Study of Magnetic Losses at Low Flux Densities in Permalloy Sheet *

By W. B. ELLWOOD and V. E. LEGG

Energy losses in ferromagnetic materials subject to alternating fields have long been considered as due solely to hysteresis and eddy currents. However, at the low flux densities encountered in certain communication apparatus, a further loss is observed which has been variously termed "residual," "magnetic viscosity," or "square law hysteresis." The search for an explanation of this loss has led to precise measurements of hysteresis loops with a vacuum ballistic galvanometer, and of a.-c. losses with inductance bridges. From these results, it appears that that part of the a.-c. effective resistance of a coil on a ferromagnetic core which is proportional to the coil current is strictly identified with the hysteresis loop area as measured by a ballistic galvanometer, or as indicated by harmonic generation in the coil. The hysteresis loop can now be constructed in detail as to size and skewness on the basis of a.-c. bridge measurements. This conclusion was reached previously on a compressed iron powder core, and is now confirmed on an annealed laminated 35 per cent nickel-iron core. Observed eddy current losses for this core exceed those calculated from classical theory by 20 per cent. This excess is ascribed to the presence of low permeability surface layers on the sheet magnetic material. The a.-c. residual loss per cycle (nominally independent of frequency, like hysteresis) is not observed by ballistic galvanometer measurements, although it indicates an energy loss some eight times the hysteresis loss for the smallest loop measured ($B_m = 1.3$ gauss). Analysis of the residual loss shows that it increases with frequency up to about 500 cycles, and remains constant at higher frequencies (to 10,000 cycles per second). Concurrently with the increase of residual loss, the permeability of the alloy is observed to decline with increasing frequency about 1 per cent below the value predicted from eddy current shielding. This effect is most noticeable at frequencies below 1000 cycles.

THE search for an explanation of the excessive magnetic losses observed at low flux densities by alternating current bridge measurement as compared with theoretical indications based on direct-current measurements has led to a more accurate review of both types of measurement.[1, 2, 3] The a.-c. energy loss per cycle which has re-

* To be published in May 1937 issue of *Jour. of Applied Physics.*

[1] W. B. Ellwood, *Physics*, **6**, 215 (1935).
[2] V. E. Legg, *Bell Syst. Tech. Jour.*, **15**, 39 (1936).
[3] W. B. Ellwood, *Rev. Sci. Inst.*, **5**, 301 (1934).

ceived most study is the "residual" or "viscosity" loss.[4] It appears related to hysteresis loss because it is nominally independent of frequency, but it differs in being proportional to B_m^2 instead of B_m^3 which would be required by Rayleigh's law for hysteresis loops. Any satisfactory investigation of this anomalous loss demands precise determination of its value, and of its variation with frequency. For this purpose, ballistic galvanometer measurements of the hysteresis loop have been made and compared with bridge measurements of a well annealed 35 permalloy laminated core.

In a previous paper, the magnetic properties of a ring of compressed powdered iron were studied at low flux densities using a sensitive vacuum galvanometer[3] and a multiple swing ballistic method.[1] Hysteresis loops were measured at flux densities B_m ranging from 1.8 to 115 gauss, which showed energy losses proportional to B_m^3 in accordance with Rayleigh's law. Alternating-current measurements agreed with the ballistic measurements as to the magnitude of the energy loss and the proportionality to B_m^3, but in addition showed a residual loss proportional to B_m^2 which was of the same order of magnitude as the Rayleigh hysteresis at these low flux densities.

The analysis of measurements made on the compressed dust core was complicated by the inhomogeneous structure, by the variety of particle shapes and thickness of insulation, and by the mechanical stresses incident to forming the core. To eliminate these objections, the present study was undertaken using a core consisting of well annealed sheet material, for which eddy current losses can be calculated by classical formulae.

Selection of Material

Considerable a.-c. data were at hand from which to select material for this experiment. The properties of a few representative materials are given in Table I. The constants are defined by the equation

$$\frac{R_f}{\mu_m f L_f} = aB_m + ef + c = \frac{8\pi W}{B_m^2}, \tag{1}$$

where R_f is the difference between the resistances measured with a.-c. and with d.-c. on a toroidal coil with inductance of L_f due to core material of permeability μ_m, when the maximum flux density is $\pm B_m$ and the frequency is f cycles per second. Here the hysteresis coefficient

[4] H. Jordan, *E. N. T.*, **1**, 7 (1924); H. Wittke, *Ann. d. Phys.* (5) **23**, 442 (1935); F. Preisach, *Zeit. f. Phys.*, **94**, 277 (1935); R. Goldschmidt, *Zeit. f. tech. Phys.*, **13**, 534 (1932).

is a, the eddy current coefficient e, and the residual loss term is c. W is the energy loss per cycle in ergs/cm³ of core material. The permeability coefficient $\lambda = (\mu_m - \mu_0)/\mu_0 B_m$.

Table I shows that annealed 35 permalloy in sheet form has the most convenient values of B_r and c/a for further study of this effect. This alloy is of the face centered cubic lattice type common to a large class of magnetic alloys. The numerals preceding the various permalloys give the nickel or alloy percentages, as classified by G. W. Elmen.[5]

The measurement of magnetic losses of 35 permalloy involved further refinements in technique. The high initial permeability required the construction of a special air core mutual inductance to simplify the

TABLE I

Material	Initial Permeability	$\lambda \times 10^4$	$c \times 10^6$	$a \times 10^6$	c/a	$B_r^* \times 10^4$.
Compressed Powder Cores						
Grade B Iron	35	7.0	110	50.	2.2	13.
81 Permalloy	75	1.8	40	5.5	7.3	3.1
Laminated Cores						
35 Permalloy, Annealed	1660	30.	60	5.0	12.	62.
38 Permalloy, Hard	100	7.0	118	9.6	12.	7.2
38 Permalloy, 800° Annealed	1330	9.0	27	1.5	18.	15.
40 Permalloy, 1000° Annealed	2060	12.0	20	1.4	14.	22.
45 Permalloy, Annealed	2550	5.4	14	.43	33.	8.2
78.5 Permalloy, Annealed	3900	8.1	0	0.6	0	18.
2.4–78 Cr Permalloy, Annealed	14600	6.4	3	.07	43.	7.6
8–79 Cr Permalloy, Annealed	3025	31.	14	2.6	5.4	60.
45–25 Perminvar, Annealed	450	0.02	0.0	.002	—	0.0

* These values of remanence were computed from Rayleigh's law as $3a\mu_0 B_m^2/16$ for $B_m = 2$.

measuring circuit and increase its stability. The high rate of change of permeability with temperature made it necessary to enclose both the specimen and the air core mutual inductance in a constant temperature box (at 37.1 ± 0.01° C.) throughout the tests.

The Specimen

The material was melted in a high-frequency furnace, cast, and cold-rolled with intermediate annealings, to strip of thickness $t = 0.0160$ cm. and width 7.62 cm. Analysis showed the following composition: Ni, 35.00 per cent; Fe, 64.25 per cent; Mn, 0.40 per cent; S, 0.030 per cent; Si, 0.02 per cent; C, 0.01 per cent. The resistivity ρ was 82.2 micro-ohm-cms. at 37.1° C. The strip was wound into a tight

[5] *Electrical Engineering*, **54,** 1292 (1935).

spiral core with successive turns insulated from each other by painting with a suspension of fine quartz powder in CCl_4 immediately prior to winding. The core had an effective magnetic diameter $d = 11.22$ cm., and cross-sectional area of alloy $A = 3.96$ cm.^{2}. It was annealed in pure hydrogen for one hour at a temperature of 1000° C.

In order to protect the annealed core from mechanical stress during subsequent winding, it was placed on felt in an annular bakelite box which held it without constraint. The box was wound with a 20-turn magnetizing coil using a flat tape composed of 28 parallel strands of insulated wires connected together at the ends. This winding practically covered the box with a single layer of wire, and gave a uniform magnetizing force. It was employed as the magnetizing coil in both the ballistic and the a.-c. bridge measurements. For the ballistic

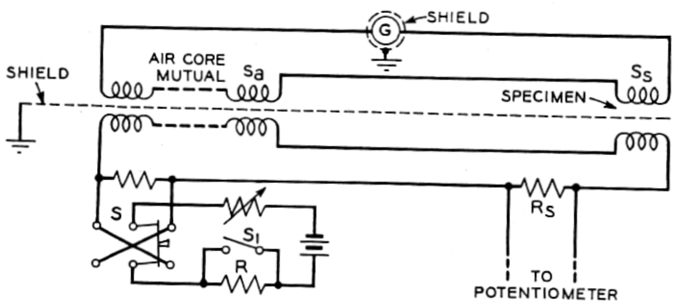

FIG. 1—Ballistic galvanometer circuit, showing adjustable air-core mutual inductance in series-opposition with the test coil.

tests, a layer of insulation was applied over the primary winding before applying the secondary winding. This insulation consisted of two wrappings of silk tape interspersed with a tinfoil sheath which formed a grounded electrostatic shield between the secondary and the primary winding. The foil was cut to avoid a short-circuited turn. The toroidal secondary winding consisted of 5000 turns No. 19 silk-covered enamelled copper wire.

D.-C. APPARATUS

In the former experiment,[1] the specimen was compared with a fixed air core mutual inductance in terms of the galvanometer deflection and the primary currents required to obtain approximate balance. For this experiment, the circuit was modified so that the same current flowed in the primaries of both the specimen and the air core mutual (Fig. 1). Thus variable thermal effects in the primary circuits were eliminated and the measuring technique simplified. This required that the mutual inductance be adjustable so as to bring the unbalance onto the galvanometer scale.

The mutual inductance (approximately 0.260 henrys) was constructed for convenience in four separate sections. Each section had a hard wood toroidal core, a low resistance toroidal secondary winding, an inter-winding shield, and sectionalized primary windings on the outside. The secondary windings were connected in series with the galvanometer by a twisted shielded pair of wires. The primary winding groups were also connected in series, and adjusted so that the combination resulted in a mutual inductance of the right value to obtain balance. To eliminate humidity as a source of error each coil was painted with cellulose acetate, covered with silk tape, painted again, baked 48 hours at 108° C. and finally potted in Superla wax in an earthenware jar with only the tops of the terminals exposed. All connections were made by soldering.

During the measurements, the maximum primary current corresponding to H_m was held constant to 0.01 per cent by comparing the voltage drop across R_s with a battery of Weston standard cells. Switching was automatically performed by a photocell and selector switch mechanism previously described [3, 1] but not shown here. These operated switches S and S_1 at the proper time and in the right order. The difference in flux turns between the air core mutual inductance and the specimen was determined in terms of the ultimate galvanometer deflection as before. From this the difference in B between the side of the hysteresis loop and a straight line drawn through its tips could be computed for a given H. A number of values of this difference ΔB were thus determined for different values of H, and plotted to give the hysteresis loop.

A.-C. Apparatus

In order to compare results obtained by the vacuum ballistic galvanometer with those obtained with alternating currents, bridge measurements of resistance and inductance were made over the same range of flux densities at a number of frequencies ranging from 35 to 10,000 cylces. The secondary winding was removed and the special 20-turn primary winding used for most of the measurements. Later an additional 60-turn winding was used for checking the measurements in the low-frequency range. In either case the inductance was low enough to depress any effect of distributed capacitance far below the precision of the measurements.

Measurements were made on a 10-ohm equal ratio arm inductance comparison bridge,[2] and were verified at low frequencies using a 1-ohm ratio arm bridge. Calibration of the bridge and standard coils was effected by making measurements over the entire frequency range on a calibrated high quality air core coil substituted for the test coil. The maximum correction required on this account was approximately 0.1 per cent of the resistance due to the magnetic core.

The source of alternating current was an oscillator-amplifier supplying approximately 0.4 watt undistorted power, calibrated for these measurements against the Laboratories' standard frequency. The current was adjusted by the insertion of resistance in series with the

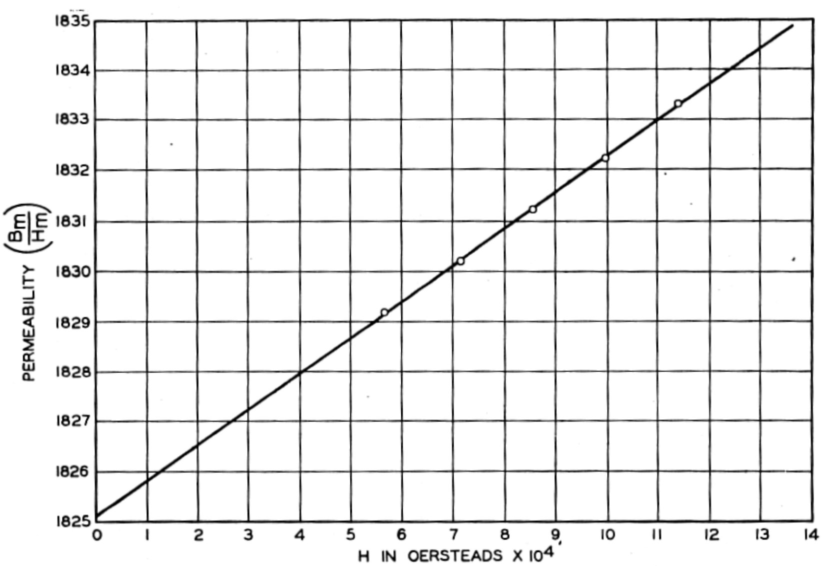

FIG. 2—Core permeability as measured by the ballistic galvanometer.

primary of the bridge input transformer, and was measured by means of a thermocouple between the transformer secondary and the bridge.

The bridge unbalance was amplified by means of an impedance coupled amplifier for the 10-ohm bridge, and by means of a resistance coupled amplifier for the 1-ohm bridge. The amplified unbalance was observed by means of head phones at frequencies above 200 cycles, and by means of a vibration galvanometer at lower frequencies. The d.-c. balance required bridge current of about 3 m.a. in the test coil winding, and had the same precision as the a.-c. balance, viz., $\pm$ 0.0002 ohm. The inductance readings were corrected for the air space within the winding, and had a relative accuracy of about 0.03 per cent, and an absolute accuracy of approximately 0.1 per cent.

D.-C. Results

The permeability $\mu = B_m/H_m$ of the specimen is shown as a function of H_m in Fig. 2, on a greatly enlarged scale in which the zero of permeability is not shown. The slope of this line gives $\lambda = 21.5 \times 10^{-4}$.

Values of ΔB are plotted against H for two different hysteresis loops in Fig. 3, from the area of which the energy loss W is computed. For

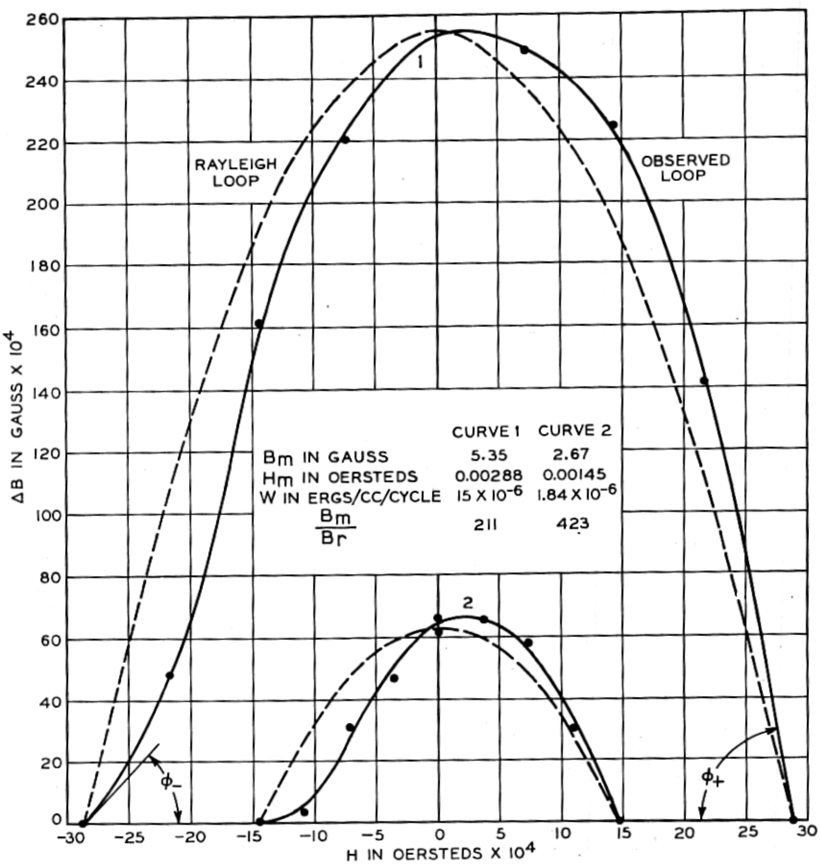

Fig. 3—Two hysteresis (half) loops plotted with reference to a straight line through their tips.

comparison, Rayleigh loops are shown as broken lines. By analysis of the interior angles ϕ_+ and ϕ_- between the ΔB vs. H curve and the H axis at points $+H$ and $-H$ respectively, it is found that ϕ_+ is larger than for the Rayleigh loop, and is given by the relation $\tan \phi_+ = \mu_0 \lambda B_m$, while ϕ_- is a corresponding amount smaller than for the Rayleigh loop. The hysteresis coefficient a and the permeability coefficient, λ are related by the equation $a = 8\lambda/3\mu$ for a loop having parabolic shape, in which case the interior angles are equal, and $\tan \phi_{\pm} = \mu_0\lambda B_m$. Since the latter equation applies for ϕ_+ only, on the observed skewed loop, it follows that the Rayleigh relation between a and λ is more or less inaccurate. In fact, the ratio of $8\lambda/3\mu$ to a is a measure of the skewness of the loop. For the present material, this ratio is about 1.15. This result is in accord with our previous data, but was evidently not noted by Rayleigh because the free poles in his magnetic circuit tended to mask the asymmetry. The fact that these hysteresis loops are slightly skewed shows that those processes which produce the familiar S-shaped loop at high flux densities are already present at these low flux densities.

Despite a skewed shape, the area of the observed loop approximates closely the area of a parabola drawn through the remanence and the tips. Hence, supplementary values of energy loss W were obtained from remanence observations at several values of H_m, using the formula $W = 2B_rH_m/3\pi$. The slenderness factor of the loops may be measured by the ratio B_m/B_r, which varies from 211 to 890 for the different loops studied.

The a.-c. resistance introduced by the hysteresis loss of the core material yields the ratio $8\pi W/B_m^2$, as noted in Eq. (1). Values of this quantity computed from the areas of the loops of Fig. 3. and from remanence determinations are plotted in Fig. 4. They agree closely with the aB_m term of Eq. (1) obtained by a.-c. measurements, as shown by the solid line in Fig. 4. The sum of $c + aB_m$ is shown by the broken line. It is evident that the ballistic galvanometer gives no indication of residual loss.

It is interesting to note that the hysteresis loop at low flux densities can now be constructed in detail using the data obtained from a.-c. measurements. The remanence is $B_r = \frac{3}{16}a\mu_0 B_m^2$ and $\tan \phi_+ = \mu_0\lambda B_m$. The angle included between the upper and lower branches of the loop at the tips is $(\phi_+ + \phi_-)$ and is given by the equation $\tan [\frac{1}{2}(\phi_+ + \phi_-)] = \frac{3}{8}\mu^2{}_0 aB_m$.

A.–C. Results

Values of R_f and L_f were measured as a function of the current at fixed frequencies. The values of $R_f/\mu_m f L_f$ are plotted in Fig. 5 as a function of current with frequency as a parameter. In order to shorten the vertical scale, the appropriate ordinates are indicated in connection with each line. These form a family of straight lines parallel to one another. This shows that the hysteresis coefficient is practically a constant over the low flux density range for all frequencies. From the slope of these straight lines the hysteresis coefficient a of eq. (1) is calculated to be 2.6×10^{-6} which agrees with the value 2.53×10^{-6}

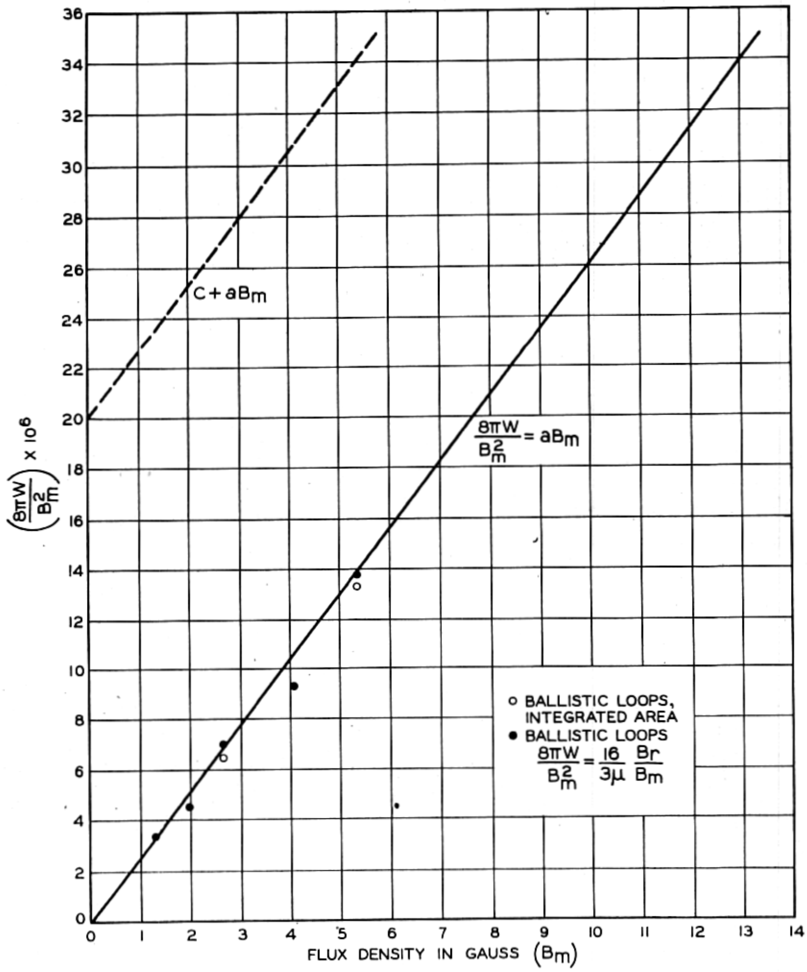

FIG. 4—Comparison of ballistic galvanometer and a.-c. determinations of hysteresis loss. Note absence of residual loss from ballistic observations.

computed from the ballistic tests. The divergence of the data from linearity at higher currents is shown by the dotted curves, which indicate divergence of the hysteresis loop from the Rayleigh form at higher flux densities.

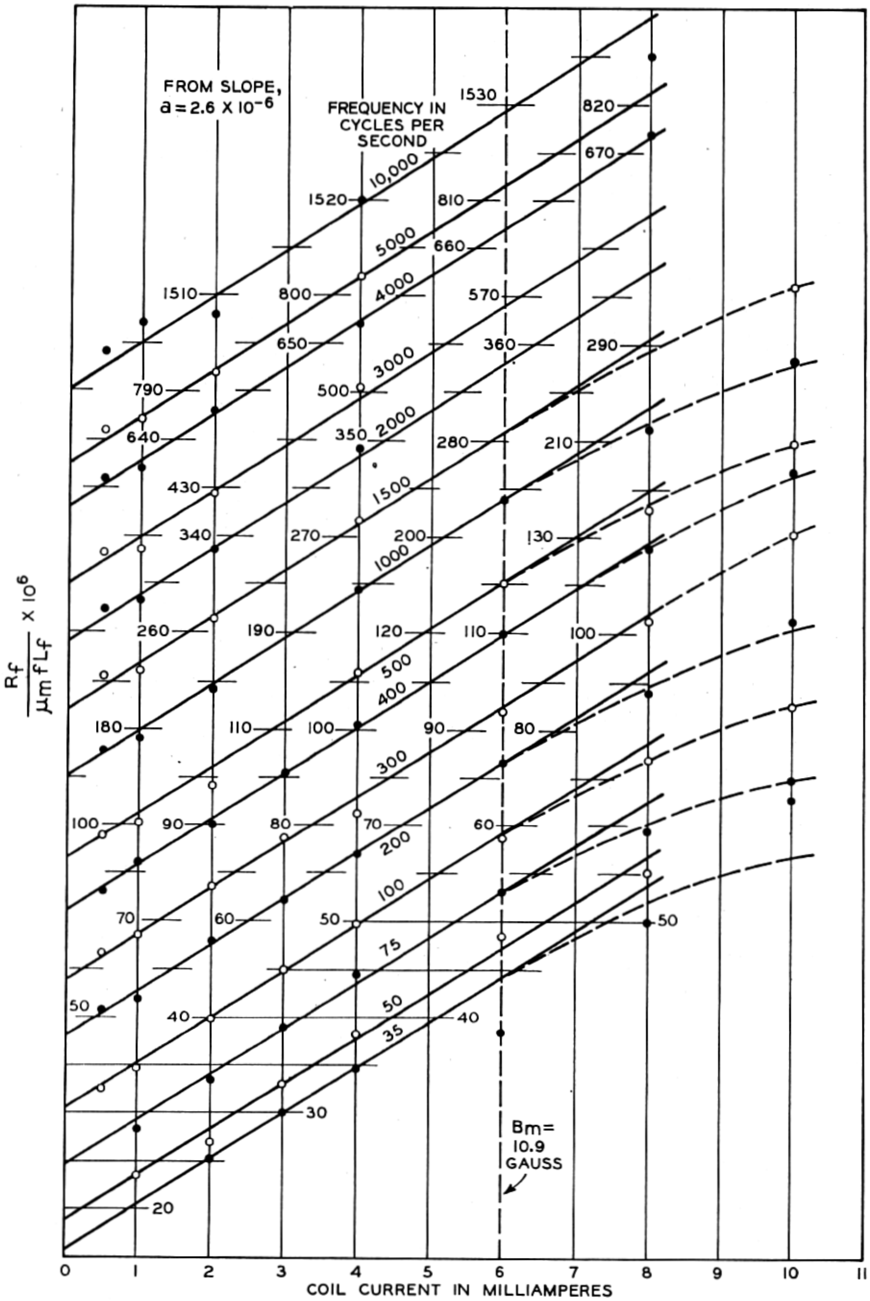

FIG. 5—A.-c. bridge values of $R_f/\mu_m f L_f$ vs. coil current of various frequencies. Slope of the straight parallel lines gives the hysteresis coefficient a.

Further information on hysteresis loss is obtained from measurements of harmonic voltages generated in the coil winding when there is current I_1 of frequency f. It has been shown [6] that the third harmonic voltage for materials with Rayleigh hysteresis loops is

$$E_3 = 0.6\, aB_m\mu_m L_m f I_1,$$

from which the hysteresis coefficient is

$$a = \frac{25\, E_3}{3\, fI_1^2}\sqrt{\frac{Ad \times 10^{-9}}{2\mu_m^3 L_m^3}}.$$

Measurements of third harmonic voltages have been made on the coil described in this paper by P. A. Reiling and the results are shown in Table II.

TABLE II

f	I_1 m.a.	E_3 m.v.	$a \times 10^6$	f	I_1 m.a.	E_3 m.v.	$a \times 10^6$
1000	2.0	.0168	2.2	100	3.0	.00447	2.6
	5.0	.133	2.8		5.0	.0106	2.2
	10.0	.55	2.9		10.0	.0473	2.5
					16.8	.1497	2.8
400	1.41	.00335	2.2				
	2.0	.00709	2.3	75	8.0	.0199	2.2
	3.0	.0158	2.3		10.0	.0335	2.3
	5.0	.0457	2.4		18.2	.112	2.4
	10.0	.195	2.5				
				50	10.0	.0359	3.7

The values of a thus obtained show no consistent variation with current or frequency. They give an average value of 2.5×10^{-6}, which is in close agreement with the ballistic and a.-c. bridge results. It therefore appears that that part of the effective resistance which is proportional to current is identifiable with hysteresis loss as obtained by ballistic means, and with that which generates harmonic voltages.

The intercepts for $I = 0$ in Fig. 5 are therefore due to eddy current and any residual losses. They have been plotted against frequency in Fig. 6. The line through these points is generally assumed to be straight, and the eddy current and residual loss coefficients are derived from its slope and intercept. It appears, however, that this line is not strictly straight, but has a somewhat steeper slope at lower frequencies, so that the ordinary graphical method of loss separation fails.

An analytical separation of losses can be made for any frequency interval by returning to the values of $R_f/\mu_0 f L_f$ as obtained from Fig. 5,

[6] E. Peterson, Bell Syst. Tech. Jour., **7**, 762 (1928).

subtracting the value at f_1 from that at f_2 and dividing by the frequency interval $f_2 - f_1$ to give the eddy current coefficient e of equation (1).[7] Figure 6 gives e thus derived, as a function of f, showing a value approximately 20 per cent higher than calculated from the relation $e = 4\pi^3 t^2/3\rho$ at frequencies above 500 cycles, and progressively higher as the frequency approaches zero.

The fact that e is larger than predictable from classical theory has been ascribed to the presence of a low permeability surface skin on

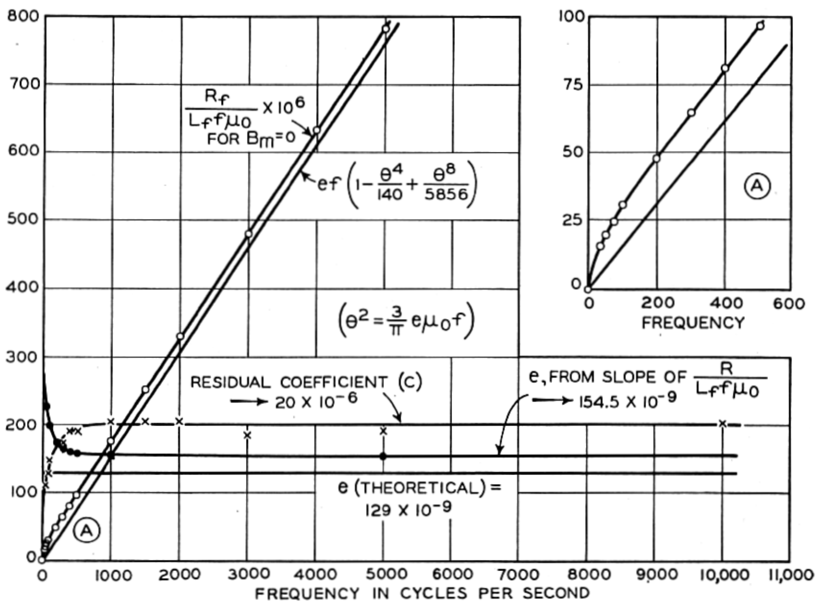

FIG. 6—Intercepts of Fig. 5 vs. frequency. Slope of the curve gives eddy current coefficient e. Residual loss coefficient c varies with frequency near $f=0$.

practically all materials which have been reduced to sheet or wire form by mechanical deformation.[8] But since the eddy current coefficient depends only on the resistivity and effective thickness of the material, any apparent variation of e with frequency can only be interpreted as an indication that the residual loss is varying with frequency.

Taking the value of $e = 154.5 \times 10^{-9}$ characteristic of the higher frequencies, the eddy current loss per cycle has been calculated, and is indicated in Fig. 6. The amount by which the observed loss exceeds

[7] Correction terms must be included at higher frequencies to take account of eddy current shielding as noted in ref. 2.

[8] E. Peterson and L. R. Wrathall, *I. R. E. Proc.*, **24**, 275 (1936).

the calculated eddy current loss gives the residual loss coefficient c. Thus, the value of c is found to be a constant 20×10^{-6} at frequencies above 500 cycles, but to decline toward zero as the frequency approaches zero, in evident accord with the ballistic galvanometer result.

The inductance due to the core shows a similar frequency effect. The observed inductances for the 20-turn winding are given in Fig. 7. The values at each frequency for the various currents fall on a straight

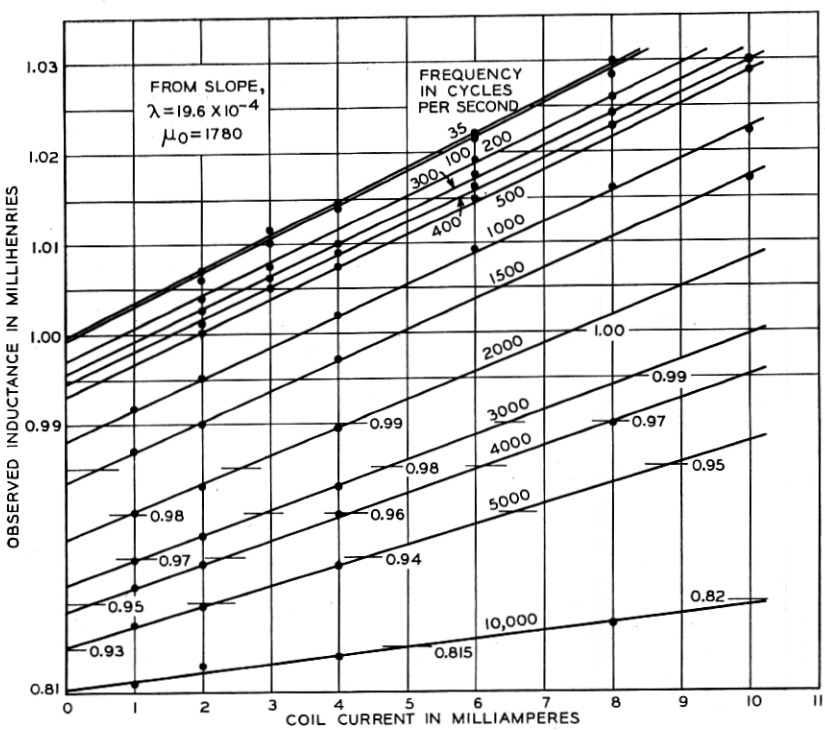

FIG. 7—Inductance observed on 20 turn coil, at various frequencies.

line, the slope of which gives the permeability coefficient, $\lambda = 19.6 \times 10^{-4}$, for the lower frequencies where eddy current shielding can be neglected.

The values of L for $I = 0$, obtained from Fig. 7, are plotted against frequency in Fig. 8. The most remarkable feature of this curve is the decline of inductance (or apparent permeability) of about 1 per cent at low frequencies, where very little decrease on account of eddy current shielding is to be expected. The characteristic shielding curve has been computed using the value of e obtained from the resistance measurements in the relation

$$\frac{L}{L_0} = \frac{1}{\theta}\frac{\sinh\theta + \sin\theta}{\cosh\theta + \cos\theta} = \left(1 - \frac{\theta^4}{30} + \frac{\theta^8}{732} - \cdots\right),^*$$

where $\theta = \sqrt{3e\mu_0 f/\pi}$. Using the values of L/L_0 thus computed, the effect of eddy current shielding was eliminated from the observed values, and the results plotted in the upper curve in Fig. 8, showing a rapid

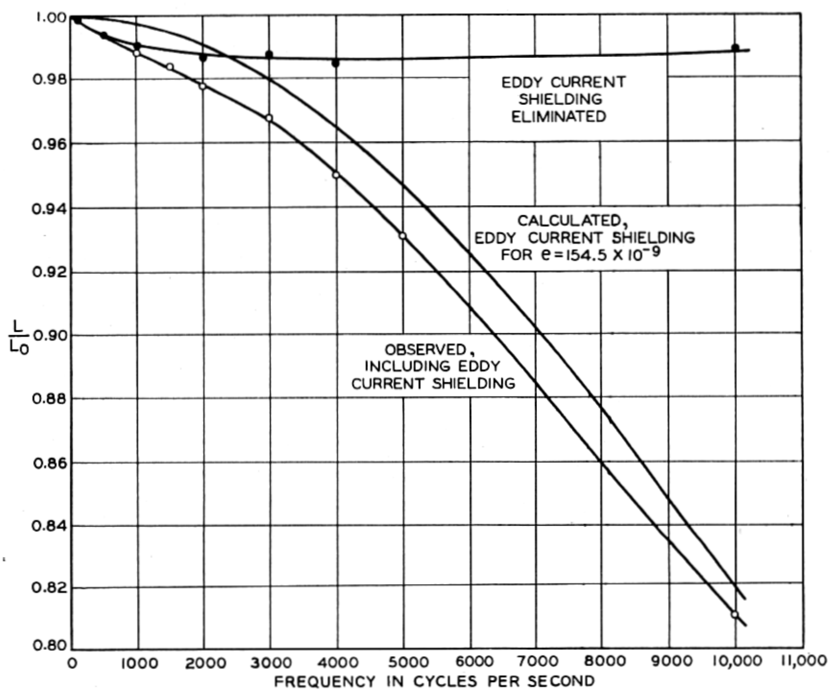

FIG. 8—L/L_0 from intercepts of Fig. 7. Observed values about 1 per cent lower than calculated for eddy current shielding.

decline of inductance or apparent permeability with increasing frequency at low frequencies, and the flattening off to a fairly constant value at higher frequencies.

The initial permeability thus obtained (average of results from the 20-turn and 60-turn windings) is 1780. This is somewhat lower than the value found by the ballistic galvanometer. The difference is due to magnetic aging which was observed to decrease the permeability of

* The series is inaccurate at frequencies above 5000 cycles.

the core at the rate of approximately 1 per cent per month. In contrast, no change of resistivity was detected in an accelerated aging test consisting of a bake at 100° C. for 150 hours.

Discussion

The ballistic data on both the present annealed 35 permalloy sheet core and the previous compressed powdered iron core show that the area of the hysteresis loop varies as B_m^3 in agreement with Rayleigh's law. The magnitude of the loss is not given by the fractional slope λ of the μ,B line as required by Rayleigh's law because the loops are not parabolic in shape. This discrepancy gives a measure of the skewness of the hysteresis loop. The B_m^3 portion of the a.-c. data agrees with the loop areas obtained with the ballistic galvanometer. The threefold agreement between the ballistic data, the harmonic measurements, and the a.-c. resistance measurements indicates that the hysteresis loop is substantially unchanged in shape over a frequency range extending from 0 to 10,000 cycles.

Since the hysteresis loop has a size and shape independent of frequency, and area strictly proportional to B_m^3, it accounts for that part of the effective resistance of a coil on a ferromagnetic core which is proportional to the alternating magnetizing current. The remainder consists of eddy current and residual losses.

The ordinary graphical method of separating these losses is excluded by the obviously non-linear relation between $R_f/\mu_0 f L_f$ and f. Using an analytical method, the eddy current loss is found to be some 20 per cent larger than computed by classical theory, indicating the presence of low permeability surface layers on the sheet material. The residual loss coefficient is found to increase with frequency up to about 500 cycles, and to remain constant at higher frequencies (up to 10,000 cycles).

The observed inductance diminishes with increasing frequency about 1 per cent below the value calculated for eddy current shielding, the most noticeable decrease occurring below 1000 cycles where eddy current shielding is practically absent.

Various theories have been advanced to account for residual loss, as noted in a previous paper.[1] Goldschmidt[9] and Dannatt[10] have attributed the loss to non-homogeneous alloy structure, or preferred axes in such directions as to give a flux component perpendicular to the sheet surface, with accompanying eddy-currents unconstrained by the sheet thickness. This theory fails to account for residual losses in com-

[9] R. Goldschmidt, *Helv. Phys. Act.*, **9**, 33 (1936).
[10] C. Dannatt, *I. E. E. J.*, **79**, 667 (1936).

pressed powder cores, where eddy-currents are confined to single particles and cannot be increased by a modified direction of magnetic flux.

The most notable feature of residual loss is its large value for unannealed materials, and its extremely small values for well annealed alloys, particularly 78.5 permalloy and 45–25 Perminvar. (See Table I.) The permeability is increased by annealing while both c and a are decreased. On the other hand B_r is slightly increased. The decreases in hysteresis and residual loss are attributed to the decrease in work done against internal strains, which also tend to limit initial permeability.[11]

This suggests that residual loss may be due to elastic hysteresis or even simple mechanical friction, with magneto-striction providing the necessary coupling between the elastic or frictional variables and the magnetizing field, as pointed out in our previous paper.[1] Thus, in addition to losses from eddy-currents and magnetic hysteresis, mechanical work is done by the alternately expanding and contracting core—work expended on itself and its supports and insulation. Since the ballistic galvanometer measures only equilibrium values of B and H, this work is not revealed in the area of the ballistic loop. However, in the a.-c. loop the magnetostriction strains produce stresses too rapidly to be relieved, so that B lags behind H with an absorption of energy into the surroundings. This results in an additional effective resistance beyond that due to magnetic hysteresis and eddy-currents. For a sufficiently slow process in well annealed material supported with minimum constraint, the stresses may relieve themselves by thermal agitation and do very little work. But for sufficiently rapid traversals of the loop, all the magnetostrictive stresses will do the same amount of work on the core and its surroundings every cycle. Unannealed materials, or materials rigidly constrained, should continue to show residual loss at very low frequencies. The magnitude of c and its variation with frequency thus should depend on the magnetostrictive constant for the material, and on the types of dissipative constraints.

[11] R. M. Bozorth, *Elec. Eng.*, **54**, 1251 (1935).