Stabilized Feedback Oscillators

By G. H. STEVENSON

The author presents a mathematical consideration of the conditions which insure constant frequency of the vacuum tube oscillator under changes of electrode potentials or of the cathode temperature. It has already been shown that the grid and plate resistances may enter into the determination of the frequency. The problem is treated here in the manner suggested in the recent studies of feedback amplifiers. The conditions necessary for stability are developed in terms which are independent of particular circuit configurations and are applicable to certain dissipative circuits as well as to purely reactive systems.

THE frequency deviations that accompany changes of the electrode potentials or of the cathode temperature in many types of vacuum tube oscillators have been recognized for some time as having their origin in the variation of the internal resistances of the tube. Llewellyn has shown [1] that both the grid and plate resistances may enter into the determination of the frequency and, by treating the problem as one of network design, has demonstrated the possibility of making the frequency substantially independent of the tube resistances. He also devised a large number of oscillator circuits stabilized in this way and established the conditions necessary for stabilization in each case.

The problem is treated here in a somewhat more general manner suggested by recent studies of feedback amplifiers.[2] The conditions necessary for stability are developed in terms which are independent of particular circuit configurations and which permit their application to certain types of dissipative circuits as well as to purely reactive systems. While no new fundamental principles are presented, it is thought that the restatement of the known principles in broader terms may be of interest.

The mathematical theory will be developed for the case of the single-tube oscillator circuit, since this is the form most generally used. The extension of the theory to multiple stage circuits presents little or no difficulty. The principal assumptions made are, first, that all of the

[1] "Constant Frequency Oscillators," F. B. Llewellyn, *Bell Sys. Tech. Jour.*, January 1932.

[2] "Regeneration Theory," H. Nyquist, *Bell Sys. Tech. Jour.*, January 1932; "Stabilized Feedback Amplifier," H. S. Black, *Bell Sys. Tech. Jour.*, January 1934.

circuit elements except the tube resistances are linear and of lumped character and, second, that modulation effects arising from the non-linearity of the tube resistances may be neglected. The validity of the second assumption is discussed in the appendix to the article by Llewellyn noted above. Its use permits the treatment of the system as though the resistances were actually linear but variable in magnitude in response to variations of the oscillation amplitude.

Theory

The essential features of the single-tube feedback oscillator are shown in Fig. 1. The feedback network B is unrestricted in its configuration

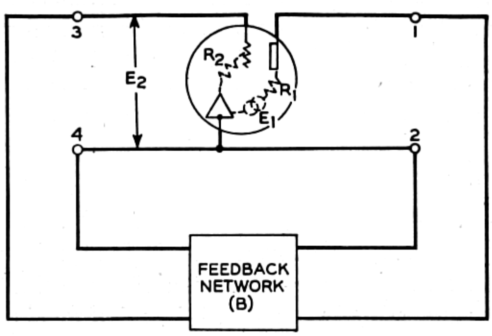

Fig. 1—Elements of a single tube feedback oscillator.

and complexity and may include the vacuum tube electrode capacitances in addition to the external elements. The impedance system of the tube is reduced to the plate and grid resistances R_1 and R_2 with unilateral coupling between them, the latter being indicated by the inclusion of a generator in series with the plate resistance. The voltage E_1 generated in the plate circuit is proportional to the voltage E_2 between the grid and the cathode and, when the system is oscillating, the latter voltage is produced entirely by E_1 as the result of the coupling through the feedback network.

The condition for the existence of self-sustained oscillations is expressed very concisely by the familiar equation

$$\mu\beta = 1, \tag{1}$$

wherein μ and β denote the voltage transfer ratios in the vacuum tube and in the feedback path respectively. The factor μ is the negative of the amplification constant of the tube, the negative sign taking account of the phase reversal inherent in a simple triode. The transfer ratio β represents the ratio of E_2 to E_1 for transmission through the feedback network.

Since the transfer ratios μ and β are both complex quantities, equation (1) expresses the two-fold requirement that the magnitude or modulus of $\mu\beta$ shall be unity and that its phase angle shall be zero. Taking the factors separately, it follows that the modulus of β must be the reciprocal of the modulus of μ and that the phase angles of the two must be equal and of opposite sign. For the single-tube oscillator, the phase angle of β must be 180 degrees since the phase angle of μ has that value.

While the relationships stated above are of simple character, they do not by themselves suffice for the calculation of the oscillation frequency from the constants of the tube and the external circuit. The reason for this is that the values of the tube resistances R_1 and R_2 enter into the determination of the frequency in the general case, and, since these are dependent upon the oscillation amplitude, they cannot be known until the final steady amplitude of the oscillations is known. What happens in an actual oscillator circuit is that, as the oscillation amplitude grows, after initiation, there is a mutual adjustment of frequency and of the resistance values until a condition is reached under which both requirements are met simultaneously. In the case of a stabilized oscillator, since the frequency is independent of the tube resistances, the conditions are simplified and the oscillation frequency can be determined directly by means of the relationships stated above. The non-linearity of the resistances affects only the amplitude of the oscillations.

The evaluation of $\mu\beta$ in terms of the impedance parameters of the circuit permits the determination of the specific circuit conditions in any case for the generation of steady oscillations. The determination is simplified by the consideration that the factor μ has a constant phase angle of 180 degrees so that the variation of the phase of $\mu\beta$ is wholly that of the factor β.

General Formulae for $\mu\beta$

The feedback path, or β circuit, is shown separately in Fig. 2, the

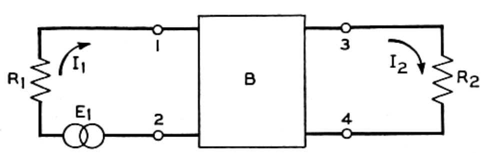

Fig. 2—Simplified schematic of an oscillator feedback circuit.

notations being the same as in Fig. 1. The network B may be of any degree of complexity, but may be assumed to be made up of lumped impedances. In writing down the equations for the mesh currents, let it be assumed that the circuit contains n meshes including the terminal meshes and that the meshes are so chosen that the resistances R_1 and R_2 do not appear as mutual impedances. Designating the meshes in which R_1 and R_2 appear as the first and second respectively, the mesh current equations take the form

$I_1,$	$I_2,$	$I_3 \cdots I_n$		
$R_1 + Z_{11},$	$Z_{12},$	$Z_{13} \cdots Z_{1n}$	E	
$Z_{21},$	$R_2 + Z_{22},$	$Z_{23} \cdots Z_{2n}$	0	
$Z_{31},$	$Z_{32},$	$Z_{33} \cdots Z_{3n}$	0	(2)
—	—	— — —	—	
$Z_{n1},$	$Z_{n2},$	$Z_{n3} \cdots Z_{nn}$	0	

The subscripts of the Z's denote self and mutual impedances in accordance with the usual conventions, the latter being subject to the reciprocal relationships characteristic of linear systems.

The solution of the above equations for the current I_2 is

$$I_2 = \frac{-E\Delta_{21}}{\Delta + R_1R_2\Delta_{11,\,22} + R_1\Delta_{11} + R_2\Delta_{22}}, \tag{3}$$

where Δ is the determinant of the coefficients of equations (2) for zero values of R_1 and R_2, and the other determinants are the minors of Δ obtained by crossing out the columns and rows indicated by the numerical subscripts. Thus Δ_{21} is obtained by crossing out the second column and the first row of Δ and $\Delta_{11,\,22}$ is obtained by crossing out the first two columns and the first two rows.

Since, by definition,

$$\beta = \frac{I_2R_2}{E},$$

equation (3) gives

$$\beta = \frac{-R_2\Delta_{21}}{\Delta + R_1R_2\Delta_{11,\,22} + R_1\Delta_{11} + R_2\Delta_{22}}. \tag{4}$$

The factor μ is the negative of the amplification constant of the tube and if the latter be denoted by α, the value of $\mu\beta$ becomes

$$\mu\beta = \frac{\alpha R_2\Delta_{21}}{\Delta + R_1R_2\Delta_{11,\,22} + R_1\Delta_{11} + R_2\Delta_{22}} \tag{5}$$

The determinants appearing in equations (4) and (5) can be expanded by the ordinary processes to give expressions in terms of the mesh impedances in any particular case. However, as they stand, they are significant parameters of the system and, for the present, need no further expansion.

Another general formula for $\mu\beta$ is obtained by making use of the image parameters of the coupling network. If the image impedances at terminals 1, 2, and terminals 3, 4, are denoted by K_1 and K_2, respectively, and the image transfer constant by θ, then

$$\mu\beta = \frac{-\alpha R_2\sqrt{K_1K_2}}{(K_1K_2 + R_1R_2)\sinh\theta + (R_1K_2 + R_2K_1)\cosh\theta}. \tag{6}$$

The two sets of parameters are related by the equations

$$\begin{aligned} K_1^2 &= \frac{\Delta_{22}}{\Delta_{11}} \cdot \frac{\Delta}{\Delta_{11,\,22}}, \\ K_2^2 &= \frac{\Delta_{11}}{\Delta_{22}} \cdot \frac{\Delta}{\Delta_{11,\,22}}, \\ \tanh^2\theta &= \frac{\Delta\Delta_{11,\,22}}{\Delta_{11}\Delta_{22}}. \end{aligned} \tag{7}$$

Equation (6) is useful in many cases because of the fact that the frequency characteristics of the image parameters are well known for a large number of circuit configurations, particularly those of wave filters.

In dealing with many practical oscillator circuits, the simplifying assumption may be made that the coupling network contains only pure reactances. The determinants in equation (5) then become either real quantities or pure imaginaries, thus making it easy to separate the real and the imaginary parts of $\mu\beta$. If the number of meshes in the β circuit, or the number of rows or columns in the determinant Δ, is even, then Δ will be real and if the number is odd Δ will be imaginary. The determinant $\Delta_{11,\,22}$ will be of the same character as Δ, but will take the opposite sign, and determinants Δ_{11}, Δ_{22}, and Δ_{21} will be imaginary when Δ is real and real when Δ is imaginary. Accordingly, equation (5) may be transformed to

$$\mu\beta = \frac{\alpha R_2 D_{21}}{(R_1D_{11} + R_2D_{22}) + j(D - R_1R_2D_{11,\,22})}, \tag{8}$$

in which the D's are determinants of the mesh reactances corresponding respectively to the Δ's of equation (5) having the same subscripts. The phase angle of $\mu\beta$, denoted by φ, is given by

$$\tan\varphi = \frac{D - R_1R_2D_{11,\,22}}{R_1D_{11} + R_2D_{22}} \tag{9}$$

and the value of $\mu\beta$, when $\tan\varphi$ is zero, by

$$\mu\beta_0 = \frac{\alpha R_2 D_{21}}{R_1 D_{11} + R_2 D_{22}}. \tag{10}$$

The angle φ may be either zero or 180 degrees when $\tan\varphi$ is zero, but which it is may be determined by the sign of $\mu\beta_0$. If this is positive, the phase angle is zero, the phase angle of β being then 180 degrees.

For the simplified case of the pure reactance coupling network the expression for $\mu\beta$ in terms of the image parameters takes different forms depending upon whether the frequency lies in a transmission band or in an attenuating band. At frequencies within a transmission band the image impedances are resistive and the transfer constant θ is a pure imaginary quantity indicating a phase shift without attenuation. Denoting this phase shift by ψ, equation (6) becomes

$$\mu\beta = \frac{-\alpha R_2 \sqrt{K_1 K_2}}{(R_1 K_2 + R_2 K_1)\cos\psi + j(K_1 K_2 + R_1 R_2)\sin\psi}, \tag{11}$$

from which

$$\tan\varphi = \frac{(K_1 K_2 + R_1 R_2)}{(R_1 K_2 + R_2 K_1)}\tan\psi. \tag{12}$$

At the cut-off frequencies, equations (11) and (12) become indeterminate. At frequencies in the attenuation bands, the transfer constant θ takes the form

$$\theta = A + j\frac{n\pi}{2}, \tag{13}$$

where A denotes the attenuation and n is an integer the value of which may be different for the different transmission bands of a complex network. The image impedances are pure imaginaries, but their product is real and may be either positive or negative. Simplified forms of equation (6) may be written down for any particular case.

Frequency Stabilization

From equation (9), which gives the value of the phase angle of $\mu\beta$, it is at once evident that an essential condition for zero phase angle is that

$$D - R_1 R_2 D_{11,\,22} = 0. \tag{14}$$

Since the quantities D and $D_{11,\,22}$ are functions of frequency, equation (14) determines the frequency or frequencies at which the phase shift is zero and hence determines the oscillation frequency. The equation may be satisfied in two distinctly different ways. In accordance with the first, both D and $D_{11,\,22}$ may be finite and of like sign, in which case the frequency depends upon the resistance product R_1R_2. Since these resistances vary with the oscillation amplitude or with the vacuum tube excitation voltages, a solution of this type is indicative of instability of the frequency.

The second way in which equation (14) may be satisfied depends on the fact that D and $D_{11,\,22}$ may each have zero values at one or more frequencies according to the degree of complexity of the coupling network. If, then, the network can be so designed that D and $D_{11,\,22}$ each have a zero at the same frequency and are of opposite sign elsewhere, the condition expressed by the equation will be satisfied at that frequency for any values of the tube resistances and will be satisfied at that frequency only. Whether or not oscillations can be sustained at the frequency so determined may be ascertained readily with the help of equation (10). Oscillations occurring under the above condition are theoretically stable. As demonstrated experimentally by Llewellyn, a very high degree of constancy of the frequency is obtained in actual circuits.

The method of stabilization described above consists in establishing a limited frequency interval within which the oscillation frequency must necessarily lie and then reducing the width of the interval to substantially zero. The establishment of the finite interval is a matter of the choice of an appropriate circuit configuration and the determination of its limits is effected by suitable proportioning of the elements.

Considered in the light of the image parameters of the feedback network, the method of stabilization consists in making the image phase angle of the network take the value 180 degrees at a frequency within a transmission band. Referring to equations (11) and (12), it will be seen that when ψ, the image phase angle of the network, takes the value 180 degrees, the quantity $\mu\beta$ becomes real and positive, indicating the possibility of self-oscillation. Since the result is independent of the values of the tube resistances, the oscillations are theoretically stable.

In an attenuation band, the transfer constant θ may include a phase angle of 180 degrees which is constant with frequency, but, because of the real component representing attenuation, neither $\cosh\theta$ nor $\sinh\theta$ in equation (6) can become zero at any frequency. To obtain an overall phase shift of 180 degrees in the feedback path it is therefore necessary that

$$K_1K_2 + R_1R_2 = 0, \tag{15}$$

which requires the product K_1K_2 to be negative and, hence, that both image impedances be reactances of the same sign. Since the condition expressed by equation (15) is dependent upon the values of the tube resistances, it follows that frequency stabilization cannot be obtained at frequencies in an attenuation band.

The problem of devising stabilized reactance type oscillator circuits therefore resolves itself into that of obtaining band-pass coupling networks which have phase constants of 180 degrees at frequencies within the pass-bands. Evidently there is a multiplicity of known filter structures that meet the requirements and also many other networks of similar character including all-pass reactance networks. Since each half-section of a filter gives a phase shift of 90 degrees in the pass-band, it follows that the coupling network should be equivalent to at least three half-sections. More complex networks may be used, but with networks equivalent to more than six half-sections oscillations may occur at more than one frequency.

Illustrative Examples

The principles discussed in the foregoing section will be illustrated by a consideration of the circuit shown in Fig. 3, in which the coupling

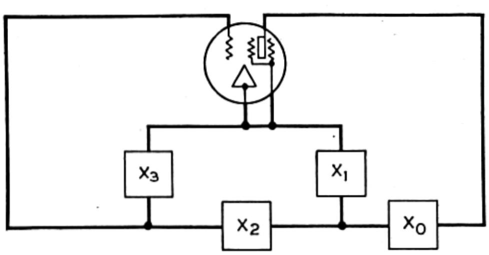

Fig. 3—Oscillator with plate circuit stabilizing impedance.

network is a simple ladder network having four reactive branches. A screen-grid vacuum tube is assumed so that the grid-to-plate capacitance is negligibly small and all feedback is confined to the coupling network. For this circuit, the determinant D has the value

$$D = X_3(X_0X_1 + X_1X_2 + X_2X_0). \tag{16}$$

The several minors are

$$D_{11,\,22} = X_1 + X_2 + X_3, \tag{17}$$

$$D_{11} = X_3(X_1 + X_2), \tag{18}$$

$$D_{22} = X_0(X_1 + X_2 + X_3) + X_1(X_2 + X_3) \tag{19}$$

and

$$D_{21} = -X_1X_3. \tag{20}$$

The condition that D and $D_{11,\,22}$ be zero at the same frequency might be met by making the reactance X_3 a simple series resonant combination and proportioning X_1 and X_2 to be resonant at the same frequency as X_3. The two determinants would then both have zeros at this frequency, but oscillations could not occur since D_{21} would also become zero and the feedback would be destroyed. The necessary condition for stabilization is, therefore, that $(X_1 + X_2 + X_3)$ and $(X_0X_1 + X_1X_2 + X_2X_0)$ become zero at the same frequency. It is readily shown that this can be achieved by making the impedance X_0 such that

$$X_0X_3 = X_1X_2 \tag{21}$$

at the frequency for which $(X_1 + X_2 + X_3)$ is zero. The addition of the plate circuit reactance X_0 provides a circuit configuration which makes stabilization possible. The character of the several branch reactances may be such that equation (21) holds at all frequencies but it need hold only at zero of $D_{11,\,22}$. This minimum restriction permits considerable diversification of the form of the coupling network.

The modified Colpitts oscillator shown in Fig. 4 is a simple case of the general circuit of Fig. 3. For this circuit the reactance determinants have the values

$$D = -\frac{L_0L_2}{\omega C_3}\left(\omega^2 - \frac{1}{L_2C_1} - \frac{1}{L_0C_1}\right), \tag{22}$$

$$D_{11,\,22} = \frac{L_2}{\omega}\left(\omega^2 - \frac{1}{L_2C_1} - \frac{1}{L_2C_3}\right), \tag{23}$$

$$D_{11} = -\frac{L_2}{\omega^2C_3}\left(\omega^2 - \frac{1}{L_2C_1}\right), \tag{24}$$

$$D_{22} = L_0L_2\left(\omega^2 - \frac{1}{L_2C_1} - \frac{1}{L_2C_3}\right) - \frac{L_2}{\omega^2C_1}\left(\omega^2 - \frac{1}{L_2C_3}\right), \tag{25}$$

$$D_{21} = -\frac{1}{\omega^2C_1C_3}. \tag{26}$$

Both D and $D_{11,\,22}$ have frequency variations corresponding to those of simple resonant circuits but, in the case of the former, with the sign reversed. The two quantities have the same sign only in the interval between the two resonance frequencies or zeros and, since these resonance frequencies are independently adjustable, the interval may be made as small as may be desired. The interval is reduced to zero and the oscillation frequency stabilized when the elements are so proportioned that

$$L_0C_1 = L_2C_3, \tag{27}$$

under which condition the oscillation frequency is determined by the equation

$$\omega_0^2 = \frac{1}{L_2}\left(\frac{1}{C_1} + \frac{1}{C_3}\right). \tag{28}$$

At the oscillation frequency the value of $\mu\beta$ (equation 10) becomes

$$\mu\beta_0 = \frac{\alpha R_2}{R_1\dfrac{C_1}{C_3} + R_2\dfrac{C_3}{C_1}} \tag{29}$$

and is equal to unity when

$$\frac{R_1}{R_2} = \frac{C_3}{C_1}\left(\alpha - \frac{C_3}{C_1}\right). \tag{30}$$

Equation (30) can be used to determine the amplitude of the stabilized oscillations if the variation of the resistance ratio with amplitude is known or can be found experimentally. At the moment of inception of the oscillations the amplitude will be infinitesimally small and the tube resistances will generally be such that the initial value of $\mu\beta$ is considerably greater than unity. As the amplitude grows, the plate resistance R_1 tends to increase and the grid resistance to diminish until at a certain amplitude the resistance ratio takes the value given by equation (25). The oscillations then remain steady at this amplitude.

It may be noted that the amplitude relationship (29) holds so long as the capacitances are maintained in the fixed ratio

$$\frac{C_3}{C_1} = \frac{L_0}{L_2} \tag{31}$$

and is independent of their absolute values. If, therefore, the capacitances are varied simultaneously while their ratio is maintained constant, the oscillation frequency will be varied, but frequency stability will be maintained for all adjustments and the oscillation amplitude will remain constant. A similar result may be obtained by varying the inductances simultaneously.

It is instructive to examine the action of the added plate circuit inductance in Fig. 4 in the light of the image parameters of the coupling network. When the values of the inductances and capacitances are unrestricted the coupling network will, in general, have three pass-bands, each pass-band being characterized by a purely imaginary value

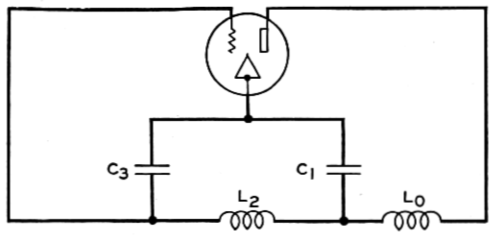

Fig. 4—Colpitts oscillator with plate circuit stabilization.

of the image transfer constant θ corresponding to a phase shift without attenuation. The phase shift is zero at zero frequency, increases by 90 degrees in each pass-band, and remains constant at 90, 180, 270 degrees in the successive attenuation bands. From the expression for tanh θ in equation (7) it follows that the image phase shift of the reactive network will be zero or an even multiple of 90 degrees at the zeros of D and $D_{11,\ 22}$ and will be an odd multiple of 90 degrees at the zeros of D_{11} and D_{22}. The general phase shift characteristic of the feedback network in Fig. 4 is shown in Fig. 5. The critical frequencies f_3 and f_4

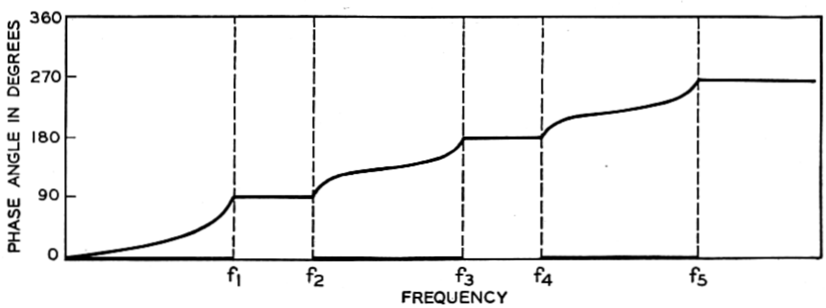

Fig. 5—Phase constant characteristic of the feedback network in an incompletely stabilized Colpitts oscillator.

marking the edges of the second attenuation band correspond to the zeros of D and $D_{11,\ 22}$. Frequencies f_2 and f_5 are the zeros of D_{22} and f_1 is the single zero of D_{11}. The pass-bands are indicated by the heavy lines on the frequency scale. An examination of the image impedances of the network will show that in the second attenuation band they are reactances of like sign and are of opposite sign in the first attenuation band.

The necessary condition for self-oscillation stated in equation (15) is satisfied in the attenuation band between f_3 and f_4. The overall phase shift in the feedback path will follow the image phase shift characteristic approximately and will be equal to 180 degrees at some frequency in this range depending upon the values of the terminal resistances provided by the tube space paths. Oscillations may result but their frequency will be unstable. By reducing the width of the attenuation band stability is increased and becomes theoretically complete when the band is reduced to zero. Proportioning the circuit in accordance with equation (27) to give stability makes the two upper pass-bands of the network confluent. If the whole network be proportioned as a one-and-a-half-section constant-k filter all three pass-bands become confluent.

Since the stabilization requirement

$$X_0X_3 = X_1X_2$$

need hold only at the oscillation frequency, various possible modifications of the circuit of Fig. 4 become readily apparent. For example, the inductance L_2 may be replaced by a series resonant combination which has the same reactance at the oscillation frequency ω_0. This

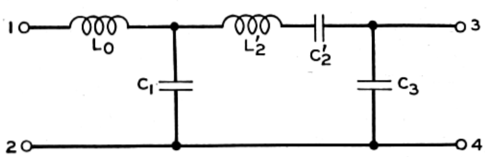

Fig. 6—Modification of the feedback network of the stabilized Colpitts oscillator by the introduction of an extra element.

gives the circuit shown in Fig. 6, in which L_2 is replaced by the combination L_2', C_2' such that

$$L_2' = L_2 + \frac{1}{\omega_0^2 C_2'}\cdot \tag{32}$$

A further possible modification is shown in Fig. 7 in which L_2 is replaced by a three-element combination comprising a series resonant

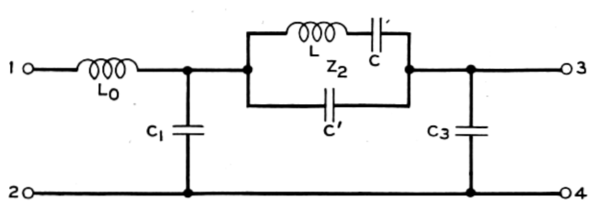

Fig. 7—Further modification of the feedback network of the stabilized Colpitts oscillator.

circuit shunted by the capacitances. At the frequency for which $(X_1 + X_2 + X_3)$ is zero the three-element combination has, as inductive reactance, jX_{20}, which can be computed. The inductance L_0 should then be such that

$$L_0 = \frac{X_{20}}{\omega_0} \cdot \frac{C_3}{C_1}. \tag{33}$$

Evidently the three-element combination in the Z_2 branch is such that it might be replaced by a piezoelectric crystal. To keep the inductance L_0 small, the capacitances C_1 and C_3 should be fairly large so that the resonance of $(X_1 + X_2 + X_3)$ lies close to the crystal resonance.

The foregoing examples are based on the constant-k low-pass filter as a prototype. Evidently high-pass or band-pass filters of the various known kinds might also be used as prototypes and diversified in similar manner. Additional forms may also be found by increasing the number of meshes in the network, but in such cases, the simplest circuits providing frequency stability appear to be the homogeneous single pass-band filter networks. The stable oscillation frequency is the frequency within the pass-band for which the phase constant is equal to 180 degrees. If the network is equivalent to six ladder-type half-sections or more, there will be two or more frequencies for which the phase constant is 180 degrees. Such networks are generally not well-suited for oscillator circuits.

Certain simple configurations which do not admit of complete stabilization in the above manner may be partially stabilized and in actual use may exhibit a very high degree of constancy. The common quartz crystal controlled oscillator with the crystal connected between the grid and cathode of the tube is an example of a partially stabilized circuit. The impedance characteristic of the crystal itself is primarily responsible for the stabilization. Usually the circuit is such as to require the crystal to exhibit an inductive reactance at the oscillation frequency and the impedance characteristic is such that this occurs only in an extremely small frequency interval. The main determinant D for this circuit has a single zero at the resonance of the crystal and complete stabilization would require that the minor $D_{11,22}$ have its zero at this frequency also. However, oscillation under this condition would be impossible since the crystal resonance would short-circuit the feedback path and reduce the magnitude of $\mu\beta$ to zero. Actually the zero of $D_{11,22}$ lies somewhere in the inductive interval of the crystal at a point fairly close to the resonance frequency. The range in which oscillation is possible is therefore only a fraction of the inductive interval of the crystal and a high degree of stability results.

Dissipative Feedback Networks

The foregoing sections deal with non-dissipative feedback networks, but the general ideas set forth are applicable to certain types, at least, of dissipative networks. For such networks the transfer constant θ is a complex quantity and may be represented by

$$\theta = A + j\psi, \tag{34}$$

where A denotes the attenuation and ψ the phase constant. When the phase constant is equal to 180 degrees the hyperbolic functions of the transfer constant take the values

$$\begin{aligned} \sinh \theta &= -\sinh A, \\ \cosh \theta &= -\cosh A, \end{aligned} \tag{35}$$

and

$$\tanh \theta = \tanh A.$$

The image impedances will generally be complex, but, if they can be made to become purely resistive at the frequency for which the phase constant is 180 degrees, the value of $\mu\beta$ at that frequency then becomes

$$\mu\beta = \frac{\alpha R_2 \sqrt{\rho_1 \rho_2}}{(\rho_1\rho_2 + R_1R_2)\sinh A + (R_1\rho_2 + R_2\rho_1)\cosh A}, \tag{36}$$

in which ρ_1 and ρ_2 denote the resistive values of K_1 and K_2. Since this is necessarily a positive real quantity the phase angle of $\mu\beta$ is zero and remains zero independently of variations of R_1 and R_2. Oscillations occurring under this condition are therefore theoretically stable.

Two Tube Oscillators

The stabilization of the single-tube oscillator depends on the circumstances that the tube itself produces a constant phase shift of 180 degrees and that feedback networks can be devised to produce a phase shift of this value which is independent of the terminal resistances. Phase shifts of 90 degrees which are independent of the termination can also be provided by means of reactive networks and this property may likewise be made use of in the design of stabilized oscillators. For this purpose it is necessary to have an amplifier which will give a uniform phase shift of 90 degrees over a fairly wide range of frequencies in the neighborhood of the oscillation frequency. A suitable amplifier may consist of two vacuum tubes coupled in tandem by a simple shunt inductance or a simple shunt capacitance. The second tube should be suitably biased to avoid drawing grid current during operation and the coupling reactance should be small in comparison with the plate resistance of the first tube. Preferably the first tube should be of the screen-grid type having a mutual conductance which is independent of the connected output impedance. The two tubes by themselves have a total phase shift of 360 degrees or zero, but the shunt coupling reactance in combination with the internal resistance of the first tube provides a further phase shift of 90 degrees which represents the total effective phase shift of the amplifier.

With a phase shift of 90 degrees in the amplifier, the oscillation frequency will be that for which the feedback path has a phase shift of 90 degrees in the reverse direction. The conditions for frequency stabilization follow readily from the principles already developed.

For a purely reactive feedback network, the expression for $\mu\beta$ may be obtained from equation (8) by substituting $\pm j\alpha$ for the amplification factor. This gives

$$\mu\beta = \frac{\pm j\alpha R_2 D_{21}}{(R_1 D_{11} + R_2 D_{22}) + j(D - R_1 R_2 D_{11,\,22})}, \tag{37}$$

from which is obtained

$$\tan \varphi = \pm \frac{R_1 D_{11} + R_2 D_{22}}{D - R_1 R_2 D_{11,\,22}}, \tag{38}$$

and

$$\mu\beta_0 = \pm \frac{\alpha R_2 D_{21}}{D - R_1 R_2 D_{11,\,22}}. \tag{39}$$

From equation (39) giving the phase angle of $\mu\beta$, it is evident that the phase angle can be zero independently of the magnitudes of the tube resistances only if D_{11} and D_{22} have zero values at a common frequency. This then is the criterion for stability of the oscillation frequency.

In terms of the image parameters of the coupling networks, the value of $\mu\beta$ becomes

$$\mu\beta = \frac{\pm j\alpha R_2 \sqrt{K_1 K_2}}{(K_1 K_2 + R_1 R_2)\sinh\theta + (R_1 K_2 + R_2 K_1)\cosh\theta}. \tag{40}$$

If it be assumed that the network is dissipative, the transfer constant has an attenuation component and may be represented by

$$\theta = A + j\psi$$

as in equation (34). When the phase constant ψ has the value $\pm$ 90 degrees, equation (40) becomes

$$\mu\beta = \frac{\pm\, \alpha R_2 \sqrt{K_1 K_2}}{(K_1 K_2 + R_1 R_2) \sinh A + (R_1 K_2 + R_2 K_1) \cosh A}. \quad (41)$$

When the feedback network is purely reactive, stabilization of the frequency requires that the phase shift component of the image transfer constant have a value $\pm$ 90 degrees within a transmission band. Under this condition the attenuation component of the transfer constant is zero and equation (41) is simplified by the reduction of sinh A to zero and cosh A to unity.

A simple example of an oscillator stabilized in the above manner is shown in Fig. 8. The two vacuum tubes are coupled by a simple shunt

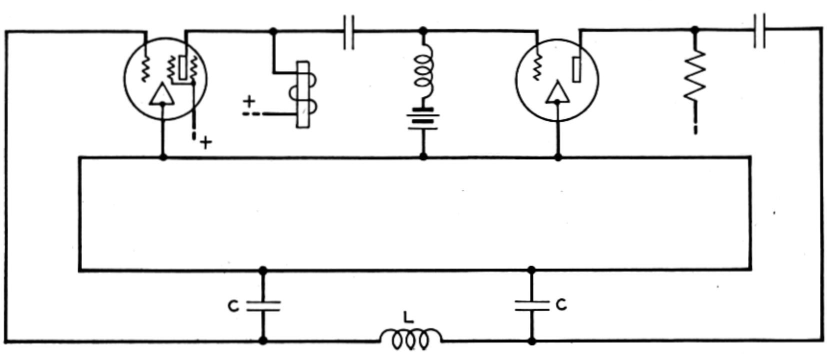

Fig. 8—Stabilized two-stage oscillator.

inductance of relatively low magnitude and low dissipation. With the high resistance of the screen-grid tube in the first stage this shunt reactance coupling provides a substantially constant phase shift of 90 degrees. A low-impedance transformer might also be used for coupling the stages or, if desired, a four-terminal dissipative network designed to provide the required phase shift in a moderately wide frequency range.

The feedback network is required to produce a phase shift of only 90 degrees and may therefore have a relatively simple configuration. The direction of the phase shift should, of course, be opposite to that of the amplifier. In the example illustrated the feedback network corresponds to that of a simple Colpitts oscillator. The condition for stabilization is that the two shunt capacitances be equal and the oscillation frequency is that of the resonance of the inductance with one of the two equal condensers.

It will be evident from the foregoing that a very large number of circuit configurations of the feedback network will provide theoretical stabilization of the oscillation frequency either with single stage or with multiple amplifiers. Naturally, not all of these will be of the same practical interest, since an undue complexity of the network may give rise to difficulties in its adjustment and may increase the problem of temperature compensation.