Magnetic Generation of a Group of Harmonics*

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A harmonic generator circuit is described which produces a number of harmonics simultaneously at substantially uniform amplitudes by means of a non-linear coil. Generators of this type have been used for the supply of carrier currents to multi-channel carrier telephone systems, for the synchronization of carrier frequencies in radio transmitters, and for frequency comparison and standardization.

A simple physical picture of the action of the circuit has been derived from an approximate mathematical analysis. The principal roles of the non-linear coil may be regarded as fixing the amount of charge, and timing the charge and discharge of a condenser in series with the resistance load. By suitably proportioning the capacity, the load resistance, and the saturation inductance of the non-linear coil, the amplitudes of the harmonics may be made to approximate uniformity over a wide frequency range. The sharply peaked current pulse developed by condenser discharge passes through the non-linear coil in its saturated state and so contributes nothing to the eddy current loss in the core. In this way the efficiency of frequency transformation is maintained at a comparatively high value for the harmonics in a wide frequency band, even with small core structures. The theory has also been adequate in establishing a basis for design, and in evaluating the effects of extraneous input components.

I. Outline of Development

THE use of non-linear ferromagnetic core coils to generate harmonics started with a simple type of circuit due to Epstein [1] which appeared in 1902. Application of the idea was not made to any great extent until it was elaborated by Joly [2] and by Vallauri [3] in 1911. The frequency multipliers thus developed were limited to doublers and to triplers, polarization being required for the doubler. In these, as well as in subsequent developments, single and polyphase circuits were used, and various arrangements were adopted for the structure of the magnetic core and for the circuit, by which unwanted components were balanced out of the harmonic output path. Later developments had to do with improvements in detail, and with the generation of higher harmonics in a single stage and in a series of stages. The applications

* Presented at the Pacific Coast Convention of A. I. E. E., Spokane, Washington, September 2, 1937. Published in *Elec. Engg.* August 1937.

of perhaps greatest importance were to high power, long-wave radio-telegraph transmitters, where the fundamental input was obtained from an alternator. Other applications of the idea of harmonic production by magnetic means have been made in the power and communication fields.[4]

It appears that these circuits were all developed primarily to generate a single harmonic. Comparatively good efficiencies were obtained, values from 60 to 90 per cent being reported for the lower harmonics. The theory of frequency multiplication was investigated by a number of workers, among whom may be mentioned Zenneck[5] and Guillemin.[6] The latter, after analysis which determined the optimum conditions for the generation of any single harmonic, found experimentally that the efficiency of harmonic production decreased as the order of the harmonic increased. He obtained efficiencies of 10 per cent for the 9th harmonic, and 3 per cent for the 13th harmonic of 60 cycles.

Where the circuits are properly tuned and the losses low, free oscillations may be developed. The frequencies of these free oscillations may be harmonic, or subharmonic as in the circuit described by Fallou;[7] they may be rational fractional multiples of the fundamental, or incommensurable with the fundamental, as in Heegner's circuit.[8] The amplitudes of these free oscillations are usually critical functions of the circuit parameters and input amplitudes, and where the developed frequencies are not harmonic, they are characterized by the fact that the generated potentials are zero on open circuit. The theory of the effect has been worked out by Hartley.[9] It is presumably this effect which is involved in the generation of even harmonics by means of an initially unpolarized ferromagnetic core, an observation which has been attributed to Osnos.[10]

II. Circuit Description

The harmonic producer circuit which forms the subject of the present paper differs from those mentioned in that it is designed to generate simultaneously a number of harmonics at approximately the same amplitude.

Harmonics developed in circuits of this type have been used for the supply of carrier currents to various multi-channel carrier telephone systems, for synchronizing carriers used in radio transmitters, and for frequency comparison and standardization. Only odd harmonics are generated by the harmonic producer when the core of the non-linear coil is unpolarized, as is the case here. To generate the required even harmonics, rectification is employed. This is accomplished by means of a well balanced copper oxide bridge, which provides the even harmonics in a path conjugate to the path followed by the odd harmonics.

A typical circuit used for the simultaneous generation of a number of odd and even harmonics at approximately equal amplitudes is shown schematically in Fig. 1. Starting with the fundamental frequency

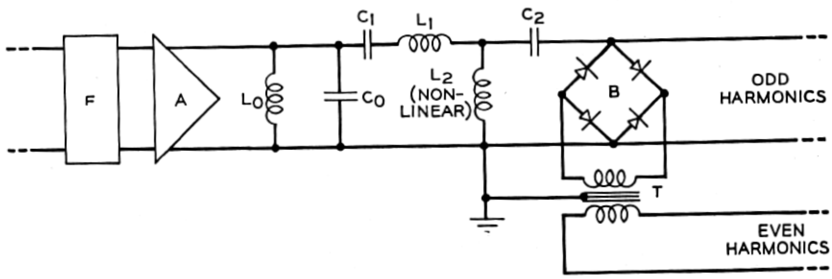

Fig. 1—Circuit diagram of channel harmonic generator.

input, a sharply selective circuit (F) is used to remove interfering components, and an amplifier (A) provides the input to the harmonic generator. The shunt resonant circuit (L_0C_0) tuned to the fundamental serves primarily to remove the second harmonic generated in the amplifier. The elements C_1L_1 are inserted to maintain a sinusoidal current into the harmonic producer proper, as well as to tune out the circuit reactance.

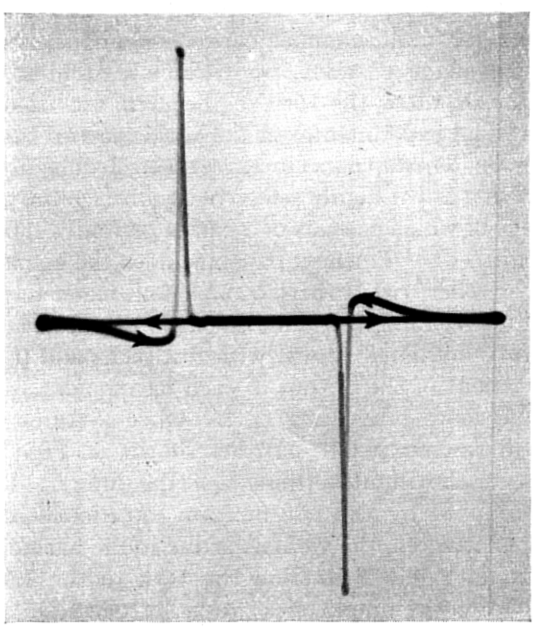

Fig. 2—Cathode ray oscillogram of output current wave form with fundamental input current as abscissa.

L_2 is a small permalloy core coil which is operated at high magnetizing forces well into the saturated region. The circuit including L_2, C_2, and the load impedance, which is practically resistive to the desired harmonics, is so proportioned that highly peaked current pulses rich in harmonics flow through it. Two such pulses, oppositely directed, are produced during each cycle of the fundamental wave, the duration of each being a small fraction of the fundamental period. The typical output wave shown in Fig. 2 was obtained by means of a cathode ray oscillograph, the ordinate representing the current in the load resistance, and the abscissa representing the fundamental current into the coil. The desired odd harmonics are selected by filters connected across the input terminals of the copper oxide bridge. The even harmonics are obtained by full-wave rectification in the copper-oxide bridge. They appear at the conjugate points of the bridge, and are connected through an isolating transformer to the appropriate filters. Thus the harmonics are produced in two groups, with the even harmonics separated from the odds to a degree depending largely upon the balance of the copper-oxide bridge, as well as upon the amount of second harmonic passed on from the amplifier. In this way the required discrimination properties of any filter against adjacent harmonics are reduced to the extent of the balance.

A particular application of the circuit described above to the generation of carriers for multi-channel carrier telephone systems uses a fundamental frequency of 4 kc., from which a number of harmonics are developed. Of these the 16th to the 27th are used as carriers. A photograph of an experimental model of this carrier supply system * is shown in Fig. 3. The top panel includes an electromagnetically driven tuning fork serving as the highly selective circuit (F), the amplifier (A), the output stage of which consists of a pair of pentodes in push-pull, and the tuned circuit L_0C_0. The next panel includes the elements L_1C_1, L_2, C_2, B, and T, together with a thermocouple and meter terminating in a cord and plug for test and maintenance purposes. The last two panels include the twelve harmonic filters, with test jacks and potentiometers for close adjustment of the output of each harmonic.

A few of the more interesting performance features are given in Fig. 4. The harmonic power outputs shown in Fig. 4*a* represent measurements at the input terminals of the filters. The variation observed is produced by the non-uniform impedances of the filters. When these are corrected, the variations due to the harmonic generator proper are less than $\pm$ 0.2 db from the 16th to the 27th harmonic. Outside this region the amplitudes gradually decrease to the extent

* Developed by J. M. West.

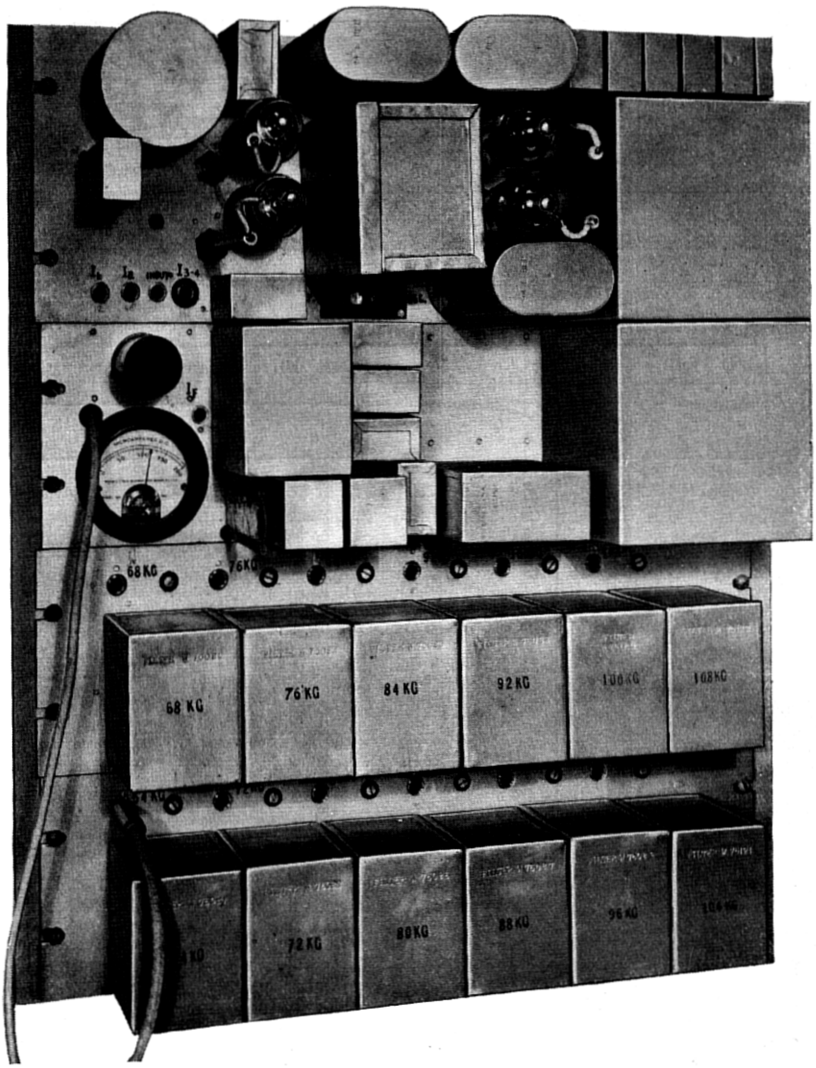

Fig. 3—Carrier supply unit, furnishing twelve harmonics of 4 kc. (experimental model).

of 4 db at the 3d and 35th harmonics, and 11 db at the fundamental and the 61st harmonic. The variation of harmonic output with change of amplifier plate potential is given for the two harmonics indicated in Fig. 4*b*. Figure 4*c* shows the 104 kc. output as a function of the 4 kc. input. Arrows are used to indicate normal operating points. The input amplifier is operated in an overloaded state so that beyond a critical input, the fundamental output of the amplifier and

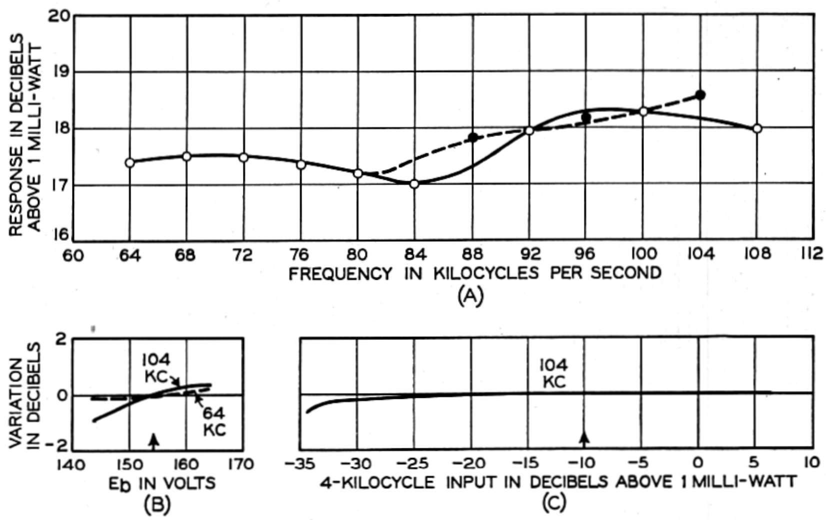

Fig. 4—Performance curves of channel harmonic generator. (*A*) Harmonic outputs; (*B*) Variation of 16th and 26th harmonics with amplifier plate potential; (*C*) Variation of 26th harmonic with fundamental input.

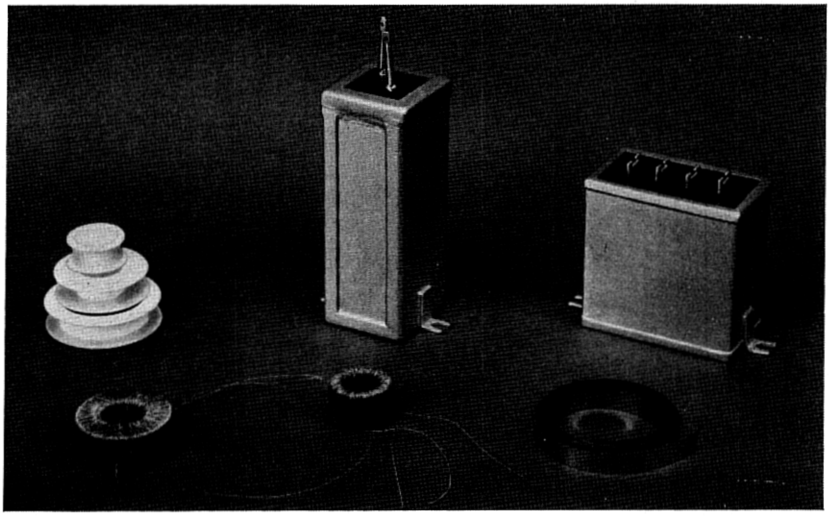

Fig. 5—Construction of experimental non-linear coils used for harmonic generation, showing core forms, magnetic tape, wound coils, and assembled units.

the harmonic output corresponding are but little affected by change of input amplitude. With a linear amplifier the harmonic output current would vary roughly as the four-tenths power of the input current.

Another application involving higher frequencies has been made to the generation of the so-called "group" carriers used in conjunction with a coaxial conductor.[11] There odd harmonics of 24 kc. from the 9th to the 45th are used. The circuit differs from Fig. 1 in that the copper oxide bridge is omitted, and the non-linear coil is provided with two windings to facilitate impedance matching. The performance of an experimental model is similar to that of the generator described above. A notion of the physical size and construction of the non-linear coils used may be had from the photographs of Fig. 5.

In both applications the required harmonics are generated at amplitudes high enough to avoid the necessity for amplification.

III. Theory of Operation

The analysis of operation of the harmonic generating circuit described above meets with difficulties, since a high degree of non-linearity is involved in working the coil well into its saturated region.

To avoid these difficulties, an expedient is adopted by which the hysteresis loop is replaced by a single-valued characteristic made up of connected linear segments [6] as shown in Fig. 6*b*. It is then possible to formulate a set of linear differential equations with constant coefficients, one for each linear segment. The solutions are readily arrived at and may be pieced together by imposing appropriate conditions at the junctions, so that a solution for the whole characteristic is thereby obtained. From this solution the wave form of current or voltage associated with any circuit element may be calculated. Resolution of the wave form into components may then be accomplished by an independent Fourier analysis.

The assumed *B-H* characteristic of Fig. 6*b* is made up of but three segments. While it is manifestly a naive representation of a hysteresis loop, it will be shown by comparison with experiment that the main performance features of harmonic generators may be reproduced by this crude model.

It will be noted on Fig. 6*b* that the differential permeability of the assumed non-linear core, a quantity proportional to dB/dH, takes on one of two values, determined by the absolute value of the magnetizing force. These are designated by μ in the permeable region and μ_s in the saturated region. The corresponding inductances are L_{20} and L_{2s}, L_{20} being many times greater than L_{2s}. The values of current through the coil at which the differential inductance changes are designated $\pm I_0$,

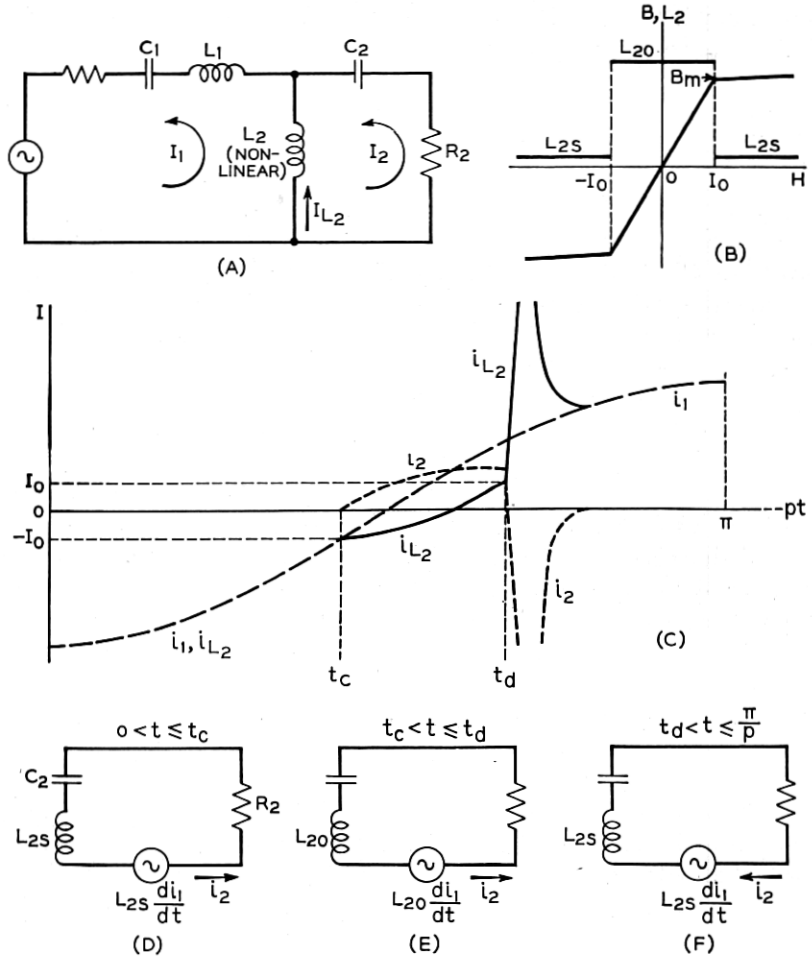

Fig. 6—Diagrams illustrating operation of the harmonic generator. (*A*) Harmonic generator circuit; (*B*) Differential inductance and flux density of assumed non-linear coil as functions of magnetizing force; (*C*) Variation with time of currents in primary and secondary meshes, and in non-linear coil; (*D*) (*E*) (*F*) Equivalent circuits of the harmonic generator for the three time intervals indicated.

corresponding to the magnetizing forces $\pm H_0$. With this simple representation of the non-linear inductance, the operation of the circuit shown in Fig. 6*a* will be described over a complete cycle of the fundamental input wave.

The current flowing in the input mesh is made practically sinusoidal by tuning L_1, C_1. If now we start at the negative peak of the sinusoidal input current of amplitude I_1 and frequency $p/2\pi$, the non-linear coil is worked in the saturated state where its inductance L_{2s} is low. Since the resistance of the winding is small, the potential drop across the coil is correspondingly small. The current i_2 which charges the condenser C_2, assuming the latter to have zero charge at the start, is therefore negligible as indicated in Fig. 6*c*. This state of affairs is maintained until the current through L_2 reaches the value $-I_0$, at time t_c. At this point the inductance of the coil increases suddenly to L_{20} and the voltage across the coil tends to increase. Hence the current i_2 increases and C_2 is charged much more rapidly than in the preceding interval. Charging continues until the current through the coil increases through I_0 at time t_d. At that time, the coil inductance returns to the low saturation value L_{2s}, and the potential across the coil decreases. The condenser potential is no longer opposed by the potential drop across the coil and the condenser discharges through R_2 and L_{2s}; i_2 reverses its direction, maintaining the coil in the saturated region. The form and duration of the sharply peaked discharge pulse characteristic of this type of harmonic generator are determined by the values of the elements just mentioned. The resistance, capacity, and saturation inductance effectively in circuit are adjusted to permit the current to rise to a high maximum, to damp the pulse, and to shorten the pulse duration to the point at which the highest harmonic required reaches the desired amplitude. Under the working conditions which will be assumed in the following, this insures that the pulse dies away before the end of the half-cycle as shown in Fig. 6*c*. At that time the currents and potentials are the same, except for reversals of sign, as those at the start, so that the current wave consists of an alternating succession of these pulses. Equivalent circuits for the three respective time intervals of a half-cycle are shown in Figs. 6*d*, 6*e*, 6*f*. The similarity of the load current wave form derived above to that experimentally observed and shown in Fig. 2, is to be noted.

The course of events described above parallels closely conclusions drawn from the mathematical analysis. This picture attributes to the coil L_2 a sort of switching property which permits the condenser C_2 in the load circuit to be charged and discharged alternately. The charge starts when the large inductance L_{20} is switched across the primary and secondary meshes, thus permitting energy to flow from the primary circuit into the condenser C_2. This corresponds to that part of the wave described above during which the load current slowly rises as the charge accumulates on C_2. Discharge starts when the large inductance L_{20} is switched out and the much smaller inductance L_{2s} is switched in. This sharply reduces the voltage across L_2, and the condenser is discharged through the load resistance and the saturation inductance.

During this interval the secondary circuit is practically isolated from the primary. The switching process is sustained by the alternations of the sinusoidal primary current and is periodic, as we have seen, since similar conditions exist at the start of each pulse. The times at which switching occurs are those at which the current through the coil passes through the critical values ($\pm I_0$) where the inductance changes.

Since the narrow discharge pulse provides the principal contribution to the higher harmonics in which we are interested, and since this discharge takes place in the secondary independently of the primary, the elements of the secondary mesh during discharge determine the form of the output spectrum. From this viewpoint we may regard the condenser as the source of energy for these harmonics and hence as a possible location for equivalent harmonic generator e.m.f.'s. In this light, the discharge circuit becomes a half-section of low-pass filter terminated in resistance R_2, with L_{2s} as the series element and C_2 as the shunt element.

IV. Quantitative Results of Analysis

To connect the three solutions which hold for the three linear regions of the *B-H* characteristic, conditions at the junctions are introduced which lead to transcendental equations. These may be solved graphically when definite values are assigned to the circuit parameters. From these may be obtained the maximum value Q_m of charge on C_2 which is reached at the end of the charging stage.

By plotting a representative group of these final charges over a range of parameters ordinarily encountered, an empirical equation has been deduced for Q_m as follows:

$$Q_m = \sqrt{2}\,\frac{I_1}{p}\left(\frac{pL_{20}}{R_2}\right)^{0.75}(pC_2R_2)^{0.65}\left(\frac{I_0}{I_1}\right)^{0.6}. \tag{1}$$

For the usual operating conditions the narrow peaked discharge part of the current pulse is most important in the determination of the higher harmonics (say beyond the 9th) with which we are concerned here. The charging interval then may be neglected in calculating the higher harmonics. The form of the discharge pulses is determined by the parameters pC_2R_2 and k, where

$$k = L_{2s}/R_2^2C_2.$$

The familiar criterion for oscillation in a series circuit containing inductance, capacity and resistance may be expressed in terms of k. If $k > \frac{1}{4}$, the discharge is an exponentially decaying oscillation; if

$k \leq \frac{1}{4}$, the discharge is an exponentially decaying pulse. This last condition is the one assumed in the description of operation given above.

If the discharge is oscillatory, and if further the second peak is large enough, the current through the coil may become less than I_0 during the discharge interval. Thus L_2 will return to its larger value, and recharging of the condenser will result. This process may lead to large and undesired variations in the amplitudes of the harmonics. To maintain the frequency distribution as uniform as possible over the frequency range of interest, the circuit parameters are usually adjusted so that recharging does not occur.

Harmonic analysis shows that the nth harmonic amplitude under the above assumptions is given by

$$I(n) = \frac{(2/\pi)pQ_m}{\sqrt{1 + (1 - 2k)(npC_2R_2)^2 + k^2(npC_2R_2)^4}}, \tag{2}$$

where n is odd. This expression neglects the contributions due to the charging stage, which are usually small for harmonics higher than the ninth.

The corresponding harmonic power output is

$$W_n = \frac{I(n)^2R_2}{2} = \frac{W_0}{1 + (1 - 2k)(npC_2R_2)^2 + k^2(npC_2R_2)^4}, \tag{3}$$

where W_0 is a convenient parameter which does not vary with n and hence serves as an indication of the power of the output spectrum. It is related to W, the total power delivered to the load resistance, by the equation,

$$W_0 = \frac{4}{\pi} pC_2R_2W.$$

For purposes of calculation, W_0 may be found from (1) and (2) to be

$$W_0 = \frac{10^{-7}}{\pi^2} pB_mAdH_1 \left(\frac{H_0}{H_1}\right)^{0.2} (pC_2R_2)^{1.3} \left(\frac{pL_{20}}{R_2}\right)^{0.5} \text{ watts}, \tag{4}$$

where

$$L_{20} = \frac{4N^2A\mu 10^{-9}}{d}, \quad H_1 = 0.4NI_1/d,$$

and N is the number of turns wound on the toroidal core of diameter d cm. and cross-sectional area A cm.^{2}

In Fig. 7 the power spectrum is shown by plotting W_n in db above or below W_0 as a function of npC_2R_2 for several values of k. These curves illustrate the degree of uniformity obtainable in harmonic am-

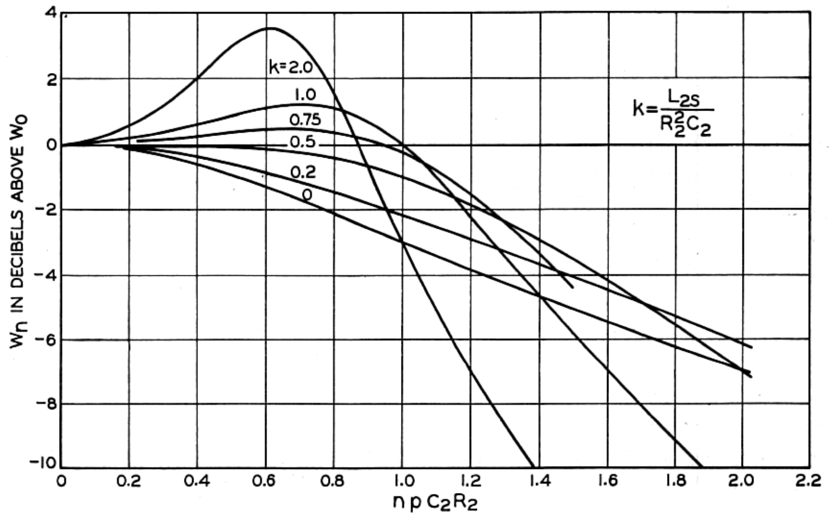

Fig. 7—Harmonic power spectrum plotted from eq. (2) as function of npC_2R_2 with k as parameter.

plitudes under different conditions. It may be shown from (3) that W_n has a maximum with respect to n when k is greater than $\frac{1}{2}$, if

$$npC_2R_2 = \frac{1}{k}\sqrt{k - \tfrac{1}{2}},$$

and that its value at this point is

$$(W_n)_{max} = W_0k^2/(k - \tfrac{1}{4}).$$

A number of relations may be derived from these equations which are useful for design purposes. Thus the form of harmonic distribution is fixed by k and pC_2R_2. The output power for a given magnetic material worked at a given fundamental magnetizing force then depends solely upon the volume of core material. Finally, the impedance is fixed by the number of turns per unit length of core. If the impedances desired for primary and secondary circuits differ, separate windings may be used for each circuit.

V. Calculated and Observed Performance

In order to make practical use of the results given above, we need some basis for deriving the assumed parameters of the non-linear coil from the physical properties of the magnetic materials used in harmonic producers.

The fact that the actual magnetization curve is a loop instead of a single-valued curve as assumed requires increased power input to the circuit to provide for the hysteresis and eddy losses in the core. Other than this, the principal remaining effect of the existence of a loop is a lag in the time at which the pulses occur, an effect which is of no great moment in determining the form or magnitude of the resulting pulses.

The next point requiring consideration is the effect introduced by the assumed abrupt change of slope contrasted to the smooth approach to saturation actually observed. While no rigorous comparisons can be drawn, the effect of the more gradual approach to saturation was approximated analytically by introducing an additional linear segment between the permeable region and each saturated region of the *B-H* characteristic, at a slope intermediate between the two, so as to form a *B-H* characteristic of five segments in place of the original three. The solutions for these two characteristics were found to yield negligibly small differences in the amplitudes of the higher harmonics. It was inferred from this result that no substantial change would be introduced by a smooth approach to saturation.

Finally, the actual *B-H* characteristic has a slight curvature in the saturated region, while the analysis considered a small linear variation. A rough approximation for the effect of this curvature, which leads to fair agreement with experiment, consists in taking for L_{2s} the average of the actual slope, from its minimum value reached during the discharge peak down to the point at which the slope is one-tenth maximum. To this is added the linear inductance contributed by the dielectric included within the winding.

To summarize then, the harmonic outputs obtained from the analysis with the assumed *B-H* characteristic may be brought into line with experimental observations by the introduction of quantities obtained from actual *B-H* loops at appropriate frequencies and magnetizing forces. In these the maximum slope found on the loop is taken for L_{20}, the average slope over the saturated region is taken for L_{2s}, and the energy corresponding to the area of the real *B-H* loop must be added to that originally supplied the harmonic generator input.

A comparison between measured and calculated harmonic distributions obtained with a 4-kc. fundamental input is shown in Fig. 8. In this case the harmonic distributions were measured for four different values of the secondary condenser C_2 as shown by the plotted points. The power output of each harmonic is plotted in terms of the quantity npC_2R_2. Calculated values are indicated by dashed lines. It is observed that while the agreement between calculation and experiment

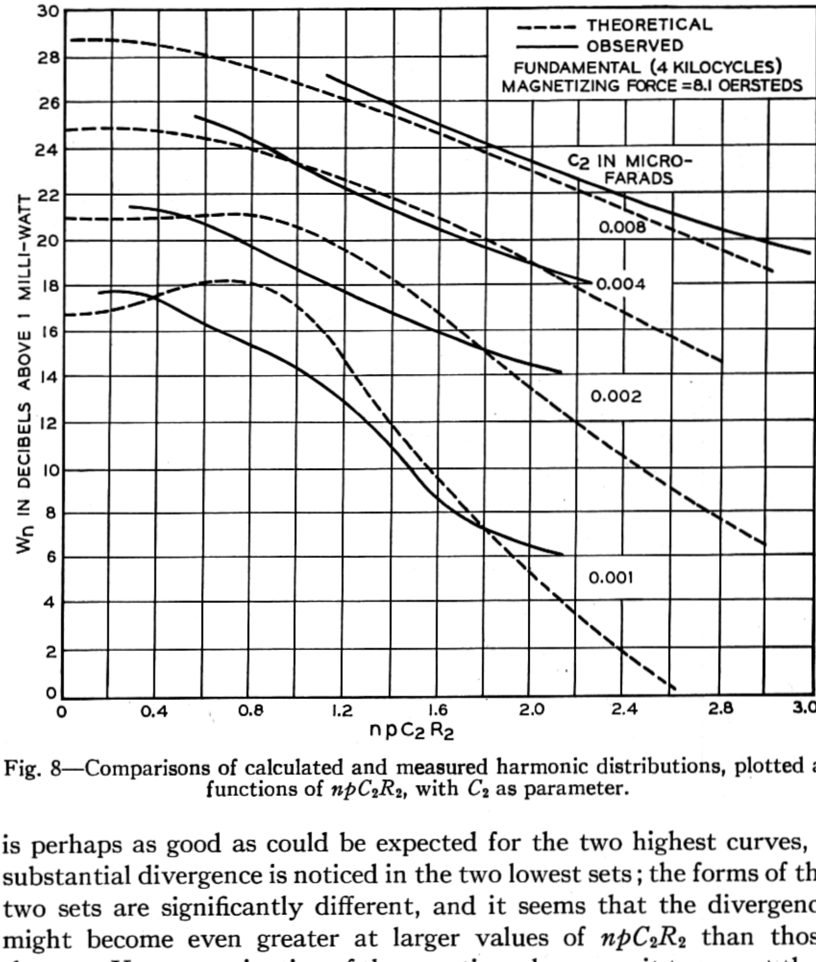

Fig. 8—Comparisons of calculated and measured harmonic distributions, plotted as functions of npC_2R_2, with C_2 as parameter.

is perhaps as good as could be expected for the two highest curves, a substantial divergence is noticed in the two lowest sets; the forms of the two sets are significantly different, and it seems that the divergence might become even greater at larger values of npC_2R_2 than those shown. Upon examination of the equations, however, it turns out that the conditions existing for the lowest pair of curves are just those for which recharging occurs, so that the conditions for which the equations were framed hold no longer. The calculated distributions might be expected to be too low for the higher harmonics, since we have taken an average value for the saturation inductance. This means that the peak of the discharge pulse will be sharper than that calculated, with a corresponding effect upon the higher harmonics.

Another comparison between calculated and observed values is shown in Fig. 9 for a fundamental input of 120 kc. with two values of resistance load. Fair agreement is observed over the greater part of the frequency range which extended to 5 MC. The distribution curve for the smaller resistance load undulates as the load resistance is reduced, since multiple oscillations and recharging are then promoted, in consequence of which the output power tends to become concentrated in definite bands of harmonics. In general, agreement within a few db is found over a wide range of circuit parameters when working into a resistance load, provided that recharging does not occur.

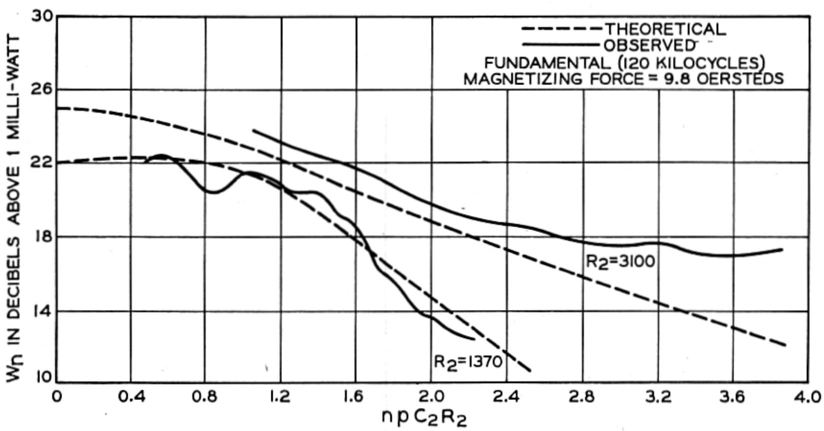

Fig. 9—Comparisons of calculated and measured harmonic distributions plotted as functions of npC_2R_2, with R_2 as parameter.

When the resistance termination is replaced by a bank of filters as it is in practice, the resistance termination is approximated over the frequency band covered by the filters. Where the band is wide the results obtained do not differ greatly from those with the pure resistance load, but when only a few harmonics are taken off by filters and the impedances to the other harmonics of large amplitude vary widely over the frequency range, then the wave form of the current pulse is substantially altered, with corresponding effect upon the frequency distribution, and the calculations for a pure resistance termination do not apply.

A difficulty sometimes arises in getting a desired value of fundamental current into the coil. Under certain circuit conditions the current amplitude is found to change rapidly as the input voltage is smoothly varied. This phenomenon has been described by various terms such as Kippeffekt, ferro-resonance, and current-hysteresis.[13] If the operating point is located close to one of these discontinuities, the fundamental input and harmonic output may vary widely with small changes in supply potentials and circuit parameters. This troublesome source of variation may be avoided in a number of different ways, of which the simplest is to increase the resistance of the resonant mesh. In the present case this is effectively accomplished without sacrificing efficiency by using pentodes, which have high internal resistances, in the amplifier stage connected to the resonant mesh.

The efficiency of power conversion from fundamental to harmonics may be found from the fundamental power input to the circuit, as derived from measurements on a cathode ray oscillograph, and from the total harmonic output measured by means of a thermocouple. The maximum efficiency obtainable with the low-power circuits described in the second section is in the neighborhood of 75 per cent, and decreases with increasing fundamental frequency because of the increased dissipation due to eddy currents. It should be noted that this figure does not include losses in the primary inductance L_1. When only a few harmonics are used, the efficiency of obtaining this useful power naturally drops to a much lower value, which for the particular cases mentioned in the second section, is between 15 and 25 per cent.

VI. Effect of Extraneous Components

In any practical case the fundamental input to the harmonic producer is accompanied by extraneous components introduced by crosstalk, by modulation, or by an impure source. Thus if the fundamental is derived as a harmonic of a base frequency, small amounts of adjacent harmonics will be present. Or if the amplifiers are a.-c. operated, side-frequencies are produced differing from the fundamental by 60 cycles and its multiples. Extraneous components of this sort in the input modulate the fundamental and produce side-frequencies about the harmonics in the output. When the harmonics are used as carriers, the accompanying products must be reduced to a definite level below the fundamental if the quality of the transmitted signal is to be unimpaired. The requirements imposed by this condition can be calculated by simple analysis, the results of which agree rather well with experimental values.

The method of analysis used is to consider the extraneous component at any instant as introducing a bias [14] to the non-linear coil. The primary effect of a small bias (b) is to shift the phase of the discharge pulse by $\mp b/H_1$ radians, H_1 being the amplitude of the fundamental magnetizing force. The sign of the shift alternates so that intervals between pulses are alternately narrowed and widened.

The effect of this shift on the harmonics produced may be found by straightforward means in which the amplitude of any harmonic is expressed in terms of the bias. Hence when the extraneous component or components vary with time, the sidebands produced may be evaluated when the bias is expressed by the appropriate time function.

If the bias is held constant, the wave is found to include both odd and even harmonics, the amplitudes of which are given by

$$\left.\begin{aligned} I_n &= I(n)\,|\cos nb/H_1|, \quad (n \text{ odd}), \\ &= I(n)\,|\sin nb/H_1|, \quad (n \text{ even}), \end{aligned}\right\} \tag{5}$$

$I(n)$ being the harmonic distribution in the absence of bias as given by eq. (2).

If the extraneous input component is sinusoidal, we have

$$b = Q \sin (qt + \varphi). \tag{6}$$

Substituting this expression for b in the equation for the harmonic components yields odd harmonics of the fundamental, and modulation products with the angular frequencies $mp \pm lq$, which may be grouped as side-frequencies about the odd harmonics. The amplitude of the nth (odd) harmonic is

$$I_n = I(n)\left|J_0\left(\frac{nQ}{H_1}\right)\right|, \tag{7}$$

and the amplitude of the modulation product $mp \pm lq$ is

$$I_{m.\,\pm l} = I(m)\left|J_l\left(\frac{mQ}{H_1}\right)\right|, (m + l \text{ odd}), \tag{8}$$

where $J_l(x)$ is the Bessel function of order l.

Considering the side-frequencies about the nth harmonic, the largest and nearest of these are $(n + 1)p - q$ and $(n - 1)p + q$, n being odd. The ratio of the amplitudes of either side-frequency to the nth harmonic is

$$\frac{I_{n\pm 1,\,\mp 1}}{I_n} = \left|\frac{J_1[(n \pm 1)Q/H_1]}{J_0(nQ/H_1)}\right|, \tag{9}$$

on the assumption that the harmonic distribution in the neighborhood of n is uniform so that $I(n \pm 1) \doteq I(n)$. If the arguments of the Bessel functions are less than four-tenths, a good approximation to the right member of eq. (9) is $(n \pm 1)Q/2H_1$. Hence with sufficiently small values of interference, the sidebands produced are proportional to the amplitude of the interference, and increase linearly with the order of the harmonic. These relations apply to harmonic generators which produce sharply peaked waves in general, and are not peculiar to the magnetic type.

Neighboring modulation products involving the interfering component q more than once have much smaller amplitudes in normal circumstances than the product considered above. Because of the tuning in the input mesh, interfering components far removed in frequency from the fundamental are greatly reduced and the most troublesome interference is likely to be close in frequency to the fundamental.

It may be noted that where the interference is produced by amplitude modulation of the fundamental, so that two interfering components enter the input, the distortion produced may be approximated by doubling the amplitudes of the side-frequencies produced by one of the interfering components. If the disturbance is the second harmonic of the fundamental, the effect is nearly the same as that for constant bias, and the relations (5) may be used if b is taken as the amplitude of the second harmonic magnetizing force.

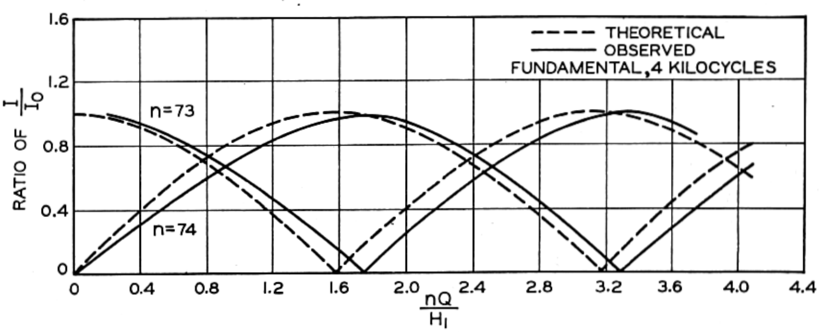

Fig. 10—73rd and 74th harmonic amplitudes as functions of direct current flowing through non-linear coil. Ordinate is ratio of harmonic amplitude with bias indicated, to that of 73rd harmonic with zero bias. Abscissa is harmonic number multiplied by the ratio of bias to fundamental. Dashed lines calculated from eq. (5), full lines measured.

To illustrate the effects of d.-c. bias, Fig. 10 shows the amplitudes of the 73d and 74th harmonics of 4 kc. as functions of the parameter nQ/H_1. The agreement between measured and calculated values indicates that the most important effects of bias have been included in the simple analysis.

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