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## Resistance Compensated Band-Pass Crystal Filters for Use in Unbalanced Circuits

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In this paper are discussed several types of crystal band-pass filters which can be used in unbalanced circuits. These types of filters are all resistance compensated, i.e., the resistances associated with the filter elements are in such a position in the filter that they can be effectively brought to the ends of the filter and combined with the terminal resistances with the result that the dissipation produces an additive loss for the filter characteristic and does not affect the sharpness of cut-off attainable. It is shown that all these types of networks can be reduced to three lattice types of crystal filters, and the formulae for these three networks are given. A comparison is given between the characteristics obtainable with resistance compensated crystal and electrical filters and a conclusion regarding their comparison given by V. D. Landon [4] is shown to be incomplete.

### I. Introduction

In a recent paper [1] a description is given of a number of wave filters employing quartz crystals as elements. Most of these filters were of the lattice type and hence were inherently balanced. For some purposes, however, such as connecting together unbalanced tubes, it is desirable to obtain a filter in an unbalanced form and it is the purpose of this paper to show several forms for constructing resistance compensated band-pass crystal filters which will give results similar to those described previously. Another purpose is to give a numerical comparison between the characteristics obtainable with resistance compensated crystal and electrical filters.

### II. A Comparison of the Performance Characteristics of Crystal vs. Coil and Condenser Filters

In order to show the properties of resistance compensated crystal filters it is instructive to give a comparison between the types of characteristics which can be obtained by using crystal and coil and

[1] "Electrical Wave Filters Employing Quartz Crystals as Elements," W. P. Mason, *B. S. T. J.*, July, 1934, p. 405.

condenser filters. The quartz crystal filter considered here is shown on Fig. 1.

By using the balancing resistance $R_x$ of Fig. 1 the crystal filter can be made entirely compensated for coil resistance; i.e. the resistance associated with the coils of the network is in such a place in the network that

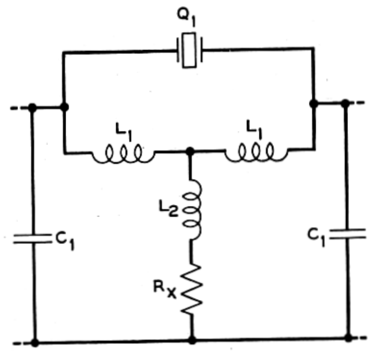


Fig. 1—A bridge T quartz crystal filter.

it can be effectively brought to the ends of the filter and combined with the terminal impedances with the result that the effect of the dissipation in the coils is only to produce an additive loss for the filter characteristic and does not affect the sharpness of cut-off attainable. In fact

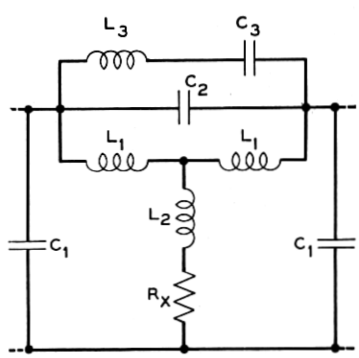


Fig. 2—Electrical network equivalent to crystal filter of Fig. 1.

if the filter works into a vacuum tube the dissipation in the coil can be used to terminate the filter completely, and introduces no loss.

For the electrical filter, however, the dissipation introduced by the electrical elements which replace the crystal is not compensated and causes a considerable distortion of the pass band which becomes more prominent as the band width is narrowed. To show this let us consider the network of Fig. 2. In analyzing such networks it is usually more convenient to reduce them to their equivalent lattice form and apply network equivalences holding for lattice type networks. This can be done by applying Bartlett's Theorem [2] which states that any network which can be divided into two mirror image halves can be reduced to an equivalent lattice network by placing in the series arms of the lattice a two-terminal impedance formed by connecting the two input terminals of one half of the network in this arm and short-circuiting all of the cut wires of the network, and in the lattice arm placing the same network with all its cut wires open-circuited. Applying this process to Fig. 1, a lattice network equivalent to the network of Fig. 1 is that shown on Fig. 3. In this network the capacitances can be considered as sub-

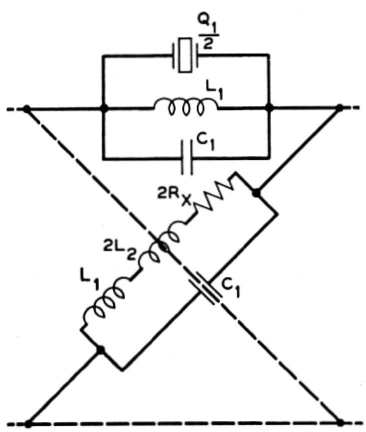


Fig. 3—Lattice equivalent of crystal filter of Fig. 1.

stantially dissipationless and if the network representing the crystal can also be considered dissipationless, the resistance introduced by the coils can be effectively brought outside the lattice and incorporated with the terminal resistances. This follows from the fact that an inductance with an associated series resistance can just as well be represented over the narrow-frequency range of the filter by an inductance paralleled by a much higher resistance. The impedance of an inductance and resistance in series and the impedance of an inductance and resistance in parallel are given by the expressions

$$R_1 + j\omega L_1 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{R_2\omega^2 L_2^2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}. \tag{1}$$

[2] "Extension of a Property of Artificial Lines," A. C. Bartlett. *Phil. Mag.*, **4**, pp. 902–907, Nov. 1927.

Defining $Q$, the ratio of reactance to resistance, as $Q = \omega L_1/R_1$, we have

$$R_2 = R_1(1 + Q^2); \quad L_2 = L_1(1 + 1/Q^2). \tag{2}$$

This relation holds strictly only for a single frequency, but over a narrow-band filter the relation holds quite accurately.

Employing this conception, the lattice network can be reduced to that of Fig. 4 in which a resistance $R$ parallels each arm of the lattice.

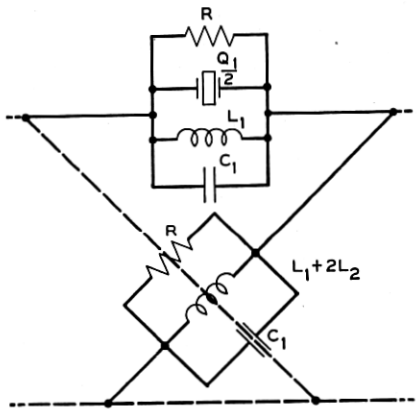


Fig. 4—Filter showing paralleling resistance.

This is made possible by the adjustable resistance $R_x$ which is fixed at such a value that the parallel resistance associated with the inductance $L_1 + 2L_2$ is equal to that associated with $L_1$. Then by employing the two lattice equivalents shown on Fig. 5, first proved by the writer,[3] it is

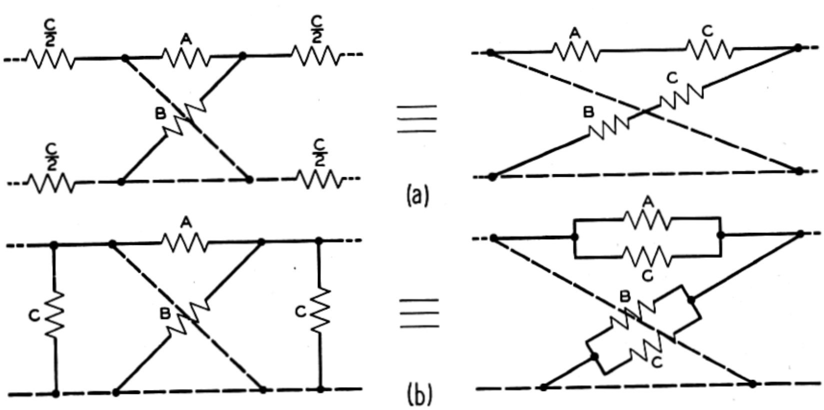


Fig. 5—Lattice network equivalences.

[3] Reference 1, page 418.

possible to take these resistances outside the lattice and combine them with the terminating impedance, leaving all the elements inside the lattice dissipationless. The two remaining arms of the lattice have the impedance characteristic shown on Fig. 6*A*. A lattice filter has a pass band when the two impedance arms have opposite signs and an attenuation band when they have the same sign. When the impedance of two arms cross, an infinite attenuation exists. Hence the characteristic obtainable with this network is that shown on Fig. 6*B*.

Next let us consider an electrical filter in which coils and condensers take the place of the essentially dissipationless crystal. In this case the dissipation due to $L_1$ and $L_2$ can be balanced as before and the only question to consider is the effect of the dissipation associated with $L_3$ and $C_3$. In a similar manner to that employed for the coil we can show

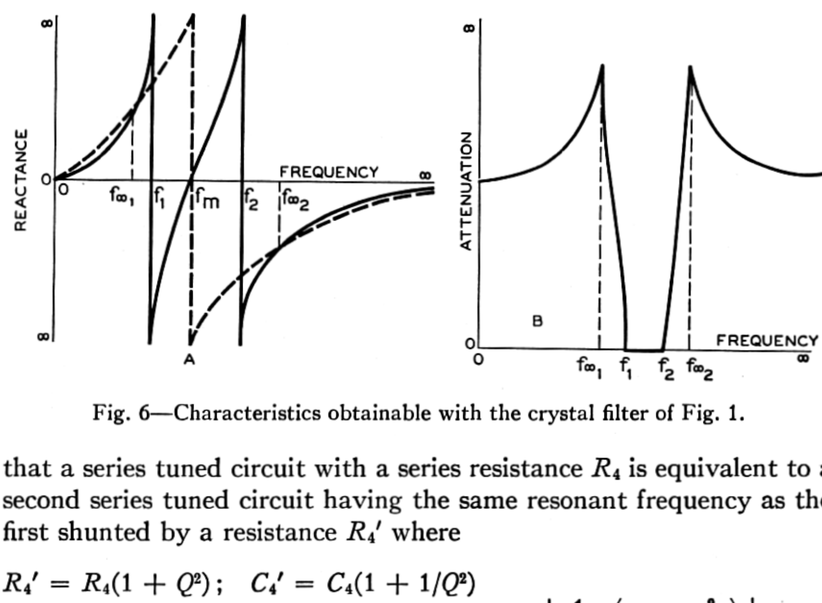


Fig. 6—Characteristics obtainable with the crystal filter of Fig. 1.

that a series tuned circuit with a series resistance $R_4$ is equivalent to a second series tuned circuit having the same resonant frequency as the first shunted by a resistance $R_4'$ where

$$R_4' = R_4(1 + Q^2); \quad C_4' = C_4(1 + 1/Q^2)$$

$$\text{where } Q = \left| \frac{\frac{1}{\omega C_4}\left(1 - \frac{\omega^2}{\omega_R^2}\right)}{R_4} \right|. \quad (3)$$

At two frequencies for which the absolute values of the reactances are the same and therefore the value of $Q$ equal, it is possible to replace the series resistance by a shunt resistance and hence compensate it by varying the resistance $R_x$. Since, however, the reactance of the tuned circuit varies from a negative value through zero to a positive value over the pass band of the filter, the value of this shunt resistance is not even approximately constant and hence the filter cannot be resistance compensated throughout the band of the filter. It can, however, be compensated at the frequencies of infinite attenuation and high losses can be obtained at these frequencies.

The effect of the lack of resistance compensation throughout the band can best be shown by a numerical computation of the loss of an electrical filter as compared to that for a crystal filter. A practical example has been taken of a filter whose band width is 12 kilocycles wide with the mean frequency at 465 kilocycles. In order to obtain the best $Q$'s with reasonably sized coils an arrangement suggested by R. A. Sykes is used. The coils $L_1$ are obtained by using the two equal windings of a coupled coil series aiding so that $L_1$ equals the primary inductance plus the mutual inductance. Since in an air core coil all of the dissipation is associated with the primary inductance and none with the mutual this gives a high $Q$ for $L_1$. The inductance $L_2$ neutralizes the negative mutual inductance $-M$ and supplies in addition a small positive inductance. The $Q$ of this combination is poor but it makes unnecessary the use of a high resistance $R_x$ for balancing purposes. By this method a much higher effective $Q$ is obtained than can be obtained by a single coupled coil or by three separate coils.

The calculated curve for the electrical filter assuming $Q$'s of 150 for all the coils is shown on Fig. 7 by the dotted lines. A similar curve for a crystal filter is shown on Fig. 7 by the full lines. As is evident the effect of the coil dissipation is to round off the edges of the pass band and to limit the effective discrimination between the passed and attenuated bands.

This result does not agree with that given by Landon,[4] who in a recent paper makes a comparison between the results obtained with crystal and electrical filters which appears to be somewhat misleading. It is stated in this paper that the electrical filter circuits given are completely resistance compensated and "in crystal filters in which the crystal is confined to the rejector meshes of the network, the limitation is about the same as for electrical filters." By referring to the curves of Fig. 7 it is readily seen that high losses can be obtained outside the pass band with resistance compensated electrical filters,[5] but that the

[4] "'*M* Derived' Band-Pass Filters with Resistance Cancellation," Vernon D. Landon, *R. C. A. Review*, Oct. 1936, Vol. 1, No. 2, Page 93.

[5] The use of resistance for compensating and balancing the attenuation in electrical filters has been worked out by H. W. Bode and S. Darlington (see U. S. patents 2,002,216, 1,955,788, 2,029,014, 2,035,258). The first work was done for low- and high-pass filters but it was later extended also to band-pass filters. Some of these results are analogous to those of Landon, while others give a better compensation within the transmitted band. The use of the resistance in the crystal filter of Fig. 1 was suggested by Mr. Darlington.

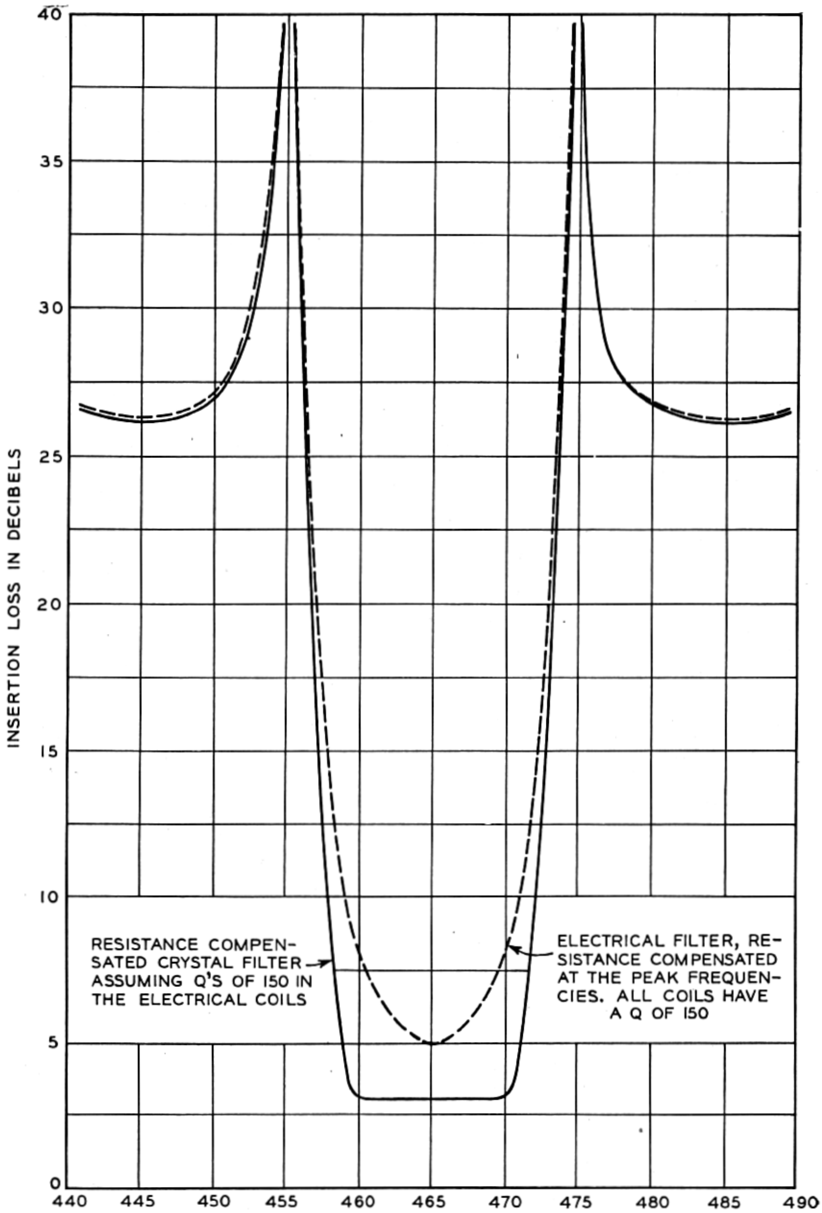


Fig. 7—Numerical comparison between the loss characteristics of a crystal filter and a coil and condenser filter.

pass band of the filter is seriously distorted unless elements, such as crystals, are used which have negligible dissipation.

## III. Band-Pass Resistance Compensated Crystal Filters

All of the wide-band resistance compensated crystal filters proposed so far can be shown to be equivalent to the two general types of lattice crystal filters shown on Fig. 8. For example the crystal filter of Fig. 1 was shown to be equivalent to the lattice type filter of Fig. 8 (b) in which the crystals in the lattice arms are left out.

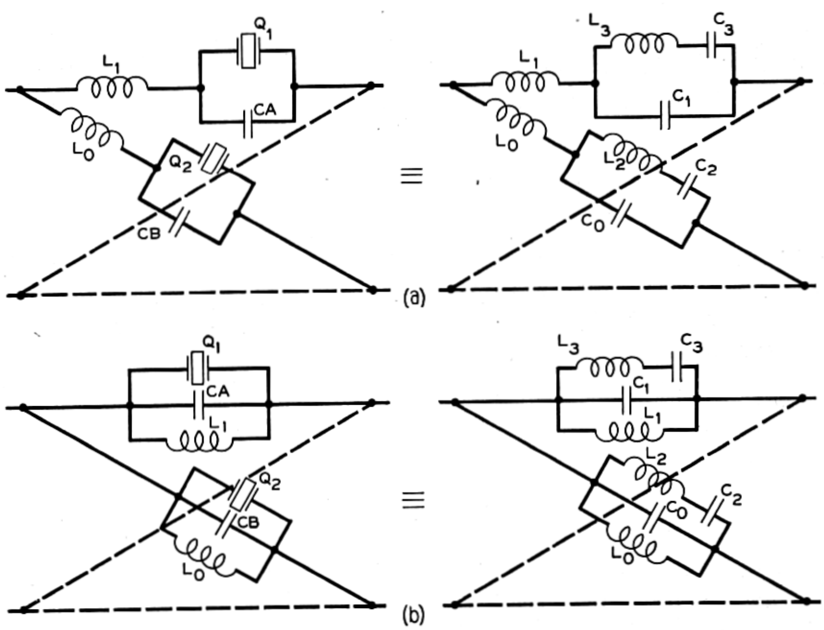


Fig. 8—Wide-band lattice crystal filters.

In the lattice filters of Fig. 8 the number of crystals employed can be cut in half by employing in two similar arms a crystal with two pair of equal plates. It can be shown that such a crystal used in similar arms is equivalent to two identical crystals of twice the impedance of the crystal used as a single plate and having the same resonance frequency. Hence the lattice filters of Fig. 8 are as economical of elements—except for two condensers—as an unbalanced type filter. For some purposes, however, such as connecting together unbalanced tubes, it is desirable to obtain a filter in an unbalanced form. Also, at high frequencies the crystals become quite small and hence it becomes difficult to divide the plating on such crystals. It is the purpose of this section to list a number of filters of the unbalanced type which are equivalent to the lattice filters of Fig. 8. They do not have as general filter character-

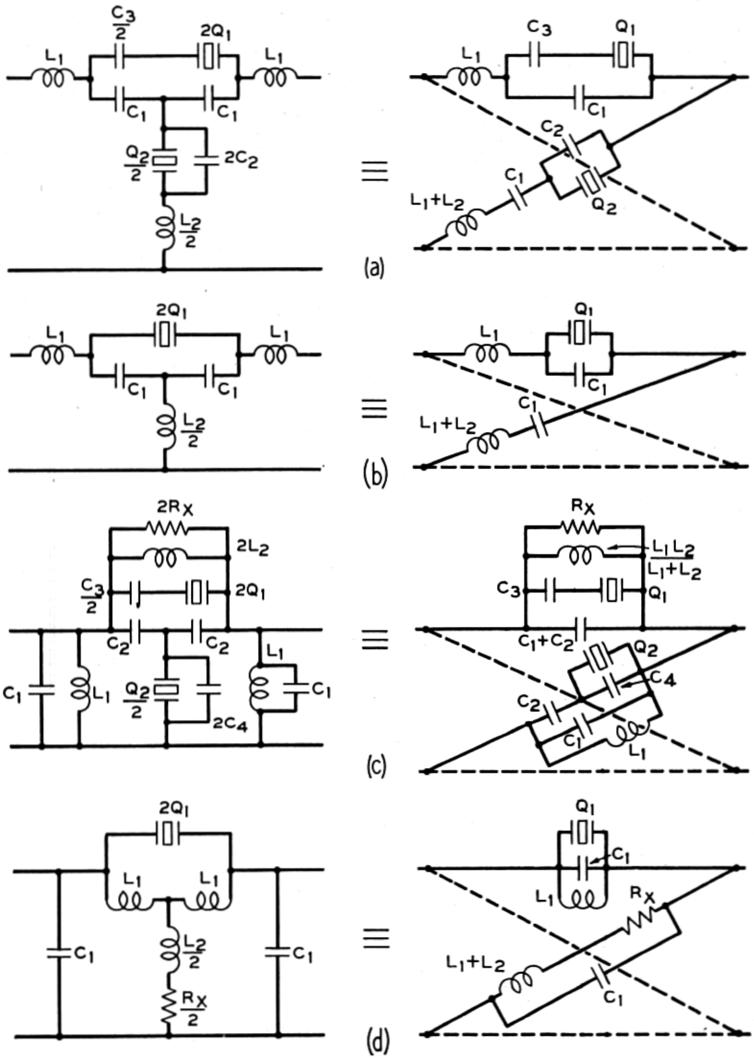


Fig. 9—Wide-band bridge T crystal filters.

istics as the equivalent lattice networks but for a number of purposes are satisfactory.

Fig. 9 shows four bridge $T$ crystal filters which are equivalent respectively to the lattice crystal filters of Fig. 8. The equivalent lattice configurations are shown on Fig. 9. The first two filters have series coils which inherently give low-impedance filters. The second of these is equivalent to the filter of Fig. 8 (a) with one pair of the crystals eliminated. If the inductance $L_2$ were eliminated from Fig. 9 (a) or (b) the filters will be resistance compensated, for all of the resistance will be on the ends of the filter. Furthermore if a small amount of coupling is allowed between the two end coils, the effect of this will be to introduce the small coil $L_2/2$ in the desired place as can be seen from the $T$ network equivalent of a coupled coil as shown on Fig. 10. Furthermore if the coils are air core, no dissipation is associated with the mutual inductance and hence if coupled coils are used the networks still have a resistance balance. Similarly the filters shown on Figs. 9 (c) and (d) are equivalent to the high-impedance type filter shown on Fig. 8 (b) with all crystals present or with crystals missing from the lattice arms. By

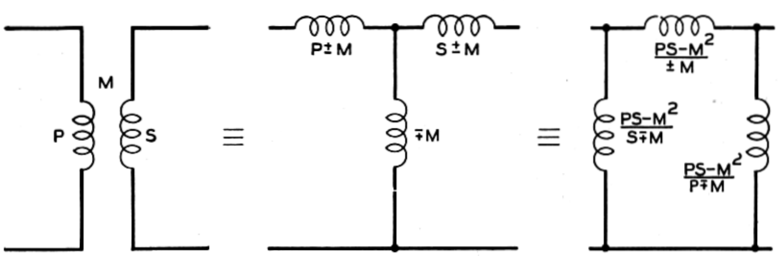


Fig. 10—T and $\pi$ network equivalences of a transformer.

employing coils with a small amount of mutual inductance these types can also be made with a resistance balance. They can also be made to balance for physical coils by employing the resistances shown. It is obvious from the equivalent lattice structures that these networks have limitations on band widths and allowable attenuation which are not present for the original lattice structures of Fig. 8. However, for filters whose pass bands are less than the maximum pass bands, useful results can be obtained.

Another method for obtaining results similar to that obtainable in a lattice network is to use a hybrid coil with series aiding secondaries which are connected to a crystal and a condenser as shown on Fig. 11. This circuit, which has been used extensively to provide a narrow band crystal filter in telegraph work, was invented first by W. A. Marrison [6] of the Bell Telephone Laboratories. Under certain circumstances this configuration can be shown to give results similar to the narrow-band

[6] Patent 1,994,658 filed June 7, 1927, granted March 19, 1935.

lattice filter of Fig. 12. A hybrid coil with series aiding windings connected to two impedances $2Z_1$ and $2Z_2$ as shown by Fig. 13*A* can be shown to be equivalent to the circuit of Fig. 13*B* in which a lattice

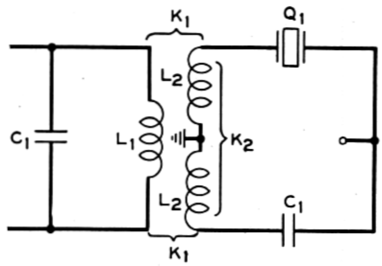


Fig. 11—A three-winding transformer crystal filter.

network with the branches $Z_1$ and $Z_2$ is placed in series with the transforming network and the series terminating inductances. Hence if the hybrid coil has nearly a unity coupling between its secondary coils and

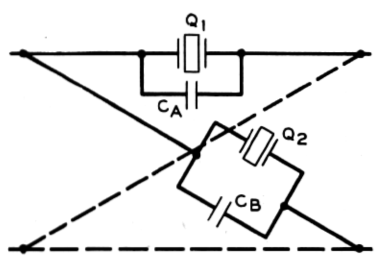


Fig. 12—A narrow-band lattice crystal filter.

the remainder of the transformer is designed to work into the impedance of the filter, the network of Fig. 11 is equivalent to the narrow-band lattice filter of Fig. 12 with crystals removed from the lattice arms, plus

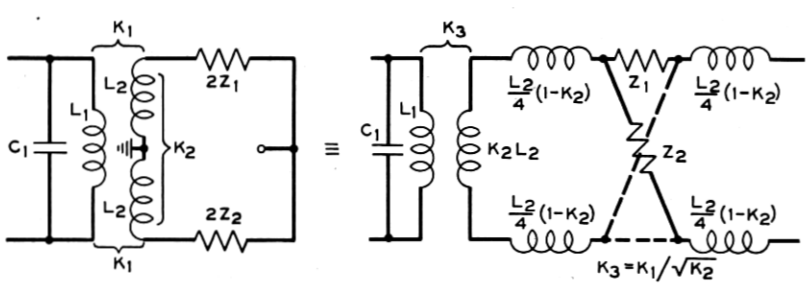


Fig. 13—An equivalent circuit for a three-winding transformer and network.

a transformer. As usually used, however, the impedance of the transformer is much lower than that of the filter and as a consequence the band-pass characteristic of the filter is lost. As a result the network passes only a single frequency and gives results similar to those obtainable with a very sharply tuned circuit. By placing a crystal in the other arm of the network as shown by Fig. 14,[7] this configuration can be made equivalent to the filter shown in Fig. 12.

It is obvious from the equivalence of Fig. 13 that the configurations of Fig. 11 and Fig. 14 can also be used to give a wide-band filter. This follows since the series inductances can be taken inside the lattice and the low-impedance crystal filter of Fig. 8 (a) results. The $Q$ of the coils included in the filter will ordinarily not be high since the inductance is obtained by a difference of primary and mutual inductances, and a better result will be obtained by making the secondary coupling high and including physical coils in series with the crystals.

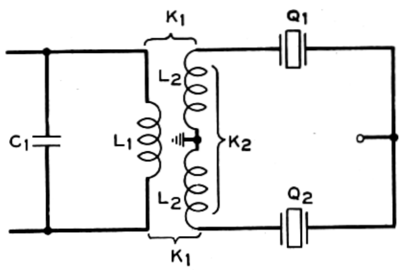


Fig. 14—A three-winding transformer crystal filter with two crystals.

We see then that all of the resistance compensated wide-band filters are equivalent to the lattice filters of Figs. 8 and 12, and all their design equations are known when the design equations of the equivalent lattices are calculated. This requires two steps, first the calculation of the spacing of the resonant frequencies of the network to give the required attenuation and secondly the calculation of the element values from the known resonances by means of Foster's theorem. Such calculations are familiar in filter theory and hence only the results are given here. The results are given in Tables I, II, and III for the network of Figs. 8 (a), 8 (b) and 12 respectively. These values are given in terms of the characteristic impedance $Z_0$ of the filter at the mean frequency, the lower and upper cut-off frequencies $f_1$ and $f_2$ respectively and the $b$'s of the network. These last are parameters which specify

[7] This configuration is covered by patent 2,001,387 issued to C. A. Hansell.

TABLE I

| Element | Formula |
|---|---|
| $L_0$ | $\dfrac{Z_0 f_2(f_2^2 + f_1^2 B)}{2\pi f_1(f_2 - f_1)(f_2^2 A + f_1^2 C)}$ |
| $L_1$ | $\dfrac{Z_0 f_1(f_2^2 A + f_1^2 C)}{2\pi f_2(f_2 - f_1)(f_2^2 + f_1^2 B)}$ |
| $L_2$ | $\dfrac{Z_0[f_2^4(A(1+B) - C) + 2f_1^2 f_2^2 C + f_1^4 BC]^2}{2\pi f_1 f_2(f_2 - f_1)^3(f_2 + f_1)^2[f_2^2 A + f_1^2 C]C(AB - C)}$ |
| $L_3$ | $\dfrac{Z_0[f_2^4 A + 2f_1^2 f_2^2 C + f_1^4(B(A + C) - C)]^2}{2\pi f_1 f_2(f_2 - f_1)^3(f_2 + f_1)^2(f_2^2 + f_1^2 B)(AB - C)}$ |
| $C_0$ | $\dfrac{(f_2 - f_1)(f_2^2 A + f_1^2 C)^2}{2\pi Z_0 f_1 f_2[f_2^4(A(1+B) - C) + 2f_1^2 f_2^2 C + f_1^4 BC]}$ |
| $C_1$ | $\dfrac{(f_2 - f_1)(f_2^2 + f_1^2 B)^2}{2\pi Z_0 f_1 f_2[f_2^4 A + 2f_1^2 f_2^2 C + f_1^4(B(A + C) - C)]}$ |
| $C_2$ | $\dfrac{(AB - C)C(f_2 - f_1)^3(f_2 + f_1)^2}{2\pi Z_0 f_1 f_2[f_2^4(A(1+B) - C) + 2f_1^2 f_2^2 C + f_1^4 BC](1 + B)}$ |
| $C_3$ | $\dfrac{(AB - C)(f_2 - f_1)^3(f_2 + f_1)^2}{2\pi Z_0 f_1 f_2[f_2^4 A + 2f_2^2 f_1^2 C + f_1^4(B(A + C) - C)](A + C)}$ |

where $A = b_1 + b_2 + b_3$; $B = b_1 b_2 + b_1 b_3 + b_2 b_3$; $C = b_1 b_2 b_3$;

$$b_n = \sqrt{\frac{1 - f_{\infty n}^2/f_1^2}{1 - f_{\infty n}^2/f_2^2}}; \qquad n = 1, 2, 3$$

TABLE II

| Element | Formula | Element | Formula |
|---|---|---|---|
| $L_0$ | $\dfrac{Z_0(f_2 - f_1)(A + C)}{2\pi f_1 f_2(1 + B)}$ | $C_0$ | $\dfrac{(f_2^2 + f_1^2 B)f_2}{2\pi Z_0 f_1(f_2 - f_1)(f_2^2 A + f_1^2 C)}$ |
| $L_1$ | $\dfrac{Z_0(f_2 - f_1)(1 + B)}{2\pi f_1 f_2(A + C)}$ | $C_1$ | $\dfrac{(f_2^2 A + f_1^2 C)f_1}{2\pi Z_0 f_2(f_2 - f_1)(f_2^2 + f_1^2 B)}$ |
| $L_2$ | $\dfrac{Z_0(A + C)(f_2^2 A + f_1^2 C)^2}{2\pi f_1 f_2(f_2 - f_1)(f_2 + f_1)^2 C(AB - C)}$ | $C_2$ | $\dfrac{(f_2 - f_1)(f_2 + f_1)^2 C(AB - C)}{2\pi Z_0 f_1 f_2(f_2^2 A + f_1^2 C)(A + C)^2}$ |
| $L_3$ | $\dfrac{Z_0(1 + B)(f_2^2 + f_1^2 B)^2}{2\pi f_1 f_2(f_2 - f_1)(f_2 + f_1)^2(AB - C)}$ | $C_3$ | $\dfrac{(f_2 - f_1)(f_2 + f_1)^2(AB - C)}{2\pi Z_0 f_1 f_2(1 + B)^2(f_2^2 + f_1^2 B)}$ |

where $A = b_1 + b_2 + b_3$; $B = b_1 b_2 + b_1 b_3 + b_2 b_3$; $C = b_1 b_2 b_3$;

$$b_n = \sqrt{\frac{1 - f_{\infty n}^2/f_1^2}{1 - f_{\infty n}^2/f_2^2}}; \qquad n = 1, 2, 3$$

TABLE III

| Element | Formula | Element | Formula |
|---|---|---|---|
| $C_0$ | $\dfrac{f_1(b_1+b_2)}{2\pi Z_0(f_2^2+f_1^2b_1b_2)}$ | $C_2$ | $\dfrac{b_1b_2(f_2^2-f_1^2)}{2\pi Z_0f_1f_2^2(b_1+b_2)}$ |
| $C_0'$ | $\dfrac{f_2^2+f_1^2b_1b_2}{2\pi Z_0f_1f_2^2(b_1+b_2)}$ | $L_1$ | $\dfrac{Z_0(f_2^2+f_1^2b_1b_2)^2}{2\pi f_1f_2^2(b_1+b_2)(f_2^2-f_1^2)}$ |
| $C_1$ | $\dfrac{(b_1+b_2)(f_2^2-f_1^2)}{2\pi Z_0f_1(1+b_1b_2)(f_2^2+f_1^2b_1b_2)}$ | $L_2$ | $\dfrac{Z_0f_2^2(b_1+b_2)}{2\pi f_1b_1b_2(f_2^2-f_1^2)}$ |

$$b_1 = \sqrt{\frac{1-f\infty_1^2/f_1^2}{1-f\infty_1^2/f_2^2}}; \qquad b_2 = \sqrt{\frac{1-f\infty_2^2/f_1^2}{1-f\infty_2^2/f_2^2}}$$

the location of the attenuation peaks of the network with relation to the cut-off frequencies and are given by the expression:

$$b_n = \sqrt{\frac{1-f\infty_n^2/f_1^2}{1-f\infty_n^2/f_2^2}}; \quad n = 1, 2, 3,$$

where $f\infty_n$ is the frequency of infinite attenuation.

These tables give the design formulae for the networks of Figs. 8 and 12. To obtain the equations for a network having crystals in the series arms alone, it is only necessary to let $b_3 = 0$. If one of the peaks of the filter of Fig. 8 (a) is placed at infinity—which results when $b_2 = f_2/f_1$—the two coils will have equal values and by the theorem illustrated by Fig. 5 can be brought out to the ends of the filter, simplifying the construction. In a similar manner if one of the peaks of the filter of Fig. 8 (b) is placed at zero frequency, i.e. $b_2 = 1$, the two shunt inductances are equal and can be brought out to the ends of the filter. The design equation of the narrow band filter of Fig. 12 with the lattice crystals replaced by condensers can be obtained from Table III by letting $b_2 = 0$.