Variable Equalizers

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THE use of equalizing structures to compensate for the variation in the phase and attenuation characteristics of transmission lines and other pieces of apparatus is well known in the communication art. Ordinarily, of course, an equalizer has a definite characteristic fixed by the apparatus with which it is to be associated. It may happen, however, that the characteristics demanded of the equalizer cannot be prescribed in advance, either because the characteristics of the associated apparatus are not known with sufficient precision, or because they vary with time. Examples are found in the equalization of transmission lines the exact lengths of which are unknown, or the characteristics of which may be affected by changes in temperature and humidity.

In recent years the problem of providing equalizers which will meet such conditions as these has assumed particular importance because of the large variations in line attenuation which may result from temperature changes in new carrier systems. In some of these the maximum change in attenuation is more than 1 db per mile. Evidently, if a reasonable standard of quality is to be maintained for systems the overall length of which may be several hundred or several thousand miles, these variations must be compensated for with great accuracy. Moreover, since the total amount of correction must necessarily be divided up into much smaller amounts appearing at many points, and since the daily cycle of temperature changes may be large, it is almost essential that the adjustments made be so simple that they can readily be performed automatically by a suitable auxiliary circuit.

The variable equalizers described here are attempts to meet this problem. In order to secure the maximum simplicity it has been assumed that the characteristics of the structure are controlled by a single variable element, which in most cases is a variable resistance. It has also been assumed that the temperature coefficient is known as a function of frequency and that it is the same at all temperatures. What is required, then, is a structure by means of which an arbitrary multiple of a given attenuation characteristic can be introduced into a circuit by changes of a single element.

For purposes of future discussion it is convenient to express this requirement in precise form. If the network functions ideally its loss must be given by an equation of the following type:

$$\theta = F_1(\omega) + F_2(\omega)F_3(R), \tag{1}$$

where R is the variable resistance. The function $F_2(\omega)$ corresponds to the temperature characteristic. It must evidently be under our control. The function $F_1(\omega)$ represents a fixed loss, analogous to that of an ordinary equalizer. It is of less importance since it can always be changed by the addition of separate fixed networks. In several of the structures to be described, however, it also is under our control, so that the networks can be used as combined fixed equalizers and temperature correcting devices. The function $F_3(R)$ expresses merely the calibration of the controlling element with respect to temperature, and its exact form is consequently of minor importance.

It is not difficult to find circuits which function broadly in the manner described by equation (1). In most instances, however, the desired proportionality in the set of variable characteristics is realized only very approximately. A simple circuit, which it is hoped is both a fair and a plausible example, may illustrate the sort of performance to be expected.[1] The structure consists of a condenser in series with a variable resistance, bridged across a resistance circuit, as shown by Fig. 1. For high values of the variable resistance we may anticipate that the attenuation will be low at all frequencies, while at lower values a characteristic rising with frequency should be obtained. The network should then behave much like a radio "tone control." An inspection of the actual characteristics, also shown on Fig. 1, indicates, however, that although this general behavior is in fact obtained, the curves change shape rapidly in every range except that corresponding to high resistances and high frequencies, where they are almost constant.

The distortion exemplified by the curves of Fig. 1 is the greatest obstacle in the design of variable equalizers to satisfy the specifications of equation (1). To a certain extent it is unavoidable. It is easily shown for example, that the transfer admittance from generator to load impedance in any network containing a single variable resistance can be written as

$$Y = \frac{ZY_s + RY_0}{Z + R}, \tag{2}$$

[1] More elaborate circuits have, of course, been devised in the past. Mention should be made in particular of the structure described in U. S. Patent No. 2,019,624, issued to Mr. E. L. Norton. For moderate ranges of variation, the characteristics of this network are somewhat similar to those of the structure exemplified later by Fig. 6. Another method of attack is shown by U. S. Patent No. 2,070,668, issued to Mr. W. R. Lundry.

where R represents this resistance, Z represents the impedance which it faces, and Y_s and Y_0, as the equation implies, are the transfer admittances obtained by short-circuiting and open-circuiting R. It is evident by inspection that log Y, which represents the θ of equation (1), cannot be written in the form which the right-hand side of (1) demands. A certain amount of distortion of the type shown by Fig. 1 must therefore always occur.

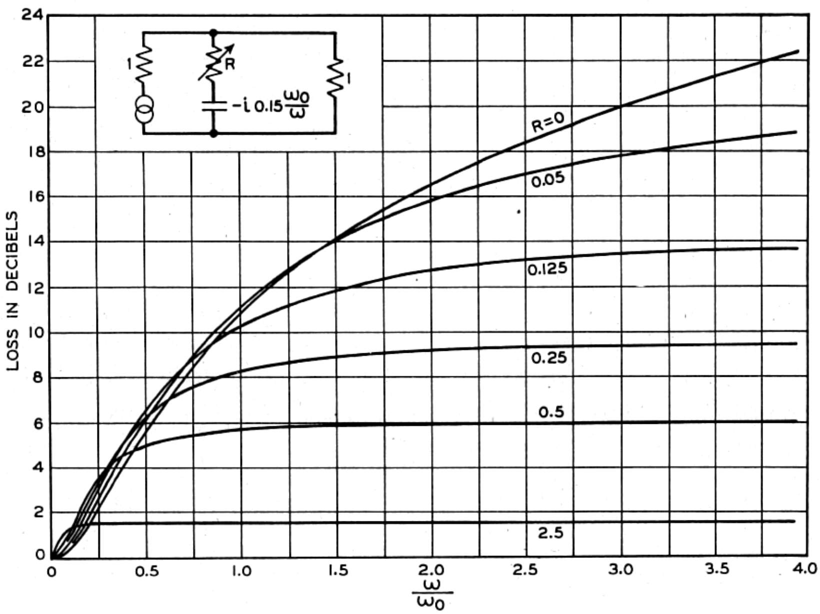

Fig. 1—Characteristics of a simple variable structure.

There still remains the possibility, however, of obtaining a network in which the distortion can be kept within tolerable limits over a given range. The quantities Y_s, Y_0, and Z, which are, of course, all functions of frequency, allow us to determine the transfer admittance at three values of R. The transfer admittances at other settings will then be fixed. If we suppose for simplicity, that the extreme characteristics, corresponding to Y_s and Y_0, are set by the engineering requirements on the structure, the problem reduces to that of so choosing Z in relation to these quantities that the distortion is as small as possible at intermediate settings of R.

Of the variety of possibilities open in the selection of Z, one in particular commends itself by the simplicity and symmetry of the results to which it leads. It is given by the condition

$$Z = \sqrt{\frac{Y_0}{Y_s}} R_0, \tag{3}$$

where R_0 is an arbitrary constant which represents, physically, a reference value for the variable resistance. With the help of this condition (2) becomes

$$Y = \sqrt{Y_0 Y_s} \frac{R_0 + \frac{R}{R_0} Z}{Z + R}, \tag{4}$$

which can be rewritten in a slightly different notation as

$$e^{-\theta} = e^{-\theta_0} \frac{1 + xe^{-\varphi}}{x + e^{-\varphi}}, \tag{5}$$

where $e^{-\theta}$, $e^{-\theta_0}$, x and $e^{-\varphi}$ stand respectively for the quantities Y, $\sqrt{Y_0 Y_s}$, $\frac{R}{R_0}$ and $\frac{Z}{R_0}$.

The significance of the assumption made in equation (3) is apparent from an inspection of equation (5). When $R = R_0$ the total loss θ of the circuit is equal to θ_0. The quantity θ_0 can therefore be described as the average or reference loss of the circuit, corresponding to the average or reference value of R. It is represented by the middle curve shown on Fig. 2. Setting $R = 0$ or $R = \infty$ gives the symmetrically located extreme curves $\theta_0 \pm \varphi$ also shown in the figure. The quantity φ is therefore the extreme change in the attenuation of the network produced by variations in R. Since the two extreme curves correspond to Y_0 and Y_s the situation can also be described by saying that condition (3) fixes the third arbitrary transfer admittance characteristic symmetrically between the first two.

It is easily shown that any other pair of characteristics which correspond to reciprocal values of x, such as those shown by the broken lines in Fig. 2, will also be symmetrically placed with reference to θ_0. This line therefore divides the complete family of characteristics into two equal halves. The departures of the intermediate characteristics from θ_0 are, of course, not strictly proportional to φ. The error can, however, readily be investigated by expanding (5) as a power series in terms of φ.[2] We find

$$\theta = \theta_0 + \frac{x - 1}{x + 1} \varphi + g_3(x)\varphi^3 + g_5(x)\varphi^5 + \cdots, \tag{6}$$

even terms being absent because of the symmetry of the original ex-

[2] A more detailed treatment of this analysis will be found in the writer's U. S. Patent 2,096,027.

pression. The particular forms of the functions $g_3(x)$ and $g_5(x)$ are not of great interest. It is important, however, to know their maximum values, which are respectively 0.03 and 0.002.

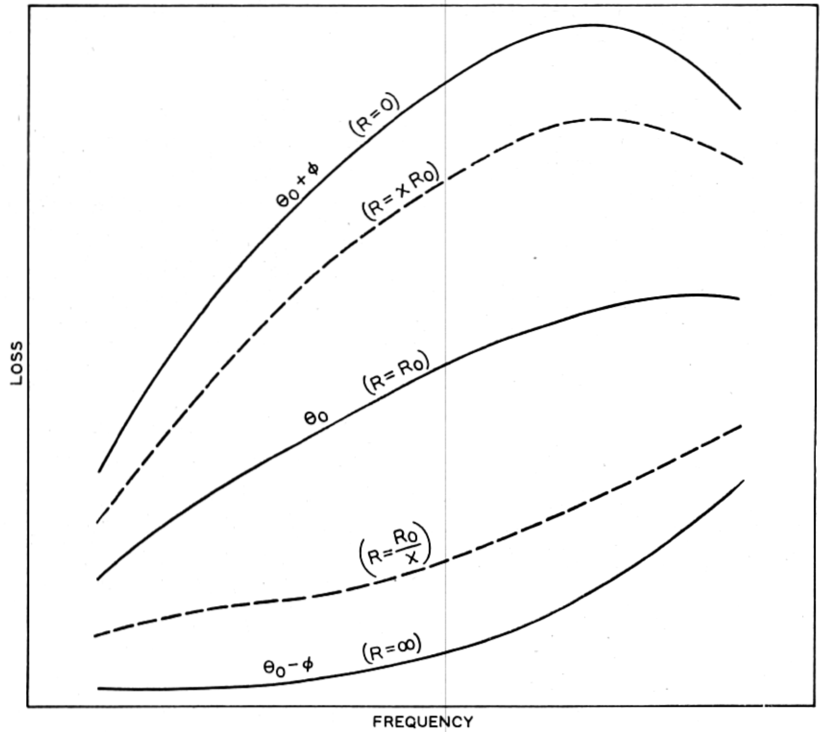

FIG. 2—Diagram to illustrate symmetrical characteristics obtained from the equalizers described in the paper.

The first two terms in equation (6) can evidently be identified with the quantities appearing on the right-hand side of the ideal regulator equation (1). So far as these terms are concerned, the variable characteristics are always strictly proportional to φ. The departure from the ideal is measured by the remaining terms of (6). For ordinary values of φ the series converges so rapidly that only the term in $g_3(x)\varphi^3$ is important. Since the maximum value of $g_3(x)$ is known an estimate of the distortion is easily made, although some allowance is necessary for the possibilities of "splitting differences" in the design process. With the help of these allowances, however, it turns out that the distortion should be about 0.1 db when the maximum value of φ is 1 neper, corresponding to a total variation in attenuation of about

18 db. For other values of φ the distortion is, of course, proportional to the cube of the total change in attenuation.

This estimate is in good agreement with the computations made on actual networks. It also covers the range of greatest practical interest, since in most communication systems changes in attenuation of as much as 18 db produce undesirable variations in the levels of the different channels with respect to one another or to interfering signals. If necessary, however, it appears to be possible to go considerably farther. This possibility arises from the fact that in actual practice φ will be complex, which means that the structure acts as a variable equalizer with respect to both phase and attenuation. Ordinarily, however, only the variation of the attenuation characteristic is of interest. Since the real component of φ^3 depends upon both the real and imaginary components of φ, it is thus possible, by choosing the proper relation between these latter two quantities, to eliminate, effectively, the third order term as well as the even order terms in the general expansion. The first disturbing term is then of the fifth order, and has a very small coefficient. If we assume that the desired relation between the real and imaginary components of φ can be obtained with sufficient precision in a physical network, it appears that this process allows us to confine the distortion to 0.1 db for total variations in attenuation as great as 30 or 35 db. Since the distortion now depends upon the fifth power of the total variation in attenuation it is, of course, very small for more moderate variations. For example, under the same assumptions it is only about 0.001 db for a total variation of 12 db.

All of these relations, of course, have no utility unless structures meeting the general conditions laid down by equation (3) can be found. The simplest structure for the purpose appears to be the Π of fixed resistances shown in Fig. 3. A set of illustrative characteristics, drawn on the assumption that the parameter a equals 2, is shown at the bottom of the figure.

At first sight, this may appear to be a trivial illustration, since θ_0 and φ are merely constants, and the structure thus has only the properties of an ordinary gain control. It is possible, however, to introduce auxiliary networks by means of which θ_0 and φ can be made prescribed functions of frequency. For example, θ_0 can be altered by adding an ordinary equalizer in tandem with either terminating resistance. The modification which allows us to vary φ may be somewhat less obvious. It consists of the introduction of a symmetrical four-terminal network having the image impedance R_0, between the variable resistance and the terminals to which it was previously connected, as shown by Fig. 4.

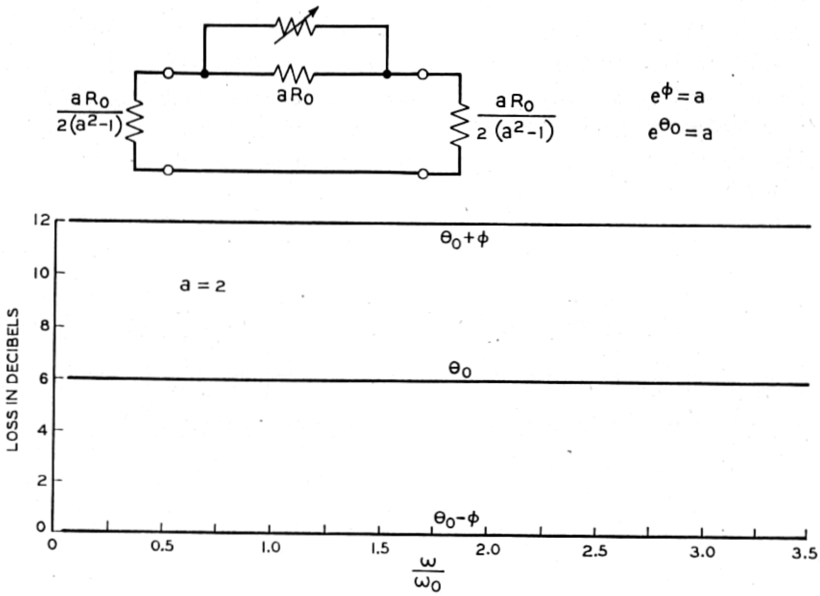

Fig. 3—The simplest type of symmetrical variable equalizer.

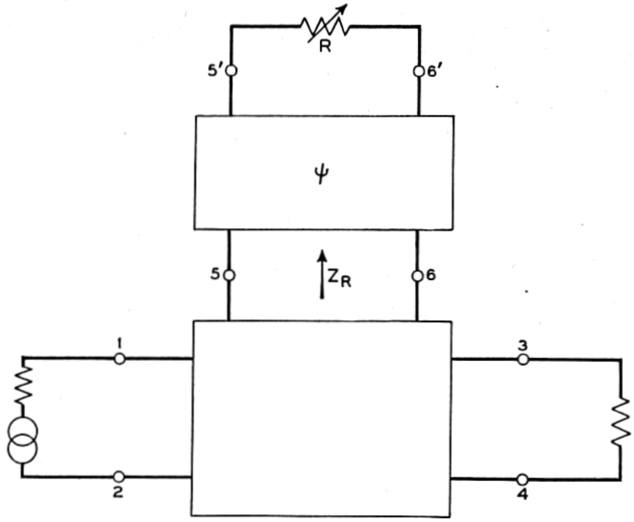

Fig. 4—Adjustment of the variable characteristic by the addition of an auxiliary network.

The effect of the added network is easily understood from the preceding equations. It will be noticed that although these equations were written under the assumption that R is a real quantity, they will still be valid if R is complex. We need therefore merely to replace R by the impedance of the auxiliary network terminated by the variable resistance. If we represent this impedance by Z_R, the appropriate expression is

$$Z_R = R_0 \frac{x + \tanh \psi}{1 + x \tanh \psi}, \tag{7}$$

where ψ is the transfer constant of the added network and x is, as before, the ratio of the variable resistance to R_0. Since reciprocal values of x still correspond to reciprocal values of $\frac{Z_R}{R_0}$, all of the preceding conditions of symmetry in the resulting family of characteristics are maintained. The simplest formulation for the new φ is secured from equation (6). Upon replacing the "x" of this expression by $\frac{Z_R}{R_0}$ we readily find that the equation becomes

$$\theta = \theta_0 + \frac{x - 1}{x + 1} e^{-2\psi} \varphi + \text{higher order terms}. \tag{8}$$

The effect of the added network is therefore merely to multiply the original φ by $e^{-2\psi}$.

An example of the use of this device in conjunction with the network of Fig. 3 is given by Fig. 5. The parameter a was chosen equal to 2, which corresponds to a maximum change in attenuation of 12 db. The auxiliary network, as will be seen, is a conventional bridged-T equalizer. The characteristics of the structure are shown by Fig. 6. The series of straight lines represents the assumed curves, while the circles show the actual computed points. The scale of the drawing is too small to show the differences between the two very clearly. The actual error, at the worst setting and frequency, amounts, however, to about 0.05 db. Of this total, about half is due to the intrinsic distortion of the structure, which in this instance is controlled by the φ^3 term, the effect of the imaginary component of φ being negligible. The remaining half is due to the failure of the bridged-T network to realize the desired ψ characteristic with sufficient precision, and could presumably be eliminated by the addition of more elements to the structure.

Since both θ_0 and ψ can be controlled by auxiliary networks, the structure of Fig. 3 is by itself theoretically sufficient to meet all requirements. There exist, however, a number of other circuits which also

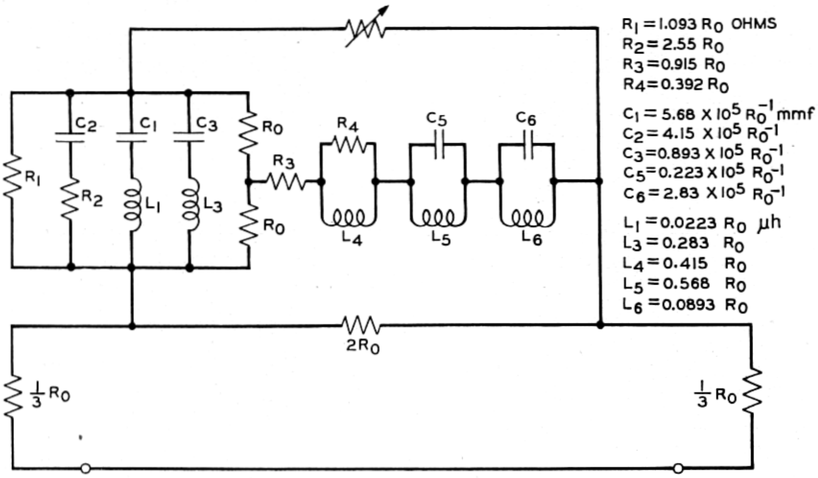

Fig. 5—An equalizer of the type shown in Fig. 3 after the addition of an auxiliary network.

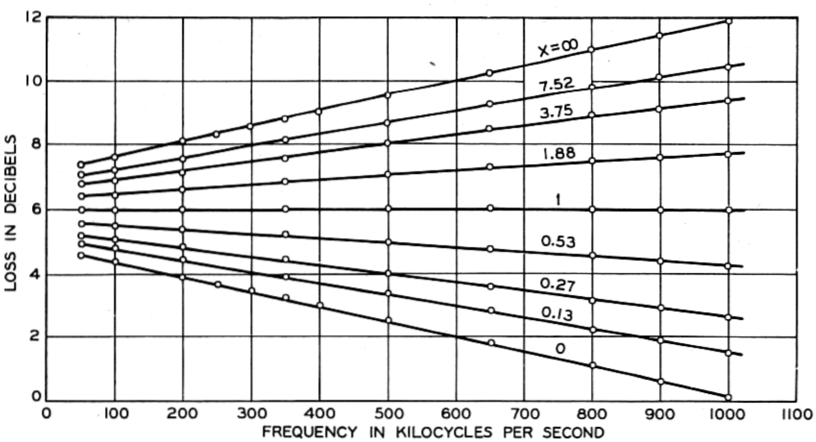

Fig. 6—Characteristics of the structure shown by Fig. 5.

satisfy the condition expressed by equation (3). For example, any structure having the general configuration of Fig. 7 will meet this condition provided the impedances Z_1, Z_2 and Z_3 are so related that when the network is considered as a 4-terminal structure transmitting from a–a' to b–b' it has a constant resistance image impedance equal to R_0 at a–a'.

In contrast to the network of Fig. 3 in which both θ_0 and φ are merely constants, most of the networks which have been found give rather complicated expressions for these two quantities. There are still a number, however, the properties of which are sufficiently simple to be of special interest. The first two are shown by Fig. 8. They have

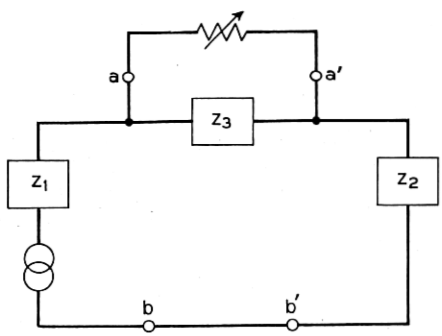

Fig. 7—Diagram to illustrate requirements on a more general form of variable equalizer.

the same design formulae, but one is terminated in a finite resistance at both ends, while an open-circuit, such as the grid of a vacuum tube, must be provided at one end of the other. Illustrative characteristics, drawn on the assumption that the impedance Z_{11} is a simple inductance are shown at the bottom of Fig. 8. It will be seen that φ is still a constant, so that an additional network must be added to control it, exactly as in the structure of Fig. 3. The reference loss θ_0, however, now varies with frequency and can be controlled by the adjustment of Z_{11}. The design impedances have been written as Z_{11} and Z_{21} in accordance with the usual convention for fixed equalizers, to emphasize the fact that the formula for θ_0 is essentially similar to the standard equalizer design formula

$$e^{\theta} = 1 + \frac{Z_{11}}{R}. \tag{9}$$

The choice of an appropriate Z_{11} therefore requires only routine design methods.[3] The same correspondence with the conventional formula will be found to hold also for most of the design equations to be given later.

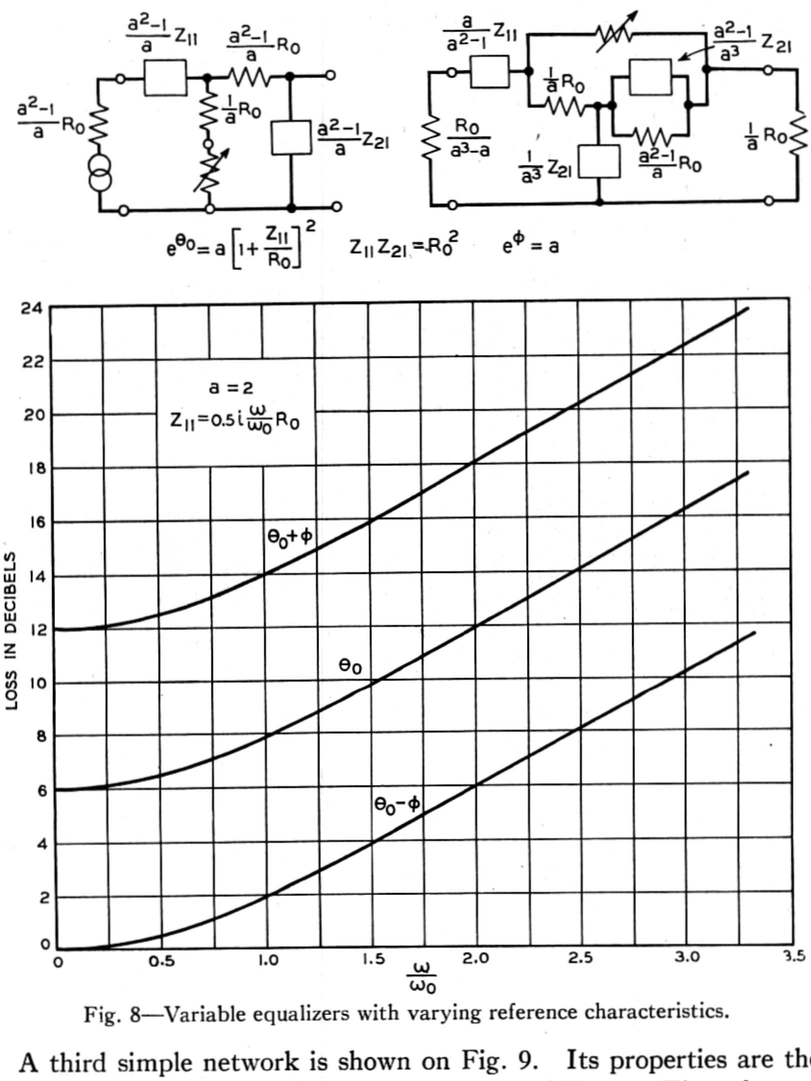

Fig. 8—Variable equalizers with varying reference characteristics.

A third simple network is shown on Fig. 9. Its properties are the converse of those secured from the structures of Fig. 8. The reference loss, θ_0, is now a constant, while φ varies with frequency in a manner

[3] See, for example, "Distortion Correction in Electrical Circuits," O. J. Zobel, *Bell Sys. Tech. Jour.*, July, 1928.

which depends upon the choice of Z. An additional control of ϕ can of course be obtained by the introduction of an auxiliary structure in front of the variable resistance. As in the previous example, the illustrative curves are drawn on the assumption that the characterizing

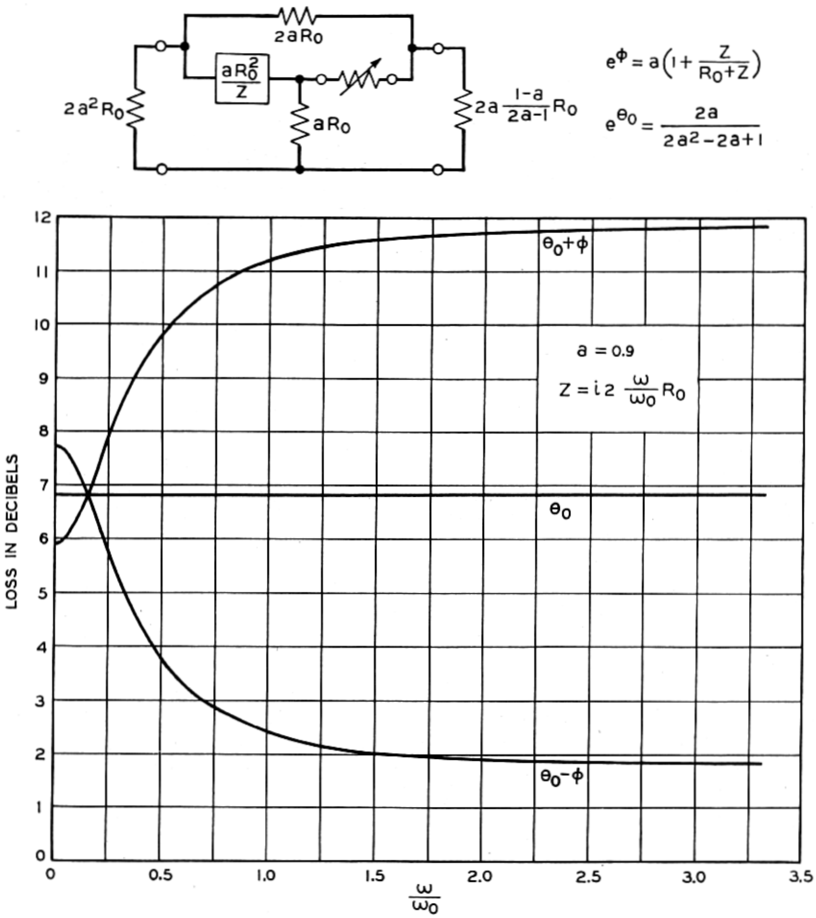

Fig. 9—A variable equalizer requiring only one general impedance branch.

impedance Z is a simple inductance. It will be observed that the curves "see-saw," the attenuation at certain frequencies increasing while that at other frequencies is decreased. This phenomenon depends upon the choice of the parameter a. It disappears when a is assigned either extreme value $\frac{1}{2}$ or 1, and becomes most pronounced at the intermediate value $a = 1/\sqrt{2}$. A similar effect can also be produced in the networks we have already considered since, as equation (8) shows, the variable attenuation will change sign if the phase shift of the auxiliary network is allowed to increase beyond 45°.

In the fourth structure, shown by Fig. 10, both θ_0 and φ are variable.

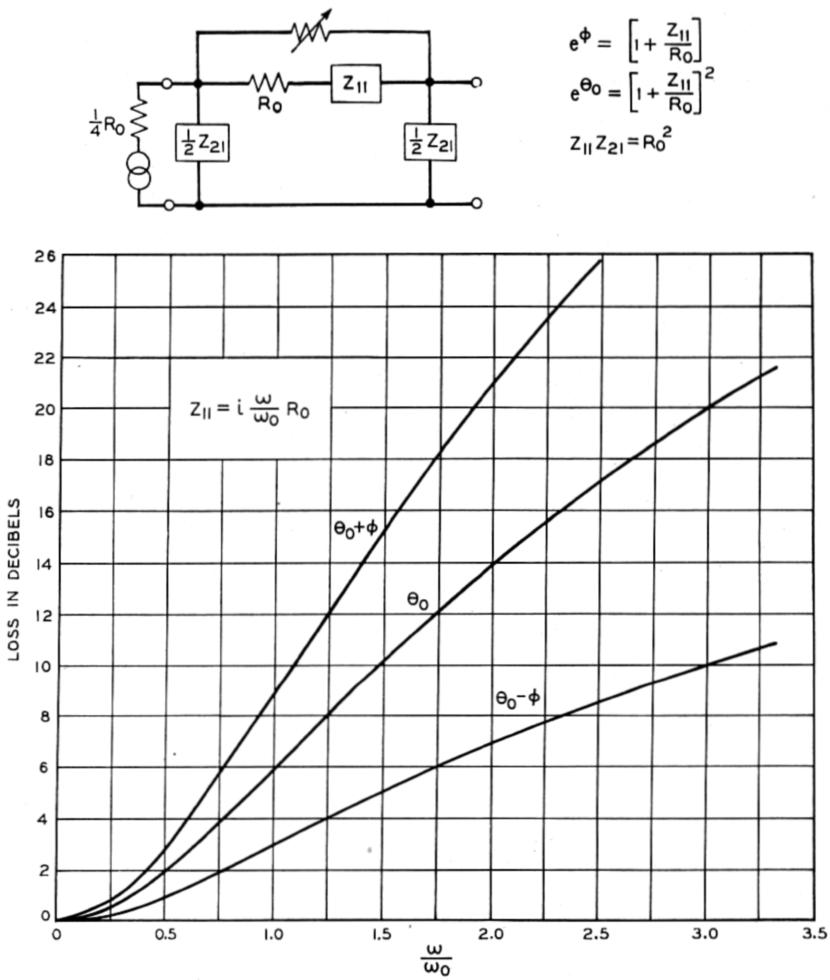

Fig. 10—A variable equalizer with a constant ratio between the reference loss and the variable characteristic.

Their ratio, however, is constant, and the overall loss characteristic of the structure is therefore proportional to some fixed characteristic at all settings of the variable resistance. This property suggests that the network may be of interest for such applications as the equalization of varying lengths of a given transmission line.

The sixth network, shown in Fig. 11, is structurally more elaborate than any which have preceded it. Its design equations are also relatively

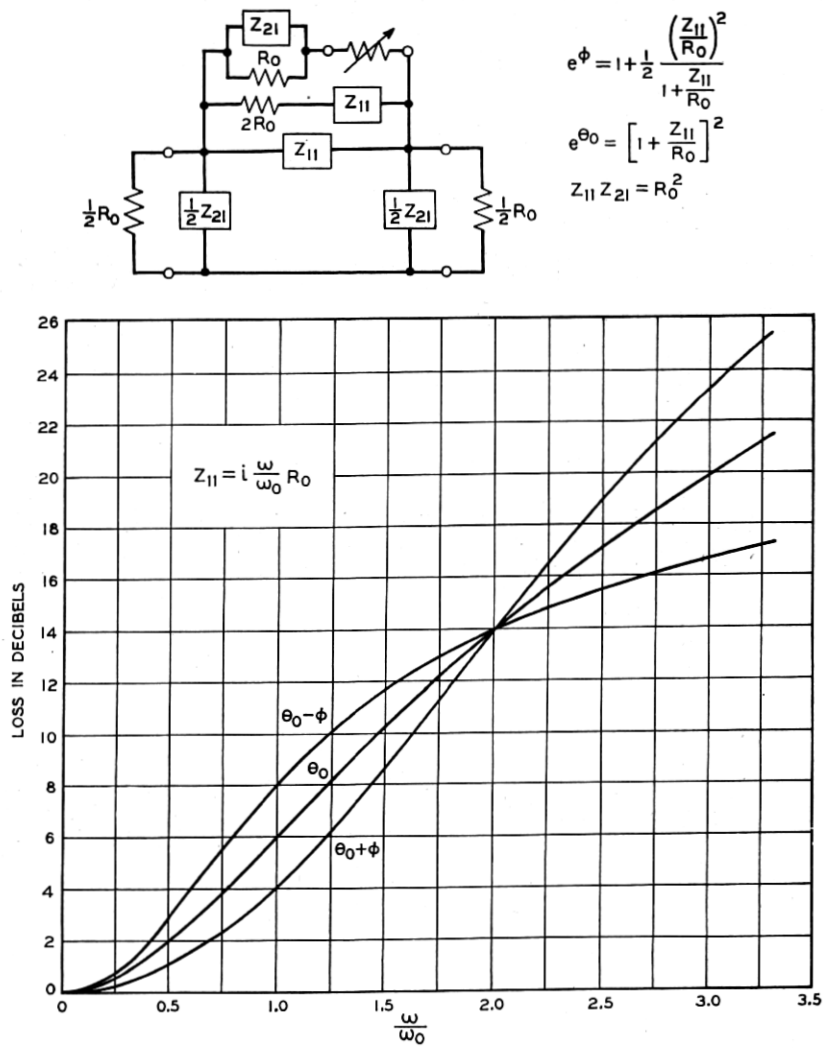

Fig. 11—A variable equalizer with zero loss at one frequency for all settings of the controlling element.

elaborate and difficult to deal with. It possesses, however, the salient property that when $Z_{11} = 0$, perfect transmission of power from one terminating impedance to the other is secured at any setting of the variable resistance. This property suggests that the network may be useful for systems where it is necessary to introduce variable equalization without attenuation of the channels having the lowest signal level.

The final structure is shown in Fig. 12. Its chief point of interest

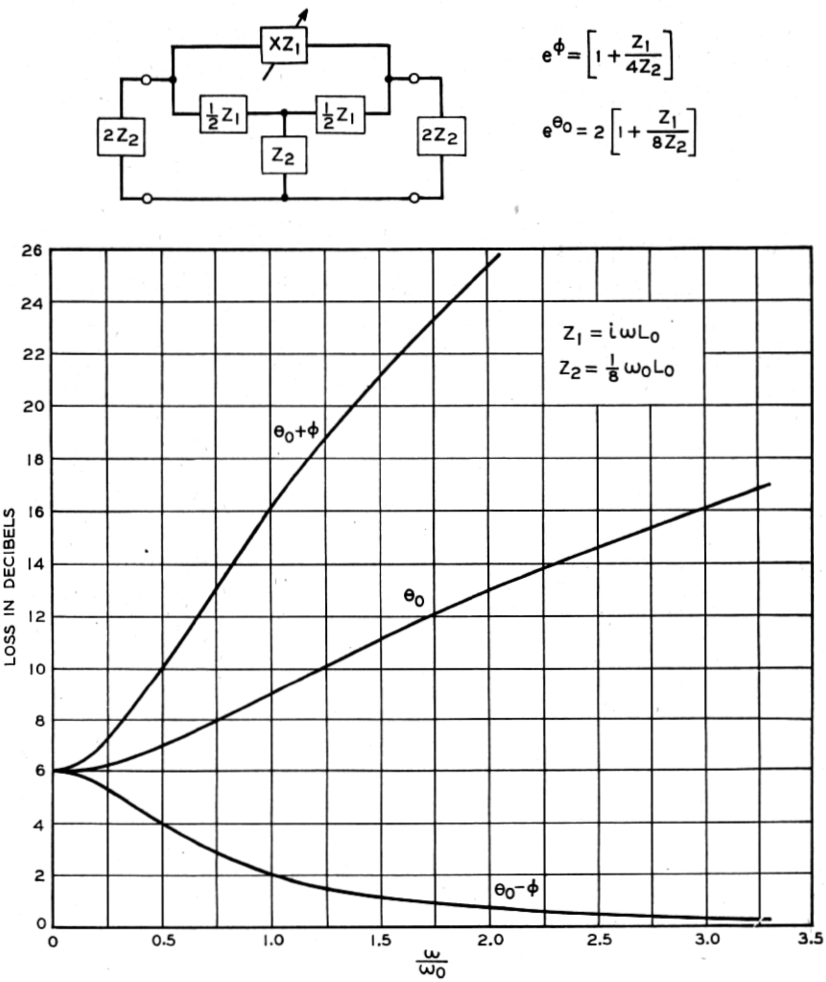

Fig. 12—A variable equalizer adapted to a general controlling impedance.

is the fact that the controlling element, instead of being a variable resistance, is a variable general impedance, which has been labelled xZ_1 in the drawing. In practical cases, of course, Z_1 will ordinarily be a simple resistance, inductance, or capacity. In addition to the variable branch the network also includes two fixed branches proportional to Z_1 and three fixed branches proportional to a second general impedance Z_2. The change from a variable resistance to a variable impedance makes little difference in the analysis. It is merely necessary to replace each R and R_0 by a Z or Z_0 in every equation. So long as equation (3), with the appropriate modification, is satisfied, as it is in this structure, the resulting family of attenuation characteristics will have the same general symmetrical form as those obtained from the resistance controlled devices. A set of curves illustrating this point is shown at the bottom of Fig. 12. They are drawn on the assumption that the Z_1 and Z_2 impedances are respectively inductances and resistances.

The structure of Fig. 12 will still function satisfactorily as a variable equalizer if it is turned inside out so that the $Z_1/2$ impedances become the terminations and the central shunt branch becomes the variable. In this event the present variable impedance must be set at its nominal value. The resulting structure is essentially the inverse of the network of Fig. 12. In the same way, of course, each of the other configurations which have been described can be replaced by its inverse.