The Physical Basis of Ferromagnetism

By R. M. BOZORTH

After an introductory review of the general nature of the theory of magnetic phenomena and the magnitudes of the atomic forces involved, there is a discussion of Ewing's theory, its results and limitations. The later theory of Weiss is then given briefly in order to fix the concept of the molecular field. In order to elucidate the nature of this field a digression is made to discuss the atomic structure of the ferromagnetic elements and elements having similar structures. With this as a basis the physical nature of the molecular field is discussed at some length. Its relation to the structure of domains, particularly the nature of the boundaries between domains, is brought out.

Finally there is a review of the gyromagnetic effect, its significance for magnetic theory, the principal experimental method for its determination, and the numerical results supporting the idea that the spin of the electron and not its orbital moment is responsible for ferromagnetism.

INTRODUCTION

IN THE last five or ten years the theory of ferromagnetism has shown indications of maturity. For the first time a plausible story can be told concerning the ultimate magnetic particle, the essential nature of the atom of a ferromagnetic substance, the kind of forces which determine the properties of magnetic crystals, the effect of strain on magnetic materials and the manner in which these various phenomena combine to determine the properties of commercial materials. It is true that the story is largely qualitative, and that there are still many points that are uncertain or missing entirely, but nevertheless it is possible to describe the major features with some confidence.

The fundamental magnetic particle is the spinning electron. One might think that the orbital motions of the electrons in the atom would also contribute to ferromagnetism, owing to their magnetic

moments, but it has now been established that when the magnetization is altered all that changes is the direction or "sense" of the spin of certain of the electrons in the atoms—the orbital motions remain practically unchanged.

The electrons that are responsible for the magnetic properties of iron, cobalt, nickel and their alloys lie in a definite "shell" in the atom. As shown 'in Fig. 1, there are four shells or regions, more or

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+ SPINS__l_____1, 3_____l, 3, 5_____l
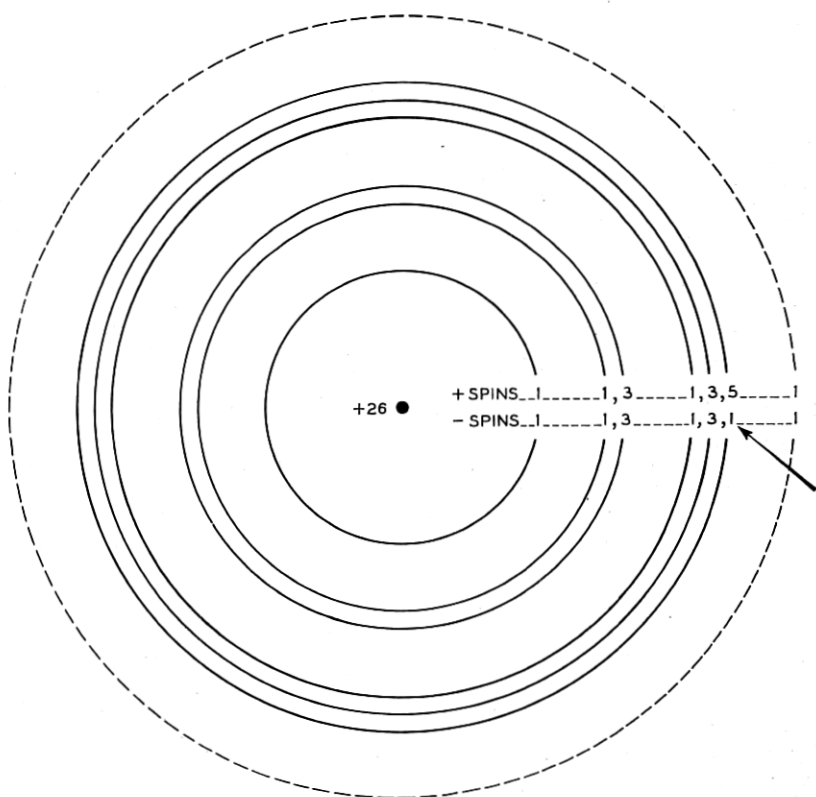− SPINS__l_____l, 3_____l, 3, l_____l

Fig. 1—Electron shells in an atom of iron. The arrow indicates the incomplete sub-shell that is responsible for ferromagnetism. The numbers specify how many electrons with each spin are in the corresponding sub-shells.

less well defined, into which all the electrons circulating about the nuclei of these atoms may be divided when the atom is separated from its neighboring atoms, as it is, for example, in a gas. Some of these shells are subdivided as shown. When the atoms come closer together as they do in a solid, the fourth or outermost shell of each becomes disrupted, and the two electrons which comprised it wander from atom to atom and are the "free" electrons responsible for

electrical conduction. The electrons in the outer part of the third shell are those responsible for the distinctive kind of magnetism found in iron, cobalt and nickel. Some of these electrons spin in one direction and some in the opposite, as indicated, so that their magnetic moments neutralize each other partially but not wholly, and the excess of those spinning in one direction over those spinning in the other causes each atom as a whole to behave as a small permanent magnet.

The well-established kinetic theory of matter tells us that if each atom were to act independently of its neighbors, the atoms would be vibrating and rotating so energetically that they could not be aligned even with the strongest field that can be produced in the laboratory. To explain the kind of magnetic properties found in iron, therefore, it is necessary that there be some internal force capable of making the magnetic moment of a group of neighboring atoms lie parallel to each other—the small atomic "permanent magnets" of each group must point in the same direction so as to provide a magnetic moment great enough to permit a realignment when subjected to external fields. Recently it has been shown by independent means that there is such a force in just those elements which are ferromagnetic, and it is from this force that the difference between magnetic and non-magnetic materials arises. The force is electrostatic in nature and is called "exchange interaction" by the atomic-structure experts, the wave mechanicians, who have shown its existence and calculated its order of magnitude. This force maintains small groups of atomic magnets parallel against the forces of thermal agitation. (When the material is heated so hot that the disordering action of the agitation becomes strong enough to overpower the forces of "exchange interaction" the material loses its ferromagnetism; in iron this happens at 770° C.)

But why then is not every piece of iron a complete permanent magnet? For some reason not understood at present, at ordinary ·temperatures the electrostatic forces of exchange interaction maintain the elementary magnets parallel only over a limited volume of the specimen. This volume is usually of the order of 10^{-8} or 10^{-9} cubic centimeters and contains a million billion atoms and is of course invisible. Such a volume is said to be saturated because the atomic magnets are all pointing in the same direction, and has been given the name "domain." Thus a magnetic material at room temperature, before it has been magnetized by subjecting it to the influence of a magnetic field, is divided into a great many domains each of which is magnetized to saturation in some direction generally different from that of its neighbors. The net or vector sum of the magnetizations is zero, and externally the material appears to be unmagnetized but in

reality the magnetization at any one point is very intense. When a magnetic field is applied by bringing near the metal a permanent magnet or a coil of wire carrying a current, the magnetization of the material as a whole is increased to a definite value. We believe that what then takes place is simply a change in the direction of the mag-netizations of the domains. If we represent the magnetization of any domain by a vector, the effect of the externally applied field will be represented by the rotation of these vectors—rotations not accom-panied by any changes of length.

DIRECTIONS
OF EASY
MAGNETIZATION

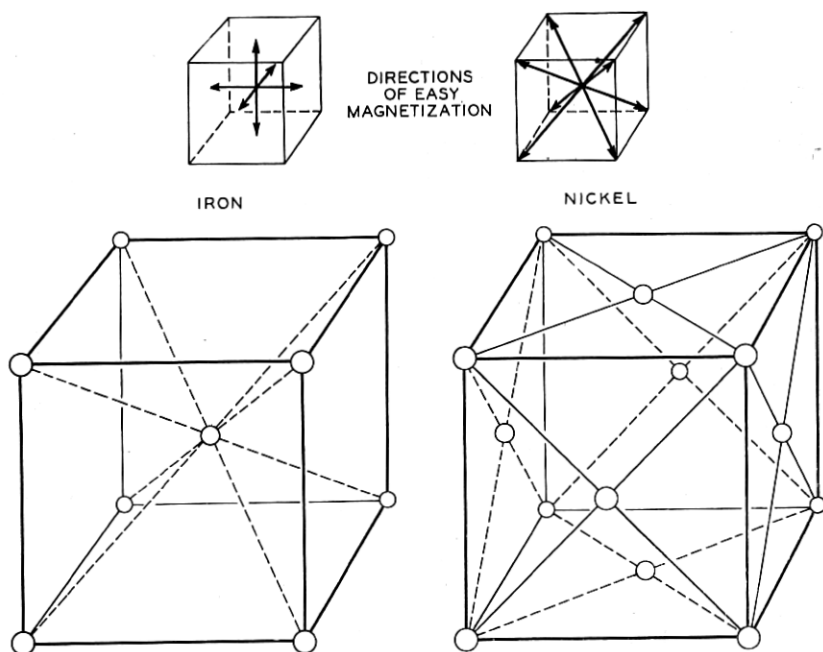IRON   NICKEL

Fig. 2—The positions of the atoms and the directions of easy magnetization
in crystals of iron and of nickel.

Recently much has been learned about the magnetic properties of materials by a study of single crystals. Ordinary metals are composed of a great many crystals often too small to be seen easily by the naked eye. But in the last few years methods have been found for making large crystals of almost all the common metals, crystals as large as the more familiar ones of rock candy and even of quartz. Experiments on such crystals of iron show that they are much more easily magnetized in some directions than in others.

This dependence of ease of magnetization on direction is illustrated in Fig. 2 for iron and nickel in relation to the positions of the atoms in

the crystals. The circles represent the positions which centers of atoms take up on an imaginary framework or lattice. Because of the smallness of atomic dimensions only a small fraction of the atoms in a crystal of ordinary size are shown, but the same pattern, the unit of which is outlined by solid lines, extends throughout the whole of the single crystal. The arrows indicate the directions of "easiest" magnetization, which are different for the two materials as may be noticed.

In order to give a notion of the absolute and relative sizes of crystals and domains and atoms with which magnetic processes are concerned, it may be pointed out that a piece of ordinary iron a cubic centimeter in volume may contain about 10,000 single crystals, and that each crystal contains on the average 100,000 domains each with from 10^{14} to 10^{15} atoms.

Although this article is not concerned primarily with the details of the changes in magnetization that occur when a magnetic field is applied, a brief description of such changes is desirable. In a crystal of iron the directions of easy magnetization are parallel to the cubic axes, that is, they are the six directions parallel to the edges of the cube which represents the structure. When such a magnetic material is unmagnetized as a whole a portion of one of the crystals in it may be represented by the highly schematic Fig. 3(a). As shown, each of the domains, represented by the arrows, circles and crosses, is magnetized in one of the directions of easy magnetization, equal numbers in each of the six directions. When a weak field is applied in the direction indicated and its strength gradually increased to a high value, the magnetizations of the domains change suddenly and their directions approach coincidence with that of the magnetic field. This is usually accomplished by the displacements of domain boundaries, these moving so that some domains grow at the expense of others in which the magnetization lies in a direction further from that of the field. When the field has been increased to such a strength that practically all the domains are oriented as shown in (b) and the crystal is really just one large domain, a second process commences: the magnetization changes slowly in direction until finally it is parallel to the field, and then changes no more. The material is then said to be saturated, as shown in (c).

Figure 3 is drawn to illustrate the changes in magnetization that occur in a single crystal of iron. Iron as we ordinarily see it is composed of a great many minute single crystals, but the changes in magnetization that occur in each one of these crystals are just those which have been described, the magnetization of the whole polycrystalline material being the sum of the magnetization of the parts.

The most definite evidence of the existence of domains is the Barkhausen effect. To produce and detect it, a piece of magnetic material is wound with wire the ends of which are connected to a vacuum tube amplifier. When the magnetization of the material is changed, as *e.g.* by moving a permanent magnet near it, a rustling sound or a series of clicks may be heard in phones or in a loud speaker

(a)—DEMAGNETIZED

(b)—SUDDEN REVERSALS COMPLETE (KNEE OF MAGNETIZATION CURVE)

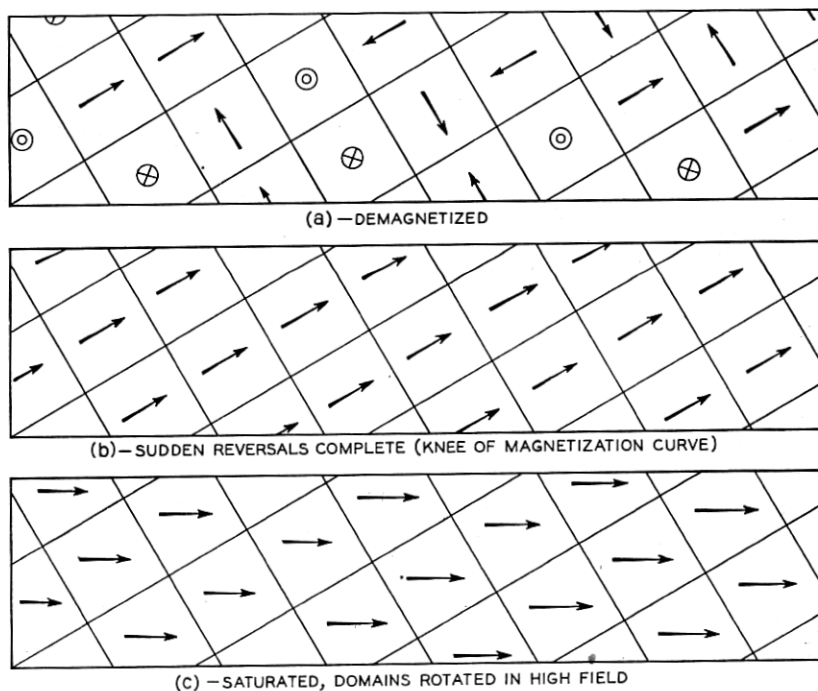(c)—SATURATED, DOMAINS ROTATED IN HIGH FIELD

CRYSTAL AXES

MAGNETIC FIELD

Fig. 3—Domains in a single crystal of iron. As the magnetic field increases in strength the magnetic moments first change suddenly (*a* to *b*) by displacement of the boundaries between them, then rotate smoothly (*b* to *c*).

connected to the output end of the amplifier. Every such click is ascribed to the sudden change in direction of magnetization in a single domain, and from measurements of the sizes of the clicks we get our best estimate of the sizes of the domains. Even more direct evidence of the existence of domains and the changes that they undergo has been obtained recently by spreading colloidal iron oxide over the surface of a magnetic material and looking at it under a microscope.

The regular pattern observed [1] is similar in nature to the familiar one obtained when iron filings are sprinkled near a permanent magnet; the fine colloidal particles are necessary in this case because the whole scale is small. This micro-pattern changes when the applied field changes, and the difference is attributed to the redistribution or reorientation of groups of domains. These patterns are obtained only on magnetic materials and are found on them even when the material is unmagnetized; such a one is shown in Fig. 4.
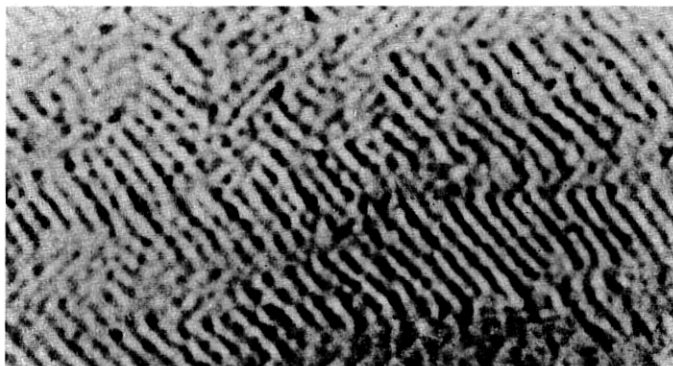

Fig. 4—The powder pattern produced by colloidal iron oxide on the surface of a demagnetized silicon-iron crystal, showing the presence of inhomogeneous magnetic fields. Magnification about 1000.

MAGNITUDES OF MAGNETIC FORCES

Ferromagnetic theory has been made difficult by the fact that the magnetic forces between the electrons in an atom are small compared to the electrostatic forces. The latter force between two electrons of charge e (in e.s.u.), a distance a apart, is equal to

$$e^2/a^2.$$

The magnetic force between the same electrons depends on the speed of the charges as well as on their magnitudes, and, when the direction of motion is perpendicular to the line joining them, is equal to

$$\frac{e^2}{a^2} \cdot \frac{v^2}{c^2},$$

where v/c is the ratio of the speed of each electron to the speed of light. Since v/c is usually of the order of 0.01, these magnetic forces

[1] L. W. McKeehan and W. C. Elmore, *Phys. Rev.*, *46*, 226–228 (1934). See also the earlier experiments by F. Bitter, *Phys. Rev.*, *41*, 507–515 (1932). See also the account by Elmore in F. Bitter's Introduction to Ferromagnetism, McGraw-Hill, New York, 55–66 (1937).

are about 10^{-4} of the electrostatic forces. The difference is even greater when electrostatic forces between electrons and nuclei, or between nuclei, are compared with magnetic forces. The magnitudes of these forces for a specific hypothetical arrangement are shown in Fig. 5.

ELECTROSTATIC FORCES IN DYNES
2 x 10⁻³
2 x 10⁻¹
ELECTRON −e
NUCLEUS +26e
ELECTRON −e

MAGNETIC FORCES IN DYNES
ORBITAL MOTIONS, 2 x 10⁻⁵
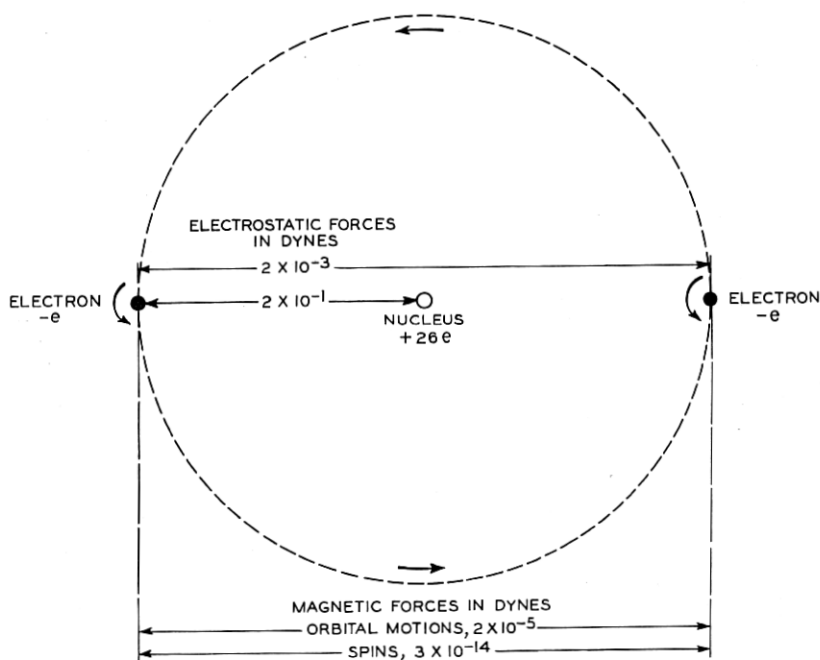SPINS, 3 x 10⁻¹⁴

Fig. 5—The magnitudes of the forces in a hypothetical iron-like atom, showing that electrostatic forces are more powerful than magnetic forces.

Consider the magnitude of magnetic forces from another point of view. The magnetic energy of a permanent magnet of moment μ_A in a field of strength H is

$$E = -\mu_A H,$$

when μ_A and H are parallel. In a magnetic substance we may regard the atomic magnets as being held parallel by a fictitious field H_i. When the material is heated to the Curie temperature, θ, the energy of thermal agitation ($\approx k\theta$) destroys the alignment of the atomic magnets by the fictitious or "internal" field H_i. Then

$$k\theta \approx \mu_A H_i.$$

For iron, $\theta = 1043°$ K. and $\mu_A = 2.04 \times 10^{-20}$ erg/gauss, thus the

energy per atom is

$$k\theta = 1.4 \times 10^{-13} \text{ erg} = 0.09 \text{ electron-volt}$$

and the internal field

$$H_i = 7,000,000 \text{ oersteds}.$$

Although this field is much stronger than any so far produced in the laboratory, the energy involved is small compared to that which controls chemical binding. For example, the energy of ionization of the helium atom is about 25 electron volts. Another way of showing that the magnetic forces are small compared to the electrostatic forces holding atoms together, is to compare the Curie temperature with the temperature of vaporization.

The calculation of magnetic forces by theory is thus extremely difficult, because they are but small additions to the electrostatic forces which themselves cannot usually be calculated with much precision.

Ewing's Theory

Ewing[2] was one of the first to attempt to explain ferromagnetic phenomena in terms of the forces between atoms. His theory will be described briefly here, since many physicists today, when thinking about magnetic phenomena, still go back to Ewing's ideas of fifty years ago. He assumed with Weber that each atom was a permanent magnet free to turn in any direction about its center. The orientations of the various magnets with respect to the field and to each other were supposed to be due entirely to the mutual magnetic forces. The I, H curve and hysteresis loop were calculated for a linear group of such magnets and were determined experimentally using models having as many as 130 magnets arranged at the points of a plane square lattice.

The calculations for a linear chain show that as the field is gradually increased in magnitude from zero there is at first a slow continuous rotation of the magnets, then a sudden change in orientation and finally a further continuous rotation until the magnets lie parallel to the field. The I, H curves calculated for such a group of magnets resemble in general form the actual curves of iron: they show a permeability first increasing then decreasing, and saturation and hysteresis.

A magnetization curve and a hysteresis loop obtained[3] with a model of 130 magnets in square array, are shown in Fig. 6. Experi-

[2] J. A. Ewing summarized in "Magnetic Induction in Iron and Other Metals," The Electrician, London, 3d ed. (1900).
[3] J. A. Ewing and H. G. Klaassen, *Phil. Trans. Roy. Soc.*, *184A*, 985–1039 (1893).

ments with the model showed a variety of other phenomena including rotational hysteresis loss and its reduction to zero in high fields, the effect of strain on magnetization, the existence of hysteresis in the strain *vs.* magnetization diagram, the effect of vibration and the existence of time lag ·and accommodation with repeated cycling of the field.

Ewing's general method may be illustrated by calculating the magnetization curve and hysteresis loop for an infinite line of parallel
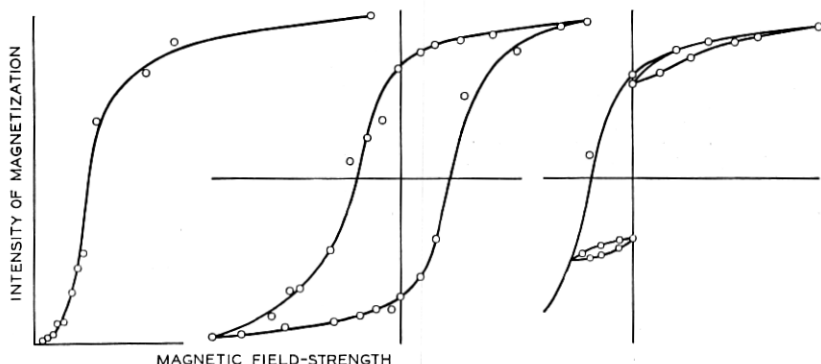

Fig. 6—A magnetization curve and hysteresis loops of a Ewing model of 130 pivoted magnets in square array.

equally spaced magnets (Fig. 7a). It is done most simply by considering first the magnetic potential energy [4] of a magnet of moment μ_A and length l, in the field of a similar magnet:

$$W = -\frac{\mu_A^2}{r^3} P_2(\theta) - \frac{\mu_A^2 l^2}{r^5} P_4(\theta) - \frac{\mu_A^2 l^4}{r^7} P_6(\theta) - \cdots. \qquad (1)$$

Here r is the distance between the centers of the magnets and the $P(\theta)$'s are Legendre functions of the angle, θ, between the direction of the moment of the magnet and the line joining the magnet centers.

$$P_2(\theta) = (1 + 2 \cos 2\theta)/4,$$
$$P_4(\theta) = (9 + 20 \cos 2\theta + 35 \cos 4\theta)/64,$$
$$P_6(\theta) = (50 + 105 \cos 2\theta + 126 \cos 4\theta + 231 \cos 6\theta)/512.$$

The potential energy per magnet, W_1, for an infinite straight row of magnets can easily be obtained by summing W for all pairs.

$$W_1 = -\frac{2\mu_A^2}{r^3}[1.20 P_2(\theta) + 1.04 P_4(\theta)(l/r)^2$$
$$+ 1.01 P_6(\theta)(l/r)^4 + \cdots]. \qquad (2)$$

[4] G. Mahajani, *Phil. Trans. Roy. Soc.*, 228A, 63–114 (1929).

The behavior of the line when subjected to a field H may be found by adding to W_1 the energy term $- H\mu_A \cos(\theta_0 - \theta)$, where θ_0 is the angle between the line of centers and the direction of the field, and

(a)

(b)

$\dfrac{l}{r} = 0.5$

$\theta_0 = 45°$
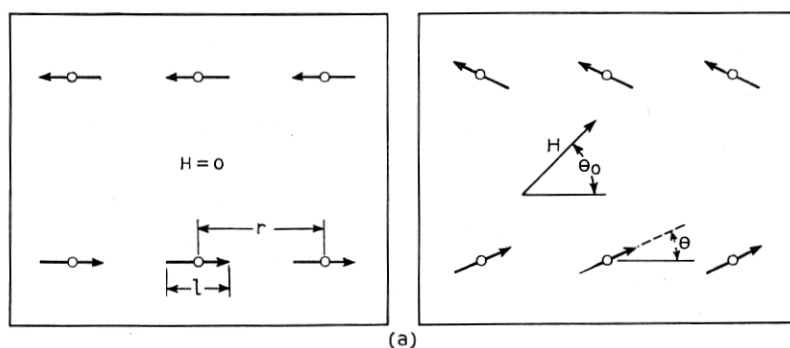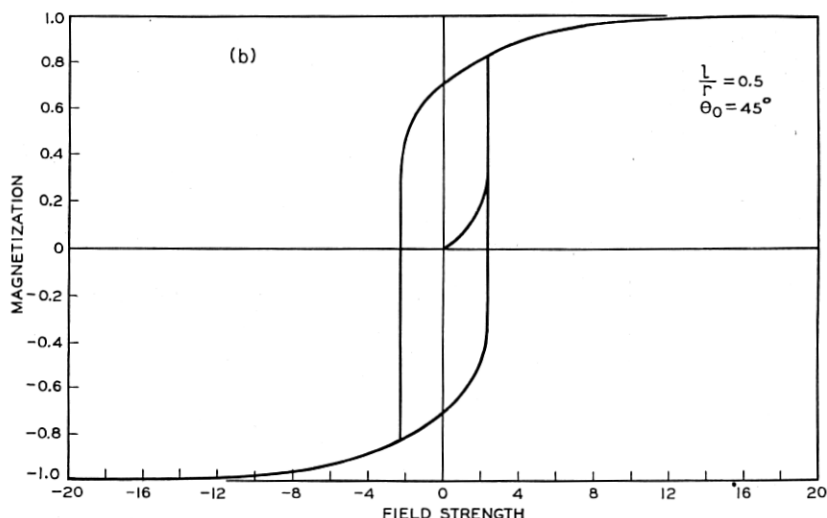
Fig. 7—A magnetization curve and hysteresis loop for an infinite line of equally spaced magnets originally "demagnetized."

finding the value of θ which makes this total energy a minimum for given values of θ_0 and H:

$$\frac{d}{d\theta}\left[W_1 - H\mu_A \cos(\theta_0 - \theta)\right] = 0.$$

This gives

$$H = \frac{(d/d\theta)\,W_1}{\mu_A \sin(\theta_0 - \theta)}.$$

The component of magnetization parallel to H is

$$I = I_s \cos (\theta_0 - \theta),$$

where I_s is the saturation magnetization. By starting with half of the line of magnets pointing in a direction opposite to that of the other half, the initial magnetization is zero and an unmagnetized or demagnetized material is simulated. Thus a magnetization curve and a hysteresis loop of this assemblage are obtained by plotting H against I. Such a plot is shown in Fig. 7(b), with the scale of H determined by the magnitudes of μ_A and r. The curves are obviously similar to those for real materials.

LIMITATIONS OF EWING'S THEORY

So far, this calculation is equivalent to what Ewing did over four decades ago. But now we know the crystal structure of iron and in particular the distances between the atoms. We also know the magnetic moment of each iron atom and know, therefore, the value of μ_A/r^3 which determines the scale of H. Using the appropriate values $\mu_A = 2.0 \times 10^{-20}$ erg/gauss and $r = 2.5 \times 10^{-8}$ cm, the coercive force H_c for $l/r = 0.1$ is found to be 4600 oersteds. This is affected somewhat by the ratio l/r, but in any case H_c is found to be of this order of magnitude unless l/r is very close to unity. This magnitude of H_c is greater by a factor of 10^5 than the lowest value obtained experimentally, 0.01. Similarly the initial permeability, μ_0, according to the model is about unity while observed values for iron range from 250 to 20,000. Adjustment of l/r to higher values decreases μ_0.

This calculation of the magnetization curve and hysteresis loop are based on a very much idealized model, and it is difficult to estimate the error to which it may lead. One factor that has been completely neglected is the fluctuation in energy. A much better approximation would be to calculate the magnetic potential energy of a group of magnets arranged in space in the same way that the iron (or nickel) atoms are arranged in a crystal. This has been done by Mahajani [4] who showed that application of Eq. (1) with $l = 0$ (but summed to account for the effects of all magnets in the structure) leads to the result that the magnetic potential of the space array is independent of θ, in other words one orientation of the dipoles is as stable as any other and the magnetization curve would go to saturation in infinitesimal fields no matter in what direction H might be applied. If l is finite, the stable positions of the magnets are parallel to the body-diagonals of the cube which is the unit of the crystal structure, and

this becomes therefore the direction of easy magnetization, a situation which is correct for nickel but decidedly not so for iron. The best correspondence between the action of the model and of iron itself is obtained if the model is made by placing a small circular current of electricity, instead of a magnet with finite length, at each lattice point of the space array. In the latter case we can explain the direction of easy magnetization in iron and the variation of magnetic energy with direction in the crystal.

In considering Ewing's model it is appropriate to estimate the energy of thermal agitation and to compare it with the magnetic potential energy as calculated from the model. Substituting in Eq. (2) the same values of μ_A and r as were used above, we obtain 10^{-16} erg per atom for the magnetic potential energy in zero field. This is to be compared with the rotational energy of a single molecule at room temperature, 2×10^{-14} erg per atom as given by the kinetic theory. Thus the energy of thermal agitation is 200 times as great as the calculated magnetic energy. Even at liquid air temperatures the thermal agitation would prevent the atomic magnets from forming stable configurations. Without some additional force the model Ewing used would behave as a paramagnetic rather than a ferromagnetic solid.

In a real material, however, it is now well established that there are very powerful forces, not contemplated when Ewing made his model and proposed his theory, which maintain parallel the dipole moments of neighboring atoms. These are the electrostatic forces of exchange (see p. 24) which Heisenberg suggested are powerful enough to align the elementary magnets against the disordering forces of thermal agitation, forces much larger than those of magnetic origin. Theory accounts only for the order of magnitude of these forces. Our best estimate of the corresponding energy of magnetization is obtained by assuming that it is equal to the energy of thermal agitation at the Curie point, $\frac{1}{2}k\theta$. For iron ($\theta = 1043$ °K) this gives 7×10^{-14} erg per atom.

THE WEISS THEORY

In order to understand how atomic forces give rise to ferromagnetism it is desirable to review briefly Weiss's theory [5] of ferromagnetism, which introduces a so-called "molecular field" that presently will be identified with the nature of these forces. This theory is an extension of Langevin's theory of a paramagnetic gas. The original Langevin theory culminated in a formula relating the magnetization, I, to the field-strength, H, and the temperature, T; this is the hyperbolic co-

[5] P. Weiss, *Jour. de physique* (4) *6*, 661–690 (1907). P. Weiss and G. Föex, "Le Magnetisme," Colin, Paris (1926).

tangent law,

$$\frac{I}{I_0} = \operatorname{ctnh}\frac{\mu_A H}{kT} - \frac{kT}{\mu_A H}.$$

In deriving this the assumptions are made that the elementary mag-
nets, each of moment μ_A, are subject to thermal agitation and momen-
tarily may have any orientation with respect to the direction of the
field, and that they are too far apart to influence each other.   Quantum
theory alters the second of those assumptions by stating that in such
an ensemble of elementary magnets (atoms) there will be only a limited
number of possible orientations, in the simplest case only two, one
parallel and the other antiparallel to the direction of the field.   In this
case the equation corresponding to Langevin's is

$$\frac{I}{I_0} = \tanh\frac{\mu_A H}{kT}. \tag{3}$$

These two theoretical relations are plotted for variable H and con-
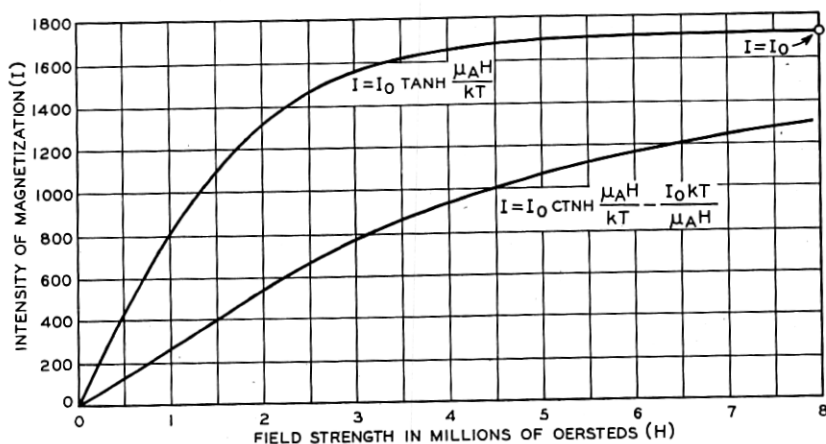stant T (room temperature) in Fig. 8, the constants being those for

Fig. 8—With no helpful mutual action between atoms, enormous fields
would be necessary to saturate a magnetic material.

iron ($I_0 = 1740$, $\mu_A = 2.04 \times 10^{-20}$ erg/gauss).   It is obvious that
with the highest fields so far attained in the laboratory (about 300,000
oersteds) the magnetization would attain only a small fraction of its
final value I_0 if this law were obeyed, and in this range I would be
sensibly proportional to the field-strength:

$$I = \frac{CH}{T},$$

where C is a constant. This relation, known as Curie's Law, is obeyed by some *paramagnetic* though not by ferromagnetic substances. It is usually written with I/H denoted by the symbol χ, representing susceptibility:

$$\chi = \frac{C}{T}.$$

Many more paramagnetic substances obey the similar "Curie-Weiss Law":

$$I = \frac{CH}{T - \theta}. \tag{4}$$

Weiss pointed out the significance of θ in this equation: it means that the material behaves magnetically as if there were an additional field, NI, aiding the true field H. This equivalence is shown mathematically by putting $\theta = NC$ in Eq. (4) with the result

$$I = \frac{C(H + NI)}{T}.$$

The quantity represented by NI is called the "*molecular field*" and that by N the "*molecular field constant.*" It is interpreted by supposing that the elementary magnet does have an influence on its neighbors, contrary to the assumptions of the simple Langevin theory.

The significance of the molecular field for ferromagnetism is now apparent if we replace the H by $H + NI$ in the more general Eq. (3) and examine the resulting equation:

$$\frac{I}{I_0} = \tanh \frac{\mu_A(H + NI)}{kT}. \tag{5}$$

This equation is perhaps the most important in the theory of ferromagnetism. It indicates that even in zero field there is still a magnetization of considerable magnitude, provided the temperature is not too high. Putting $H = 0$ and

$$\theta = \mu_A N I_0/k,$$

Eq. (5) reduces to

$$\frac{I}{I_0} = \tanh \frac{I/I_0}{T/\theta}. \tag{6}$$

This purports to specify the magnetization at zero applied field by a function that is the same for all materials, when the magnetization is expressed as a fraction of its value at absolute zero and the temperature as a fraction of the Curie temperature on the absolute scale. This magnetization *vs.* temperature relation, plotted as the solid line of Fig.

9, means that *at all temperatures below θ the intensity of magnetization has a definite value even when no field is applied.*

How is it then that a piece of iron can apparently be unmagnetized at room temperature? The answer, given by Weiss, is that below the Curie point all parts of the iron are magnetized to saturation but that different parts are magnetized in different directions so that the overall
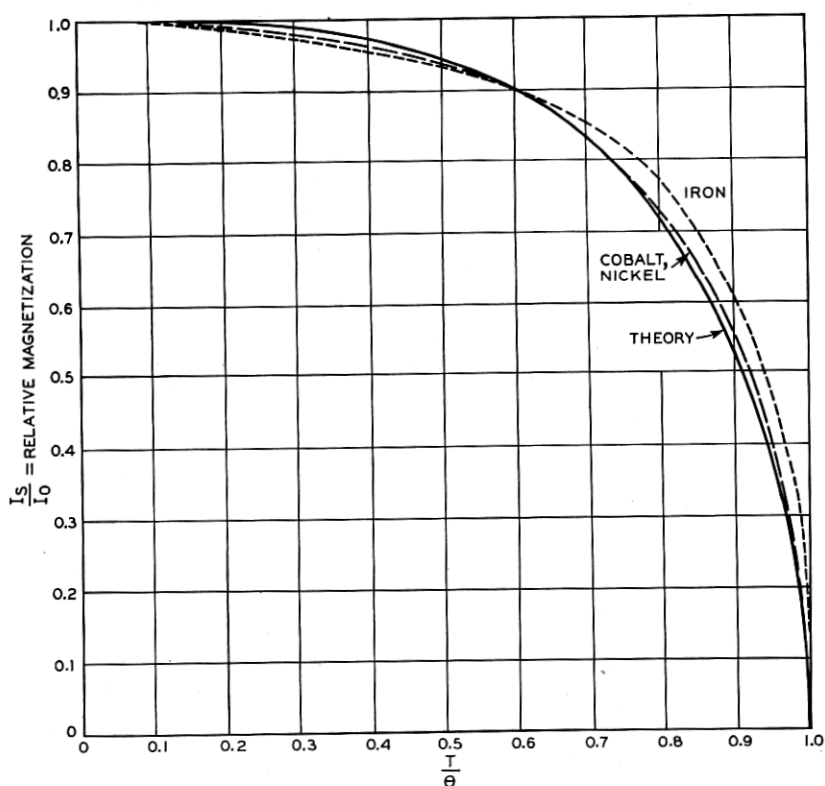

Fig. 9—Dependence on the temperature of the saturation magnetization of iron, cobalt and nickel, as compared with theory.

effect is zero. This is the concept of the domain, already discussed. According to this conception the I of Eq. (5) is that of a domain and is determined experimentally by measuring the magnetization of a specimen when all domains are parallel, *i.e.*, at (technical) saturation ($I = I_s$). Eq. (6) should then be written

$$\frac{I_s}{I_0} = \tanh \frac{I_s/I_0}{T/\theta}.$$

It is a problem of theoretical physics to determine the nature of the molecular field. Before discussing what progress has been made in doing this it will be necessary to review some of our knowledge of the structure of the atoms with which we are concerned.

ATOMIC STRUCTURE OF FERROMAGNETIC MATERIALS

The structure of an isolated iron atom has already been shown in Fig. 1. The twenty-six electrons are divided into four principal "shells," each shell a more or less well defined region in which the electrons move in their "orbits." The first (innermost) shell contains two electrons, the next shell eight, the next sixteen, and the last two. As the periodic system of the elements is built up from the lightest element, hydrogen, the formation of the innermost shell begins first, and when completed the numbers of electrons in the first four shells are two, eight, eighteen, and thirty-two, but the maximum number in each shell is not always reached before the next shell begins to be formed. For example, when formation of the fourth shell begins, the third shell contains only eight electrons instead of eighteen; it is the subsequent building up of this third shell that is intimately connected with ferromagnetism. In this shell some electrons will be spinning in one direction and others in the opposite, and these two senses of the spins may be conveniently referred to as positive and negative. The numbers on the circles show how many electrons with + and − spins are present in each shell in iron and it will be noticed that all except the third shell contain as many electrons spinning in one direction as in the opposite. The magnetic moments of the electrons in each of these shells mutually compensate one another so that the shell is magnetically neutral and does not have a permanent magnetic moment. In the third shell, however, there are five electrons with a positive spin and one with a negative so that four electron spins are unbalanced or uncompensated and there is a resultant polarization of the atom as a whole. The existence of a permanent magnetic moment for each atom obviously satisfies one of the requirements for ferromagnetism.

In the free atom the orbital motions of the electrons also contribute to the magnetic moment. When the iron atom becomes part of metallic iron the electron orbits become too firmly fixed in the solid structure to be influenced appreciably by a magnetic field. The corresponding moments do not change when the intensity of magnetization changes—this is shown by the gyromagnetic experiments discussed later—and it is supposed that the orbital moments of the electrons in various atoms neutralize one another.

In the solid structure neighboring atoms influence the motion and distribution of electrons, particularly in the third part of the third shell (3d shell) and the first part of the fourth shell (4s shell). In Fig. 10 the difference between a free atom and one that is part of a metal is illustrated. Each of the ten places for electrons in the 3d shell is represented by an area which is shaded if that place is occupied. The distribution corresponds in (a) to an isolated atom of nickel, in (b) to a nickel atom in a metal; in the latter situation there is *on the average* 0.6 electron per atom in the 4s shell (these electrons are loosely bound and are the free electrons responsible for electric conduction) and a vacancy or hole of 0.6 electron per atom in the 3d-shell.[6] In the 4s shell the number of electrons with + and with − spin are almost exactly equal, but in the 3d shell all of the spaces for + spin are filled. The difference between the numbers of + and − spins is equal to the net magnetic moment per atom. Experimentally the difference in the number of + spins and − spins in an atom is determined from the saturation intensity of magnetization at absolute zero. When this difference is one the atom has a moment of one Bohr magneton,

$$\mu_B = 9.2 \times 10^{-21} \text{ erg/gauss}$$

consequently the number of Bohr magnetons can be calculated from the atomic weight, A, and the density, d:

$$\text{Bohr magnetons/atom} = \beta = \frac{I_0 A}{\mu_B d}.$$

In Fig. 10 (f) the diagram for nickel is repeated, this time with the tops of the unfilled positions on the same level to bring out an analogy with the filling of vessels with water. Diagrams for manganese, iron, cobalt, nickel and copper are shown in parts (c) to (g). In each case the 18 electrons in closed shells are not shown. In iron the situation is somewhat different from that in nickel, neither the 3d+ nor the 3d− shell is filled. This follows from the relative constancy of the number of electrons in 4s, from the excess of holes in 3d+ over those in 3d− ($\beta = 2.2$), and from the total number, 26, of extra-nuclear electrons.

The distribution in space of electrons belonging to the 3d and 4s shells is known approximately [7] and is depicted in Fig. 11. In (a) the ordinate shows the number of electrons there are at various distances from the nucleus. The 3d shell is thus seen to be a rather dense ring

[6] E. C. Stoner, *Phil. Mag.*, *15*, 1018–1034 (1933); N. F. Mott, *Proc. Phys. Soc.*, *47*, 571–588 (1935); L. Pauling, *Phys. Rev.*, *54*, 899–904 (1938).
[7] Calculations were based on the equation given by J. C. Slater, *Phys. Rev.*, *36*, 57–64 (1930).
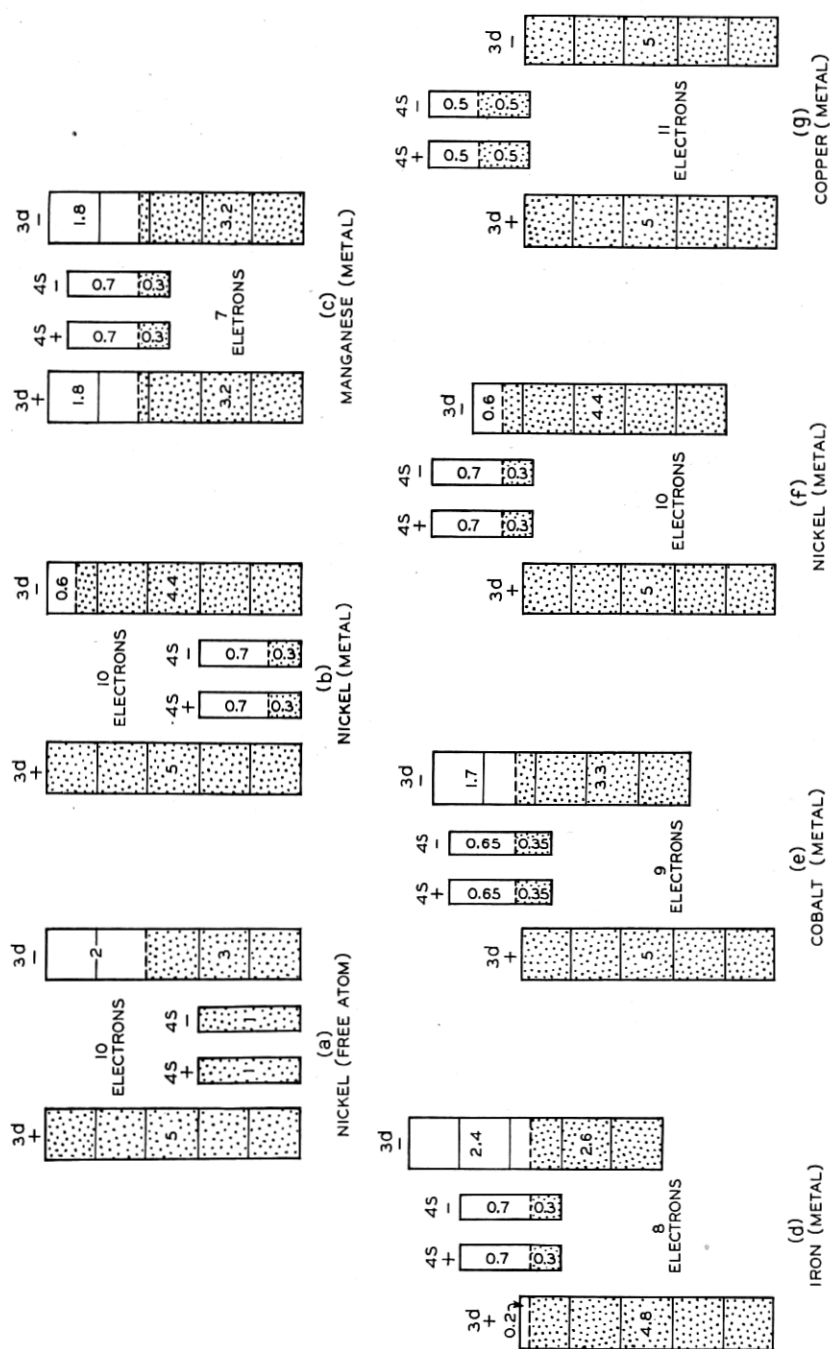
Fig. 10—The distribution of electrons among the possible electron positions in a free atom of nickel, and in manganese, iron, cobalt, nickel and copper atoms that form part of a metal.

of electrons, as contrasted with the 4s shell which extends farther from
the nucleus, so far that in the solid the shells of neighboring atoms over-
lap considerably.   In (b) the number of electrons having energy be-
tween E and $E + dE$ is plotted against the energy E; this representa-
tion is similar to that of Fig. 10 but now the squares and rectangles
are replaced by the more appropriate curved surfaces.   If (b) is turned
90° relative to (a) the two pairs of curves bear some resemblance to
each other.   This is so because the energy of binding is generally less
at greater distances from the nucleus.   The $3d+$ level is represented
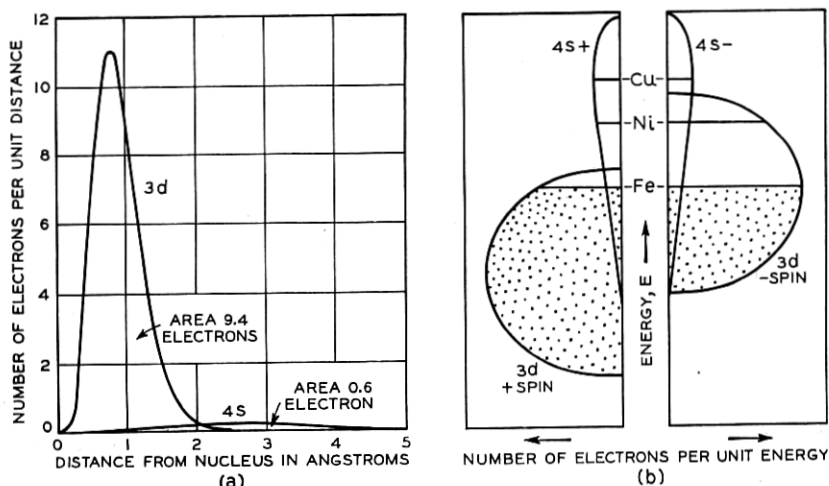as lower in energy than the $3d-$ since one of these bands is preferred.

Fig. 11—The filling of electron positions in iron, and some elements near it in the
periodic table.   Electron positions for closed shells, containing 18 electrons, are not
shown.

The area enclosed by each $3d$ curve corresponds to 5 electrons while
that enclosed by the $4s$ corresponds to 2.

The line "Fe" in Fig. 11(b) represents the limit to which the $3d$ and
$4s$ shells are filled in iron; neither $3d+$ nor $3d-$ is completely full.   The
lowest energy levels are filled first, and the picture is drawn so that the
analogy with the filling of connected vessels with water is apparent.
In cobalt and nickel the extra one and two electrons completely fill $3d+$
but not $3d-$, as indicated by the line "Ni" for nickel.   Since the
range of energy in the $3d$ "bands" is much greater than in the $4s$
bands the additional electrons do not alter greatly the number in $4s$,
and from the saturation intensity of nickel we estimate this number
as 0.6.   In copper the additional electron is sufficient to fill both $3d$
shells with one electron to spare, and this electron must go into the

$4s$ shell which then becomes half full as shown by the line "Cu" as well as by (g) of Fig. 10. The diagram does not show changes in the relative levels of the $3d+$ and $3d-$ bands that occur in going from one element to another; when both $3d$ bands are filled, as in copper, these levels are the same. The numbers of electrons and "holes" in metals near iron in the periodic table are given in Table I. A more

TABLE I

NUMBER OF ELECTRONS AND VACANCIES (HOLES) IN VARIOUS SHELLS
IN METAL ATOMS NEAR IRON IN THE PERIODIC TABLE

Element	Number of electrons in following shells				Total	Holes in		Excess holes in $3d-$ over $3d+$
	$3d+$	$3d-$	$4s+$	$4s-$		$3d+$	$3d-$	
Cr	2.7	2.7	0.3	0.3	6	2.3	2.3	0
Mn	3.2	3.2	0.3	0.3	7	1.8	1.8	0
Fe	4.8	2.6	0.3	0.3	8	0.2	2.4	2.22
Co	5	3.3	0.35	0.35	9	0	1.7	1.70
Ni	5	4.4	0.3	0.3	10	0	0.6	0.61
Cu	5	5	0.5	0.5	11	0	0	0

accurate determination of the form of the $3d$ and $4s$ bands for copper is given in Fig. 12, due to Slater.[8]

An especially simple and interesting illustration of the atom-model described is afforded by the alloys of nickel and copper. The substitution of one copper for one nickel atom in the lattice is equivalent to adding one electron to the alloy. This electron seeks the place of lowest energy in the alloy and finds it in the $3d$-shell of a nickel atom rather than in the copper atom to which it originally belonged. This lowers the magnetic saturation of the alloy by one Bohr unit, since the added electron in the $3d-$ band just neutralizes the moment of one in the $3d+$ band. Addition of more copper to nickel decreases the average moment until the empty spaces in the $3d-$ band are just full; this occurs when 60 per cent of the atoms are copper, and then the magnetic saturation at $0°$ K will be just zero. This is the explanation of the experimental results[9] shown in Fig. 13. There are shown also the saturation moments for other alloys of nickel; it is evident that zinc with two $4s$ electrons fills up the $3d$ band twice as fast as copper, aluminum three times as fast, silicon and tin four times and antimony five, in good accord with theory. In each of these cases the added

[8] J. C. Slater, *Phys. Rev.*, 49, 537–545 (1936).

[9] V. Marian, *Ann. de Physique* (11), 7, 459–527 (1937). Some of the data for the other alloys shown in Fig. 12 are taken from C. Sadron, *Ann. de Physique*, 17, 371–452 (1932). The interpretation of these results is due to E. C. Stoner, ref. 6.

atoms have filled up 3d bands, losing their more loosely bound 4s electrons when there are available places of lower energy. The data for palladium indicate that this element has the same number of outer
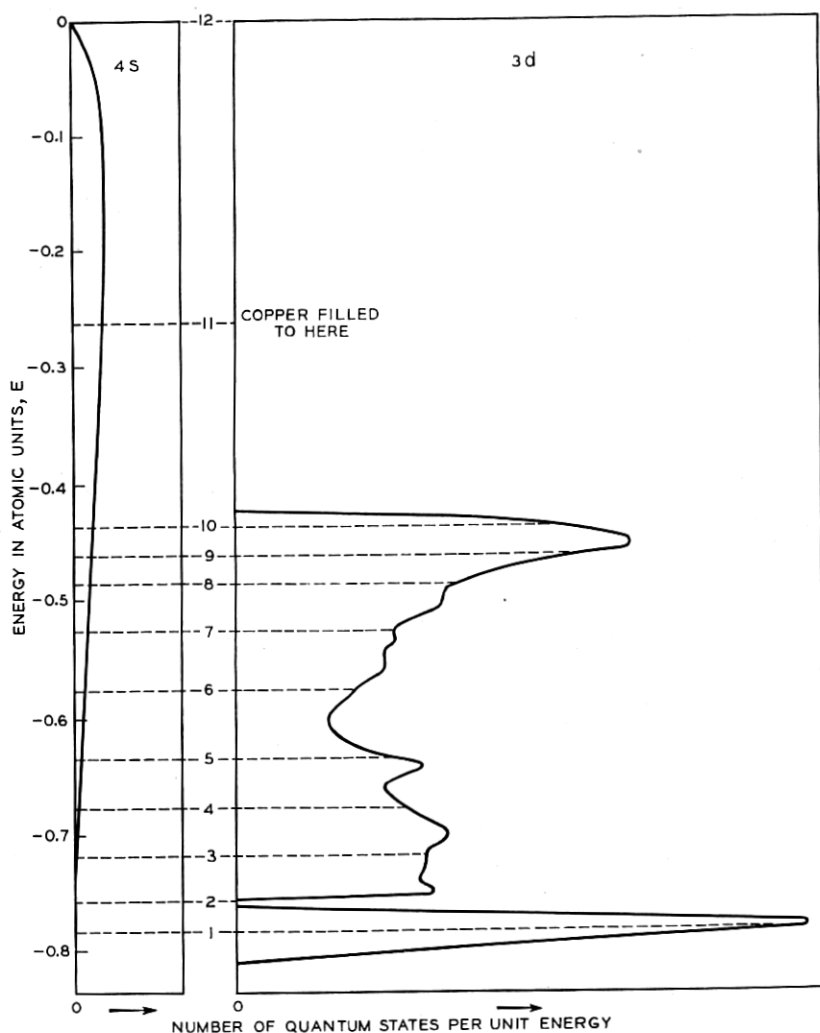

Fig. 12—Energy levels in the 3d and 4s shells in copper, according to Slater. Similar levels are believed to exist in nickel and cobalt with the levels filled to "10" and "9" respectively.

electrons as nickel; this might be expected since palladium lies directly below nickel in the periodic table. When the similar but heavier platinum is added to nickel, the decrease in average atomic moment

indicates that some of the outer electrons of platinum go into the $3d$ band of nickel, but that they do not fill this level as rapidly as the outer electrons of copper do when this element is added.

Electron shells that are completely filled behave more like hard elastic spheres than those which are only partially filled.   In solid copper with
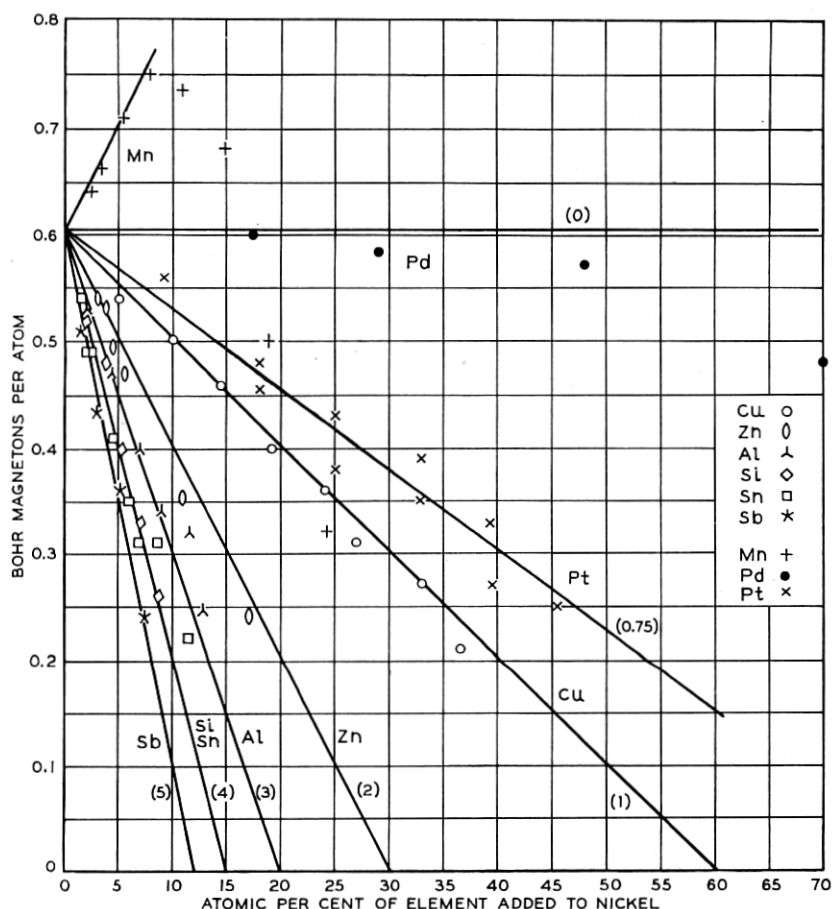

Fig. 13—The saturation magnetization of nickel decreases upon the addition of other elements having 1, 2, 3, ... electrons in the outermost shell.

a complete $3d$ shell and a $4s$ shell just begun, the $4s$ electrons "overlap" those of neighboring atoms so much that their connection with any one atom is lost; the $3d$ shells on the other hand have very little overlap with neighboring atoms.   In the ferromagnetic metals the $3d$ shells are incomplete and the overlap is greater than in copper; this affects the interaction responsible for the Weiss molecular field, now to be

discussed. But copper would not be ferromagnetic even if the interaction were large, because the completed shell means that the saturation magnetization is zero; in reality copper is diamagnetic.

A more detailed discussion of the atomic structure of metals, particularly of the band picture of the ferromagnetic metals, is given in a recent article in this journal by W. Shockley.[10]

INTERPRETATION OF THE MOLECULAR FIELD

It was shown by Heisenberg[11] that the molecular field can be explained in terms of the quantum mechanical forces of exchange acting between electrons in neighboring atoms. Imagine two atoms some distance apart, each atom having a magnetic moment of one Bohr magneton due to the spin moment of one electron. A force of interaction has been shown to exist between them, in addition to the better-known electrostatic and (much weaker) magnetic forces. It is known that, as one would expect, such forces are negligible when the atoms are two or three times as far apart as they are in crystals. It is supposed also, on the basis of calculations by Bethe,[12] that as two atoms are brought near to each other from a distance these forces cause the electron spins in the two atoms to become parallel (positive interaction). As the atoms are brought nearer together the spin-moments are held parallel more firmly until at a certain distance the force diminishes and then becomes zero, and with still closer approach the spins set themselves antiparallel with relatively strong forces (negative interaction). In the curve of Fig. 14 the energies corresponding to these forces are shown as a function of the distances between atoms.

Bethe's curve was drawn originally for atoms with definite shell radii and varying internuclei distances. It may equally well be used for a series of elements if we take account of the different radii of the shell in which the magnetic moment resides. The criterion of overlapping or interaction for the metals of the iron group is the radius, R, of the atom (half the internuclear distance in the crystal) divided by the radius, r, of the $3d$ shell. In Fig. 14 this ratio R/r has been used as abscissa and the elements iron, cobalt and nickel have been given appropriate positions on the curve. The recently discovered ferromagnetism of gadolinium[13] is apparently associated with a large R/r and small interaction, as compared to nickel. It is placed on the curve accordingly. Slater[7] has shown that the ratio R/r is larger in the

[10] W. Shockley, *Bell System Technical Journal*, 18, 645–723 (1939).
[11] W. Heisenberg, *Z. f. Physik*, 49, 619–636 (1928).
[12] H. Bethe, *Handbuch der Physik*, 24, pt. 2, 595–598 (1933).
[13] G. Urbain, P. Weiss, and F. Trombe, *Compt. Rend.*, 200, 2132–2134 (1935).
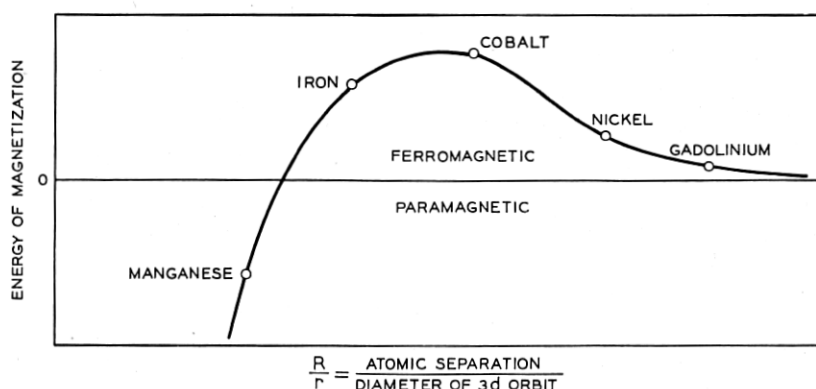
Fig. 14—Bethe's curve relating the energy of magnetization to the distance between atom-centers, with a fixed diameter of the unfilled inner shell that has the magnetic moment.

ferromagnetic elements than in other elements having incomplete inner shells, and that the point at which the curve crosses from the non-ferromagnetic to the ferromagnetic region is near $R/r = 1.5$. Values of $2R$, $2r$ and R/r, as calculated by Slater for some of the elements with incomplete inner shells, are given in Table II.

TABLE II

INTERNUCLEAR DISTANCES $(2R)$ AND DIAMETERS $(2r)$ OF INCOMPLETE INNER SHELLS OF SOME ATOMS, IN ANGSTROMS

	Atom $2R$	Inner Shell $2r$	Ratio R/r	Incomplete Inner Shell	Curie Temperature θ, °K.
Mn	2.52	1.71	1.47	$3d$	
Fe	2.50	1.58	1.63	$3d$	1040
Co	2.51	1.38	1.82	$3d$	1400
Ni	2.50	1.27	1.97	$3d$	630
Cu-Mn	2.58	1.44	1.79	$3d$	600
Mo	2.72	2.94	0.92	$4d$	
Ru	2.64	2.33	1.13	$4d$	
Rh	2.70	2.11	1.28	$4d$	
Pd	2.73	1.93	1.41	$4d$	
Gd*	3.35	1.08	3.1	$4f$	290
W	2.73	3.44	0.79	$5d$	
Os	2.71	2.72	1.02	$5d$	
Ir	2.70	2.47	1.09	$5d$	
Pt	2.77	2.25	1.23	$5d$	

* Calculated using Slater's formula.

The energy of interaction, J—the positive ordinate of Fig. 14—can be estimated from the value of the Curie temperature, θ, in a manner suggested by Stoner.[14]

Let $2J$ be the difference in the energy of interaction between two atoms when their moments are respectively parallel and antiparallel. The total energy of these two atoms is therefore

$$2E = 2E_0 \pm J$$

where E_0 is the energy of an isolated atom. The negative sign applies when the spins are parallel, the positive when they are antiparallel. Imagine a crystal in which each atom of moment μ_A is surrounded at equal distances by z other atoms of which x have their spins parallel and y antiparallel. Then turning one atom from the parallel to antiparallel position produces a change of $(y - x)$ in the number of parallel pairs and $(x - y)$ in the number of antiparallel pairs and, therefore, requires an energy

$$\epsilon = 2J(x - y). \tag{5}$$

Since in each atom the moment must be parallel or antiparallel to the field, the magnetization of the material as a whole will depend on the average value of $x - y$:

$$I/I_0 = \overline{(x - y)}/z. \tag{6}$$

According to Boltzmann's equation an atom will have the following probabilities of being parallel and antiparallel

$$P_p = 1/[1 + \exp(-\epsilon/kT)]$$

$$P_a = \exp(-\epsilon/kT)/[1 + \exp(-\epsilon/kT)].$$

Since all atoms behave in the same way on the average $\bar{x}$ and $\bar{y}$ must be zP_p and zP_a. Hence we have

$$I/I_0 = (\bar{x} - \bar{y})/z = P_p - P_a = \tanh(\epsilon/2kT)$$

or using (5) and (6)

$$\frac{I}{I_0} = \tanh\left(\frac{zJ}{kT}\frac{I}{I_0}\right).$$

Comparing this with the modified Weiss equation, Eq. (4),

$$\frac{I}{I_0} = \tanh\frac{\mu_A NI}{kT} = \tanh\frac{I/I_0}{T/\theta}$$

we have J in terms of the molecular field constant or the Curie temperature:

$$J = \mu_A NI_0/z = k\theta/z.$$

For iron, $z = 8$, $J = k\theta/8 = 1.8 \times 10^{-14}$ erg or 0.01 electron volt.

This derivation indicates that J is proportional to θ, and that the constant of proportionality depends on the number of nearest neighbors. The number of neighbors has not been taken into account in the following discussion of Fig. 14.

The interaction curve is substantiated in a qualitative manner by the observed variation of the Curie points of the iron-nickel alloys.[15]

[14] E. C. Stoner, *Phil. Mag.*, *10*, 27–48 (1930). Stoner's original work appears to have been in error by a factor of two; the modified treatment given here is due to W. Shockley and follows closely the method employed in dealing with order and disorder in alloys (see e.g. Eqs. 1.11, 1.12, 2.2 and 2.16 in the article by F. C. Nix and W. Shockley, *Rev. Mod. Phys.* 10, 1–71 (1938)).

[15] Summarized by J. S. Marsh, *Alloys of Iron and Nickel*, v. 1, pp. 45 and 142, McGraw-Hill, New York (1938).

shown in Fig. 15. The maximum in the curve near 70 per cent nickel apparently corresponds to the maximum of the interaction curve of Fig. 14. In alloys of higher nickel content the curve indicates that the Curie point should be increased if the material is compressed. The opposite should be true of the face-centered alloys having less than this amount of nickel. These contentions are borne out by the fact
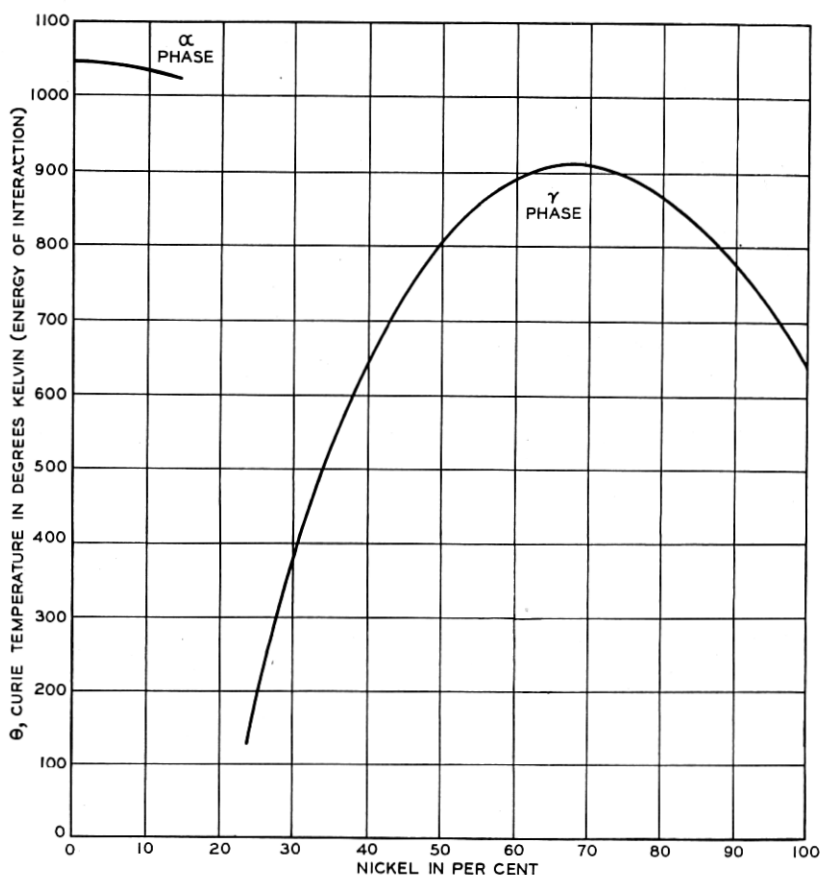

Fig. 15—The Curie temperatures for iron-nickel alloys, showing a maximum corresponding to the maximum of Bethe's curve of Fig. 14.

that under a hydrostatic pressure of 10,000 atmospheres the 30 per cent nickel alloy becomes practically non-ferromagnetic [16] at room temperature (permeability is independent of field-strength and equal to 1.7). On the other hand the effect of the pressure on the phase equilibrium is unknown so that the data might be explained also by a change of phase

[16] R. L. Steinberger, *Physics*, 4, 153–161 (1933).

brought about by the change of pressure. More data are needed to clarify the theory.

There is an anomalous expansion of the high nickel alloys (due to loss of magnetism) as the alloy is heated through the Curie point, a contraction of the low nickel alloys, and no anomaly in the alloys having about 70 per cent nickel, as indicated by the data [15] of Fig. 16 on the expansion of these alloys in the range of temperatures including the Curie points. Bethe's curve represents the change of interaction energy with volume as a material is expanded or contracted, and it is to be expected that there will be a reciprocal effect, a change in volume
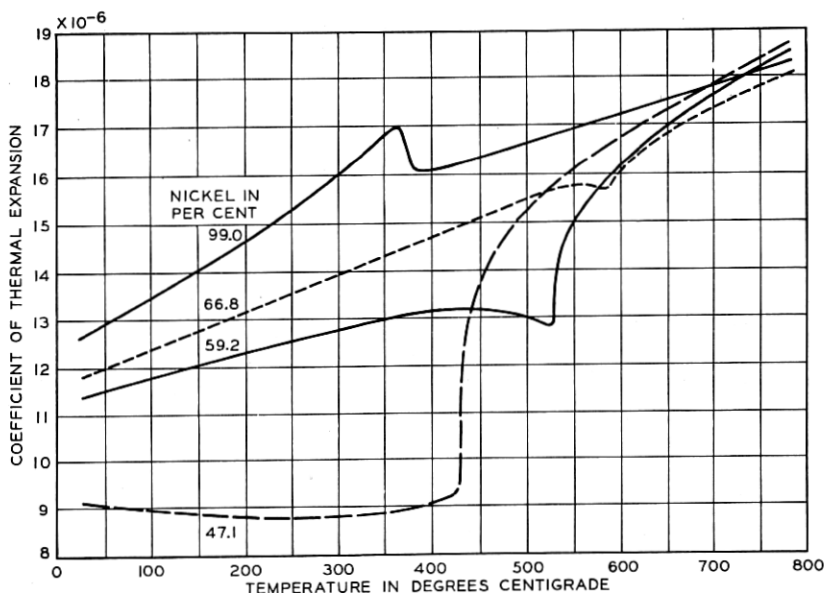

Fig. 16—The expansion coefficient of some iron-nickel alloys, showing the magnetic anomoly and its change in sign at about 70 percent nickel.

as the material passes through the Curie temperature. More careful consideration of the theory [10] shows that the effect to be expected does agree in sign with experiment. Also the disappearance of the anomalous expansion occurs as expected at the same composition as the maximum Curie temperature.

Iron lies to the left of the maximum, as indicated by its expansion curve. Calculations by Kornetski [17] indicate that the interaction energy doubles for a 2 per cent increase in lattice constant. The behavior of cobalt, nickel, and alloys of cobalt-nickel and of nickel-

[17] M. Kornetzki, *Z. f. Physik, 98,* 289–313 (1935).

copper, indicates that all of these substances should lie to the right of the maximum. It should be expected that iron-cobalt, like iron-nickel, alloys should lie in the region including the maximum. This is not observed; instead, the Curie point continually decreases as iron or nickel is added to cobalt—in this case, however, the change of Curie point with composition is obscured by a change of phase so that no easy test of the theory is possible.

SIZES OF DOMAINS AND WIDTHS OF DOMAIN BOUNDARIES

The quantum mechanical interaction in ferromagnetic materials tends to make the magnetic moments of neighboring atoms parallel. One infers that the whole ferromagnetic specimen should be one single large domain; nevertheless in actual fact the parallelism extends over much smaller regions only. This behavior is attributed to strains, crystal boundaries, temperature vibrations, impurities, etc. The fact that a specimen can be demagnetized so that no residual magnetization can be observed by ordinary means, indicates that the domains are not larger than microscopic in size; while the occurrence of heat effects at the Curie point shows that the magnetic unit is larger than a single atom.

A direct measure of the *domain size* is obtained from experiments on the Barkhausen effect;[18] the volume is found to be of the order of 10^{-9} cm.^{3}, so that it contains about 10^{14} atoms. The Barkhausen data give little information concerning the shape of a domain, but this has been made evident by the powder patterns of Bitter and others;[1] a typical domain is long and slender, either rod-like or plate-like with a thickness of the order of one micron (10^{-4} cm.) and a length of perhaps 10 microns. The volume thus agrees with the results of the Barkhausen effect within one or two orders of magnitude. No explanation has been given for the occurrence of domains of this particular size.

There is at present no experimental evidence regarding the nature of the *transition region* between domains, and in the schematic Fig. 3 no transition region is shown. It is believed that the boundary will not be sharp on an atomic scale, but will be spread over a region a considerable number of atoms wide. Calculation indicates that less energy is required if the electron spins change direction gradually from atom to atom as indicated in Fig. 17. The spreading of the transition region over many atoms instead of over one, is analogous to the separation of similar electric charges; the mutual forces tend to spread them over a region as large as possible and they are held together

[18] R. M. Bozorth and J. F. Dillinger, *Phys. Rev.*, 35, 733–752 (1930).

only by some other forces such as those imposed by an electric field.
The expression for the energy of interaction in a boundary layer has
been derived by Bloch,[19] and found to be *inversely proportional* to the
thickness of the layer,

$$\gamma_0 = \frac{k\theta}{a} \cdot \frac{1}{\delta}$$

per unit area of boundary. Here k is Boltzmann's constant, θ the
Curie temperature, a the distance between atoms and δ the thickness
of the layer; since the layer has no sharp limit, δ is measured between
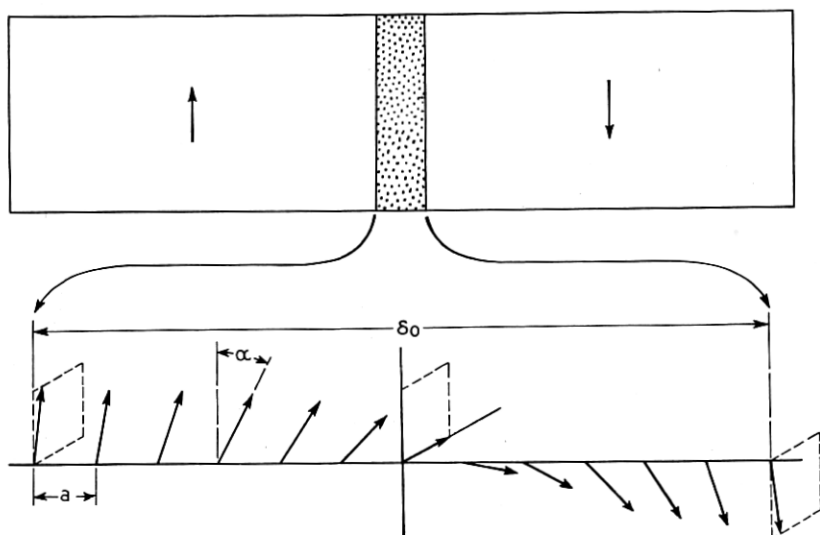

Fig. 17—The nature of the domain boundary. The transition region between two
domains is believed to be about 1000 atom diameters thick.

points at which the spins are inclined at a certain small angle (α almost
0° or 180° as shown) to the spins in the middle of the domains.

The forces of interaction are opposed by forces (e.g. of crystal
anisotropy or strain) which correspond to fixed values of energy *per
unit volume*. This opposing energy is thus *directly proportional* to the
thickness of the boundary,

$$\gamma_1 = C\delta.$$

The minimum energy occurs when

$$\frac{d}{d\delta}(\gamma_0 + \gamma_1) = 0$$

[19] F. Bloch, *Z. f. Physik, 74,* 295–335 (1932). See also the more recent article by
H. Kersten in "Probleme der Technischen Magnetizurungskurve" (R. Becker, ed.)
42–72, Springer, Berlin (1938).

or

$$\delta = \sqrt{k\theta/(aC)} = \delta_0.$$

In iron and similar materials free from any considerable strain the value of C is determined by the crystal anisotropy and is about 10^5 ergs/cm.^{3}, $\theta \approx 10^3\,°K$, $a \approx 10^{-8}$ cm. and the thickness of the boundary layer comes out to be about 1000 atom diameters. This value, probably correct as to order of magnitude, indicates that the volume of the domain proper is much larger than that of the boundary or transition region.

At present it is not clear why application of an indefinitely small field will not cause continual progression of the 180° boundary in one
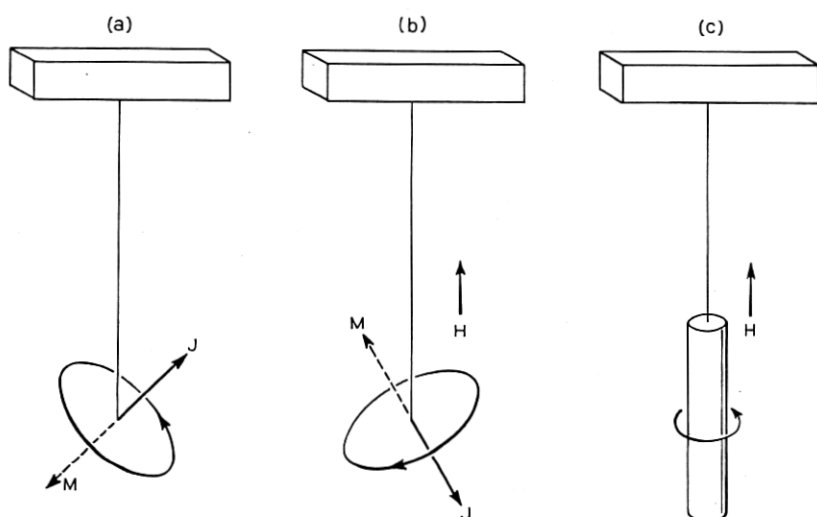

Fig. 18—The magnetic moment, M, and the moment of momentum, J, of an electron in its orbit about the nucleus. A change in one moment entails a change in the other, the (gyromagnetic) ratio remaining constant.

direction so that one domain will disappear completely. The reason for the non-occurrence of this progression except under certain circumstances is probably connected with the existence of strain gradients.

GYROMAGNETIC EFFECT

In the discussion of the structure of ferromagnetic atoms, use was made of the concept of electron spin. This section will review the evidence for the existence of this spin, its experimental determination, and its relation to magnetic phenomena.

Theory. In principle, the ratio of the moment of momentum to magnetic moment may be determined as illustrated in Fig. 18. An

electron of mass m and negative charge e revolves about its nucleus f times per second in an orbit of radius r. The magnetic moment due to the circulating current is at right angles to the plane of the orbit and is

$$M_0 = ef\pi r^2/c.$$

The moment of momentum is in the opposite direction and its magnitude is

$$J_0 = 2mf\pi r^2.$$

The ratio of the moments for this orbital motion is then

$$\rho_0 = \frac{J_0}{M_0} = \frac{2mc}{e}.$$

Imagine now that the atom is suspended in space by a fibre as shown in (a). If a strong magnetic field is applied the vector M representing the magnetic moment will rotate around the axis of the suspension, and J will rotate with it, as the electron precesses. As long as there is no external force or friction the angle between M and the axis will not change but only the speed of its rotation will vary. On the other hand if there is an exchange of energy with other atoms as there is in a real material subject to temperature agitation, then M approaches parallelism with H as shown in (b), and the components of M and J parallel to the axis change in the same ratio. Consequently the change in the magnetic moment about the axis of the suspension may be said to cause a change in the moment of momentum about the same axis. As a result of the concerted action of all of the atoms composing a rod (c), and the recoil of the rod as a whole, the suspension is subject to a torque equal to the (negative) time rate of change of the moments of momentum of the constituent electrons:

$$L = -dJ/dt.$$

Thus a rod suspended as shown in Fig. 17 (c) may be magnetized a known amount, its resulting rotation measured, and its gyromagnetic ratio M/J so determined. The same ratio may be found also by measuring the magnetic moment M caused by rotating a similar rod with a known angular acceleration; this is the inverse effect.

The existence of a magnetic moment and an angular momentum associated with an electron apart from its orbital motion in the atom, was postulated in 1925 by Goudsmit and Uhlenbeck [20] primarily to explain the structure of atomic spectra. The magnetic moment

[20] S. Goudsmit and G. E. Uhlenbeck, *Nature*, *117*, 264–265 (1926).

assigned to this spin of the electron about its own center was equal to one Bohr magneton which by definition is that of the smallest electron orbit on the Bohr theory.

$$\mu_B = \frac{eh}{4\pi mc} = 9.2 \times 10^{-21} \text{ erg/gauss.}$$

The unit of angular momentum was taken as *one-half* of that for the smallest Bohr orbit or as

$$J_s = \frac{h}{4\pi}.$$

The ratio for the spin motion, denoted by ρ_s, is

$$\rho_s = \frac{J_s}{\mu_B} = \frac{mc}{e} = \frac{\rho_0}{2},$$

and is thus twice the gyromagnetic ratio for the orbital motion of the electron. Dirac has shown that these results are consequences of relativistic quantum theory.

In general the ratio M/J is

$$\rho = \frac{mc}{e} \cdot \frac{2}{g}$$

where g is known as the Landé splitting factor. For spin moment, $g = 2$; for orbital moment, $g = 1$. When the moment of an atom is the resultant of finite spin and orbital moments, g may be found in terms of the quantum numbers, s and l, expressing the angular momenta of the spin and orbital components:

$$g = 3/2 + \frac{s(s+1) - l(l+1)}{2j(j+1)}.$$

Here s may have any of the half-integral values $0, 1/2, 1, 3/2, \cdots$ and l any of the integral values $0, 1, 2 \cdots$, while the number, j, representing the angular momentum of the resultant may be any positive number equal to the sum or difference of s and l. (The actual value of the resultant angular momentum is

$$J = \frac{h}{2\pi} \sqrt{j(j+1)},$$

and that of the magnetic moment is

$$M = \frac{eh}{4\pi mc} \cdot g\sqrt{j(j+1)},$$

but the components parallel to the applied field are $jh/2\pi$ and $gjeh/(4\pi mc)$, respectively.) For some values of s, l and j, e.g. 4, 2 and 2, g is greater than 2, and for some values it is less than 1.

The sign as well as the magnitude of the rotation is of importance. All experiments are consistent with the idea that the magnetic moment is due to the spinning or circulation of negative electrons rather than of positive charges.

The results to be described below show that in ferromagnetic materials generally the value of g has nearly the value two and not at all the value one, so we conclude that ferromagnetic processes are concerned primarily with the spins of the electrons and not their orbital motions. When a *change in magnetization* takes place we therefore attribute it to a change in the *direction of spin* of some of the electrons, and believe that the orientations of the orbits are disturbed but slightly. This change is illustrated in Fig. 19. In some
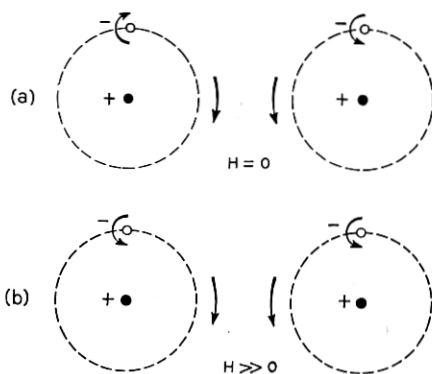

Fig. 19—In the common ferromagnetic materials a change in magnetization is effected by a change in the direction of electron spin, not in the direction of motion of the electron in its orbit.

paramagnetic materials, on the other hand, the reorientation of orbits plays an important part.

Gyromagnetic Experiments. The first gyromagnetic experiment to be performed successfully was magnetization by rotation. After an unsuccessful trial by Perry [21] in 1890, the experiment was considered independently in 1909 by Barnett [22] who in 1914 obtained the result, then inexplicable, that g was approximately twice the classical value one. Richardson,[23] in 1907, was the first to propose rotation by

[21] J. Perry, as quoted by Barnett, ref. 27.
[22] S. J. Barnett, *Science*, *30*, 413 (1909); *Phys. Rev.*, *6*, 239–270 (1915). An accidental error in the calculation of the results was corrected in *Jour. Wash. Acad. Sci.*, *11*, 162 (1921). Magnetization by rotation.
[23] O. W. Richardson, *Phys. Rev.*, *26*, 248–253 (1908).

magnetization, and Einstein and de Haas [24] performed the experiment in 1915. It was repeated in 1918 by Stewart [25] who for the first time obtained a result consistent with Barnett's, and has been confirmed since by a number of others.
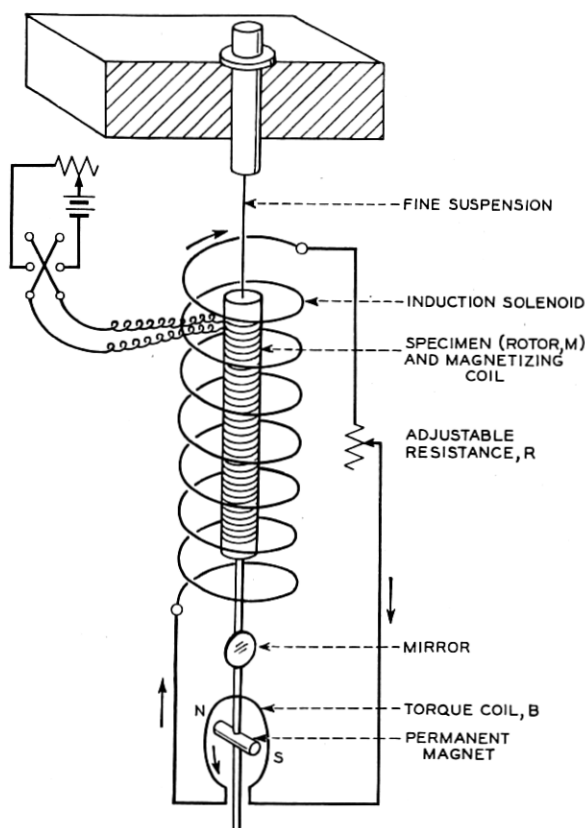

Fig. 20—Schematic diagram of the method of determining the gyromagnetic ratio.

In recent years the method most often used (rotation by magnetization) is that due to Sucksmith and Bates.[26] As modified by Barnett,[27] it is shown diagrammatically in Fig. 20. A rod of the material under

[24] A. Einstein and W. J. de Haas, *Verh. d. D. Phys. Ges.*, *17*, 152–170 (1915); *18*, 173–177 (1916); *18*, 423–443 (1916).

[25] J. Q. Stewart, *Phys. Rev.*, *11*, 100–120 (1918).

[26] W. Sucksmith and L. F. Bates, *Proc. Roy. Soc.*, *104A*, 499–511 (1923). W. Sucksmith, *Proc. Roy. Soc.*, *108A*, 638–642 (1925).

[27] S. J. Barnett, *Rev. Mod. Phys.*, *7*, 129–166 (1935). This article and the one in *Phys. Zeits.*, *35*, 203–205 (1934) give a good account of the history, methods and results to date.

investigation (the "rotor," M) is wound with a magnetizing coil and suspended by a fine quartz fibre in a second (induction) coil A. The leads from the latter are connected in series with an adjustable resistance R and a third coil B, inside of which is a small permanent magnet (moment m) mounted below the rotor and connected rigidly to it. A change in the moment of the rotor is produced by changing the current in the magnetizing coil. This causes a gyromagnetic rotation of the rotor and at the same time induces a voltage in coils A and B. R is adjusted so that the current flowing is of such strength that the field produced by it in B acts on the permanent magnet to annul the gyromagnetic torque of the rotor. The magnetizing current is alternated with a period equal to the natural period of rotation of the rotor assembly and the final deflection δ noted for various values of R. R is plotted against δ and its value, R_0, determined for zero deflection by interpolation.

Let

$$L_A = - \, dJ/dt$$

be the torque due to the gyromagnetic effect. The current induced in coils A and B by a change in the moment M of the rotor is

$$i = E/R = (dM/dt)(K_A/R),$$

where K_A is a constant of coil A. This current produces a torque on the magnet m in B:

$$L_B = miK_B,$$

K_B being a constant of coil B. When $R = R_0$, $L_A = - \, L_B$ and

$$\rho = \frac{dJ}{dM} = \frac{mK_AK_B}{R_0}.$$

The value of ρ is calculated by this formula after finding the values of the coil constants, the resistance R_0 and the moment of the permanent magnet. Barnett has taken great care to eliminate various errors caused mainly by the presence of undesirable fields such as the earth's and by asymmetry and magnetostriction of the rotor.

EXPERIMENTAL VALUES OF g

The results of gyromagnetic experiments are given preferably in terms of g:

$$g = (M/J)(2mc/e),$$

and are collected in Table III. Here a g-value of two means that

TABLE III

VALUES OF g FOR SOME FERROMAGNETIC SUBSTANCES ACCORDING TO VARIOUS AUTHORS

Gyromagnetic ratio $\rho = (mc/e)(2/g)$

Substance	B[22] 1915	E & d.H[24] 1915-6	S[25] 1918	B & B[23] 1917-25	B[29] 1919	A[30] 1920	C & B[31] 1923	S & B[32] 1923-5	B[33] 1931-4	C & S[33] 1932-3	C[34] 1932-5
Iron	2.1	1	2.0	1.91	1.89	2.1	1.99	1.99	1.94	2.01	—
Cobalt	—	—	2.1	1.83	—	—	—	1.94	1.82	—	—
Nickel	—	—	—	1.96	1.75	—	1.98	2.00	1.90	—	—
Fe-Co (34% Co)	—	—	—	1.88	—	—	—	—	1.98	—	—
Fe-Ni (25% Ni)	—	—	—	1.97	—	—	—	—	1.97	—	—
Fe-Ni (75 to 80% Ni)	—	—	—	1.91	—	—	—	—	1.92	—	—
Co-Ni (54% Co)	—	—	—	1.86	—	—	—	—	1.84	—	—
Co-Cu (92% Co)	—	—	—	—	—	—	—	—	1.87	—	—
Fe-C (steel)	—	—	—	1.91	—	—	—	—	—	—	—
Mn-Al-Cu (Heusler alloy)	—	—	—	1.96	—	—	—	2.00	—	—	—
Fe_2O_3	—	—	—	—	—	—	—	—	—	—	1.96
Fe_3O_4	—	—	—	—	—	—	—	2.02	—	—	1.96
$NiFe_2O_4$	—	—	—	—	—	—	—	—	—	—	1.94
$CuFe_2O_4$	—	—	—	—	—	—	—	—	—	—	1.94
$MnFe_2O_4$	—	—	—	—	—	—	—	—	—	—	1.94
Zn_2FeO_{11}	—	—	—	—	—	—	—	—	—	—	1.92
FeS	—	—	—	—	—	—	—	—	—	0.63	—

22, 24, 25 Loc. cit.

28 S. J. and L. J. H. Barnett, *Proc. Am. Acad.*, *60*, 127-216 (1925).   Magnetization by rotation.

29 E. Beck, *Ann. d. Physik*, *60*, 109-148 (1919).

30 G. Arvidson, *Phys. Zeit.*, *21*, 88-91 (1920).

31 A. P. Chattock and L. F. Bates, *Phil. Trans. Roy. Soc.*, *223A*, 257-288 (1922).

32 S. J. Barnett, *Proc. Am. Acad.*, *66*, 274-348 (1931); *69*, 119-135 (1934).

33 F. Coeterier and P. Scherrer, *Helv. Phys. Acta*, *5*, 217-223 (1932).   Pyrrhotin, F. Coeterier, *Helv. Phys. Acta*, *8*, 522-564 (1935).

34 D. P. Ray Chandhuri, *Indian J. Phys.*, *9*, 383-414 (1935).

electron spin only is operative; the ratio would be one if change in orbit orientation were the only effect. The apparent slight difference of most of the values from two, indicates that there is some small but definite change in orbit-orientation in ferromagnetic materials when they are magnetized. In the weakly ferromagnetic pyrrhotite (FeS) the experimental value 0.63 is in harmony with the theoretical value, 0.67, for a possible state of the iron atom ($s = -1/2, l = 2, j = 3/2$) in which orbital moment is of importance.

Gyromagnetic ratios for paramagnetic materials have been determined by Sucksmith [35] and are given in Table IV. The departures

TABLE IV

VALUES OF g FOR SOME PARAMAGNETIC SUBSTANCES (SUCKSMITH)

Substance	g-value		Substance	g-value	
	obs.	calc.		obs.	calc.
Nd_2O_3	0.78	0.76	$FeSO_4$	1.89	<2.00
Gd_2O_3	2.12	2.00	$CoCl_2$-$CoSO_4$	1.54	<2.00
Dy_2O_3	1.36	1.33	$CrCl_2$	1.95	<2.00
Eu_2O_3	>4.5	6.56	$MnCO_3$-$MnSO_4$	1.99	2.00
			Ni-Cu(56% Ni)	1.9	2.00

from the values 1 and 2 show that changes in both spin and orbital moments occur during magnetization. In the last column are added theoretical values deduced from spectroscopic data.

SUMMARY

In this paper the author has discussed some of the difficulties encountered in the interpretation of the fundamental phenomena of ferromagnetism, and some of the successes that have been attained by applying our recent knowledge of the structure of atoms in solids. The difficulties are large because the atomic forces controlling the magnetism are small compared to those that hold the atoms together in a solid. The successes have come largely as a result of the quantum theory which has explained, mainly in a qualitative way, many of the phenomena previously correlated by the empirical Weiss theory of the molecular field.

In some ways magnetic studies have aided materially in clarifying our picture of the atom; this has been brought out in a discussion of

[35] W. Sucksmith, *Proc. Roy. Soc.*, *133A*, 179–188 (1931); *135A*, 276–281 (1932); *Helv. Phys. Acta, 8*, 205–210 (1935).

(1) the atomic magnetic moment (determined from the saturation magnetization at $0°$ K), which gives directly the numbers of electrons in certain shells in the atom, and (2) the gyromagnetic effect, experiments on which give results characteristic of an electron spinning about an axis passing through its center.

ACKNOWLEDGMENT