The Use of Coaxial and Balanced Transmission Lines in Filters and Wide-Band Transformers for High Radio Frequencies

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At the high radio frequencies, filters and transformers become difficult to construct from conventional electrical coils and condensers, on account of the small sizes of the elements, the large effects of the interconnecting windings and the low ratios of reactance to resistance realizable in coils. It is shown in this paper that selective filters and wide-band transformers can be constructed using transmission lines and condensers as elements. The ratio of reactance to resistance in these elements can be made very high; consequently very selective filters and transformers with small losses, can be constructed. The effect of the distributed nature of the elements is taken account of in the design equations and methods are described for obtaining single-band filters and transformers. Experimental measurements of such filters and transformers are shown. The experimental loss curve is shown of a coaxial filter used in the Provincetown-Green Harbor short-wave radio circuit for the purpose of connecting a transmitter and receiver to the same antenna.

I. Introduction

At the higher radio frequencies, coil and condenser networks become difficult to construct on account of the small sizes of the elements and the large effects of the interconnecting windings. The Q realizable in high-frequency coils is about the same as can be obtained at the lower frequencies but due to the smaller percentage band widths, it is desirable to obtain a higher Q. There has been a tendency to replace coils by small lengths of transmission lines, and these have been used to some extent as tuned circuits, and as single-frequency transformers.[1,2,3]

It is the purpose of this paper to describe work which has been done in constructing selective filters and wide-band transformers from lengths of transmission lines and condensers. Due to the high ratio of reactance to resistance obtainable in both of these types of elements,

[1] "Transmission Lines for Short-Wave Radio Systems," E. J. Sterba and C. B. Feldman, *B. S. T. J.*, Vol. XI, No. 3, July 1932, page 411.

[2] "Resonant Lines for Radio Circuits," F. E. Terman, *Elec. Engg.*, Vol. 53, pp. 1046–1053, July 1934.

[3] "A Unicontrol Radio Receiver for Ultra-High Frequencies Using Concentric Lines as Interstage Couplers," F. W. Dunmore, *Proc. I. R. E.*, Vol. 24, No. 6, June 1936.

very selective networks can be obtained at the high radio frequencies. The effect of the distributed nature of the elements is considered and methods are described for obtaining single-band filters and transformers. Experimental measurements of such filters and transformers are shown, and these results indicate that such structures should be of use in short-wave radio circuits.

II. Characteristics of Transmission Lines

To facilitate an understanding of the following discussion, the equations of transmission lines as they apply to filter structures will be briefly reviewed first. The equations of propagation for any uniform transmission line can be expressed in the form of equations between the output voltage e_2, the output current i_2, the input voltage e_1, and the input current i_1 by the relations

$$\begin{aligned} e_2 &= e_1 \cosh Pl - i_1 Z_0 \sinh Pl, \\ i_2 &= i_1 \cosh Pl - \frac{e_1}{Z_0} \sinh Pl, \end{aligned} \tag{1}$$

where l is the length of the line, P the propagation constant, and Z_0 the characteristic impedance of the line. In terms of the distributed resistance R per unit length of line, L the distributed inductance, G the distributed conductance, and C the distributed capacitance, P and Z_0 can be expressed by the relations

$$P = \sqrt{(R + j\omega L)(G + j\omega C)}; \qquad Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \tag{2}$$

where ω is 2π times the frequency.

The distributed conductance G is usually very low and can be neglected for coaxial or balanced transmission lines in dry atmospheres. For copper coaxial lines, the values of R, L, and C have been calculated[4] to be

$$\begin{aligned} R &= 41.6 \times 10^{-9} \sqrt{f} \left(\frac{1}{a} + \frac{1}{b} \right) \text{ ohms per centimeter}, \\ L &= 2 \log_e \frac{b}{a} \times 10^{-9} \text{ henries per centimeter}, \\ C &= \frac{1.11 \times 10^{-12}}{2 \log_e \frac{b}{a}} \text{ farads per centimeter}, \end{aligned} \tag{3}$$

where b is the inside radius of the outer conductor and a the outside radius of the inner conductor. If we define the Q of the conductor as

[4] See reference 1, page 415 and page 417.

the ratio of the series inductive reactance to the series resistance, the ratio will be

$$Q = .302b\sqrt{f}\,(\log_e k)/(k+1), \tag{4}$$

where $k = b/a$. It will be noted that this is the value of Q measured for a short-circuited conductor for low frequencies. As an example, a conductor 3 inches in diameter with the optimum ratio of $k = 3.6$ will have a Q of 3,200 at 100 megacycles. The value of the characteristic impedance Z_0 of a coaxial line is

$$Z_0 \doteq \sqrt{\frac{L}{C}} = 60 \log_e \frac{b}{a}\cdot \tag{5}$$

For a balanced transmission line the values of R, L, and C are,[4] if D is much larger than a,

$$\begin{aligned} R &= \frac{83.2 \times 10^{-9}\sqrt{f}}{a} \text{ ohms per centimeter,} \\ L &= 4 \log_e \frac{D}{a} \times 10^{-9} \text{ henries per centimeter length,} \\ C &= \frac{1.111 \times 10^{-12}}{4 \log_e \frac{D}{a}} \text{ farads per centimeter,} \end{aligned} \tag{6}$$

where D is the spacing between wires, and a the radius of one of the pair of wires. With these values, the expressions for Q and Z_0 become

$$Q = .302\sqrt{fa}\log_e \frac{D}{a}\,; \qquad Z_0 = 120 \log_e \frac{D}{a}\cdot \tag{7}$$

Another combination of some interest is obtained by using the inside conductors of two coaxial conductors adjacent to each other. Such a construction results in a balanced and shielded transmission line. All of the constants are double those given by equation (3), except for the capacitance which is halved.

III. Filters Employing Transmission Lines as Elements

One of the first uses of transmission lines as elements in wave filters is described in a patent of one of the writers.[5] In this patent are considered the characteristics obtainable by combining sections of trans-

[5] U. S. Patent 1,781,469 issued to W. P. Mason. Application filed June 25, 1927; Patent granted Nov. 11, 1930. Wave filters using transmission lines only are very similar to acoustic wave filters as pointed out in an article, "Acoustic Filters," *Bell Laboratories Record*, April 1928. Most of the equations and results of a former paper on acoustic filters, *B. S. T. J.*, April 1927, p. 258, are also applicable to transmission line filters.

mission lines in ladder filter structures. The results obtained are briefly reviewed here.

One of the simplest filters considered is shown on Fig. 1. The filter consists of a length $2l_1$ of transmission line shunted at its center by a short-circuited transmission line of length l_2. To determine the transmission bands of a filter, it is necessary to neglect the dissipation occurring in the elements and hence we assume that R and G are zero. In any case these values are small for transmission lines since they have a high Q. Neglecting R and G, equations (1) become

$$e_2 = e_1 \cos \frac{\omega l}{v} - j i_1 Z_0 \sin \frac{\omega l}{v},$$
$$i_2 = i_1 \cos \frac{\omega l}{v} - j \frac{e_1}{Z_0} \sin \frac{\omega l}{v}, \tag{8}$$

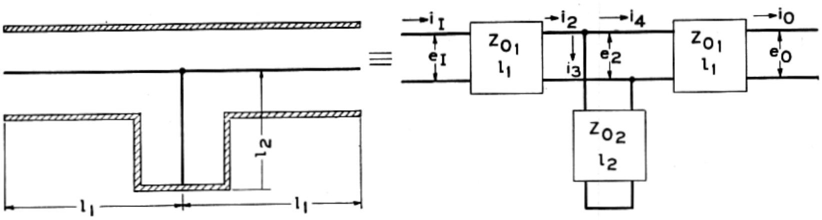


Fig. 1—Band-pass filter constructed from coaxial conductors.

where v, the velocity of propagation, and Z_0, the characteristic impedance, have the values

$$v = \frac{1}{\sqrt{LC}}; \qquad Z_0 = \sqrt{\frac{L}{C}}. \tag{9}$$

Using these equations the characteristics of the filter illustrated by Fig. 1 are easily calculated. With reference to Fig. 1 and equations (8) we can write the equations of the network as

$$e_2 = e_I \cos \frac{\omega l_1}{v} - j i_I Z_{0_1} \sin \frac{\omega l_1}{v},$$
$$i_2 = i_I \cos \frac{\omega l_1}{v} - j \frac{e_I}{Z_{0_1}} \sin \frac{\omega l_1}{v}, \qquad i_2 = i_3 + i_4,$$
$$e_0 = e_2 \cos \frac{\omega l_1}{v} - j i_4 Z_{0_1} \sin \frac{\omega l_1}{v}, \qquad i_3 = \frac{e_2}{j Z_{0_2} \tan \frac{\omega l_2}{v}}. \tag{10}$$
$$i_0 = i_4 \cos \frac{\omega l_1}{v} - j \frac{e_2}{Z_{0_1}} \sin \frac{\omega l_1}{v},$$

Combining these equations and eliminating all terms except the input and output voltages and currents we have

$$e_0 = e_I \left[\cos\frac{2\omega l_1}{v} + \frac{Z_{0_1}}{2Z_{0_2}} \frac{\sin\frac{2\omega l_1}{v}}{\tan\frac{\omega l_2}{v}} \right] - j i_I Z_{0_1} \left[\sin\frac{2\omega l_1}{v} + \frac{Z_{0_1}}{Z_{0_2}} \frac{\sin^2\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}} \right];$$

$$i_0 = i_I \left[\cos\frac{2\omega l_1}{v} + \frac{Z_{0_1}}{2Z_{0_2}} \frac{\sin\frac{2\omega l_1}{v}}{\tan\frac{\omega l_2}{v}} \right] - j\frac{e_I}{Z_{0_1}} \left[\sin\frac{2\omega l_1}{v} - \frac{Z_{0_1}}{Z_{0_2}} \frac{\cos^2\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}} \right]. \tag{11}$$

The properties of symmetrical filters such as considered here are usually specified in terms of the propagation constant Γ and the iterative impedance K. These are similar to the line parameters used in equation (1) and it can be shown that the same relations exist between the input and output voltages and currents that exist in equation (1). Hence comparing equation (1) with equation (11) we find

$$\cosh \Gamma = \cos\frac{2\omega l_1}{v} + \frac{Z_{0_1}}{2Z_{0_2}} \frac{\sin\frac{2\omega l_1}{v}}{\tan\frac{\omega l_2}{v}},$$

$$K = Z_{0_1} \sqrt{\frac{1 + \frac{Z_{0_1}}{2Z_{0_2}} \frac{\tan\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}}{1 - \frac{Z_{0_1}}{2Z_{0_2}} \frac{\cot\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}}}. \tag{12}$$

The propagation constant Γ has the same significance as for a uniform line, namely that

$e^{-\Gamma} = \frac{e_n}{e_{n-1}} = \frac{i_n}{i_{n-1}}$ = ratio of currents or voltages between any two sections in an infinite sequence of sections, while K, the iterative impedance, is the impedance measured looking into an infinite sequence of such sections. The propagation constant Γ is in general a complex number $A + jB$. The real part is the attenuation constant and the complex part the phase constant. For a pass band, A must be zero, which will occur when $\cosh \Gamma$ is between $+1$ and -1. Hence to locate the pass band of the dissipationless filter, the value of the first equation of (12) must lie between $+1$ and -1.

In order to point out what types of filters result from a combination of transmission lines, two simplifying cases are considered. The first case is when $l_1 = l_2$. Then the equation for the propagation constant becomes

$$\cosh \Gamma = \left(1 + \frac{Z_{01}}{Z_{02}}\right)\left(\cos^2 \frac{\omega l_1}{v}\right) - \sin^2 \frac{\omega l_1}{v}$$
$$= \cos \frac{2\omega l_1}{v} \times \left[1 + \frac{Z_{01}}{2Z_{02}}\right] + \frac{Z_{01}}{2Z_{02}}. \quad (13)$$

The edges of the pass band occur when

$$\tan \frac{\omega l_1}{v} = \sqrt{\frac{Z_{0_1}}{2Z_0}} \quad (14)$$

and the centers of the bands occur when

$$\sin \frac{\omega l_1}{v} = 1; \qquad \frac{\omega l_1}{v} = \frac{(2n+1)\pi}{2} \qquad \text{and} \qquad f_m = \frac{(2n+1)v}{4l_1}, \quad (15)$$

where $n = 0, 1, 2$, etc. A plot of the propagation constant for several ratios of Z_{01}/Z_{02} is shown in Fig. 2. As is evident the filter is a multi-band filter with bands centered around odd harmonic frequencies. The ratio of Z_{0_1} to Z_{0_2} determines the band width of the filter. This ratio cannot be made very large because the characteristic impedance of a coaxial line or a balanced line cannot be widely varied from the mean value. For example when the ratio of b/a varies from 1.05 to 100 the characteristic impedance of a coaxial conductor changes from about 3 ohms to 275 ohms. This represents about as extreme a range as can be obtained. For a balanced conductor the impedance may range from 90 to 1100 ohms by taking extreme values of the ratio D/a. Taking 100 as the extreme range between Z_{0_1} and Z_{0_2} the band width for this case cannot be made less than 20 per cent.

We next consider the case given by taking $Z_{0_1} = 2Z_{0_2}$. For this case the filter parameters take on the simple form

$$\cosh \Gamma = \frac{\sin \dfrac{\omega(2l_1 + l_2)}{v}}{\sin \dfrac{\omega l_2}{v}} \text{ and } K = Z_{01}\sqrt{-\tan\frac{\omega l_1}{v}\tan\frac{\omega(l_1+l_2)}{v}}. \quad (16)$$

For this case the lower side of the pass band occurs when $\cosh \Gamma = +1$ and the upper side when $\cosh \Gamma = -1$. Hence the frequency limits

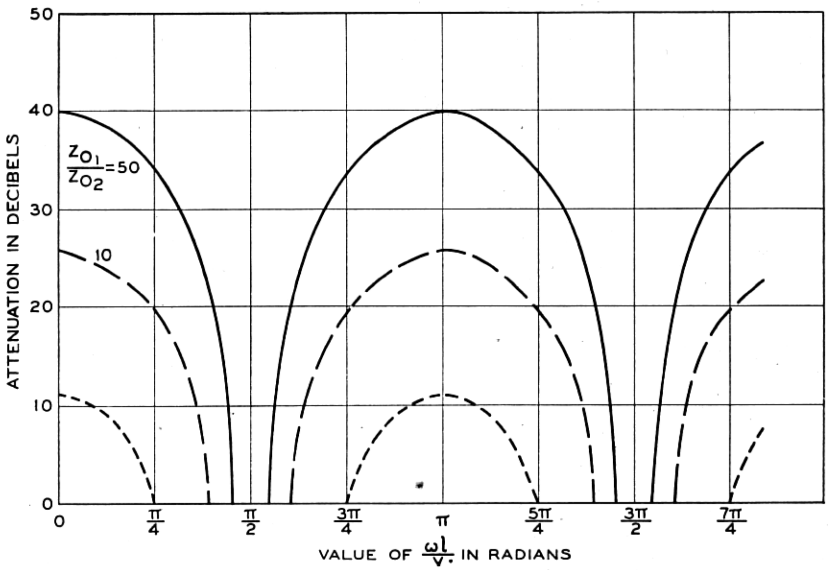


Fig. 2—Attenuation characteristic of band-pass filter.

of the first pass band can be obtained by solving the transcendental equation

$$\sin\frac{\omega(2l_1+l_2)}{v} \mp \sin\frac{\omega l_2}{v} = 0,$$

which has the solutions

$$f_1 = \frac{v}{4(l_1+l_2)}; \qquad f_2 = \frac{v}{4l_1}. \quad (17)$$

Hence by making the shunting line very short, it is possible to obtain a narrow band with this type of filter. The mid-frequency of the band occurs when

$$\sin\frac{\omega(2l_1+l_2)}{v} = 0 \qquad \text{or when} \qquad f_m = \frac{v}{4l_1+2l_2}.$$

The characteristic impedance of this type of filter becomes very high for narrow-band filters as is shown by equation (16). At the mean frequency f_m the impedance of the filter is

$$K = Z_{0_1}\sqrt{-\tan\left[\frac{\pi/2}{1+\frac{l_2}{2l_1}}\right]\tan\left[\frac{\pi}{2}\left(\frac{1+\frac{l_2}{l_1}}{1+\frac{l_2}{2l_1}}\right)\right]}$$

$$\doteq \frac{4l_1}{\pi l_2} Z_{0_1} \text{ for narrow bands.} \qquad (18)$$

Such a filter would be useful for an interstage coupler to couple together the plate of one screen grid or pentode tube to the grid of another one. One such arrangement is shown in Fig. 3. This method of coupling together two stages of vacuum tubes has an advantage over using a coaxial conductor or a coil and condenser as a tuned circuit on several counts. In the first place the width of the band passed can be accurately controlled and a flatter gain characteristic is obtained. As will be shown later, distributed capacity in the plate and grid of the

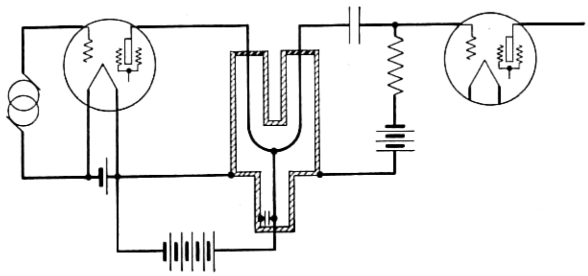
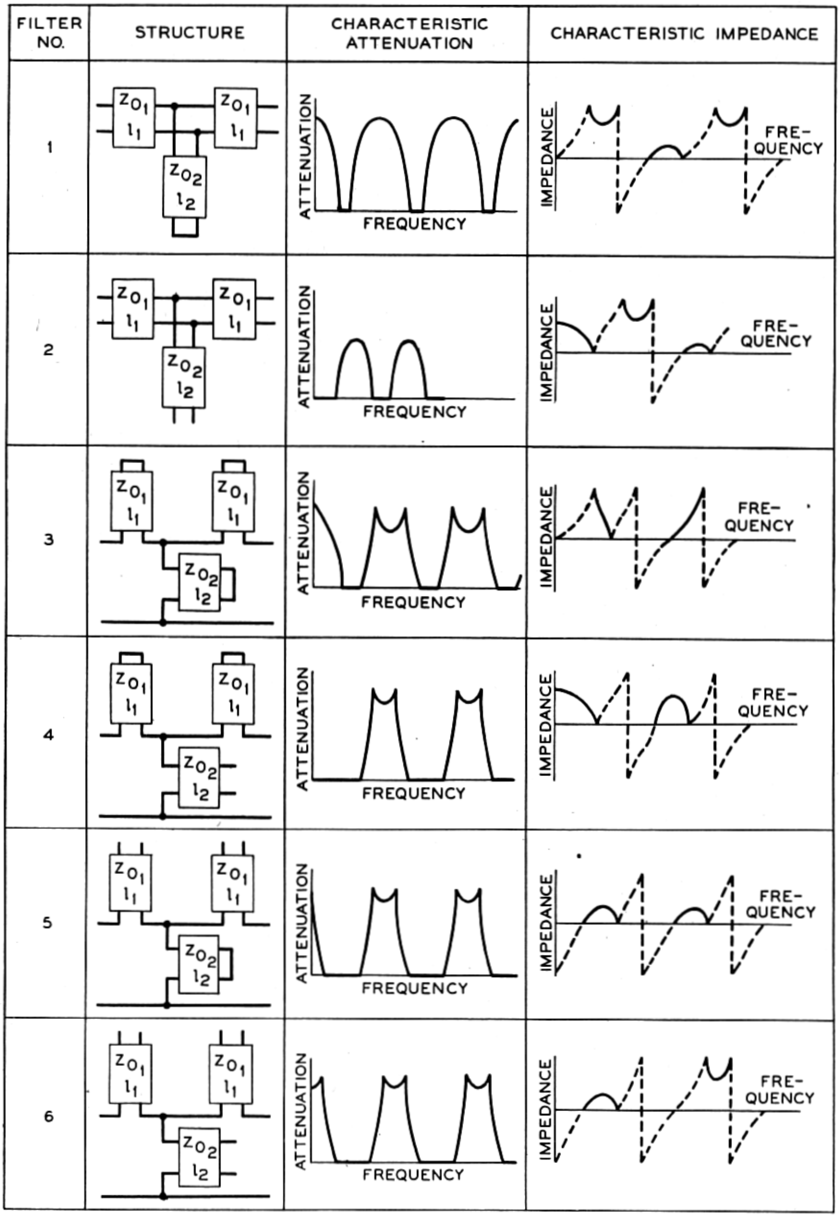


Fig. 3—Coaxial band-pass filter used to couple vacuum tubes.

vacuum tube can be absorbed in the filter by making the line length l_1 shorter. Since only half of the total distributed capacity has to be absorbed on each end of the filter, a higher impedance can be built up in the filter for the same band width, and hence more gain per section can be obtained than with a tuned circuit.

The filter will have other pass bands at $2f_m$, $3f_m$, etc., but these do not usually cause any trouble at the short-wave frequencies because the gain in the vacuum tube is falling off very rapidly and no appreciable signal is passed. If desired, however, another band-pass filter can be added which has the same fundamental bands but different overtone bands, and this will eliminate the effect of the additional pass bands. For very narrow bands, the line length l_2 can be made longer and hence more realizable by making the characteristic impedance Z_{0_2} lower. If anything is to be gained by making the impedance on the plate side different from that on the grid side this can be accomplished by making the filter an impedance transforming device, as discussed in the next section.



NOTE:
DOTTED LINES INDICATE REACTIVE IMPEDANCE. SOLID LINES INDICATE RESISTIVE IMPEDANCE.

Fig. 4—A list of filters constructed from transmission lines.

The filter discussed above shows some of the possibilities and limitations of filters constructed from transmission lines. Many other types are also possible. Figure 4 lists a number of these and the types of filter characteristics they give. The design equations for a number of them are considered in detail in the patent referred to above and hence will not be worked out here.

IV. Impedance Transforming Band-Pass Filters Employing Transmission Lines as Elements

In a good many cases it is desirable to transform from one impedance to another over a wide range of frequencies. Examples of such uses are when antennas are connected to transmission lines, or when transmission lines are to be connected to vacuum tubes, etc. Transforming band-pass filters constructed out of transmission lines which will transform over wide frequency ranges are therefore of practical interest. Previously two types of single-frequency transformers, constructed

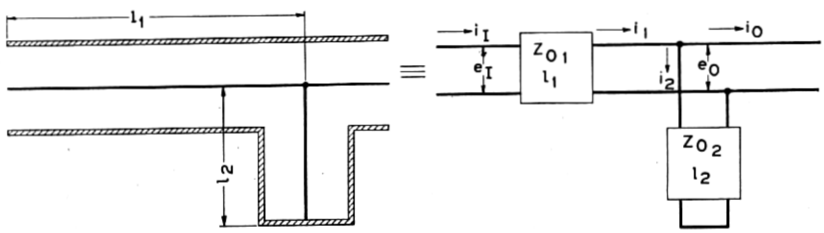


Fig. 5—A wide-band transformer—constructed from coaxial conductors.

from transmission lines, have been suggested [6] but these differ from the types investigated here in that they have a specified ratio of impedances for a single frequency only.

One of the simplest types of band transforming filters is shown by Fig. 5. It consists of a series transmission line of characteristic impedance Z_{0_1} and a shunt line having the characteristic impedance Z_{0_2}. Since the impedance of the short-circuited line is

$$jZ_{0_2} \tan \frac{\omega l_2}{v},$$

the equations for the structure are

$$e_1 = e_I \cos \frac{\omega l_1}{v} - j i_I Z_{0_1} \sin \frac{\omega l_1}{v}; \qquad i_1 = i_2 + i_0; \qquad e_1 = e_0;$$
$$i_1 = i_I \cos \frac{\omega l_1}{v} - j \frac{e_I}{Z_{0_1}} \sin \frac{\omega l_1}{v}; \qquad i_2 = - \frac{j e_0}{Z_{0_2} \tan \frac{\omega l_2}{v}}. \tag{19}$$

[6] See reference 1, pages 430 and 431.

Combining these equations we have

$$e_0 = e_I \cos\frac{\omega l_1}{v} - j i_I Z_{0_1} \sin\frac{\omega l_1}{v},$$

$$i_0 = i_I \cos\frac{\omega l_1}{v}\left[1 + \frac{Z_{0_1}\tan\frac{\omega l_1}{v}}{Z_{0_2}\tan\frac{\omega l_2}{v}}\right] - j e_I \frac{\sin\frac{\omega l_1}{v}}{Z_{0_1}}\left[1 - \frac{Z_{0_1}\cot\frac{\omega l_1}{v}}{Z_{0_2}\tan\frac{\omega l_2}{v}}\right]. \quad (20)$$

In order to interpret this equation in terms of transformer theory, it can be shown that equations (20) are identical to the equations for a perfect transformer and a symmetrical filter. To show this, consider the circuit of Fig. 6, which consists of two half-sections of a symmetrical

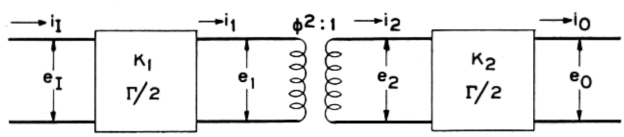


Fig. 6—Transformer and filter.

filter separated by a perfect transformer having an impedance step-down φ^2 to 1. The characteristic impedance of the first filter is φ^2 times that of the second filter. The equations for the first section, the transformer, and the last section are respectively

$$e_1 = e_I \cosh\frac{\Gamma}{2} - i_I K_1 \sinh\frac{\Gamma}{2}; \quad i_1 = i_I \cosh\frac{\Gamma}{2} - \frac{e_I}{K_1}\sinh\frac{\Gamma}{2};$$

$$i_2 = \varphi i_1; \quad e_2 = e_1/\varphi; \quad (21)$$

$$e_0 = e_2 \cosh\frac{\Gamma}{2} - i_2 K_2 \sinh\frac{\Gamma}{2}; \quad i_0 = i_2 \cosh\frac{\Gamma}{2} - \frac{e_2}{K_2}\sinh\frac{\Gamma}{2}.$$

Combining these equations on the assumption that $K_1/K_2 = \varphi^2$, we have

$$e_0 = \frac{1}{\varphi}\left[e_I \cosh\Gamma - i_I K_1 \sinh\Gamma\right];$$

$$i_0 = \varphi\left[i_I \cosh\Gamma - \frac{e_I \sinh\Gamma}{K_1}\right]. \quad (22)$$

Similar results can be obtained by using the image impedance parameters of a dissymmetrical filter. For a dissymmetrical filter the most general relationship between the input and output voltages and currents can be written in the form

$$e_2 = e_1 A - i_1 B; \quad i_2 = i_1 C - e_1 D \quad \text{where} \quad AC - BD = 1. \tag{23}$$

In terms of these parameters the image transfer constant θ and the image transfer impedances are given by the relationships [7]

$$\cosh \theta = \sqrt{AC}; \qquad K_1 = \sqrt{\frac{BC}{AD}}; \qquad K_2 = \sqrt{\frac{AB}{CD}}. \tag{24}$$

The transformation ratio between the two ends of the network is then

$$\frac{K_1}{K_2} = \frac{C}{A} = \varphi^2 \tag{25}$$

in agreement with the results of equation (22).

Applying these results to equation (20) we find for the most general case

$$\cosh \theta = \cos \frac{\omega l_1}{v} \sqrt{1 + \frac{Z_{0_1} \tan \frac{\omega l_1}{v}}{Z_{0_2} \tan \frac{\omega l_2}{v}}};$$

$$K_1 = Z_{0_1} \sqrt{-\tan \frac{\omega l_1}{v} \left[\frac{\tan \frac{\omega l_1}{v} + \frac{Z_{0_2}}{Z_{0_1}} \tan \frac{\omega l_2}{v}}{1 - \frac{Z_{0_2}}{Z_{0_1}} \tan \frac{\omega l_1}{v} \tan \frac{\omega l_2}{v}} \right]};$$

$$\varphi^2 = 1 + \frac{Z_{0_1} \tan \frac{\omega l_1}{v}}{Z_{0_2} \tan \frac{\omega l_2}{v}}. \tag{26}$$

Two special cases are of interest here: the first when $Z_{0_1} = Z_{0_2}$, and the second when $l_1 = l_2$. The first case corresponds to the transformer disclosed by P. H. Smith,[8] which can be used to transform over a wide

[7] These relations were first proved for wave transmission networks in a paper of the writer's, *B. S. T. J.*, April 1927. See equations 67 and 68, page 291. They are proved by other methods in "Communication Networks," E. A. Guillemin, p. 139.

[8] See reference 1, p. 431.

range of impedance for a single frequency. For this case

$$\cosh \theta = \cos \frac{\omega l_1}{v} \sqrt{1 + \frac{\tan \frac{\omega l_1}{v}}{\tan \frac{\omega l_2}{v}}};$$

$$K_1 = Z_{0_1} \sqrt{- \tan \frac{\omega l_1}{v} \tan \frac{\omega(l_1 + l_2)}{v}}; \tag{27}$$

$$\varphi^2 = 1 + \frac{\tan \frac{\omega l_1}{v}}{\tan \frac{\omega l_2}{v}}.$$

The pass band of the filter lies between the values

$$f_1 = \frac{v}{4(l_1 + l_2)} \quad \text{and} \quad f_2 = \frac{v}{4l_1} \tag{28}$$

and hence between these frequencies the structure will act as a transformer. The ratio of the transformer, however, varies with frequency, and hence two given impedances can only be matched at one frequency. By adjusting the values of l_1 and l_2 it is possible to transform between any two resistances at a given frequency.

For a number of purposes it is desirable to transform a wide band of frequencies between two constant impedances. This requires a transformer with a constant transformation ratio over the whole band of frequencies. As can be seen from equation (26) this can be accomplished with the structure considered above if we let $l_1 = l_2 = l$. For this case

$$\cosh \theta = \sqrt{1 + \frac{Z_{0_1}}{Z_{0_2}}} \cos \frac{\omega l}{v};$$

$$K_1 = Z_{0_1} \varphi \sqrt{\frac{1}{1 - \frac{Z_{0_1}}{Z_{0_2}} \cot^2 \frac{\omega l}{v}}}; \tag{29}$$

$$\varphi^2 = 1 + \frac{Z_{0_1}}{Z_{0_2}}.$$

The mid-band frequency occurs when $\cosh \theta = \cos (\omega l/v) = 0$. Hence l is $\frac{1}{4}$ wave-length at the mid-band frequency. At the mid-band frequency the impedance of the transformer is

$$K_{1_0} = Z_{0_1} \varphi. \tag{30}$$

This is the impedance that the transformer should be connected to since this is the characteristic impedance of the filter. The width of the pass band is determined by

$$1 \geqq \cosh \theta \geqq -1. \tag{31}$$

The cut-off frequencies of the filter transformer are given by the formula

$$\cos \frac{\omega l}{v} = \pm \frac{1}{\varphi}. \tag{32}$$

For relatively narrow bands, the ratio of the band width to the mean frequency is given by the simple formula

$$\frac{f_2 - f_1}{f_m} \doteq \frac{4}{\pi \varphi}, \tag{33}$$

as can readily be shown from equation (32) by using the approximation formula for the cosine in the neighborhood of the angle $\pi/2$.

Hence the structure shown in Fig. 5 is equivalent to a perfect transformer whose ratio is $\varphi^2 = 1 + (Z_{0_1}/Z_{0_2})$ to 1 and a filter whose band width is given by equation (33). Such a filter will have a flat attenuation loss over about 80 per cent of its theoretical band when it is terminated on each side by resistances equal to $Z_{0_1}\varphi$ and Z_{0_1}/φ on its high- and low-impedance sides respectively. Due to the high Q obtainable in the transmission lines, the loss in the band of the filter can be made very low and hence such a transformer will introduce a very small transmission loss. Furthermore, since it is constructed only of transmission lines, it can carry a large amount of power. The complete design equations for the transformer are

$$\varphi^2 = 1 + \frac{Z_{0_1}}{Z_{0_2}}; \qquad Z_{0_1} = \frac{R_I}{\varphi}; \qquad Z_{0_2} = \frac{R_I}{\varphi(\varphi^2 - 1)};$$
$$l = \frac{v}{4f_m}; \qquad R_0 = \frac{R_I}{\varphi^2}, \tag{34}$$

where R_I and R_0 are respectively the input and the output resistances that the transformer works between.

Many other types of transforming networks containing only transmission lines are also possible. Another simple network which is the inverse of the one considered above is shown in Fig. 7. It consists of a length of line l_1 and characteristic impedance Z_{0_1} in series with a balanced open-circuited line of length l_2 and characteristic impedance Z_{0_2}. It is easily shown by employing the equations for a line that when $l_1 = l_2 = l$

$$e_0 = e_I\left(1 + \frac{Z_{0_2}}{Z_{0_1}}\right)\cos\frac{\omega l}{v} - j i_I Z_{01}\sin\frac{\omega l}{v}\left[1 - \frac{Z_{0_2}}{Z_{0_1}}\cot^2\frac{\omega l}{v}\right];$$

$$i_0 = i_I\cos\frac{\omega l}{v} - j\frac{e_I}{Z_{0_1}}\sin\frac{\omega l}{v}. \tag{35}$$

For this case

$$\varphi^2 = \frac{1}{1 + \frac{Z_{0_2}}{Z_{0_1}}}; \quad \cosh\theta = \frac{\cos\frac{\omega l}{v}}{\varphi}; \quad K_1 = Z_{0_1}\varphi\sqrt{1 - \frac{Z_{0_2}}{Z_{0_1}}\cot^2\frac{\omega l}{v}}. \tag{36}$$

The band width of the filter for narrow bands is given approximately by

$$\frac{f_2 - f_1}{f_m} \doteq \frac{4\varphi}{\pi}.$$

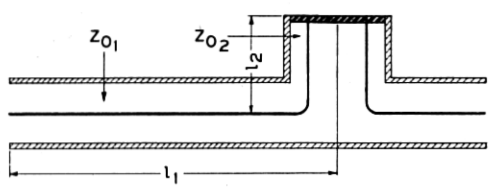


Fig. 7—A wide-band transformer constructed from coaxial conductors.

The design equations for the transformer are

$$\varphi^2 = \frac{1}{1 + \frac{Z_{0_2}}{Z_{0_1}}}; \qquad Z_{0_1} = R_I/\varphi; \qquad Z_{0_2} = R_I(1 - \varphi^2)/\varphi^3;$$

$$l = v/4f_m; \qquad R_0 = R_I/\varphi^2. \tag{37}$$

The first transformer discussed is a step-down transformer while the one considered here has a step up from the input of the line to the output. The first filter had a mid-shunt impedance characteristic on each end, i.e., the impedance of the band is infinity at the two edges; whereas the transformer with the series open-circuited line has a mid-series impedance, since the impedance given by the last expression in equation (36) goes to zero at the two cut-off frequencies. The range of transformation is about the same for each and hence one type has no particular advantage over the other.

It is often desirable in filter work to be able to have the impedance of one end of the filter somewhat different from that of the other, i.e., to have the filter act as a transformer of a moderate ratio. An example of this occurs when using a structure composed of short lengths of line to connect two high-frequency pentodes. As shown by Salzberg and Burnside,[9] the output impedance of a high-frequency pentode at 100 megacycles may be in the order of 30,000 ohms due to the high-frequency shunting loss of the tube. On the other hand, due to active grid loss, the impedance looking into the grid of the next tube may be in the order of 20,000 ohms. Hence in order to obtain the most gain from such tubes, the coupling circuit should be able to work from 30,000 ohms when connected with the output to 20,000 ohms for the input of the next tube. If we employ the simple coupling circuit shown in Section III, Fig. 1, this can be made impedance transforming by making the second-series conductor of a different characteristic impedance from the first-series conductor. Such a combination is shown in Fig. 8. The equations of the combination are easily solved

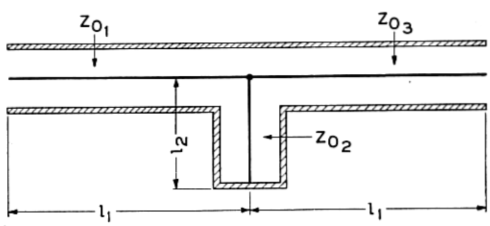


Fig. 8—A narrow-band transformer for coupling vacuum tubes.

with the result that

$$i_c = i_I\left[\cos^2\frac{\omega l_1}{v} - \frac{Z_{0_1}}{Z_{0_3}}\sin^2\frac{\omega l_1}{v} + \frac{Z_{0_1}}{Z_{0_2}}\frac{\sin\frac{\omega l_1}{v}\cos\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}\right] - je_I\left[\frac{Z_{0_1}+Z_{0_3}}{Z_{0_1}Z_{0_3}}\sin\frac{\omega l_1}{v}\cos\frac{\omega l_1}{v} - \frac{\cos^2\frac{\omega l_1}{v}}{Z_{0_2}\tan\frac{\omega l_2}{v}}\right];$$

$$e_0 = e_I\left[\cos^2\frac{\omega l_1}{v} - \frac{Z_{0_3}}{Z_{0_1}}\sin^2\frac{\omega l_1}{v} + \frac{Z_{0_3}}{Z_{0_2}}\frac{\sin\frac{\omega l_1}{v}\cos\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}\right] - ji_I\left[(Z_{0_1}+Z_{0_3})\sin\frac{\omega l_1}{v}\cos\frac{\omega l_1}{v} + \frac{Z_{0_1}Z_{0_3}}{Z_{0_2}}\frac{\sin^2\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}\right]. \quad (38)$$

[9] "Recent Developments in Miniature Tubes," *Proc. I. R. E.*, Vol. 23, No. 10, p. 142, Oct. 1935.

For this case the image transfer constant and the impedance ratio of the transforming filter are given by the equations

$$\cosh\theta = \sqrt{\cos^2\frac{2\omega l_1}{v} + \left(\frac{Z_{0_1}+Z_{0_3}}{2Z_{0_2}}\right)\frac{\sin\frac{2\omega l_1}{v}\cos\frac{2\omega l_1}{v}}{\tan\frac{\omega l_2}{v}} + \sin^2\frac{2\omega l_1}{v}\left[\frac{Z_{0_1}Z_{0_3}}{4Z_{0_2}^2\tan^2\frac{\omega l_2}{v}} - \frac{(Z_{0_1}-Z_{0_3})^2}{4Z_{0_1}Z_{0_3}}\right]};$$

$$\varphi^2 = \left[\frac{\cos^2\frac{\omega l_1}{v} - \frac{Z_{0_1}}{Z_{0_3}}\sin^2\frac{\omega l_1}{v} + \frac{Z_{0_1}}{Z_{0_2}}\frac{\sin\frac{\omega l_1}{v}\cos\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}}{\cos^2\frac{\omega l_1}{v} - \frac{Z_{0_3}}{Z_{0_1}}\sin^2\frac{\omega l_1}{v} + \frac{Z_{0_3}}{Z_{0_2}}\frac{\sin\frac{\omega l_1}{v}\cos\frac{\omega l_1}{v}}{\tan\frac{\omega l_2}{v}}}\right]. \tag{39}$$

When we solve the expression for $\cosh\theta$ for the cut-off frequencies, we find that one of them is given by the expression

$$f_2 = \frac{v}{4l_1} \tag{40}$$

and the other by

$$\tan\frac{2\omega l_1}{v} = \frac{-2\left(\frac{Z_{0_1}+Z_{0_3}}{Z_{0_1}Z_{0_3}}\right)Z_{0_2}\tan\frac{\omega l_2}{v}}{1-\frac{(Z_{0_1}+Z_{0_3})^2}{Z_{0_1}^2Z_{0_3}^2}Z_{0_2}^2\tan^2\frac{\omega l_2}{v}}. \tag{41}$$

If we consider the special case $Z_{0_2} = Z_{0_1}Z_{0_3}/(Z_{0_1}+Z_{0_3})$ equation (41) reduces to the simple form

$$\tan\frac{2\omega l_1}{v} = -\tan\frac{2\omega l_2}{v} \tag{42}$$

or for narrow bands

$$\frac{2\omega_1 l_1}{v} = \pi - \frac{2\omega_1 l_2}{v} \quad \text{and} \quad f_1 = \frac{v}{4(l_1+l_2)} \tag{43}$$

in agreement with equation (17).

At the two cut-off frequencies, it can be shown that

$$\varphi^2 = \frac{Z_{0_1}^2}{Z_{0_3}^2} \tag{44}$$

and throughout the band the value of φ does not differ much from this value for narrow-band filters.

Hence the structure of Fig. 8 acts as a narrow-band coupling unit which introduces a transformation from input to output. For narrow bands it is easily shown that the image impedance at the middle of the pass band is given by the expression

$$K_1 = \frac{4}{\pi}\frac{(f_m)}{(f_2 - f_1)}\sqrt{Z_{0_1}Z_{0_3}}\frac{Z_{0_1}}{Z_{0_3}}. \tag{45}$$

The design equations for this transforming filter are

$$l_1 = \frac{v}{4f_2}; \qquad l_2 = \frac{v}{4}\left[\frac{1}{f_1} - \frac{1}{f_2}\right]; \qquad Z_{0_1} = \frac{\pi}{4}\frac{R_I(f_2 - f_1)}{\sqrt{\varphi}f_m};$$

$$Z_{0_3} = \frac{\pi}{4}\frac{R_I(f_2 - f_1)}{f_m\varphi^{\frac{3}{4}}}; \qquad Z_{0_2} = \frac{\pi R_I(f_2 - f_1)}{4f_m\varphi^{\frac{1}{4}}(1 + \varphi)}; \qquad \frac{R_I}{R_0} = \varphi^2. \tag{46}$$

V. Filters and Transformers Employing Transmission Lines and Condensers

Condensers can be constructed for high radio frequencies which have little dissipation and hence they can be combined with short sections of transmission lines to produce filters and transformers. Combina-

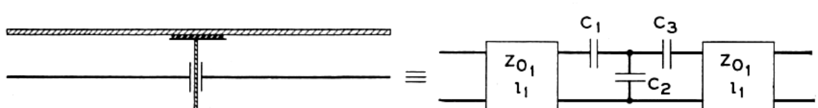


Fig. 9—A transformer or filter using condensers and coaxial conductors.

tions of lines with condensers have the advantages that much more isolated bands can be obtained, in general narrower pass bands can be obtained, and at the lower frequencies shorter sections of lines can be employed if they are resonated by capacities. Also, when such structures are used as interstage coupling units working between vacuum tubes, they usually will have to incorporate the grid to filament and plate to filament capacities as part of the coupling circuit. Hence it is desirable to consider combinations of transmission lines and condensers as filters and transformers.

One of the simplest and most useful types of band-pass filter using transmission lines and condensers is shown in Fig. 9. This structure has been used as a wide-band transformer and as a very narrow-band filter, and experimental curves are given in the next section. The equations connecting the output voltage and current with the input voltage and current can easily be calculated by the methods given above and are

$$e_0 = e_I\left[\frac{C_2 + C_3}{C_3}\cos^2\frac{\omega l}{v} - \frac{Z_{0_2}}{Z_{0_1}}\left(\frac{C_1 + C_2}{C_2}\right)\sin^2\frac{\omega l}{v} + \frac{\sin\frac{2\omega l}{v}}{2}\left(\frac{C_1 + C_2 + C_3}{\omega Z_{0_1}C_1C_3} - \omega C_2 Z_{0_2}\right)\right] - j i_I Z_{0_1}\left[\frac{\sin\frac{2\omega l}{v}}{2}\left[\frac{C_2 + C_3}{C_3} + \frac{Z_{0_2}}{Z_{0_1}}\left(\frac{C_1 + C_2}{C_1}\right)\right] - \left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_1}C_1C_3}\cos^2\frac{\omega l}{v} + Z_{0_2}\omega C_2\sin^2\frac{\omega l}{v}\right]\right];$$

$$i_0 = i_I\left[\frac{C_1 + C_2}{C_1}\cos^2\frac{\omega l}{v} - \frac{Z_{0_1}}{Z_{0_2}}\left(\frac{C_2 + C_3}{C_3}\right)\sin^2\frac{\omega l}{v} + \frac{\sin\frac{2\omega l}{v}}{2}\left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_2}C_1C_3} - Z_{0_1}\omega C_2\right]\right] - \frac{j e_I}{Z_{0_1}}\left[\frac{\sin\frac{2\omega l}{v}}{2}\left[\frac{C_1 + C_2}{C_1} + \frac{Z_{0_1}}{Z_{0_2}}\left(\frac{C_2 + C_3}{C_3}\right)\right] + \sin^2\frac{\omega l}{v}\left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_2}C_1C_3}\right] + \omega C_2 Z_{0_1}\cos^2\frac{\omega l}{v}\right]. \quad (47)$$

In order that the structure shall transform uniformly over a band of frequencies we must have

$$\varphi^2 = \left(\frac{C_1 + C_2}{C_1}\right)\left(\frac{C_3}{C_2 + C_3}\right) = \frac{Z_{0_1}}{Z_{0_2}} = \text{impedance transformation ratio.} \quad (48)$$

With this substitution, equations (47) simplify to

$$\begin{aligned} e_0 = e_I &\left[\frac{C_2 + C_3}{C_3} \cos \frac{2\omega l}{v} + \frac{\sin \frac{2\omega l}{v}}{2} \left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} - \omega C_2 Z_{0_2} \right] \right] \\ & - j i_I Z_{0_1} \left[\left(\frac{C_2 + C_3}{C_3} \right) \sin \frac{2\omega l}{v} \right. \\ & \qquad \left. - \left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} \cos^2 \frac{\omega l}{v} + Z_{0_2} \omega C_2 \sin^2 \frac{\omega l}{v} \right] \right]; \\ i_0 = \varphi^2 &\left[i_I \left[\frac{C_2 + C_3}{C_3} \cos \frac{2\omega l}{v} \right. \right. \\ & \qquad \left. + \frac{\sin \frac{2\omega l}{v}}{2} \left(\frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} - Z_{0_2} \omega C_2 \right) \right] \\ & - \frac{j e_I}{Z_{0_1}} \left[\left(\frac{C_2 + C_3}{C_3} \right) \sin \frac{2\omega l}{v} + \frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} \sin^2 \frac{\omega l}{v} \right. \\ & \qquad \left. \left. + \omega C_2 Z_{0_2} \cos^2 \frac{\omega l}{v} \right] \right]. \end{aligned} \tag{49}$$

Comparing these equations with equations (24) we have for the image parameters

$$\begin{aligned} \cosh \theta &= \varphi \left[\frac{C_2 + C_3}{C_3} \cos \frac{2\omega l}{v} + \frac{\sin \frac{2\omega l}{v}}{2} \left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} - \omega C_2 Z_{0_2} \right] \right]; \\ K_1 &= Z_{0_1} \sqrt{\frac{\frac{C_2 + C_3}{C_3} \sin \frac{2\omega l}{v} - \left[\frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} \cos^2 \frac{\omega l}{v} + Z_{0_2} \omega C_2 \sin^2 \frac{\omega l}{v} \right]}{\frac{C_2 + C_3}{C_3} \sin \frac{2\omega l}{v} + \frac{C_1 + C_2 + C_3}{\omega Z_{0_1} C_1 C_3} \sin^2 \frac{\omega l}{v} + Z_{0_2} \omega C_2 \cos^2 \frac{\omega l}{v}}}; \\ K_2 &= \frac{K_1}{\varphi^2}. \end{aligned} \tag{50}$$

These equations give the image parameters for a general transforming band-pass filter. The two uses to which such a structure will ordinarily be put are either to obtain a transformer with as wide a pass band as possible for a given impedance transformation or else to obtain a filter without transformation ratio. For the transformer case it can be shown that the widest pass-band occurs when $C_3 \to \infty$, or in other words the condenser C_3 is short-circuited. In order to obtain a simple design, it is assumed that each conductor is an eighth of a wave-length at the mid-band frequency or that

$$\frac{\omega_m l}{v} = \frac{\pi}{4}. \tag{51}$$

The mid-band frequency occurs when $\cosh \theta = 0$. Upon substituting the relation $Z_{0_1} = l/vC_0$ where C_0 is the total distributed capacity of the input line of the transformer, $\cosh \theta$ vanishes when

$$\frac{C_0}{C_1} = \frac{\pi^2}{16}\frac{C_2}{\varphi^2 C_0}. \tag{52}$$

Solving for the frequencies for which $\cosh \theta = \pm 1$, it is easily shown that the ratio of the band width to the mean frequency is given by the expression

$$\frac{f_2 - f_1}{f_m} \doteq \frac{\dfrac{4}{\pi\varphi}}{1 + \dfrac{C_2}{2\varphi^2 C_0}}. \tag{53}$$

The image impedance K_1 at the mid-frequency of the band is from equation (50)

$$K_{1_0} = Z_{0_1}\sqrt{\frac{1 - \dfrac{\pi}{4}\dfrac{C_2}{\varphi^2 C_0}}{1 + \dfrac{\pi}{4}\dfrac{C_2}{\varphi^2 C_0}}}. \tag{54}$$

From the above equations and noting that $\varphi^2 = 1 + C_2/C_1$, the design equations of the transformer become

$$Z_{0_1} = R_1\sqrt{\frac{\varphi + \sqrt{\varphi^2 - 1}}{\varphi - \sqrt{\varphi^2 - 1}}}; \qquad Z_{0_2} = \frac{Z_{0_1}}{\varphi^2}; \qquad l = \frac{v}{8f_m};$$

$$C_0 = \frac{33.3l}{Z_{0_1}} \text{ in } \mu\mu\text{f}; \qquad C_1 = \frac{4C_0}{\pi}\sqrt{\frac{\varphi^2}{\varphi^2 - 1}}; \tag{55}$$

$$C_2 = \frac{4C_0}{\pi}\sqrt{(\varphi^2 - 1)\varphi^2},$$

where R_1 is the input impedance from which the transformer must work.

When the structure of Fig. 9 is used as a filter without transformation, we have $C_1 = C_3$; $Z_{0_1} = Z_{0_2} = Z_0$. For this case the image parameters become

$$\cosh\theta = \frac{C_1 + C_2}{C_1}\cos\frac{2\omega l}{v} + \frac{\sin\frac{2\omega l}{v}}{2}\left[\frac{2C_1 + C_2}{\omega Z_0 C_1^2} - \omega C_2 Z_0\right];$$

$$K_1 = K_2 = K = Z_0\sqrt{\frac{1 - \left[\dfrac{(2C_1 + C_2)\cot\frac{\omega l_1}{v}}{2\omega Z_0 C_1(C_1 + C_2)} + \dfrac{Z_0\omega C_1 C_2}{2(C_1 + C_2)}\tan\frac{\omega l_1}{v}\right]}{1 + \dfrac{(2C_1 + C_2)\tan\frac{\omega l_1}{v}}{2\omega Z_0 C_1(C_1 + C_2)} + \dfrac{Z_0\omega C_1 C_2}{2(C_1 + C_2)}\cot\frac{\omega l_1}{v}}}. \quad (56)$$

For narrow-band filters it is easily shown that

$$\frac{f_2 - f_1}{f_m} = \Delta = \frac{\frac{4}{\pi}}{1 + \frac{C_2}{C_1} + \frac{C_2}{2C_0}}; \qquad \frac{(2C_1 + C_2)C_0}{C_1^2} = \frac{\pi^2}{16}\frac{C_2}{C_0}. \quad (57)$$

At the mid-band of the filter, since the constants were worked out on the assumption that each conductor was an eighth of a wave-length at the mid-band frequency, the mid-band filter impedance can be obtained from the last part of (56) by setting $\omega l/v = \pi/4$, giving

$$K_0 = Z_0\sqrt{\frac{1 + \frac{C_2}{C_1} - \frac{\pi}{4}\frac{C_2}{C_0}}{1 + \frac{C_2}{C_1} + \frac{\pi}{4}\frac{C_2}{C_0}}}. \quad (58)$$

Hence solving for the constants of the filter on the assumption that Δ is a small quantity, we find

$$C_1 = \frac{4C_0}{\pi}; \qquad C_2 = \frac{16C_0}{\pi(2 + \pi)\Delta}; \qquad Z_0 = \frac{8R}{(\pi + 2)\Delta};$$
$$l = \frac{v}{8f_m}; \qquad C_0 = \frac{33.3l}{Z_0}\text{ in }\mu\mu\text{f}, \quad (59)$$

where R is a resistance equal to K at the mean-frequency of the filter. For narrow bands this gives a very large value for C_2, the shunt capacity. A more practical arrangement is to replace the two series condensers C_1 and the shunt condenser C_2 by a π network consisting of two shunt condensers C_A separated by a series condenser C_B. These have the values

$$C_A = \frac{C_1 C_2}{2C_1 + C_2}; \qquad C_B = \frac{C_1^2}{2C_1 + C_2}. \tag{60}$$

With this arrangement we find that for narrow bands $C_A \doteq C_1$ and C_B is a very small capacity. This can readily be obtained physically by inserting a partition with a small hole in it at the middle of the section. Then C_A will be the capacity of the inside conductors to the partition, and C_B will be the capacity of one inside conductor to the other looking through the small hole. By adjusting the size of this hole, this capacity can be made as small as desired.

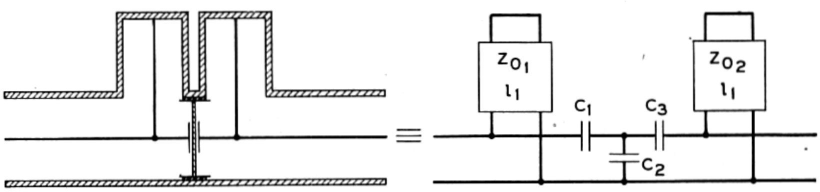


Fig. 10—A shunt terminated transformer.

The transformer discussed above is suitable for transforming from line impedances down to very low impedances, but cannot be used to transform from line impedances up to very high impedances such as the impedance of a vacuum tube. This generally requires a shunt type of termination rather than the series type discussed above. One such transformer is shown in Fig. 10. It consists of two shunt lines connected together by a T or π network of capacities. The constants of such a transformer are readily calculated, and for the condition of maximum transformation for a given band width—which occurs when $C_3 \to \infty$, and for eighth wave-length conductors on each end—these have been found to be

$$\varphi^2 = 1 + \frac{C_2}{C_1} = \frac{Z_{01}}{Z_{02}}; \qquad Z_{01} = \frac{K_{10}}{\varphi}; \qquad Z_{02} = \frac{K_{10}}{\varphi^3};$$
$$C_1 = \frac{4C_0}{\pi}; \qquad C_2 = \frac{4}{\pi} C_0(\varphi - 1); \qquad l = \frac{v}{8f_m}. \tag{61}$$

The theoretical band width for this type transformer is given approximately by the expression

$$\frac{f_2 - f_1}{f_m} \doteq \frac{4}{\varphi(\pi + 2)}. \tag{62}$$

Such a transformer is also suitable for connecting together vacuum tubes of high impedance.

Another type of transformer of some interest is one which will transform from very high impedances to very low impedances. Such a transformer is shown in Fig. 11. It has a shunt conductor on the high-impedance end and a series conductor on the low-impedance end. Such a transformer does not have a constant transformation ratio over the whole band, but for about 80 per cent of the theoretical band width

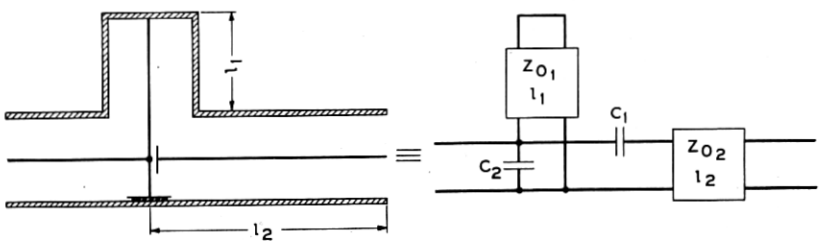


Fig. 11—A series shunt type transformer suitable for wide bands.

the transformation ratio is approximately constant. The design equations for such a transformer are

$$\frac{K_{10}}{K_{20}} = \varphi^2; \quad f_m = \sqrt{f_1 f_2}; \quad Z_{01} = \frac{K_{10}(f_2 - f_1)}{f_m}; \quad l_1 = \frac{v}{8f_m};$$

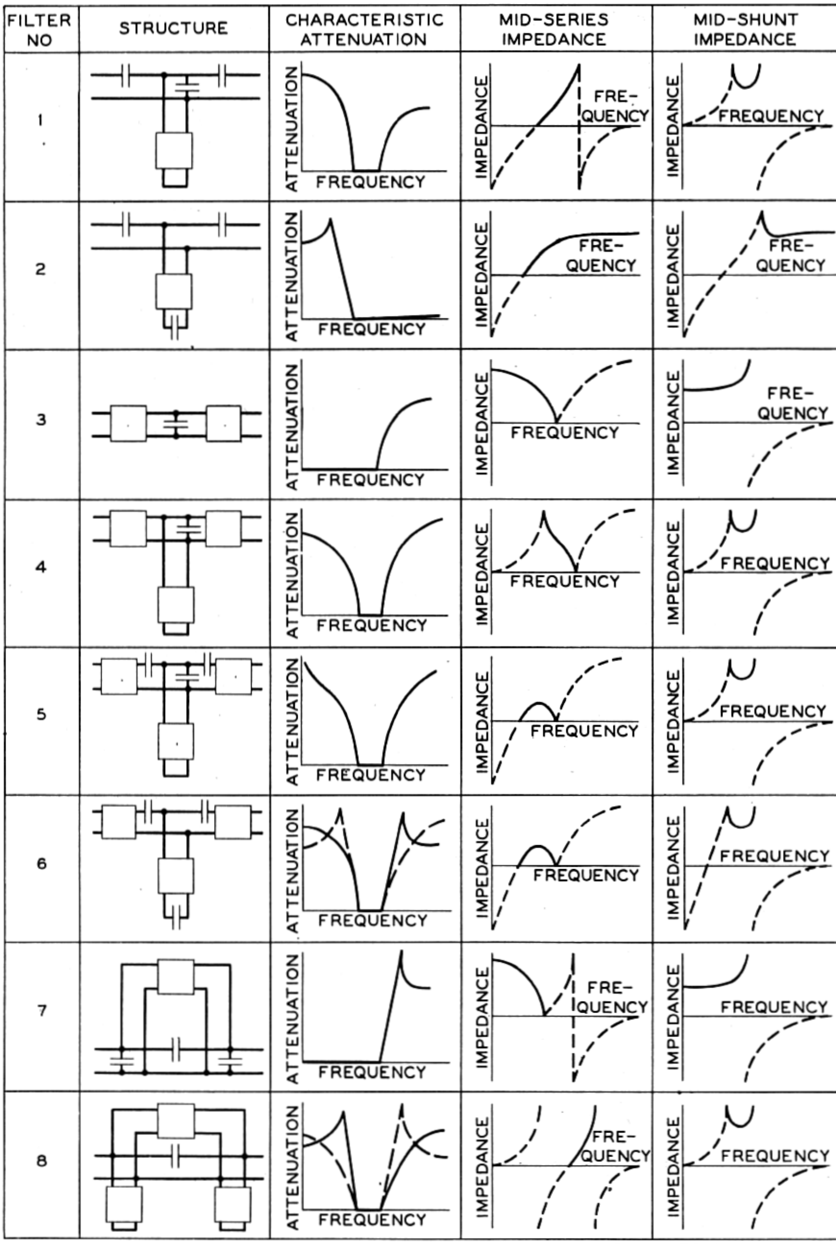
$$Z_{02} = \frac{K_{20} f_m}{(f_2 - f_1)\sqrt{1 - 1/\varphi}}; \quad l_2 = \frac{v\sqrt{1 - 1/\varphi}}{2\pi f_m}; \tag{63}$$

$$C_1 = \frac{(f_2 - f_1)\varphi}{2\pi f_m^2 K_{10}};$$

$$C_2 = \frac{1}{2\pi K_{10}(f_2 - f_1)}\left[1 - \frac{(f_2 - f_1)^2(\varphi - 1)}{f_m^2}\right].$$

The transformer is especially useful since it will give the widest transmission band for a given transformation ratio of any of the transformers discussed.

Many other types of filters, transforming and nontransforming, are also possible using transmission lines and condensers. A partial list of such filters is shown in Fig. 12 together with their attenuation and



NOTE DOTTED LINES INDICATE REACTIVE IMPEDANCE. SOLID LINES INDICATE RESISTIVE IMPEDANCE

Fig. 12—A list of filter structures employing transmission lines and condensers.

iterative impedance characteristics. All of the band-pass filters can be made impedance-transforming by varying the ratios of the impedance elements for the two ends.

VI. Experimental Results

Several filters and transformers of the types discussed above have been tested experimentally and have been found to give operating characteristics in accordance with the calculated results. Figure 13

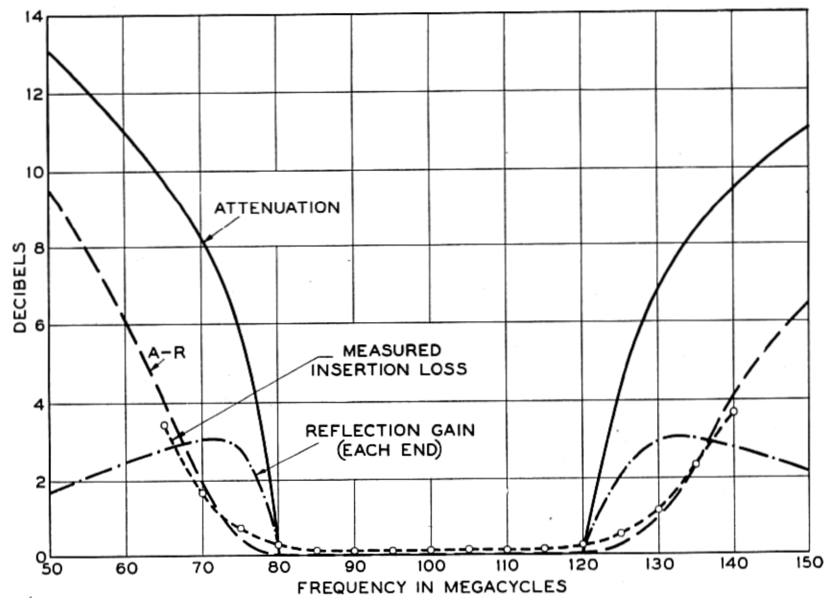


Fig. 13—Measured insertion loss of a wide-band transformer.

shows the measured insertion loss of a wide-band low-impedance transformer which transforms from 70 ohms down to an impedance of 17.5 ohms. The useful transformation band is from 80 megacycles to 120 megacycles. The measuring circuit is shown in Fig. 14. It consists

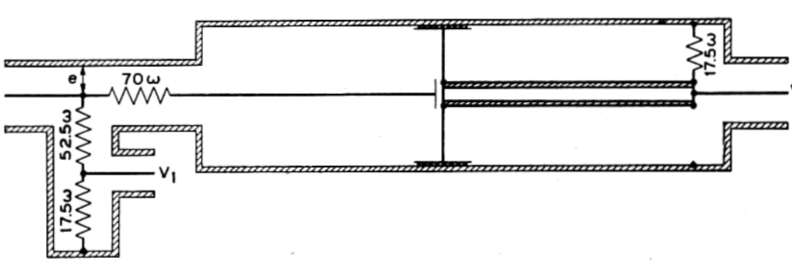


Fig. 14—Measuring circuit for a transformer.

of a source of high-frequency voltage impressed on a divided circuit. In one branch is a series resistance of 70 ohms connected to the 70-ohm side of the transformer. The output of the transformer is connected

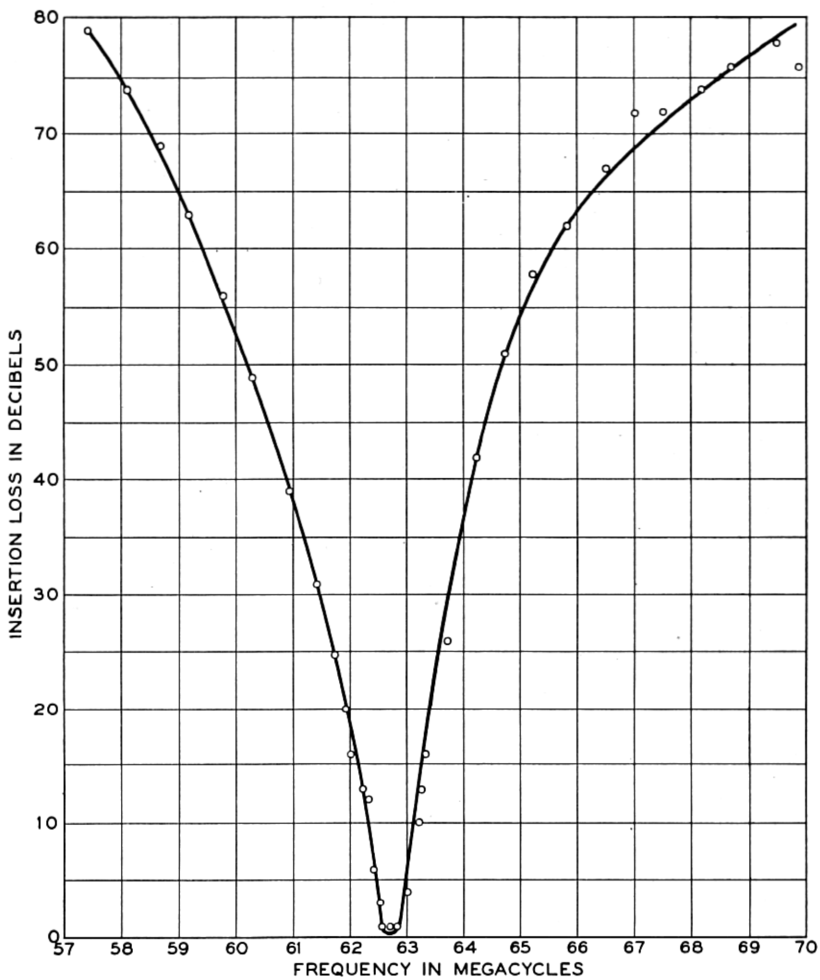


Fig. 15—Measured insertion loss of filter used on Green Harbor–Provincetown radio link.

to a 17.5 ohm resistance and a high-impedance voltmeter is connected across it. The other branch contains a 52.5-ohm and a 17.5-ohm resistance in series, with a high-impedance voltmeter shunted across the 17.5 ohms. If the transformer were a perfect transformer, the current from the output of the transformer would be

$$i_0 = \frac{e}{140} \times \sqrt{\frac{70}{17.5}} = \frac{e}{70}, \tag{64}$$

where e is the voltage applied to ground at the common point. The voltage across the output should be

$$e_0 = \frac{e}{70} \times 17.5 \doteq \frac{e}{4}. \tag{65}$$

But this is just the voltage that should occur across the voltmeter V_1. Hence the difference in reading between V_2 and V_1 will be a measure of the loss introduced by the transformer. From Fig. 13 we see that this is in the order of 0.1 db, which represents a small loss for a transformer.

Several of the narrow-band filters of the type shown in section V, Fig. 9, have also been constructed and tested. One of these has been used on an experimental radio system at Green Harbor, Massachusetts, since 1935, for the purpose of connecting a transmitter and receiver on the same antenna. This filter has been constructed and tested by Messrs. F. A. Polkinghorn and N. J. Pierce using the design data developed here. The filter used consisted of three sections of the type shown in Fig. 9 connected in tandem. The resulting insertion loss of the filter and associated transformers is shown in Fig. 15. The loss at mid-band is in the order of 1 db and an insertion loss of over 50 db is obtained 2 megacycles on either side of the center of the pass band.