# Addendum to "Radio Propagation Over Plane Earth—Field Strength Curves"

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IN the paper of the above title in the January 1937 issue of the *Bell System Technical Journal*, an approximation which was not explicitly pointed out was made in deriving equation (17). A note from Mr. K. A. Norton* of the Federal Communications Commission points out that equation (17) does not give a reasonable result when $\tau = 1$. The explanation is that two terms which are unimportant except near the transmitter when the ground is a perfect dielectric were deleted. The complete equation is

$$\frac{E}{2E} = \frac{W}{1+\tau^2} + \frac{1}{1-\tau^4}\left[\frac{1-\tau e^{(2\pi id/\lambda)(1-1/\tau)}}{2\pi id/\lambda} + \frac{1-\tau^2 e^{(2\pi id/\lambda)(1-1/\tau)}}{(2\pi id/\lambda)^2}\right]. \quad (17)$$

When $\tau = 1$ by virtue of equation (13) $W$ must equal 1/2 and accordingly the first term on the right of equation (17) is 1/4. The second term gives $1/4 + \dfrac{1/4}{2\pi id/\lambda}$ and the last term gives $\dfrac{1/4}{2\pi id/\lambda} + \dfrac{1/2}{(2\pi id/\lambda)^2}$. Hence when $\tau = 1$ equation (17) gives the following relation for the field strength in free space,

$$\frac{E}{2E_0} = \frac{1}{2} + \frac{1/2}{2\pi id/\lambda} + \frac{1/2}{(2\pi id/\lambda)^2},$$

as it should.

The terms added to equation (17) produce oscillations in the curves of Fig. 3 as shown on the following page. For any physical dielectric the conductivity is not zero and the oscillations disappear at the greater distances giving curves like those of the original Fig. 3.

Equation (19) should read

$$E \to \left[\frac{1}{1-\tau^4}\frac{1+\tau^2}{2\pi\tau^2 id/\lambda}\right]\left[1 - \tau^3 e^{\frac{2\pi id}{\lambda}\left(1-\frac{1}{\tau}\right)}\right] 2E_0. \quad (19)$$

This increases the deviation of the second factor on the right from unity but if the ground is not a perfect dielectric the exponential reduces the second factor to unity at the greater distances irrespective of the value of $\epsilon$.

* See the note at the end of " The Propagation of Radio Waves over the Surface of the Earth and in the Upper Atmosphere," *Proc. I.R.E.*, **25**, 1203–1236, September, 1937.

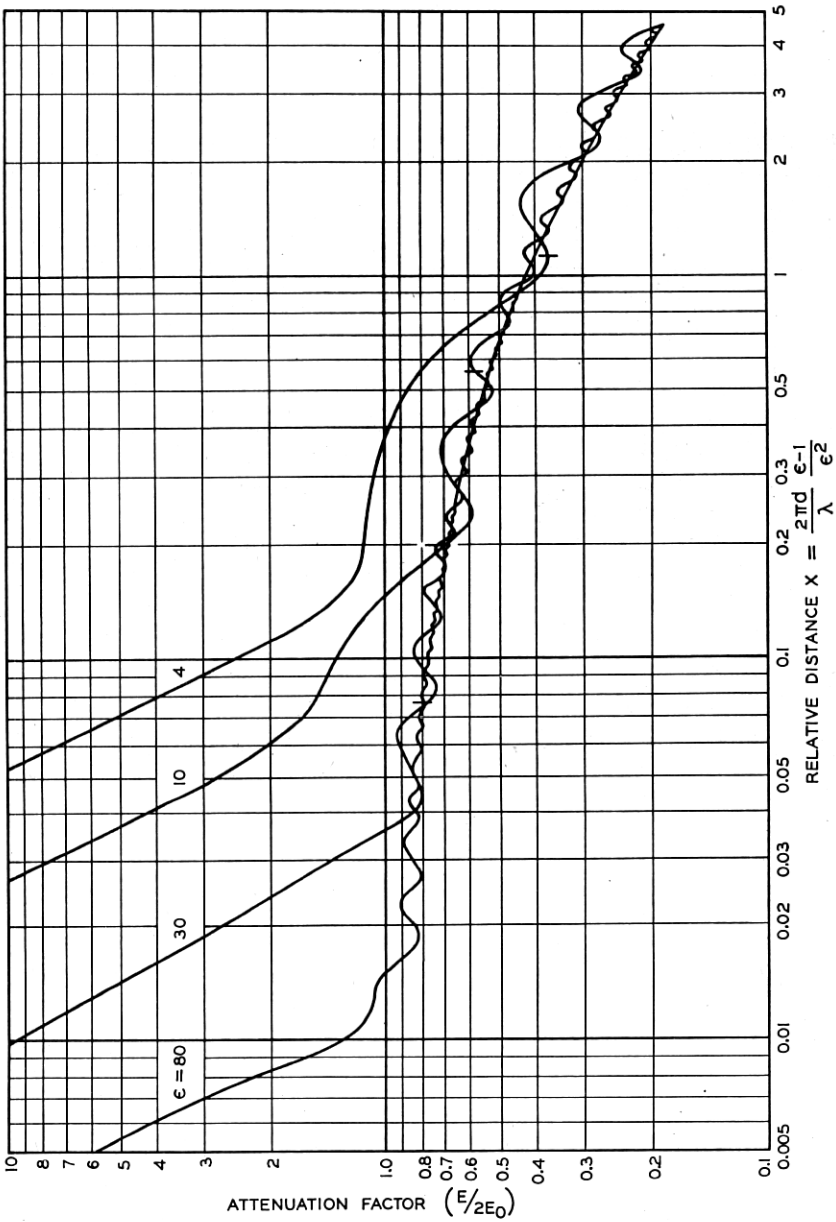


Fig. 3—Attenuation factor for radio propagation over a dielectric plane. The number on each curve gives the value of the dielectric constant to which it applies.

The situation in the immediate vicinity of the antenna is more clearly represented in Fig. 3A in which the attenuation factor is plotted against distance in wave-lengths. This allows inclusion of curves for $\epsilon = 1$ (i.e. for the earth replaced by air) and $\epsilon = \infty$ (which is equivalent to perfectly conducting earth). Comparison of these curves with the broken lines which are replotted from Fig. 2 shows that for distances greater than a wave-length the main effect of using the curves of Fig. 2 is to ignore the presence of the oscillations in the curves. For a perfect dielectric the amplitudes of these oscillations do not decrease below $\pm 1/\epsilon^{3/2}$ even at great distances as can be seen from equation (19). The presence of some conductivity causes these oscillations to be damped out. For example, a $Q$ of 5 reduces the amplitudes of these oscillations within the first four wave-lengths to a value too small to show on the figure.

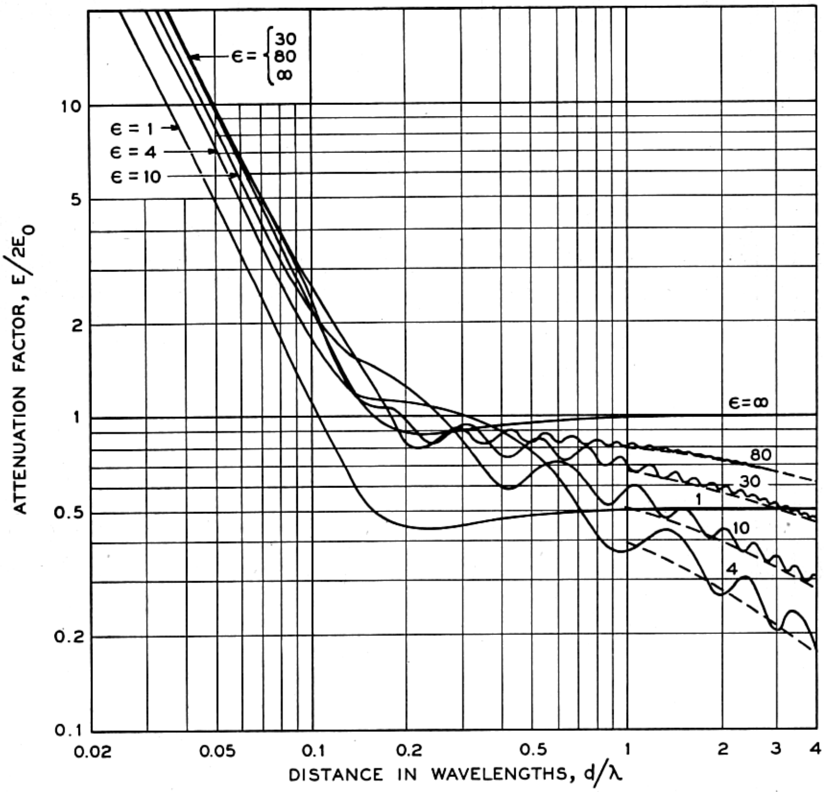


Fig. 3A—Variation of attenuation factor with distance in wave-lengths for transmission over a dielectric plane. For $d/\lambda$ small,

$$E/2E_0 = \left(\frac{\epsilon}{\epsilon + 1}\right) \Big/ \left(\frac{2\pi d}{\lambda}\right)^2.$$

The broken curves are replots of the curve for $Q = \infty$ from Fig. 2.

The second paragraph of the footnote referring to equation (17) should read:

The differential equation given by Wise for $\mathbf{A\Pi}_0$ becomes

$$-\frac{\lambda^2}{4\pi^2}\left(\frac{\partial^2}{\partial d^2}+\frac{1}{d}\frac{\partial}{\partial d}\right)\mathbf{A\Pi}_0=\left(\frac{A}{1+\tau^2}+\frac{1}{1-\tau^4}\left[\frac{1}{2\pi id/\lambda}+\frac{1}{(2\pi id/\lambda)^2}\right]\right)\mathbf{\Pi}_0,$$

when the value of $y=(1+\tau^2)\mathbf{A\Pi}_0$ is substituted in his equation (7) and the result divided by $1+\tau^2$. The $i$ of this paper is equal to $-i$ in Wise's paper. By interchanging $k_1$ and $k_2$ in Wise's equation (7) and proceeding along parallel lines the corresponding equation of $\mathbf{D\Pi}_0=y/(1+\tau^2)$ is found to be

$$-\frac{\lambda^2}{4\pi^2}\left(\frac{\partial^2}{\partial d^2}+\frac{1}{d}\frac{\partial}{\partial d}\right)\mathbf{D\Pi}_0=\left(\frac{D}{1+\tau^2}-\frac{1}{1-\tau^4}\left[\frac{\tau}{2\pi id/\lambda}+\frac{\tau^2}{(2\pi id/\lambda)^2}\right]\right)\mathbf{\Pi}_0.$$

Adding these two relations gives an expression for $\left(\frac{\partial^2}{\partial d^2}+\frac{1}{d}\frac{\partial}{\partial d}\right)\mathbf{\Pi}$, where

$$\mathbf{\Pi}=2(A+D)\mathbf{\Pi}_0,$$

which when substituted in the above equation for $\mathbf{E}$ and the result divided by $2\mathbf{E}_0$, where $\mathbf{E}_0=-240i\pi^2\mathbf{\Pi}_0/\lambda$, gives equation (17). Since $\mathbf{E}_0$ is the inverse distance component of the free space field, this relation for $\mathbf{E}_0$ follows from equation (11).

In the last line of the footnotes on page 51 read $2/(1+\tau^2)$ for $2/(1-\tau^4)$.