Load Rating Theory for Multi-Channel Amplifiers *

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The amplifiers of multi-channel telephone systems must be so designed with regard to output capacity that interchannel interference caused by amplifier overloading will not be serious. Probability theory is applied to this problem to determine the maximum single frequency output power which a multi-channel amplifier should be designed to transmit as a function of N, the number of channels in the system. The theory is developed to include the effects of statistical variations in the number of simultaneous talkers, in the talking volumes, and in the instantaneous voltages from speech at constant volume.

Introduction

In a perfect multi-channel carrier telephone system, each channel would be entirely free from interference produced by the energy present in the other channels. Since all the channels are amplified by the same repeaters, which as a practical matter cannot have perfectly linear characteristics, this is an ideal that may be approached but not completely realized. The interchannel interference must be kept down to a value which will be satisfactory for the grade of transmission concerned, further reduction being uneconomic. To do this the repeaters must meet definite load capacity requirements and modulation (non-linearity) requirements. The load capacity requirement is most conveniently specified in terms of the maximum single frequency sine wave power which a multi-channel amplifier must transmit without appreciable overloading. The modulation requirement pertains to the performance of the amplifier for impressed loads equal to or smaller than the load capacity, and specifies the allowable power in the modulation products resulting from such loads. Because of the numerous factors which affect these requirements, their determination is a rather complicated matter and the present discussion will be restricted solely to a determination of the load capacity requirement. The object is to determine this quantity as a function of N, the number of channels in the system.

The criteria ordinarily used for determining the load capacity of single-channel amplifiers are of little use here because of two funda-

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mental differences between single-channel and multi-channel systems. In the first place, the modulation produced in a single-channel amplifier depends only upon the input to that channel and occurs only when the channel is energized. In addition, the most important frequencies resulting from modulation fall directly back upon frequencies already impressed and the net effect appears as a distortion of the original input, rather than as noise. The situation is entirely different in a multi-channel system. In this case, the modulation products falling into one particular channel are in the main unrelated either to the impressed frequencies or to the volume of impressed speech in that channel. Thus it is no longer possible to think of the interference as distortion; the effect must rather be considered as that of a particular kind of noise whose level depends upon the load on the other channels of the system. For a given grade of service, the ratio of signal to noise must be much larger than the ratio of signal to modulation products resulting in distortion; thus it is to be expected that the non-linearity requirements will be more stringent for multi-channel operation than for single-channel operation.

The second fundamental difference between single-channel and multi-channel systems arises from the character of the load which each system must be designed to handle. A single-channel amplifier must be capable of handling one channel at the maximum volume normally expected. Inasmuch as the amplifier will be loaded only about one-fourth of the time, even in the busiest hour, and as the average impressed volume will be some 15 db below the maximum that must be provided for, the ratio of maximum to average load of such an amplifier is inherently very high. In a multi-channel system, however, the several channels will very rarely be heavily loaded simultaneously. There is thus a favorable diversity factor, increasing with the number of channels, and multi-channel amplifiers may accordingly be worked successfully at lower ratios of maximum to average load.

Occasionally, of course, there will be short periods of excessive loading during which the interchannel interference in multi-channel systems will rise above the value normally permitted. This sort of thing often occurs when it is desired to make economical use of facilities of any kind in common. In machine switching systems, for example, it is common practice to associate a large number of lines with a smaller number of switches and trunks. The number of switches and trunks provided is sufficient to ensure a satisfactory service, with a very small probability of requiring more facilities than are available. The multi-channel amplifier problem presents a situation identical in principle, though the methods of solution are necessarily very different.

The application of probability theory is evidently indicated as the method of attack.

Those characteristics of multi-channel amplifiers which are important to the problem will be described first. Then a description will be given of the variables which must be taken into account in computing load capacity. Finally, the combined effects of these variables will be determined on a statistical basis to establish the required load capacity as a function of the number of channels in the system.

Characteristics of the Multi–Channel Amplifier

At the present time, multi-channel systems of primary interest employ single sideband transmission; the carrier frequencies are largely suppressed and different amplifiers are used for the two directions of transmission. For such systems negative feedback amplifiers have outstanding advantages, particularly with respect to stability of gain and reduction of modulation effects, and are thus being used almost exclusively in present day multi-channel systems. The following discussion is related particularly to such systems, although many of the calculations are also applicable to less common types.

At light loads the principal modulation products in a negative feedback amplifier increase approximately as the square or the cube of the fundamental output power. Beyond a certain critical point, however, the modulation increases very rapidly and the total output of the amplifier soon becomes practically worthless for communication purposes. This critical point will be called the "overload" point. For most tube circuits it is either the point at which grid current begins to flow, or that at which plate current cutoff occurs. This point obviously defines the instantaneous load capacity.

Below the overload point the higher order modulation products are negligible in comparison with second and third order products, and the interference may be regarded as due to the latter sources alone. Beyond the overload point, however, the higher order products become important very rapidly and the resultant disturbances appear in most, if not all, of the channels. With given tubes, the interference below the overload point may be altered by changing the amount of feedback. The interference above the overload point, however, may be little changed in this way because of the rapid loss of feedback as the amplifier overloads. Accordingly, in designing an amplifier, the necessary load capacity may be determined solely by insuring that the output will rarely rise above the overload point, afterwards adjusting the amount of feedback so that the inteference below the overload point will be tolerable. There are thus two problems which may be handled separately, at least for negative feedback amplifiers, it being understood that the results are combined in the final design. As previously stated, only the load capacity problem will be considered in detail here but many of the methods used have been applied successfully to the interchannel modulation problem.

The Load on a Single Channel

The total load applied to a multi-channel amplifier varies rapidly between widely separated limits. A complete knowledge of the variations in the load applied to a single channel is necessary first; these variations arise from several recognizable causes which may be discussed separately.

Number of Active Channels

First of all, a single channel at a given instant may or may not be carrying speech; if not, it contributes nothing to the multi-channel load. A channel will be called "active" whenever continuous speech is being introduced into it; i.e., a channel is active during the time it is actually carrying speech power, and also during the short pauses that occur between words and syllables of ordinary connected speech. A channel is said to be "busy" when it is not available to the operator for completing a new call. Busy time is by no means all active time, for a busy channel is inactive during much of the time the connection is being completed, during pauses in the conversation, and finally during the time the other party is talking. The fraction of time during the busiest hour that a channel may be busy depends on the size of the group of circuits of which it is a member and on the methods of traffic operation. Measurements on circuits in large groups, made by Mr. M. S. Burgess, indicate that the largest fraction of the busiest hour that a channel may be active is about $\frac{1}{4}$. For channels in small circuit groups, this figure may become considerably smaller but it is unlikely that any probable increase in group size or improvement in operating practices will increase it appreciably. This figure, which will be represented by τ, may accordingly be taken as a conservative estimate of the limiting probability that a channel will be active in the busiest hour.

The number of channels that are active at a given instant in an N-channel system may be anything from zero to N. Inasmuch as the channels are independent, it is possible to write down at once the probability that exactly n of them are simultaneously active. This probability is

$$p(n) = \frac{N!}{n!(N-n)!}\tau^n(1-\tau)^{N-n}. \qquad (1)$$

Talking Volumes

A second source of variation in the load on a given channel is that the impressed volume may have any value within rather wide limits when a channel is active. By "volume" is meant the reading of a volume indicator of a standard type. Its importance in the present problem arises from the fact that the volume is an approximate measure of the average speech power being introduced into the channel. Although some other instrument might give a better measurement of the latter quantity, only the volume indicator has been used sufficiently widely in the plant to give data on the distribution of average speech power per call under commercial conditions. The average speech power is dependent on the type of instruments, the character of the speech, and the time interval over which the average is determined. From an analysis of phonograph records of continuous speech it is found that the average speech power of a reference volume talker may be taken as 1.66 milliwatts, and the relationship between volume[1] and average power may be expressed by the following equation:

$$\text{Volume } (db) = \frac{10 \log_{10} \text{Average Speech Power in Milliwatts}}{1.66}. \quad (2)$$

This equation is based on the long average speech power. It will be understood that for purposes other than load rating computations, a different relation might be found more suitable.

The use of equation (2) to relate volume to average speech power is applicable to speech in a single channel. It is convenient to refer to a quantity related in the same way to the total average power contributed by a number of channels as the "equivalent volume."

The single-channel volumes on commercial circuits are conveniently measured at the transmitting toll test board, which will be taken as a point of "zero transmission level." Henceforth it is assumed that there is no gain or loss between this point and the output of the amplifier, so that the latter is also a point of zero transmission level. While this will seldom be the case in an actual system, the necessary change in the load capacity is easily computed. The volumes at this point are found to be distributed approximately according to the "normal" law; that is, the probability that the volume will be between V and $V + dV$ is given by

$$p(V)dV = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(V-V_0)^2}{2\sigma^2}} dV. \quad (3)$$

[1] Subsequent to the preparation of this paper, a new volume indicator was standardized for use in the Bell System. With the new volume indicator, volume is expressed in *vu*, + 8*vu* being approximately equal to reference volume (0 *db*) as used herein.

For calls on typical toll circuits, the best present values for the parameters are $V_0 = -16.0$ db and $\sigma = 5.8$ db. These parameters depend, of course, upon the character of the local plant and upon the habits of telephone users, and changes in either will affect their values. Curve A of Fig. 1 shows this distribution of talker volumes at a point of zero transmission level. Curve B of Fig. 1 is the talker volume distribution used for load rating computations when a particular amount of peak amplitude limiting occurs in the terminal equipment. This will be discussed later. Although the mean volume is V_0, the

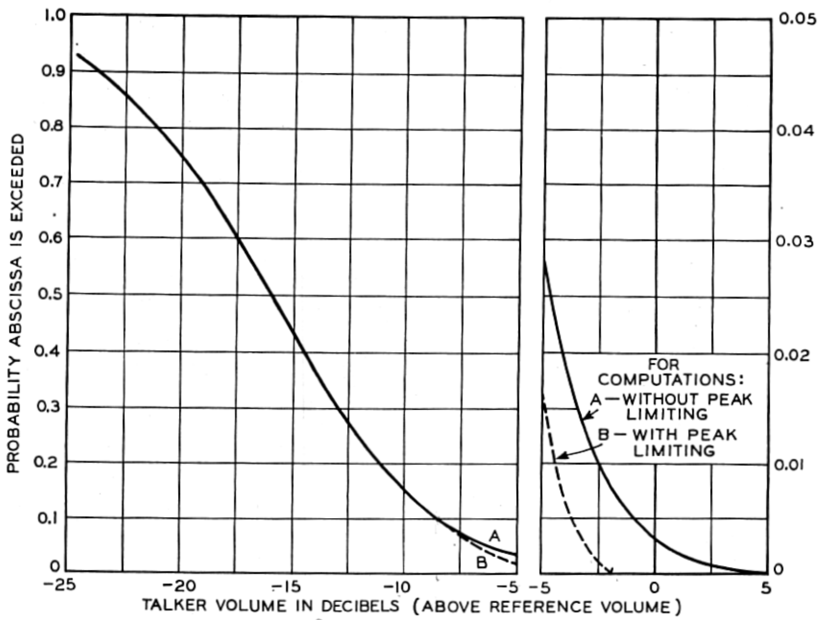

Fig. 1—Talker volume distribution.

volume corresponding to the mean power of the distribution (3) is equal to $V_0 + .115\sigma^2$, as may be seen by converting the volume scale of the distribution to power ratios, averaging, and reconverting the average to volume in db. For the values of the parameters given above, $V_0 + .115\sigma^2 = -12.1$ db.

Instantaneous Voltage Distribution

Finally, the voltage in an active channel fluctuates widely even at constant volume. Not only the differences between successive syllables and the differences between vowel sounds and consonants, but also the fine structure of single sounds, are important in this connection. The total voltage impressed on the amplifier is the quantity which determines whether or not it will overload, and the phases as well as the amplitudes of the frequency components in the several channels must be considered in determining this. It is most convenient for analysis to work directly with instantaneous voltages of speech, the frequency of occurrence of the magnitudes being expressed in the form of a distribution function.

This distribution function has been measured by Dr. H. K. Dunn, using apparatus which measures 4 samples per second of the instantaneous voltage out of a commercial subset and typical loop. By operating the apparatus until about 1000 successive samples have been measured, usable distribution curves of instantaneous voltage are obtained; this is readily checked by making repeated runs comprising the same number of samples on speech recorded on high quality phonograph records. It is, of course, known that commercial transmitters have considerable asymmetry as regards positive and negative voltages but the poling referred to the toll board is expected to be random. As the measurements were considerably simplified by doing so, it appeared desirable to average out this asymmetry by arranging a linear rectifier ahead of the sampling apparatus to obtain equal samples of positive and negative voltages.

Such measurements have been made for a number of different talkers, different commercial subsets, and different volumes, with the speech input held at substantially constant volume in each test. The various subsets now in commerical use all give essentially the same distribution curve. The resulting distributions, if they are considered as functions of the ratio of instantaneous to rms voltage, are also nearly independent of the speech volume at the subset. Specifically, the only important effect of volume is that which may be ascribed to amplitude limiting in the transmitter; i.e., to the fact that the transmitter itself has a limited load capacity. However, this effect does not appear until the volume is 10 db or more above the mean of the volume distribution curve, and is only of importance for talkers at still higher volumes. For all lower volume talkers, the instantaneous voltage distribution may be considered as the same for all volumes when expressed as a ratio of instantaneous to rms voltage. The cumulative distribution curve of the quantity E/U, where E is the rectified instantaneous voltage and U the rms voltage, is shown by the curve $n = 1$ of Fig. 2.

Voltage Limiting

While this curve of Fig. 2 is accurate for the bulk of the talkers, it changes for the high volume talkers who overload the subset transmitters. It is also the custom to provide a certain amount of amplitude limiting in each channel by suitable circuit design of the channel terminal equipment. This limiting alters the shape of the instantaneous voltage distribution curve for a range of voltages below the maximum, the extent of the modification depending on the talker volume and the characteristics of the limiting device. Its effect on the load capacity will be considered later.

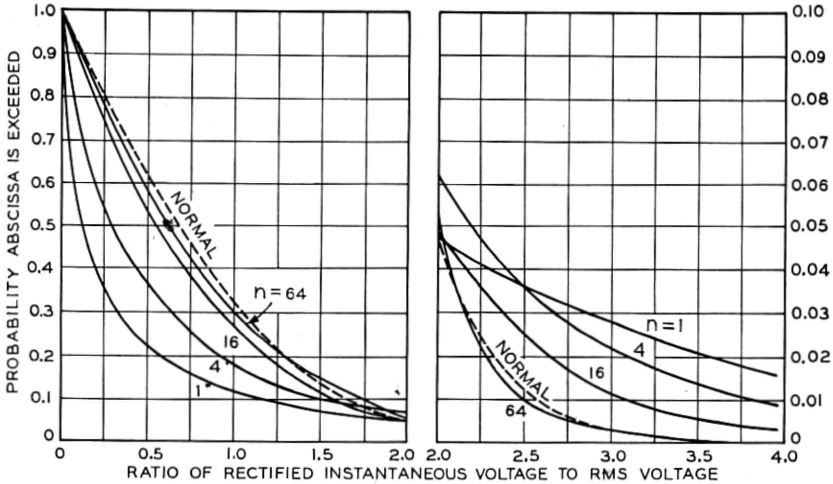

Fig. 2—Instantaneous voltage distributions for *n* talkers.

Multi-Channel Instantaneous Voltage Distributions

The number of variables with which it is necessary to deal makes the general load capacity problem rather a complicated one. The analysis will be easier to follow if the effects of the different variables are taken up one at a time, thus building up a complete theory in successive steps. To do this, it is advantageous to start with a case so simplified that it rarely, if ever, occurs in ordinary practice; i.e., that in which the volumes in all the channels are regulated to a common constant value, and in which the number of *active* channels is also kept constant. For this condition, it is necessary to consider only the effects of the distribution of instantaneous voltages in each channel. This distribution curve is the same for all of the channels, since all are at the same volume, but the voltage in any channel at a particular instant is entirely independent of the condition of the other channels.

Overload Expectation

The total voltage impressed on the amplifier by a number of channels at a given instant is the sum of the instantaneous voltages in the separate channels. Since disturbances will be produced in many of the channels when the applied voltage goes beyond the overload point, it will be useful to know the fraction of the time that this may be expected to occur; this fraction will be called the overload expectation and denoted by ϵ. It is important to notice that this quantity ϵ is not necessarily the fraction of the time during which the performance of the amplifier will be unsatisfactory. This might perhaps be the case for a device having an instantaneous cutoff characteristic, but for an ordinary amplifier the time constants (among other things) affect the results of overloading. The interpretation of the overload expectation will be discussed further later; consideration must be given first to how it is obtained.

The n-Channel Voltage Distribution

The load in each channel is applied at voice frequency to the input side of a modulator, the voice frequency instantaneous voltage distribution being as shown by the curve $n = 1$ of Fig. 2. The overloading of the amplifier is determined, however, by the distribution of the sum of n such voltages after each has been shifted by the modulator to the appropriate carrier frequency, one side-band being suppressed. It may be shown that if the phases of the various components of the voice frequency input were random, the distribution of instantaneous voltage at side-band frequency would be identical with that measured at voice frequency. It is known, however, that the phases at voice frequency are not entirely random, and there may thus be differences between the two distributions. The results of a number of tests bearing upon this point indicate that any error resulting from the use of the distribution measured at voice frequency will be small for systems of few channels, and will rapidly disappear as the number of channels is increased.

Theoretically, the resultant n-channel voltage distribution can be derived from the single-channel distribution by straightforward analytical methods; in the present case, however, expression of the result in useful form is very difficult because of the form of the single-channel curve. This difficulty might be resolved by using graphical or numerical methods, as applied later to the volume distribution curves; fortunately, the fact that the voice frequency voltage distributions may be used throughout permitted the resultant n-channel distributions to be obtained much more easily. Since the addition of voltages from the several carrier channels does not depend materially upon the frequencies at which the channels appear in the system, the addition of n channels at voice frequency will give the desired n-

channel distribution directly. Mr. M. E. Campbell effected this addition by the use of phonograph records, the n-channel distributions being determined by means of the instantaneous voltage sampling apparatus previously mentioned.

As material for this process, 16 high-quality phonograph records were made of the outputs of commercial subsets through representative subscriber loops. Both male and female voices were used. The speech was furnished by reading magazine stories containing considerable conversational material, due precautions being taken that the volume on each record was substantially constant throughout. A calibrating tone was cut on each record to enable it to be played at any desired volume and most of the volumes recorded were well below the point at which the transmitter began to act as a voltage limiter.

These individual records were then combined in groups of four, with all records adjusted to the same volume by means of the calibrating tones, and re-recorded. Several such 4-voice records were made; by combining them again in the same way, 16-voice records and finally 64-voice records were obtained. The instantaneous voltage distributions were measured before and after each re-recording to insure that the recording process introduced no errors. A few minor discrepancies were found, but all were small enough to be disregarded. Each single-voice record appeared several times in a 64-voice record, but since the phases of its different appearances were random, this had no appreciable effect on the resultant voltage distribution. This was verified by comparing the voltage distributions of the various possible 16-voice combinations. By this process n-channel voltage distributions were obtained for $n = 1, 4, 16$ and 64.

These distributions, together with a normal curve, are shown in Fig. 2 in cumulative form. To show the curves conveniently to the same scale, it has been necessary to plot for each case not the distribution of E, the rectified instantaneous voltage, but that of E/U, where U is the rms voltage. The rms voltage, it will be remembered, is directly related to the equivalent volume by equation (2). The figure shows clearly the gradual transition from the single-channel distribution to the normal one for large n, and also indicates that for 64 active channels the curve is normal within the precision of the measuring apparatus. Hence, the normal distribution may justifiably be used for any value of $n > 64$.

Further significance is accorded the above data by plotting the ratio of the voltage exceeded a fraction ϵ of the time to the single-channel rms voltage, as a function of the number n of active channels, for several fixed values of ϵ. From the data given, points on such curves can be obtained for $n = 1, 4, 16, 64$; furthermore, the fact that the distribution for $n > 64$ is normal permits drawing the asymptote for large values of n. The points read from Fig. 2 and replotted in this way give the full lines shown in Fig. 3.

In order to make practical use of these curves, it is necessary to know what value of ϵ corresponds to satisfactory performance of the amplifier. Experiments have been conducted on a number of different multi-channel amplifiers, each loaded by various numbers of active channels all at the same volume. It has been found that for low enough values of ϵ, no audible disturbance is produced but that as ϵ is increased by increasing the load on the amplifier, the disturbance falling into a channel not energized increases rapidly to a large value. Two different amplifiers having the same computed load capacity may show noticeable differences in performance in this respect when subject to identical fixed loads of the type being considered, thus indicating the influence of circuit design on the value of ϵ. In general, however, the allowable values of ϵ measured for all of the amplifiers that have been tested lie in a relatively narrow band on either side of the curve for $\epsilon = 0.001$. The broken curve of Fig. 3 represents the

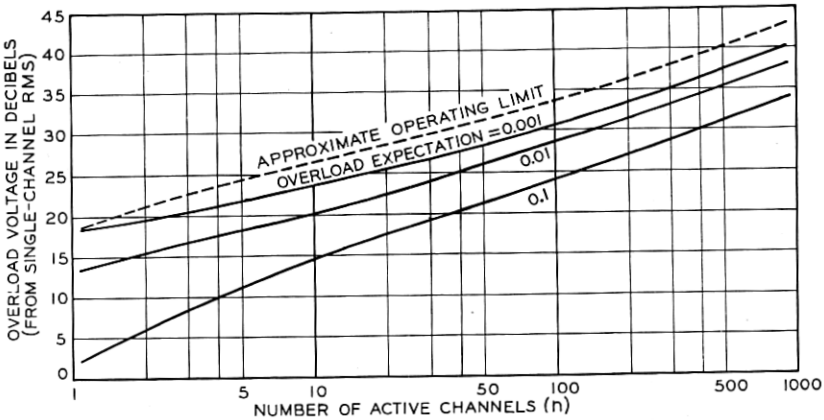

Fig. 3—Overload voltage for n active channels.

approximate upper limit of the observations, extrapolated parallel to the $\epsilon = 0.001$ curve above $n = 14$. It is possible that some amplifiers would overload even if operated in accordance with this curve, but for the great majority of amplifiers of types thus far tested the operation would be satisfactory, with perhaps a small margin.

Multi-Channel Peak Factor

It is useful at this point to introduce the concept of "multi-channel peak factor," which is defined as the limiting ratio of the overload voltage to the rms voltage for a given number of active channels at constant volume. The ratio of the overload voltage for n active channels to the rms voltage of one active channel is given directly by the broken curve of Fig. 3, and the rms voltage for n channels is simply $\sqrt{n}$ times that for one channel. A simple computation then gives the multi-channel peak factor. This is plotted in Fig. 4 as a function of the number of active channels n. The reduction in multi-channel peak factor as the number of active channels increases reflects the transition from the single-channel distribution curve to the normal curve, as depicted in Fig. 2.

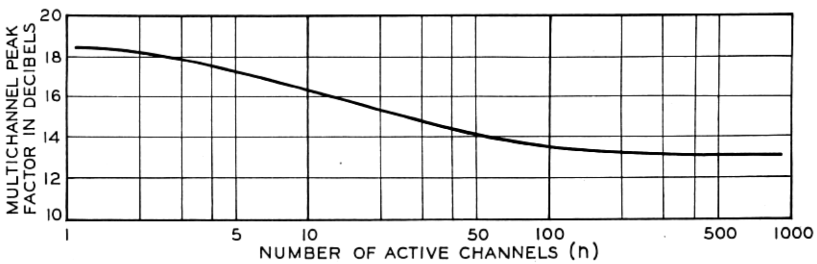

Fig. 4—Multi-channel peak factor for n active channels.

The Distribution of Equivalent Volume

The multi-channel peak factor deals only with the effects of changes in the instantaneous voltages of the channels, all other variables being fixed. It is next necessary to extend the treatment to include the effects of the other load variations that occur in practice—those in number of active channels and in channel volumes. It is important, first of all, to notice that the instantaneous-voltage variations occur very rapidly, while changes in the other two quantities are, in comparison, very slow. In the experiments described above, the loads were so fixed that the equivalent volumes could be changed only by changing the operating transmission level of the amplifier; in practical cases the amplifier transmission level is kept fixed, but the equivalent volume is constantly changing because of changes in number of active channels and in channel volumes.

The amplifier is thus loaded with a constantly changing equivalent volume but because of the great difference in the time-scales of the two classes of variations the load may be regarded as a succession of equivalent volumes, each constant for a small interval of time that nevertheless is long enough to include a representative sample of the resultant instantaneous voltage distribution. If the distribution function for equivalent volume is computed, and then corrected by the multi-channel peak factor, the fraction of such intervals during which the amplifier will be unsatisfactory from the standpoint of overloading may be determined. For a particular amplifier, the operating transmission level must be so chosen that this fraction will be small enough to make any adverse effects on transmission unimportant. For systems of very many channels the proper value of this fraction is probably about 1 per cent. During the busiest hour, this corresponds to 36 seconds during which audible interference *may* occur and as this will be broken up into many very short intervals, the total effect should be slight. For systems of very few channels, the equivalent volume may reach objectionably high values during these intervals and it might be necessary to make this fraction smaller than 1 per cent to secure good performance. For illustrative purposes, the 1 per cent figure will be used in what follows without implying that it may not need alteration in some cases. The methods used are applicable no matter what value is chosen for the fraction of time overloading is permitted.

Controlled Volumes

As the simplest case to which the above procedure may be applied, and one that may occasionally be of practical interest, consider a commercial system with all the channels controlled to the same volume. If there are N channels in the system, the probability that exactly n channels will be active at any given time is given by equation (1), with $\tau = 0.25$. By computing the value of $p(n)$ for all values of n, and taking the cumulative sum, the value of n which makes the sum 0.99 (or the next greater n) is readily determined. This determines the number n of active channels that is exceeded 1 per cent of the time. A plot of these values of n is given by the curve of Fig. 5, as a function of N, the number of channels in the system. For small values of N this curve has been drawn in a manner to smooth out the steps introduced because n must of necessity be an integer and when the value of n read from the curve is not an integer, the next higher integral value is to be used. It is of interest to compare this curve with the two straight lines of the figure. The lower straight line represents the asymptote for sufficiently large N and the upper straight line is for the condition where all channels are active simultaneously ($n = N$).

The average power for n channels is n times that of one channel, and the equivalent volume expressed in db is $10 \log_{10} n$ above that in one channel. The equivalent volume may thus be computed as a function of n, and by means of Fig. 5, as a function of N. Curve A of Fig. 6 shows the values of equivalent volume so determined as a function of N, the number of channels in the system; it applies specifically to the

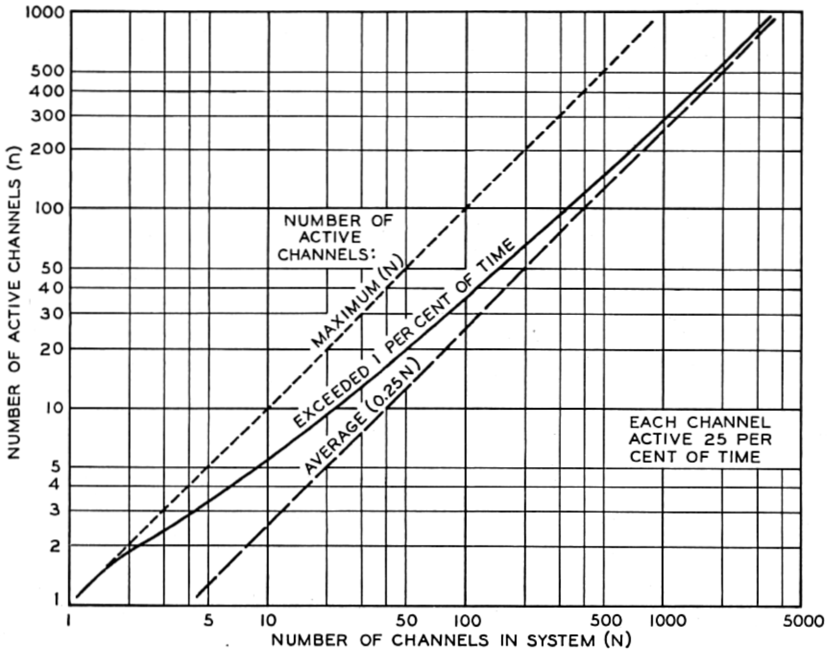

Fig. 5—Number of active channels as a function of the number of channels in the system.

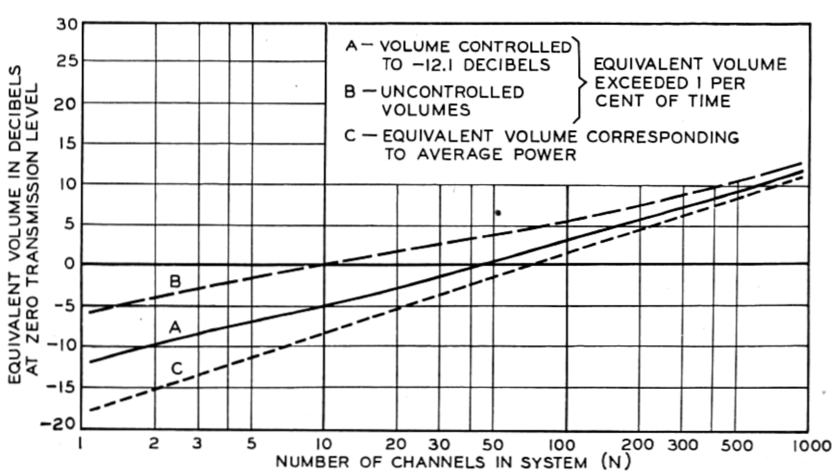

Fig. 6—Equivalent volume for systems of N channels.

case where the volume of each of the active channels is controlled so as to be 12.1 db below reference volume. The choice of this particular volume is purely arbitrary, but it corresponds to the average power of the single talker volume distribution.

The equivalent volumes given by curve A of Fig. 6 are a measure of the average power of the N channels, as computed by means of equation (2). To determine the required instantaneous load capacity of the system, the average power must be corrected by the multi-channel peak factor which is read directly from Fig. 4, using for the number of active channels the values read from Fig. 5.

For design purposes, it is more convenient to use the rms power of the single frequency test tone whose peak value represents the

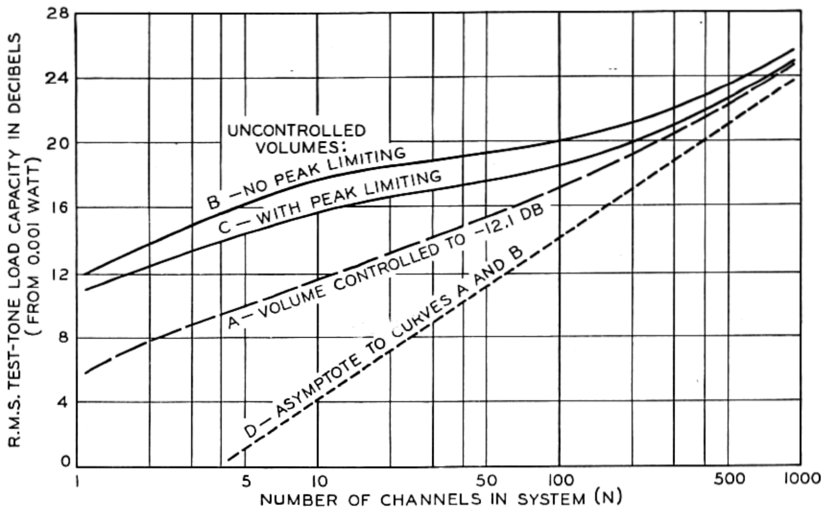

Fig. 7—Load capacity for systems of N channels.

instantaneous load capacity. As the ratio of the peak to rms power of a single frequency tone is 3 db, this test power is obtained by subtracting 3 db from the instantaneous load capacity. This required test-tone capacity is plotted as a function of N in curve A of Fig. 7, which gives the output capacity required for an N-channel system with volume control as specified above.

Uncontrolled Volumes

For systems in which volume control is not used, the application of this procedure becomes more involved. To study this more general case, it is convenient first to interchange the conditions of the preceding section, letting the number of active channels be fixed at any value n

and examining how the distribution curve of equivalent volume may be obtained for this fixed number of channels. The relation between volume and average speech power given in equation (2) may be rewritten for this case in the form

$$V_i = 10 \log_{10} \frac{W_i}{W_0} \text{db},$$

where $W_0 = 1.66$ milliwatts, W_i is the average speech power in milliwatts, and V_i is the volume in db for any one of the active channels, all at a point of zero transmission level.

Likewise, the relation between equivalent volume and average speech power for n active channels is given by the expression

$$V = n\text{-channel equivalent volume} = 10 \log_{10} \frac{\sum_1 W_i}{W_0} \text{ db}.$$

Since the distribution of the channel volumes V_i is known and the volumes of the various channels are independent, the straightforward procedure to obtain the distribution of the n-channel equivalent volume V would involve the following steps: (1) the obtaining of the distribution function of W_i by a transformation of that of V_i; (2) the calculation of the distribution function for the quantity $Y(n) = \sum_1^n W_i$; (3) the transformation of the $Y(n)$ distribution to that of V by inverting the process used in step (1).

The difficulties in this process are all in the second step, where, having given $p_1(W)$, the distribution of average powers for a single channel, it is required to obtain $p_n(Y)$, the distribution for n active channels, with Y defined in terms of W by the relation given immediately above. The formal solution requires the evaluation of integrals of the following type:

$$p_n(Y) = \int_0^Y p_{n-k}(W)\, p_k(Y - W)\, dW.$$

By successive calculation of such integrals for $n = 2, 4, 8 \ldots$, taking k each time equal to $n/2$, the required distributions may be obtained for the necessary range of values of n.

As in the case of the instantaneous voltage distributions, it has not proved feasible to perform the integrations analytically. It was necessary to resort to numerical evaluation of these integrals; by combining the transformations in steps (1) and (3) with the process of evaluating the integral, the process was somewhat shortened. In this way equivalent volume distributions have been obtained for $n = 2, 4, 8, 16 \ldots$; needed points on the distribution curves for intermediate values of n are obtainable by interpolation.

The accuracy of such a process depends upon the number of division points used in the numerical integration and this as a practical matter must be kept fairly small. When the process must be repeated many times, the errors introduced at each step may accumulate and lead to inaccuracies for large n. It is thus desirable to have some control over the accuracy other than by repeating the calculation with a larger number of division points. This is provided by calculating the moments of $p_n(Y)$ from those of $p_1(W)$ without the use of numerical integration.

The moments S_k of $p_1(W)$ are defined by

$$S_k = \int_0^\infty W^k p_1(W)\, dW,$$

and the moments $T_k^{(n)}$ of $p_n(Y)$ similarly. By the use of the semi-invariants of Thiele,[2] it may be shown that

$$T_1^{(n)} = nS_1,$$
$$T_2^{(n)} = nS_2 + n(n-1)S_1^2, \text{ etc.}$$

By comparing the moments of the distributions obtained by numerical integration with those calculated in this way, and making occasional minor alterations in the curves to bring the first and second moments into agreement, assurance was obtained that all the distributions used are reasonably accurate, with no accumulation of error as n becomes large.

Examples of the cumulative distribution curves of equivalent volume for 1, 4, 16 and 64 active channels are given in Fig. 8. The decrease in standard deviation which occurs as n increases is of interest for it indicates how the fluctuations in load due to talker volume variations are reduced by combining a large number of channels in one system.

Having now n-channel equivalent volume curves for a range of values of n, the resultant equivalent volume curves may be calculated when the restriction to a fixed number n of active channels is removed. Let $p_n(V)$ denote the probability that, with n channels active, the equivalent volume lies between V and $V + dV$ and let $p(n)$ denote the probability that just n channels *will* be active. Then the total proba-

[2] T. N. Thiele, "The Theory of Observations," 1903; reprinted in *Annals of Mathematical Statistics*, Vol. 2, 1931. See especially Sections 22, 29.

bility that the equivalent volume will be between V and $V + dV$ is given, for an N-channel system, by

$$p(V) = p(1)p_1(V) + p(2)p_2(V) + \cdots + p(N)p_N(V).$$

The $p_n(V)$ are given by equivalent volume curves such as those in Fig. 8 and the $p(n)$ by equation (1). Examples of curves thus computed are given in Fig. 9, which shows the equivalent volume distributions at a point of zero transmission level for 3, 12 and 240 channel systems. The equivalent volume that is exceeded 1 per cent of the time, read from such curves, is plotted as curve B of Fig. 6.

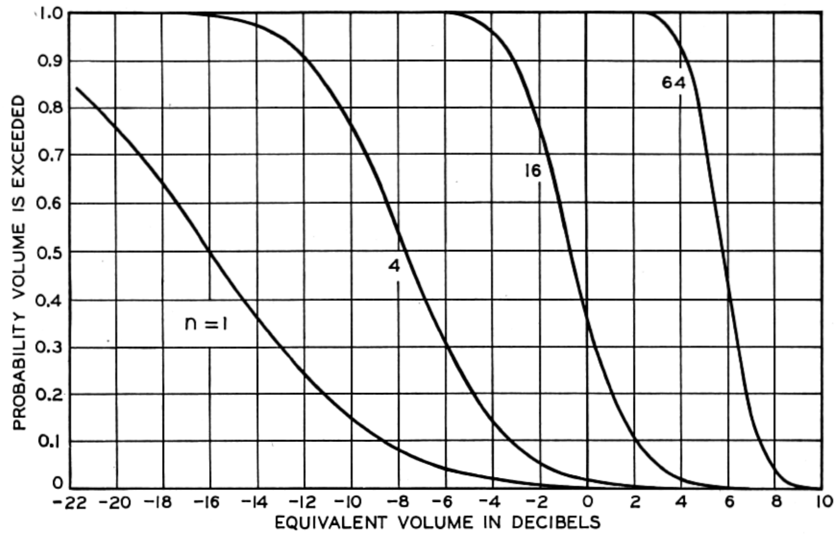

Fig. 8—Equivalent volume distributions for n active channels.

This curve gives, for any number of channels having uncontrolled volumes, the equivalent volume which will be exceeded just 1 per cent of the busy hour. To obtain the necessary load capacity, this must be corrected for the multi-channel peak factor. In the controlled volume case, for a given number N of channels in the system, there was no difficulty in deciding the value of n, the number of simultaneously active channels, for which the multi-channel peak factor should be taken. Now, however, there is no unique relation between equivalent volume and the number n; in addition, the multi-channel peak factors were measured with all n channels at the same volume, which represents a condition rarely holding on a system without volume control. It is apparent, however, that in the majority of cases in which the equivalent volume approaches values on curve B of Fig. 6, the number of simultaneously active channels will be greater than the average number N_T of active channels. Since the multi-channel peak factor decreases as n increases, the peak factors for $n = N_T$ active channels may be safely used. A more detailed analysis, feasible only for very small systems but avoiding the use of this approximation, shows that its effect is small and tends to give load capacities slightly higher than actually required, but the difference diminishes rapidly as the size of the system is increased.

For the uncontrolled volume condition, therefore, the multi-channel peak factors are read from Fig. 5 for values of $n = N_T$. They are added to the equivalent volumes obtained from curve B of Fig. 6, and

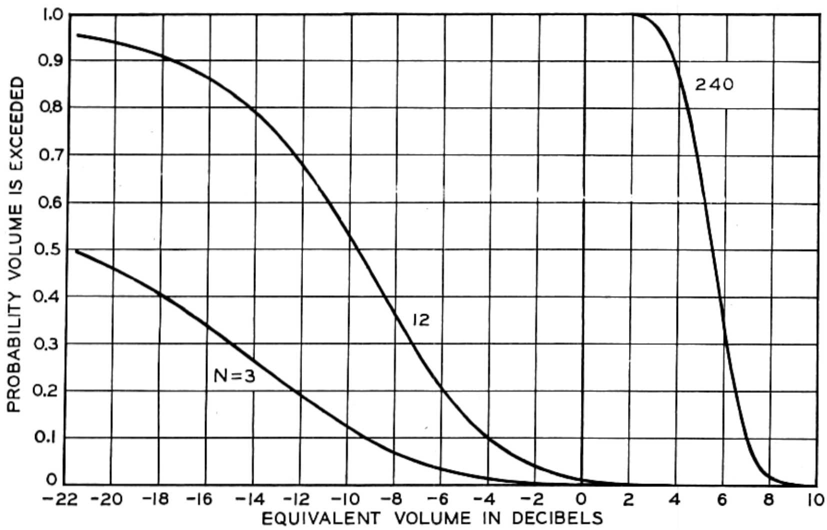

Fig. 9—Equivalent volume distributions for systems of N channels.

reduced to single frequency power as previously described for the volume controlled case. Curve B of Fig. 7 is obtained in this manner and shows the load capacity required in an amplifier for an N-channel system in which the volumes of each channel are distributed in accordance with curve A of Fig. 1. The load capacity which is approached asymptotically as N increases indefinitely is represented by curve D of Fig. 7.

The load capacities given by Fig. 7 are valid only for systems for which the basic single-channel data apply. As these may not hold in specific cases, and may be subject to modification in the future, estimates of the effects of small changes in these data are useful. These effects cannot be described simply for moderate numbers of channels but for large numbers of channels the effects are readily estimated from the change in the location of the asymptote shown on Fig. 7. The equation of this asymptote is as follows:

$$L = 10 \log_{10} N_T + (V_0 + .115\sigma^2) + MPF + P_0 - 3 \text{ db},$$

where L = test tone load capacity,
MPF = asymptotic multi-channel peak factor,
P_0 = long average power of a reference volume talker in db above .001 watt.

The other quantities are as defined before.

Peak Voltage Limiting

The curves referred to in the preceding discussion have so far neglected the effects of peak voltage limiting in the transmitters and in the channel terminal equipment. Fundamentally, the effect of such limiting is to modify the distribution of instantaneous voltages in the individual channels. The extent of the modification, however, depends on the volume. For single-channel systems it is obvious that the improvement in load capacity due to limiting will be substantially equal to the reduction in the maximum peak voltage. For a large number of channels the improvement will approach the reduction in the rms voltage per channel. An approximate method of accounting for these complicated reactions is to consider that peak voltage limiting modifies the upper end of the single-channel volume distribution. Strictly the amount of such modification is a function of the number of channels as well as of the characteristics of the limiters. Curve B of Fig. 1 represents a compromise between the different effects which is believed to give reasonably accurate results for both small and large numbers of channels for the limiting characteristic of present terminals.

With the talker volume distribution modified in accordance with curve B of Fig. 1, computations of the load capacity with voltage limiting present may be made in a manner identical with that previously described. Curve C of Fig. 7 shows the results obtained for this amount of limiting.

All of the load capacity curves of Fig. 7 are based on the equivalent volume which would be exceeded 1 per cent of the time, irrespective of the number of channels in the system. Where voltage limiting is used, it appears reasonable to consider this percentage as fixed because the action of the limiters serves to restrict the range of voltages above the overload point, thus reducing the severity of any overloading effects. When there is no limiting, and particularly for a small number of channels, the range of overloading voltage is not so restricted and overloading effects may become undesirably severe during the 1 per cent of time when the overload voltage is exceeded. If voltage limiting is not provided in some form, it may be important to reduce the percentage of time during which overloading may occur for small numbers of channels. This is a matter to be determined by experience and, if necessary, would require modification of curve *B* of Fig. 7 in the direction of requiring more load capacity for a small number of channels, thus increasing the spread between curves *B* and *C*.

Operating Margins, etc.

The curves which have been given for output capacity versus number of channels apply to a single amplifier, or to a system in which all amplifiers are identical and work at the same output level without appreciable impairment of overall performance. In practice, the number of amplifiers in tandem in a long system may be very large and problems of equalization and regulation may make it difficult to maintain exactly the same level conditions at all amplifiers. In addition, aging of tubes, and other effects will introduce some impairment. It is important, therefore, to allow a margin for these effects in the design of an amplifier for a multi-channel system. The proper margin is essentially a matter of system design and it is often economical to build a liberal margin into the amplifiers in order to allow greater latitude and economy in the design of equalizing and regulating arrangements.

In addition to the speech loads, there are also impressed on the amplifiers various signaling and pilot frequencies, carrier leaks, etc. It is not always possible in practice to make these negligibly small and the load capacity requirements must be corrected to allow for their presence. Multi-channel telephone systems are also required to transmit other types of communication circuits, such as program channels and voice-frequency telegraph systems, superposed on one or more telephone channels. Modifications of the methods applied to speech loads may readily be made to determine the effect of these on the amplifier load capacity.

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