sense, unstable; and this suggests that it might be relatively easy to disorganize, to disrupt, to explode them by a fitting agency coming from without.

Now to any physicist the term "fitting agency coming from without" suggests at once the bombarding particles by which transmutation was first effected: alpha-particles, protons, deuterons—the positively-charged nuclei of the elements helium and hydrogen at the other end of the periodic table from uranium. Should one not project these nuclei against uranium nuclei, and see what happens? Well, it has often been done, and nothing has happened;¹ and an adequate reason is supplied by the second important quality of these five nucleus-types, their greatness of atomic number. All of them are so highly charged with positive electricity, that the proton, the deuteron and the alpha-particle, however fast they are when they start, cannot approach them closely enough to do them any harm. (What with the current progress in cyclotrons that statement may soon be out of date!) Even with our

¹ Until in October of this year Gant reported strong indications of fission of uranium produced by very energetic (8-Mev) deuterons.

present resources of energy we could not tamper with any of these nuclei, had we not at our disposal those chargeless particles the "neutrons" with which to assail them.

One might of course foretell that mighty powers of transmutation would be possessed by a particle which is *not* repelled as it approaches a nucleus. Actually the transmuting powers of the neutron are greater than, I should think, anyone can have expected; nor can many people, if any, have foreseen that the *slow* neutron—the neutron having no more speed and kinetic energy than a molecule of air at room-temperature—would prove to be more potent than the fast one. Yet so it is. When the other agents of transmutation were first applied—alpha-particle, proton, deuteron, photon, fast neutron—it took years to get proof of the transmutation of even a few elements; but when the slow neutron was first applied, Fermi and his half-a-dozen colleagues at Rome managed to do something to almost every element in a very few months! Let me recall that neutrons mostly are what we call fast—i.e., they have energies of millions of electron-volts—when they start their careers. Slow neutrons are initially-fast ones which have been sent through layers of paraffin or water, and have lost nearly the whole of their initial energy by making elastic impacts with hydrogen nuclei. We shall later have to distinguish between the fast neutrons and the slow as agents of nuclear fission.

Now to supply a fitting historical background to the discovery of fission, I must draw attention to a theorem which until the end of 1938 was believed to govern the whole of transmutation, and which still governs nearly the whole of the field. It is this: with the exceptions presently to be related, *no transmutation ever produces a change in atomic number greater than 2 or a change in mass-number greater than 4*. I am going to illustrate this theorem by writing in symbolic form three of the reactions of transmutation produced by neutrons and recognized before the end of 1938. Here I use E as the general symbol for element; Z and A as the general symbols for atomic number and mass-number; and α , p , d , n , and ϕ for alpha-particle, proton, deuteron, neutron and photon; and I recall that the mass-numbers of these five particles are 4, 1, 2, 1, 0 respectively.

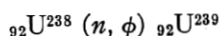
$${}_Z E^A (n, \alpha) {}_{Z-2} E^{A-3}; \quad {}_Z E^A (n, p) {}_{Z-1} E^A; \quad {}_Z E^A (n, \phi) {}_Z E^{A+1}.$$

The first of these (for example) is to be read: a neutron enters a nucleus (Z, A) and an alpha-particle comes out, leaving behind a nucleus ($Z-2, A-3$). There was a similar (not identical) rule setting a limit on the changes of atomic number and mass-number suffered by radioactive bodies. Every radioactive nucleus emits either a positive

electron or a negative electron, or an alpha-particle; the corresponding changes in Z are -1 , $+1$, -2 , those in A are 0 , 0 , 4 .

Now it is high time that we get on to uranium. This, like thorium, was one of the elements that Fermi exposed to slow neutrons, and to which he observed that something was happening. As he continued these researches, and as the great institutes of nuclear physics of Hahn in Berlin and the Jolios in Paris followed suit, it became evident that a great deal was happening. With nearly all of the other elements, there occurred just one of the reactions which I symbolized above, or maybe one reaction with some of the nuclei and another with others. Sometimes the reaction would lead to a stable nucleus-type; in such a case, when the neutron-bombardment ceased all the excitement was instantly over. Sometimes it would lead to a radioactive nucleus-type; in such a case, the radioactivity would continue after the bombardment ceased, but it would steadily die away to nothing, in accordance with the well-known law. But with uranium and thorium there was a swarm of radioactive products, so many that to this day they have not all been separated and identified. Moreover, some of these were descendants of others, for after the bombardment ceased they grew in strength for a while before declining. All of them were emitters of electrons, and the electrons (in every case in which their sign is known) were *negative*.

Owing to the theorem which has just been stated, it was taken for granted that the immediate effect of the neutron entering the nucleus of uranium was to provoke one of the three reactions which I lately listed. Of these the one most commonly assumed was the reaction,



the so-called "reaction of pure neutron-capture"—called pure, because no massive particle goes forth. I mention it here because it is still believed in, and we shall meet it later. Uranium 239 is radioactive, and some of the other radioactive products were believed to be direct or indirect descendants of it. Well, every one of the radioactive substances resulting from the reaction or reactions $U(n)$ —for so I will symbolize them in general—is an emitter of *negative* electrons. Therefore each has a greater positive charge on its nucleus than does its predecessor; therefore by this theory, each descendant from U^{239} must have a greater positive charge than the $92e$ of the uranium nucleus. But uranium is the final element of the periodic table; therefore by this theory the radioactive bodies in question had to be isotopes of new elements beyond uranium.

These so-called "trans-uranic elements" were for several years the principal study of Hahn and Meitner and their colleagues at the great institute in Berlin-Dahlem. The groups at Paris and at Rome contributed also—not very much, but enough to signify their full adhesion to these concepts. Other physicists scarcely entered the field, but had the fullest reliance in views sustained by such authorities. Yet now, the trans-uranic elements are gone! This is regrettable, because it was pleasant to think that human artifice had succeeded even in lengthening the list of the elements. It is regrettable for the chemists especially, because they were looking forward to getting information about the chemical properties of elements beyond 92. Whether on balance there is regret among physicists I doubt, because the knowledge that has replaced the trans-uranic elements seems even more spectacular than they did. Let us see how this knowledge was attained.

Some time in 1938, Hahn observed that three of the radioactive substances resulting from the exposure of uranium to neutrons had some of the chemical properties of barium—enough to follow barium in certain of the distinctive precipitations which are known to chemists. Now this is a statement which is true of radium. Hahn assumed that he had three new isotopes of radium, and this was entirely natural, for two reasons. First, radium and its isotopes already known are all radioactive, suggesting that any which remained to be discovered should also be so; and second, the atomic number of radium is 88, so that radium isotopes could conceivably come into being through the reaction $U(n, \alpha)$ followed by the spontaneous emission of an alpha-particle from the residue. Yet (and this is the fact which came out on the 6th of January 1939) these substances were much too much like barium! When Hahn and Strassmann used some of the procedures which separate radium from barium, the novel substances declined to be separated. In a typical experiment, one of them together with some well-known isotope of radium would be introduced into a solution of some salt of barium. Fractional crystallization being performed, it was found as usual that the relative concentration of the radium isotope was greatly changed in the first-to-be-formed of the crystals; but not the relative concentration of the new substance, which entered into the crystals in just the same proportion as the barium itself.²

There are people who have revolutionary and false ideas about questions of science, and who irritate the scientists by their overconfidence, their often arrogant ways of offering those ideas to the world.

² The salts were the bromide and the chromate (perhaps also the chloride and carbonate) of barium; the isotopes of radium were ThX and MsTh_1 .

Listen now to men of science having a revolutionary and true idea, and expressing it in a befitting way:

"Now we must speak of some more recent investigations, which we publish only with hesitation because of the strange results. . . . We come to the conclusion: our 'radium isotopes' have the properties of barium; as chemists, we really ought to say that these new substances are barium, not radium. . . . As chemists, we ought to use the symbols Ba and La and Ce where we have been using Ra and Ac and Th. But as 'nuclear chemists' closely associated with physics, we cannot yet bring ourselves to make this leap, in contradiction to all previous lessons of nuclear physics. Perhaps, after all, our results have been rendered deceptive by a series of strange accidents. . . ."

Here I ought to mention another famous group of nuclear physicists who at an earlier date might have taken the leap, but recoiled before it so vehemently that they could not even bring themselves to mention in print the possibility of making it. These were the physicists of the Institut du Radium at Paris: Irene Curie and Savitch discovered in early 1938 that one of the products of $U(n)$ was indistinguishable, by all the tests that they applied, from the rare-earth element lanthanum ($Z = 57$). Later on they said that at a certain moment they had envisaged what we now call the fission of the uranium nucleus, but had preferred to believe that they had before them one of the transuranic elements resembling lanthanum more closely than anyone as yet had foreseen.

Now we come on to the middle of January 1939, and I must introduce the grand idea which with the force and suddenness of revelation burst upon several people far apart in the world, as soon as they heard of the experiments of Hahn and Strassmann and of the leap which these two had dared to envisage and publish if not quite to take.

Let us be audacious enough to take the leap, and let us further imagine that after the entry of the neutron the nucleus divides itself into two pieces or "fragments" of which one shall be barium. I must say directly that this assumption is more specific than need be, and that the same conclusions would be reached if we assumed that one of the fragments is some other element close to barium in the Periodic Table. It will be simpler, however, to be definite: let us assume barium, and for still greater definiteness let us suppose that the isotopes concerned are 238 of uranium and 139 of barium. The neutron, then supposedly enters a nucleus ${}_{92}\text{U}^{238}$ and with it forms the transitory "compound nucleus" ${}_{92}\text{U}^{239}$, and from this there splits off a nucleus ${}_{56}\text{Ba}^{139}$. What is left behind must be (if in a single piece) the nucleus

of which the charge added to $56e$ makes $92e$, and of which the mass-number added to 139 makes 239—that is to say, the nucleus ${}_{36}\text{Kr}^{100}$.

This is an example of the type of process which has been named by borrowing the word “fission” from biology. The biologists seem not to have found a specific verb to correspond (I am told that they use “divide”) and the physicists have had no better inspiration. The dictionaries, however, authorize the use of “fissure” as a verb both transitive and intransitive, and I will henceforth so use it.³

Now a difficulty looms up, or rather what seems to be a difficulty but is really a great advantage, for the grandeur of the idea depends on it. Mass-numbers are only approximations to true masses, and the true mass of the nucleus U^{239} is greater than the sum of the masses of Ba^{139} and Kr^{100} . There is a superfluity of mass, and by classical ideas this superfluity might have to vanish, which would indeed be a stumbling-block. However, that stumbling block does not exist, because of something I have now to introduce. *It is the rest-mass, in the sense of relativity, of U^{239} which exceeds the sum of the rest-masses of Ba^{139} and Kr^{100} .* Now U^{239} before the explosion is practically at rest, but we are not obliged to make the same assumption about the fragments, and in fact we can assume that *the fragments fly apart at just such speeds that their relativistic increase of mass with speed brings up the sum of their masses to exactly the right value.* If so, their kinetic energies must be 50 to 100 Mev apiece. These on the nuclear scale are immense amounts of kinetic energy, and particles possessing it must be easy to isolate and easy to detect. This is why the idea is a grand one.

As it might occur to some reader to go to the tables of constants and look up the mass-values of U^{239} and Ba^{139} and Kr^{100} , I must say at once that he will not find them. Generally speaking, the mass-spectrograph cannot be used on radioactive and unstable atoms because one cannot get enough of them together for the experiment (exception being made for very long-lived ones like U^{238} and Th^{232}). All those three belong in that category, and therefore we have to estimate their masses by extrapolation from those of stable isotopes. The extrapolations for Ba^{139} and U^{239} are so small that the uncertainty is trivial, but Kr^{100} is no less than fourteen units heavier than the heaviest stable isotope of krypton, and this is serious. However, one is not so much concerned about conceivable defects in grand ideas when the ideas have already done their work by leading with success to grand experiments. I lay emphasis again, for a reason later to appear, on the extent to which Kr^{100} is out of line with the stable krypton isotopes; and now we pass to the experiments.

³ I am indebted to Dr. Elizabeth Patterson of Bryn Mawr College for this solution.

There are actually two grand experiments, which I tried to distinguish above in a sentence by saying that the energetic particles must be *easy to isolate* and *easy to detect*. "Isolate" is not a very happy word: the fact is, that if so energetic they must be able to fly right out of the bombarded sheet of uranium (unless they start too deep beneath its surface)—thus, if some sort of a collector is placed across from the uranium and not too far away, they must assemble on it and there they should be found together with all their descendants. Joliot published this experiment before the end of January. He found radioactive substances on his collector even when more than two centimetres of air⁴ had separated it throughout from the uranium.

The experiment has been performed by many, some introducing new refinements. Meitner and Frisch for instance used a bowl of water for collector, and then could concentrate the radioactive bodies by letting the water evaporate, or by precipitating various salts which in advance they had dissolved in it. This last is the chief technique for finding out the chemical nature of the radioactive products, to wit, the elements of which they are unstable isotopes; but we have not space for entering into the details of the technique, already practiced these five years. Glasoe, McMillan and others modified the method by piling very thin foils of very light substances—aluminium, filter-paper, cigarette-paper—on the uranium. Some of the radioactive matter is found embedded in each of the first few foils, and one may study thus their "distribution-in-range," an almost self-explanatory term. In McMillan's experiment the utmost perceptible range was slightly above 2.2 cm of air.

Already in the first experiment Joliot observed that in respect of its decay in time, the radioactivity on the collector was very like that remaining on the uranium. Later more accurate work has merely strengthened that conclusion, and Segré in particular affirms that out of many radioactive bodies there are only two which are found in the bombarded uranium itself and not on the distant collector also. On the distant collector there are found, in particular, the substances once classed as "trans-uranic elements." This is very important, for in the theory of the trans-uranic elements there occurs no stage in which the fragments of the uranium nucleus (or any other) are thrown apart with so tremendous energies. Were these elements trans-uranic, they should not be able at all to escape from the bombarded uranium

⁴To give the thickness of air (of the density corresponding to 15° C. and one atmosphere of pressure) which can just be traversed by a charged particle is the ordinary way of stating the "penetrating power" of the particle. Often some other substance than air is used in the tests; it is then not the actual thickness of the substance, but the "air-equivalent" thereof, which is ordinarily stated. Joliot appears to have used actual air in the experiments.

target. When in defiance of this the radioactivity crossed over to the collector, the trans-uranic elements were doomed.

In these experiments, then, the fragments of the initial explosion are found *en masse* together with their descendants upon the distant plate. In those of the other grand type, they are detected each by itself *en route*. Being charged particles of great momentum, they cleave through any gas in nearly linear paths, along which quantities of ions stay behind.⁵ The Wilson chamber may be used to make these visible, and has indeed already been so used (by Joliot, and by Corson and Thornton); but another device gave the first and as yet most instructive results. This is the ionization-chamber equipped with linear amplifier and oscillograph. In the first, the ions due to the passage of a single particle are drawn to a collector and their charges united; in the second, the united charge is multiplied by a large fixed factor; in the third, the multiplied charge produces a sharp sidewise motion of the oscillograph-beam and the spot which this last produces. On the photographic plate the moving spot produces a line, the length of which is measured. Instances of these lines appear in Fig. 1.

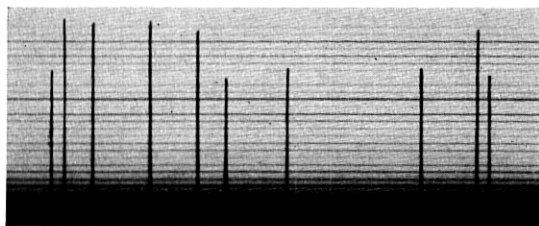


Fig. 1—Records of fission-fragments obtained with ionization chamber, linear amplifier, and oscillograph. The short lines due to alpha-particles are lost in the hazy dark band beneath. (Courtesy of J. R. Dunning)

Uranium is a spontaneous emitter of alpha-rays (this is how the radioactivity of U^{238} and U^{234} is manifest) and so the apparatus will show "kicks" even when neutrons are absent. This is an advantage really, since when the neutrons are admitted and the kicks due to the fragments appear they are so much the larger that there is no danger of confusing them with alpha-particle kicks, while these last may be pressed into service for calibrating the device. The calibration reposes upon a theorem of very great value in physics: viz., the (average) amount of energy expended by a fast charged particle in producing an ion-pair is fixed and constant, whatever the charge and mass and

⁵ This is correct whether they travel as isolated nuclei, or are attended by some though not the full quota of orbital electrons which would environ them were they already the nuclei of completed atoms. Capture of electrons along the course is almost certain (it has been proved to occur with alpha-particles).

speed of the particle. So, the ratio of the kicks caused respectively by a fragment and an α -particle is the ratio of their initial energies, provided the chamber is so "deep" that they both run their courses completely to the end in the gas thereof. If on the other hand the chamber is so shallow or "thin" that fragment and α -particle shoot across it and only a small part of the total course of each is comprised within it, then the ratio of the kicks may be the ratio of the densities-of-ionization along the two tracks. Both the initial energy and the density-of-ionization are known for the α -particles, permitting the calibration. Also the constant value of the energy-expended-per-ion-pair is known (it is about 30 ev.) so that if the experimenter can measure the actual amount of charge set free in his ionization-chamber he need not bother with the α -particles.⁶ In Fig. 1, by the way, the alpha-particle tracks are quite lost in the black band of the "background."

The second grand experiment, then, consisted in showing that when the neutrons were falling upon the uranium, there instantly appeared among the smallish kicks due to the α -particles others which were much greater—ten- and twenty-fold greater. This was done in four places⁷ at least in America in the closing days of January 1939; in Copenhagen, however, a fortnight earlier.

The greatness of the kicks when the ionization-chamber is deep signifies the greatness of the initial energies of the fragments: I shall presently quote the latest data of these. But when the chamber is thin, the kicks due to the fragments again stand out very much over those due to the α -particles; and this signifies that the ionization-density along the fragment-tracks is great. (Take note, by the way, that one and the same chamber may be thin or thick, according as the density of the gas within is low or high—a very convenient fact.) The fragments, then, not only have remarkably great energy to start with, but also spend it at a remarkable rate in ionization along their courses. The course or "range" of a fragment must therefore be much shorter than would be that of an α -particle of the same energy. This is a verifiable fact, the ranges being easily measured by this method. We have just seen how Joliot was able to estimate them earlier, wiping out by this observation the possibility that each of the great kicks may be due to many α -particles starting off together. From the ion-

⁶ All this is contingent upon the ions being completely gathered in by the collector of the ionization-chamber before any serious fraction of them is annulled by recombination, or (failing that) upon the loss by recombination being the same in proportion for α -particles and for fragments. Owing to the (unprecedentedly) great density of ion-pairs along the tracks of the fragments, this is by no means sure.

⁷ New York (Columbia), Baltimore (Johns Hopkins), Washington (Carnegie Institution), Berkeley (University of California). In these cases the suggestion originated with Fermi.

ization-density and the range and the energy all taken together, it may be inferred that in both charge and mass these particles much exceed the α -particles; but here, better data and fuller theory are urgently required.⁸

Now we will consider the energies of the particles according to the data of Kanner and Barschall of Princeton.

If the immediate products of the fission are really just a pair of fragments nearly but not quite identical, we may expect a distribution-

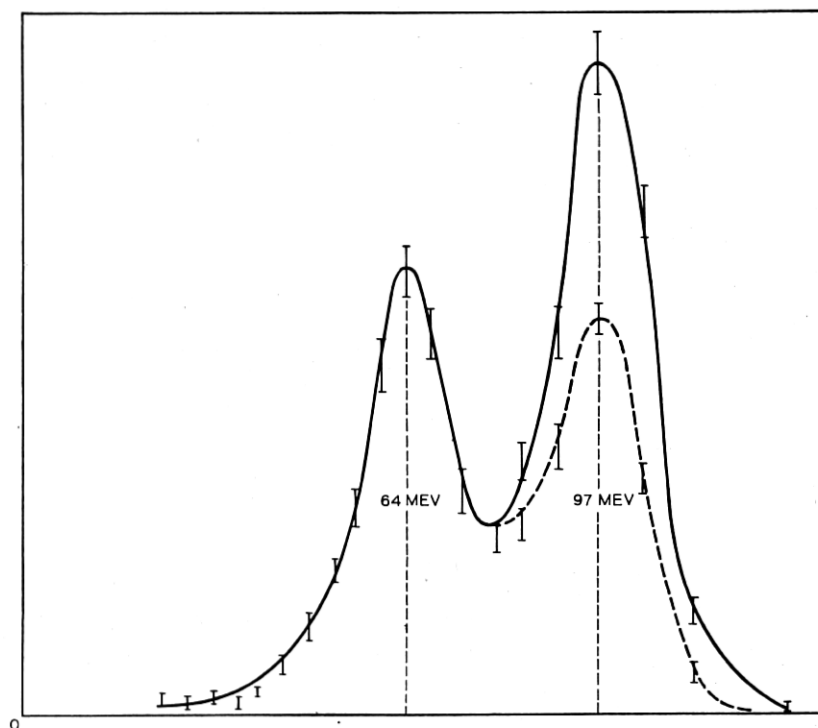


Fig. 2—Distribution-in-energy of fission-fragments of uranium.
(Kanner and Barschall; *Physical Review*)

in-energy curve with two sharp peaks. If different fissions result in different fragment-pairs, the peaks must be broadened. If three or more particles are formed at a fission, there should be a broad continuous distribution of energies. This third of the possibilities is well excluded by the curve of Fig. 2; it remains to be seen whether the breadths of the humps speak for the second over the first.

⁸ The inferring of charge and mass from energy, range and ionization-density is much practiced in the field of cosmic-ray research, in which, however, the particles usually have charge e and masses between the proton-mass and the electron-mass.

In giving the data for Fig. 2, as my words have implied, only a single fragment from each fission escaped into the gas of the ionization-chamber; this was arranged by laying down a very thin film of uranium upon a thick sheet of another metal. Figure 3 was obtained by laying down the uranium film upon a foil so very thin, that from most of the fissions both of the fragments entered into the gas. The great peak of Fig. 3 therefore indicates the sum of the energies of the fission-fragments, the peaks of Fig. 2 the components of that sum. Making al-

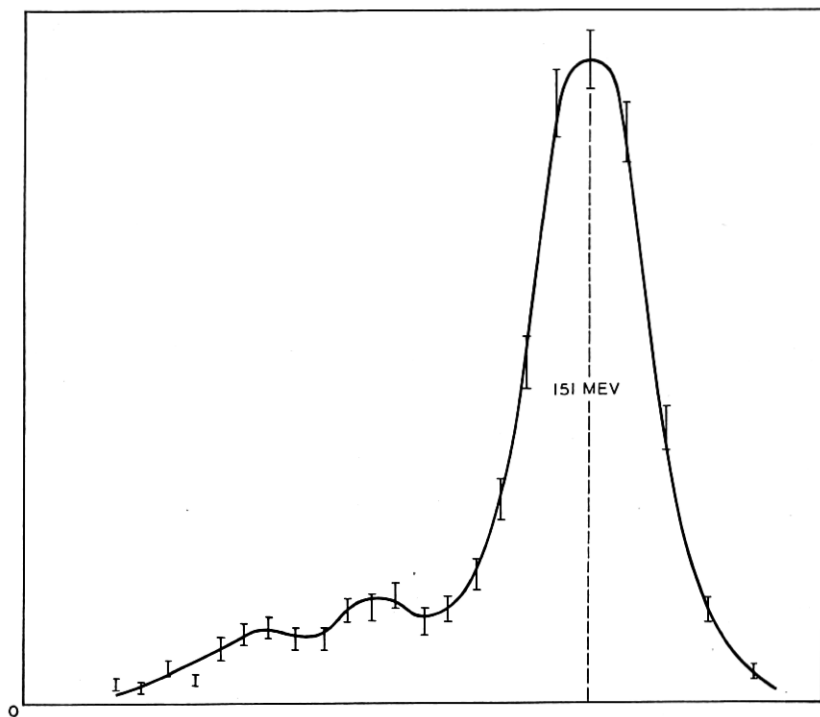


Fig. 3—Distribution-in-energy of pairs of fission-fragments from uranium. (Kanner and Barschall)

lowance for the average energy-loss suffered by the fragments in passing through solid matter before they escape to the gas, Kanner and Barschall decide on 159 Mev. for the sum, 98 and 65 Mev. for the components: the discrepancy between 159 and $(98 + 65)$ lies within the uncertainty of experiment. By the law of conservation of momentum, the ratio of the component energies is the ratio of the fragment-masses; if one of these is about 100, the other is therefore about 150—and the uncertainty implied by "about" is broad enough to permit the hypotheses which we have made and are to make about the nature

of the fragments. The Columbia school has done the same experiment, with like results.

Another way of ascertaining the energy released by fission was adopted by Henderson of Princeton; it is the oldest and most unimpeachable of all the methods of measuring energy, for he determined the rate at which heat was being developed in a uranium target and a container surrounding it while the fissuring was going on. His value was 175 Mev. per fission, with an uncertainty of some ten per cent. As some of this energy belonged not to the fragments but to the electrons emitted after the fission, the agreement is better than passable.

Now we come back to the question of the masses of the initial fragment-pair; and I will develop a second consequence of these masses, entirely different from the first. I revert to the use of mass-numbers, since the corrections needed for converting these into actual masses have not the slightest bearing on the point which is now to occupy us.

If the members of the initial fragment-pair are Ba^{139} and Kr^{100} , then the second of these two is fourteen units heavier than the heaviest stable isotope of $_{36}\text{Kr}$. It is therefore much too massive for its charge. This suggests that it may be able to shed neutrons, and so bring down its weight to the highest value compatible with its charge. But one may also say that Kr^{100} is too feebly charged for its mass. One has to go no fewer than six steps along the periodic table—to $_{42}\text{Mo}$ —to find an element with a stable isotope of mass-number 100. Yet there is nothing to prevent us from assuming that the nucleus $_{36}\text{Kr}^{100}$ may shoot out six negative electrons, and so increase its charge to the minimum value compatible with its mass. The six might come out *seriatim*, in which case there would be a chain of six radioactive substances comprising all the elements from $_{36}\text{Kr}$ to $_{41}\text{Nb}$. Again, the nucleus might conceivably eject any number of neutrons under fourteen and some number of negative electrons under six, arriving at a sort of compromise pair of values of mass and charge compatible one with the other. One guesses already a mighty number of possible radioactive bodies resulting from the fission!

But now let us discard the assumption that Ba^{139} and Kr^{100} are the actual fragments of the fission, replacing these with any two nuclei which (a) lie in the middle region of the Periodic Table and (b) have atomic numbers adding up to 92 and mass-numbers adding up to 239. What, then, will happen to our two inferences from the masses? Essentially, *nothing*. Whichever such pair we take, one at least of its members must be too heavy for its charge and too feebly charged for its mass. (With most conceivable pairs, this will be true of both the members!) This derives from one of the fundamental facts of nuclear

physics: the fact that when mass-number is plotted against atomic number, the points representing the stable nuclei cluster about a concave-upward curve. Moreover, whichever pair of fragments we assume, there will be a superfluity of rest-mass which will manifest itself in a high kinetic energy of the two fragments. This derives from another fundamental fact: when the percentage of excess of mass-number over true mass is plotted against the mass-number (or for that matter the atomic number) the points representing the nuclei cluster along an upward-trending curve.

And so, the kinetic energy of the fragments and the facts of the emerging negative electrons tell us neither which is the initial fragment-pair, nor even whether the initial pair is in all cases the same! Can these questions be answered out of the study of radioactive substances? Some of these we can indeed exclude by observing that they grow out of others; but as to these others, we shall never be able to exclude the possibility that they grow out of still others so short-lived, as to be quite unidentifiable. The half-period of a radioactive substance must be appreciable, if the substance is to be detected and its chemical character recognized; and "appreciable" thus far has signified, among the products of fission, "several seconds or more." It is true that certain *tours de force*, whereby much shorter half-periods have been measured among the natural radioactive bodies, have not yet been applied to the fission-products (so far as publications tell); they might prove workable.¹⁰

Thus it may be necessary for the nonce to lay aside the problem of deciding which is the true initial fragment-pair (or pairs) and be contented with identifying as many as possible among the radioactive substances and tracing their interrelations. Of these—hereafter to be called "the fission-products"—there is indeed a multitude. Among them, chemical elements have been recognized as follows: $_{34}\text{Se}$, $_{35}\text{Br}$, $_{36}\text{Kr}$, $_{37}\text{Rb}$, $_{38}\text{Sr}$, $_{39}\text{Y}$, $_{40}\text{Zr}$, $_{41}\text{Nb}$, $_{42}\text{Mo}$, $_{52}\text{Te}$, $_{53}\text{I}$, $_{54}\text{Xe}$, $_{55}\text{Cs}$, $_{56}\text{Ba}$, and $_{57}\text{La}$. Yet by counting these one does not count all of the distinguishable products; experimenters say that they can tell apart three isotopes of barium, three of strontium, four of iodine and no fewer than seven of tellurium! Some of these agree in their half-periods with radioactive isotopes of those same elements already formed by the older ways of transformation, and frequently we can thus identify their mass-numbers with a fair degree of certainty. (Ba^{139} , which I introduced into the hypothetical reaction of page 272, is such a one; but we shall see

¹⁰ I refer particularly to the use of a rapidly-turning wheel to carry a target swiftly from a place where it is under bombardment (or receiving a deposit of radioactive nuclei) to another place where it is opposite a detector; and the measurement of the radioactivity of a beam of fast-flying nuclei at various points along the beam (the method applied by Jacobsen to RaC').

that certainly it is not always and possibly it is never an initial fragment.) Others were unknown till 1939.

So numerous are these and the other fission-products still unrecognized, that the "decay-curve" for a piece of bombarded uranium or for the deposit on a nearby collector, due to all of them conjointly, looks like the resultant of contributions practically limitless in number and with a random distribution of half-periods. Not only is this also true when neutrons impinge on thorium, but the curves for the two elements cannot be told apart! Only after chemical separations have been made can individual half-periods be sorted out from among the welter; and if there are some characteristic differences between the results of the fission of uranium and those of the fission of thorium, they have not yet been proved.

Special interest attaches to the fission-products which are gaseous. They can be separated physically from the rest: the fission-products are received or dissolved into water (or indeed the uranium may be exposed to neutrons while in aqueous solution) and through the water a stream of air is bubbled, which takes along these particular ones to distant points in the system of tubing where they and their descendants can be studied. They cannot themselves be identified, but among their descendants are found (radioactive) isotopes of $_{37}\text{Rb}$ and $_{55}\text{Cs}$; therefore the gases comprise unstable isotopes of krypton ($_{36}\text{Kr}$) and xenon ($_{54}\text{Xe}$). Could these be initial fragments of various types of fission? If so, their mates are $_{56}\text{Ba}$ and $_{38}\text{Sr}$. Now, barium and strontium are found indeed among the fission-products, which seems to sustain this idea. But barium and strontium may also be the immediate descendants of the caesium and the rubidium aforesaid. This alternative idea is testable; and according to Hahn and Strassmann, two among the three barium isotopes (Ba^{139} being one of the two) are surely descendants of caesium, while the third may be an initial fragment.

Many other such "genetic" relationships have been published, but it would be lengthy and might be premature to quote them. I will mention at least that several sequences have been traced by Abelson in greater or less detail among the many fission-products which are isotopes of the three consecutive elements $_{51}\text{Sb}$, $_{52}\text{Te}$, and $_{53}\text{I}$. A special interest attaches to one of these bodies, the "77-hour tellurium"; for it has been identified as tellurium not only by its chemical properties but also by its X-rays. Let us pause to consider this.

The ordinary way of evoking an X-ray spectrum is to use the element in question as the target, or a constituent of the target, of an X-ray tube. This means that the atoms are excited by projecting electrons

against them. They may also be excited by projecting photons against them, and this is sometimes done. Both of these ways are completely out of the question as yet with any artificial radioactive substance, for the greatest amount yet produced of any of these is so small that if its atoms were placed in a target, the hits made upon them by electrons or photons projected in streams of any feasible strength would not be numerous enough to produce detectable X-rays. If, however, the necessary photons proceed from the nuclei of the atoms themselves, then the whole situation is changed, because now the efficiency of excitation is so great. Such is the case with many of the natural radioactive substances, and now also (it appears) with the "77-hour tellurium." Excited presumably by photons proceeding from their nuclei,¹¹ the atoms emit X-rays, and these have been found (by Abelson in Berkeley, by Feather and Bretschér in England) to be the characteristic rays of the K-series of iodine. "Iodine" here is *not* a misprint for tellurium! When the nucleus is radioactive by virtue of the emission of an electron, the photon (if any) leaves after the electron is gone, by which time the atom is already an atom of the daughter-substance.¹²

Now we take up the yield of the fission-process: how does it depend on the energy of the incident neutrons?

Here uranium sets itself apart from the two other fissurable elements. Thorium and protactinium respond to fast neutrons only, uranium both to slow and to fast (but not to intermediate) neutrons. It is, however, believed that with uranium, one isotope is sensitive only to fast and another both to fast and to slow (or possibly only to slow). There are good theoretical grounds for this belief, and also for choosing the respective isotopes; but as yet there is not the certainty to be expected from some future and probably imminent experiment on separated isotopes.¹³ Accepting nevertheless the current belief, we sup-

¹¹ Another mode of excitation is now known: an electron may fall into a nucleus, and by quitting its place in the orbital electron-family create the condition for the emission of an X-ray photon. Whether this or the other or both be the mode of excitation of the 77-hour tellurium is not yet certainly known.

¹² As this is likely to cause confusion, I emphasize that when the chemical separation is made, the atoms which have not yet emitted nuclear electrons are still tellurium atoms, and when they manifest themselves by that emission it appears among the tellurium; from then on they are iodine atoms among the tellurium, but no longer manifest themselves except through these X-rays. One may wonder whether the "transuranic elements," to which the 77-hour body was formerly thought to belong, would have been discredited if this measurement on the X-rays had been made earlier. Well, the measurement *was* earlier made (though not so precisely) and the rays were interpreted as characteristic X-rays of the L-series of a transuranic element. It is hard to make a guess as to whether further and better measurements would have destroyed this possibility.

¹³ As these pages start for the press I am authorized to say that the separation has been achieved by Nier and the experiment performed by Dunning, Booth and von Grosse. The "light fraction," consisting of U^{235} with a small proportion of the very rare isotope U^{234} is definitely sensitive to slow neutrons; U^{238} is definitely *not*.

pose that of four known types of nuclei three (Th^{232} , Pa^{231} , U^{238}) are fissurable by fast neutrons only, one (U^{235}) by slow neutrons and probably also by fast. The mass-numbers just given are those of the nuclei awaiting the invading neutrons. If one prefers (as many do) to think of the transient composite nuclei formed by the neutron-invasions, one must write Th^{233} , Pa^{232} , U^{239} and U^{236} .

To speak of "fast" neutrons is vague, but not much vaguer than the state of knowledge, which as yet is rudimentary. Fission has been detected of thorium at neutron-energy of about 2 Mev, of protactinium at about one Mev, of uranium at about 0.5 Mev. It thus appears that the "threshold," or least contribution of energy demanded for fission, declines as the end of the Periodic Table is approached; and this seems natural. (Remember always that even with the fastest neutrons ever used, the contribution is very small compared with the energy released.) Values given in the literature for the "cross-section for fission by fast neutrons" include: $0.5 \cdot 10^{-24} \text{cm}^2$ and $0.1 \cdot 10^{-24}$ for uranium and for thorium bombarded by 2.4-Mev neutrons (Princeton) and $0.1 \cdot 10^{-24}$ for uranium bombarded by the "RnBe" neutrons.¹⁴

If between the source of neutrons and a target of uranium a screen of paraffin or water is inserted, the fissions become more abundant; but if now between the paraffin or water and the uranium a shield of cadmium is placed, the fissions become very rare. Now, paraffin and water convert fast neutrons into slow or "thermal" ones,¹⁵ and cadmium is a very efficient absorbent for slow neutrons. We recognize, therefore, a specific effect of slow neutrons, peculiar to uranium. "Slow" or "thermal" signify in this usage: having kinetic energies of the very modest magnitudes possessed by molecules of air (or anything else) at ordinary temperatures: fractions of one electron-volt, and rarely more. Clearly then it is not the energy of motion of the neutron which is the insignificant spark setting off the mighty explosion; it is the mere presence of the neutron within the nuclear system.

A beam of slow neutrons falling upon a thin uranium layer produces many more fissions than does a fast-neutron beam of identical strength. Twenty-to-one was the ratio of yields found by the Columbia school, in the same experiment as gave them the value $0.1 \cdot 10^{-24}$ for the cross-section for fission by the fast "RnBe" neutrons. If, however, we put $2 \cdot 10^{-24}$ for the cross-section appropriate to the thermal neutrons, we

¹⁴ Cross-section for fission, σ_f , is so defined that if N neutrons strike a thin layer comprising M nuclei per unit area, $MN\sigma_f$ fissions occur.—The "RnBe" neutrons, *viz.*, those released when α -particles from radon and its descendants impinge on beryllium, have a very broad energy-range extending at least from 14 Mev indefinitely downward (cf. Dunning, *Phys. Rev.* **45**, 586; 1934).

¹⁵ The neutrons lose their great kinetic energies in repeated elastic impacts with hydrogen nuclei.

are in effect assuming that all the nuclei in the layer are equally liable to being fissured by these. To remain faithful to the well-grounded assumption that only the nuclei U^{235} are liable thus, we must multiply by 140, since only one nucleus in one hundred and forty is of this isotope.¹⁶ The resulting value is large-sized for the nuclear scale, though not unprecedented: there are elements which absorb thermal neutrons so voraciously (without however suffering fission) that the cross-section for absorption is found to be hundreds of times more extensive.

Now in conclusion we turn to the particles other than nuclei, which go forth into space when or after the fission occurs. These comprise photons, electrons, and newborn or "secondary" neutrons; and the last are by far the most sensational.

Of the electrons, almost all has been said that should find place in this account. I recall that by virtue of the second argument from the masses (page 279) the nuclei of the fission-products should go from instability over to stability by emitting electrons which are negative. Observation shows that the emitted electrons are negative indeed (and yet there must be many among the products for which the sign has not been ascertained). Unstable nuclei emitting positive electrons are not at all unknown; indeed they are formed in many transmutations; their absence from among the fission-products is therefore significant. Many of the electrons coming forth are of "secondary origin," i.e. released by photons from the electron-families of the atoms. When classified with the many radioactive bodies formed by other modes of transmutation, some of the fission-products are found to be identical with some of those others, and the rest are in no wise peculiar.

Of the photons, some are X-ray photons engendered as I have recently described (page 282). Others are of the gamma-ray type, i.e., they spring from unstable nuclei among the fission-products. Their existence not being in the least surprising, they have in the main been left for future study.

Coming now to the secondary neutrons, I will begin by dividing them into the "delayed" and the "instantaneous." The former come forth and are detected during an appreciable time—a few seconds up to a few minutes—after the fissions cease. Here then are radioactive bodies, of which the radioactivity consists in the emission of neutrons! Nothing of the sort had ever been known, and the discovery (made at the Carnegie Institution of Washington) created a sensation. In number they are much fewer than the "instantaneous" neutrons, define

¹⁶ The figure is from Nier, *Phys. Rev.* 55, 150 (1939), who gives 139 ± 1 as the abundance-ratio of U^{238} and U^{235} .

(for the present) as those which come out within a few thousandths of a second of the moment of fission. The ratio, according to the Columbia school, is about one to sixty. Delayed electrons and delayed photons have also been observed. Most observations on secondary neutrons are made while the target is being bombarded, and therefore relate to a mixture of the instantaneous with a small proportion of the delayed.

In energy the secondary neutrons differ greatly from the primary, a remarkable contrast! This is shown by several neat and pretty experiments, in which the secondaries manifest themselves by acting on detectors which cannot perceive the primaries at all. Thus if the primaries are thermal neutrons, an expansion-chamber or an ionization-chamber full of gas can be set among them without showing any sign of them,¹⁷ since they cannot strike hard blows against the molecules therein. Let, however, a piece of uranium be set nearby, and the chamber will show dense trains of ions, produced by nuclei struck very hard and driven out of the molecules by neutrons which are fast. In this manner Halban and Joliot and Kowarski in Paris detected secondaries running in energy up to 11 Mev and beyond, while Zinn and Szilard of Columbia mapped the energy-spectrum up to 3.5 Mev. But also there are detectors able to discriminate between fast and faster neutrons: e.g. phosphorus, which becomes radioactive when bombarded by neutrons if, but only if, these have energy greater than 2 Mev. Dode and others in Paris prepared a source producing neutrons of energy one Mev; placed it next to a uranium target; surrounded source and target with a tank of liquid carbon disulphide, in which phosphorus was dissolved; and the liquid grew radioactive.

But how many neutrons are released per fission? This is a question of singular and perhaps of devastating importance, as will presently appear.

The obvious way to answer it seems to be that elected by Zinn and Szilard, who measured the number of fissions and also estimated, from the number of recoiling nuclei observed in their expansion-chamber, that of the secondary neutrons. Most of the trials have been made by a different method, in which all of the secondary neutrons are reduced to thermal energies before they are detected. (Incidentally there is the advantage, that if neutrons are released with low initial energies they will be counted by this way but not by the other.)

In this more customary method, the neutron-source and target are close together in the midst of a great tank of water, as large as can

¹⁷ Except that if nitrogen is contained in the chamber, the thermal neutrons will react with the nitrogen nuclei so as to release protons (Zinn and Szilard).

conveniently be made. Paraffin may surround¹⁸ the target and the source, to slow down the primary neutrons; or the water itself may perform this office. Again, the target may be diffused throughout the entire water-mass, in the form of a soluble salt of uranium. The detector is a substance becoming radioactive when exposed to slow neutrons. It may itself be spread throughout the water in the form of a soluble salt, or it may be in the form of a thin foil which can be moved from place to place in the water. In the former case, the water is thoroughly stirred after the exposure is over, and then a sample is taken, the activity of which is a measure of the average density—and therefore of the total quantity—of thermal neutrons in the entire tank during the exposure. In the latter case, the foil is used for mapping out the density of thermal neutrons in the water as function of the distance r from the target in the middle, and what is usually plotted is the " Ir^2 curve," I standing for the strength of the activity of the foil.

The total quantity of thermal neutrons, existing at any moment dispersed throughout the water, is greater in the presence of the uranium than in the absence thereof¹⁹ (Anderson, Fermi and Hanstein); this is the simplest proof of the fundamental result. When the Ir^2 curves are compared, it is found that the presence of the uranium lowers the curve in the close neighborhood (within 13 or 14 cm) of the neutron-source, but raises it further out. Presumably this is because the uranium swallows up the slow primary neutrons, and those which it gives out in exchange are themselves not slow until they have gone a long way onward in the water. In tanks of sufficient size, the increase farther out more than balances the diminution nearer in, and the total quantity of thermal neutrons is augmented by the presence of uranium (Halban, Joliot and Kowarski); this agrees with the other result. It is therefore established what when the primaries are slow, the fission-process delivers more neutrons than it consumes. The same holds true when the primaries are fast, for when a beam of RnBe neutrons is sent through a plate of uranium oxide the detector beyond reveals a greater quantity of rapid neutrons than when the plate is absent (Haenny and Rosenberg, experimenting with a plate 8 cm thick).

How many neutrons then emerge, for every one which is spent in producing a fission? This is a remarkably difficult question to put to

¹⁸ It is not necessary that the "slowing-down" substance be actually between the target and the source, since slowed-down neutrons come out of it in all directions.

¹⁹ To make the situations strictly comparable, the uranium is replaced in the control experiment by some substance possessing an equal absorbing-power for neutrons, but not liable to fission.

the test of experiment. About all that the several answers have in common is, that *more neutrons emerge than are spent*. Zinn and Szilard say, two or three times as many; Anderson, Fermi and Szilard say, between one and two; the Paris school, three or four. A yet higher value (eight) published from Paris seems to comprise some "tertiary" neutrons produced by the secondaries.

But if every fission produces a fresh neutron to replace the one which caused it, and then some extras in addition, must we not anticipate a self-sustaining, nay even a self-amplifying effect? Must we not fear, in fact, a cataclysmic explosion?

Were anything of the sort to happen, we may take it for granted that the world would know of it, though in all probability the experimenter would not himself survive to report it. Evidently then it has not happened, and there must be a brake or brakes in Nature which impede the slide toward the catastrophe, and have thus far averted it. In other words, there must be ways in which neutrons are made harmless by some innocuous type of capture, before they ever produce a fission.

Some of these other ways are known already. If the uranium is mixed with other elements—as, in Nature, it invariably is—the nuclei of these can take up some of the neutrons. Whether the composite nuclei so formed are stable or radioactive is in this connection not important; they give no neutrons out in exchange for the ones absorbed, and so the chain is broken. But if all other elements are carefully extracted, do any brakes remain?

Two surely do, and one is the fact that the newborn neutrons are rapid, and cannot be efficacious as agents of fission until they are slowed down to thermal energies. In pure uranium the slowing-down can only be extremely gradual, so unfavorable is the huge mass-ratio—238 to 1—for the energy-transfer in the elastic impacts. Yet if the volume of purified metal were great enough, this brake would relax. Thus the durability of small-size pieces of uranium made chemically pure, well attested as it is, is not by itself a proof that much larger pieces would be safe. Those who are trying to approach the catastrophe, while hoping not to provoke it, are engaged in piling up uranium in greater and greater masses.

The other brake is supplied by the "reaction of pure neutron-capture," which I mentioned on an early page (p. 270). Every now and then, when a neutron enters a nucleus of uranium, the composite nucleus finds itself able to live on without fissure. It survives for a time, then emits a negative electron of energy trivial compared with

fission-energies, and relapses into permanent stability. This is especially likely to happen when the neutron-energy is about 10 ev. Suppose then a volume of pure uranium so great, that the rapid neutrons released within it can make collisions numerous enough to bring their energies down to the thermal range where they are dangerous. Before they reach this range they must pass successfully through that other where they are liable to be disarmed—or put away in prison, rather. This second brake does not diminish in its strength as the volume of uranium is raised.

Perhaps the second by itself is powerful enough to avert the explosion. In this case there is no danger of incurring the cataclysm by piling up uranium, however pure. There remains however the chance of separating the two isotopes 235 and 238, verifying that the fission by thermal neutrons occurs only in the one and the reaction of pure neutron-capture only in the other, and then accumulating the dangerous one by itself. Enough has just now been separated, as I said in an earlier footnote, for the verification to begin. To separate enough for the dangerous trial will take a good deal longer in the doing. After it is done, there is yet another brake which may avert catastrophe. When the cumulative processes begin, the heating of the metal may and probably will so affect the energies of the neutrons, that their efficiency for fissuring the nuclei will be greatly abated and so the processes find a natural limit. Otherwise it is to be hoped that those who build up great masses of sensitive uranium will recognize preliminary signs that the danger-point is close, before they actually attain it.

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