The Physical Reality of Zenneck's Surface Wave

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The first part of the paper shows that a vertical dipole does not generate a surface wave which at great distances behaves like Zenneck's plane surface wave. In Parts Two and Three it is shown that it is not necessary to call upon the Zenneck wave to explain the success of the wave antennas.

IN 1907 [1] Zenneck showed that a plane interface between two semi-infinite media could support, or guide, an electromagnetic wave which is exponentially attenuated in the direction of propagation along the interface and vertically upwards and downwards from the interface. Zenneck did not show that an antenna could generate such a wave but, because this "surface wave" seemed to be a plausible explanation of the propagation of radio waves to great distances, it was commonly accepted as one of the components of the radiation from an antenna.

After Sommerfeld [2] formulated the wave function for a vertical infinitesimal dipole as an infinite integral and noted that the integral around the pole of the integrand is the wave function for a surface wave, which at great distances is identical with the Zenneck wave, no one questioned the reality of Zenneck's surface wave.

There has been recently pointed out by C. R. Burrows [10] the lack of agreement between various formulas and curves of radio attenuation over land when the dielectric constant of the ground must be taken into account. The values of Sommerfeld [2] and Rolf [5] are stated to differ from those of Weyl [7] and Norton [9] by an amount just equal to the surface wave of Zenneck. Burrows [10] presents experimental data supporting the correctness of the Weyl-Norton values and raises a question as to whether a surface wave really is set up by a radio antenna. A vertical current dipole does not generate a surface wave which at great distances behaves like Zenneck's plane surface wave. Theoretical and numerical evidence leading to this conclusion is presented in Part One of this paper. A contemporary theoretical investigation by S. O. Rice * leads to the same conclusion.

The reader familiar with wave antennas will at once ask why the wave antennas seem to justify the Zenneck surface wave theory by means of which they were conceived and designed if there is no surface

* "Series for the Wave Function of a Radiating Dipole at the Earth's Surface," this issue of the *Bell Sys. Tech. Jour.*

wave. In Part Two of this paper it is shown that a plane electromagnetic wave, polarized with the electric vector in the plane of incidence and in the wave front, impinging on a plane solid at nearly grazing incidence produces a total field in which the horizontal electric field near the solid has very nearly the same ratio to the vertical electric field as in the Zenneck surface wave. In Part Three of this paper it is shown that the wave tilt near the ground at a great distance from a vertical dipole is almost the same as that found for the plane wave at nearly grazing incidence.

Part One—The Evidence Against the Surface Wave

The following discussion centers around the surface wave wave-function P and the series (5), (6), (8) and (9) of paper 3 in the bibliography.* These series and P follow

$$P = -\frac{\pi s \tau}{1-\tau^2} H_0^{(2)}(sr) e^{i\tau s z},\dagger \tag{12}$$

$$Q_1 + P/2 = \frac{1}{r}\frac{e^x}{1-\tau^2}\sum_{n=0}^{\infty} A_n(-x)^n, \tag{5}$$

$$Q_2 + P/2 = \frac{1}{r}\frac{1-\tau^2 e^{x_2}}{1-\tau^2}\sum_{n=0}^{\infty} B_n(-x_2)^n, \tag{6}$$

$$Q_0 = Q_1 + P \sim \frac{1}{r}\frac{e^x}{1-\tau^2}\sum_{n=1} C_n x^{-n}, \tag{8}$$

$$Q_1 \sim \frac{1}{r}\frac{1-\tau^2 e^{x_2}}{1-\tau^2}\sum_{n=1} D_n x_2^{-n}, \tag{9}$$

where r = horizontal distance, $x = -ik_1 r$, $x_2 = -ik_2 r$, $\tau = k_1/k_2$, $s = k_1/\sqrt{1+\tau^2}$, $k^2 = \epsilon\mu\omega^2 - 4\pi\sigma\mu i\omega$, $k_2^2 = k_1^2(\epsilon - i2c\lambda\sigma)$, $k_1 \approx 2\pi/\lambda$ in air, $a = \tau^2/(1+\tau^2)$, $a_2 = 1/(1+\tau^2)$,

$$\begin{aligned}
A_0 &= 1, \quad A_1 = \sqrt{a}\tanh^{-1}\sqrt{a}, \quad A_2 = A_1 - a,\\
A_n &= [(2n-3)A_{n-1} - aA_{n-2}]/(n-1)^2,\\
B_0 &= 1, \quad B_1 = \sqrt{a_2}\tanh^{-1}\sqrt{a_2}, \quad B_2 = B_1 - a_2,\\
B_n &= [(2n-3)B_{n-1} - a_2B_{n-2}]/(n-1)^2,\\
C_1 &= -1/a, \quad C_2 = -3/a^2 + 1/a,\\
C_n &= [(2n-1)C_{n-1} - (n-1)^2C_{n-2}]/a,\\
D_1 &= -1/a_2, \quad D_2 = -3/a_2^2 + 1/a_2,\\
D_n &= [(2n-1)D_{n-1} - (n-1)^2D_{n-2}]/a_2.
\end{aligned}$$

* Sommerfeld's time factor $e^{-i\omega t}$ which was used in paper 3 has been replaced by $e^{i\omega t}$.

† z, the height above ground, is zero in paper 3.

The left hand side of (8) has been altered to correspond with the facts as now known.

P is the wave-function for a surface wave which at great distances behaves like Zenneck's plane surface wave.

The series (5) and (6) constitute the complete wave-function for a unit vertical dipole centered on the interface between air and ground.

The series (8) and (9) are the asymptotic expansions of (5) $+ P/2$ and (6) $- P/2$.

The series (5), (6), (8) and (9) are exact and it is from them that the attenuation charts in a paper by C. R. Burrows in this issue of the *Bell System Technical Journal* were computed.

Since interchanging k_1 and k_2 in (5) gives (6) and interchanging k_1 and k_2 in (8) gives (9) but interchanging k_1 and k_2 in P changes its sign it follows that if (6) $\sim$ (9) $+ P/2$ then (5) $\sim$ (8) $- P/2$. Hence the complete wave-function $\Pi_z =$ (5) + (6) $\sim$ [(8) $- P/2$] + [(9) $+ P/2$] = (8) + (9) and P does not appear in the asymptotic expansion of the wave-function.

The series (5) and (6) have been computed and found to be respectively equal to (8) $- P/2$ and (9) $+ P/2$.* These computations show again that $\Pi_z =$ (5) + (6) $\sim$ (8) + (9) or putting it in words, that there is no surface wave wave-function P in the asymptotic expansion of the complete wave-function.

As a further check S. O. Rice has derived the series (5) and (6) in an entirely different manner and verified that their asymptotic expansions are indeed $Q_0 - P/2$ and $Q_2 + P/2$.

In order to get a direct numerical check on the series the wave-function integral was computed by mechanical quadrature for two cases. Van der Pol's transformation of the wave-function integral with the path of integration deformed upward along the lines $Im(ihru)$ constant was used.[6]

1. With $r/\lambda = 1/4\pi$ and $\epsilon - i2c\lambda\sigma = 12.5 - i\,12.5$ mechanical quadrature gave $\Pi_z = (.800 - i\,.578)/r$ while the series (5) and (6) gave $(.9247 - i\,.4334)/r$ and $(-\,.1242 - i\,.1438)/r$ respectively which add up to $(.8005 - i\,.5772)/r$. This is a good check on the series (5) and (6).

2. With $r/\lambda = 50$ and $\epsilon - i2c\lambda\sigma = 80 - i\,.7512$ mechanical quadrature gave $\Pi_z = (.094 - i\,.178)/r$ while the series (8) and (9) gave $Q_0 \approx (.086 - i\,.187)/r$ and $Q_2 \approx 1.2 \times 10^{-11} \overline{|13^{\circ}}/r$. Since $P = (4.47 - i\,1.92)/r$ there can be no doubt that it must be omitted in computing Π_z asymptotically. This is a good check on the above stated relation $\Pi_z =$ (5) + (6) $\sim$ (8) + (9) or $\Pi_z \sim Q_0 + Q_2$. Because the asymptotic series Q_0 here starts to diverge at the third term

* Eq. (1) in paper 4 says that (5) $\sim$ (8) $- P/2$.

it is not possible to determine Q_0 with an accuracy better than the discrepancy between the values given for Q_0 and Π_z.

The above cited facts prove that on the ground the wave-function for a vertical dipole centered on the interface between air and ground is

$$\Pi_z = (5) + (6) \sim [(8) - P/2] + [(9) + P/2] = (8) + (9)$$

or

$$\Pi_z = (Q_1 + P/2) + (Q_2 + P/2) \sim Q_1 + Q_2 + P = Q_0 + Q_2.$$

The function P can only be thought of as follows. The convergent series (5) and (6) comprising the wave function can not be directly expressed as inverse power series; but if the function $P/2$ is respectively added and subtracted the resulting sum and difference do have the asymptotic inverse power series expansions (8) and (9).

Part Two—Superseding the Surface Wave

It has now been shown by theory, by numerical studies and by crucial experiment that Zenneck's surface wave is not a component in the asymptotic expansion of the wave-function for a vertical dipole.

Since the wave antennas were designed to utilize the horizontal component of the Zenneck wave electric field and do pick up radio signals it is desirable that we explain the success of the wave antennas in some other way at the same time that we throw away the Zenneck wave.

The object of this part of the paper is to show that the success of the wave antennas can be well accounted for by means of a plane wave theory. It will be shown that if a plane electromagnetic wave polarized with the electric vector in the plane of incidence and in the wave front impinges on a plane solid at a large angle with the normal to the surface then near the surface the ratio of the horizontal to the vertical component of the total electric field is very nearly the same as though the total field were that of a Zenneck surface wave.

Since the electric and magnetic fields of an antenna ultimately lie in the wave front and since the wave front at any considerable distance is effectively plane for a structure the size of a wave antenna and since the radiation coming down from the ionosphere consists chiefly of that which has been subjected to the minimum number of reflections and the angle at which the radiation arrives at the receiving wave antenna is usually rather low this plane wave theory easily accounts for the success of the wave antennas.

A plane electromagnetic wave polarized with its electric vector in the plane of incidence falls upon a plane semi-conducting surface. We are interested in the total field.

Let the incident electric field be

$$E_{xi} = e^{i\omega t - ik_1(x \sin \theta - z \cos)} \cos \theta,$$
$$E_{zi} = e^{i\omega t - ik_1(\sin \theta - z \cos \theta)} \sin \theta.$$

The reflected field is

$$E_{xr} = - e^{i\omega t - ik_1(x \sin \theta + z \cos \theta)} \cos \theta \cdot R,$$
$$E_{zr} = e^{i\omega t - ik_1(x \sin \theta + z \cos \theta)} \sin \theta \cdot R,$$

where

$$R = \frac{\cos \theta - \tau\sqrt{1 - \tau^2 \sin^2 \theta}}{\cos \theta + \tau\sqrt{1 - \tau^2 \sin^2 \theta}},$$
$$\tau = k_1/k_2 = 1/(\epsilon - i2c\lambda\sigma)^{1/2}.$$

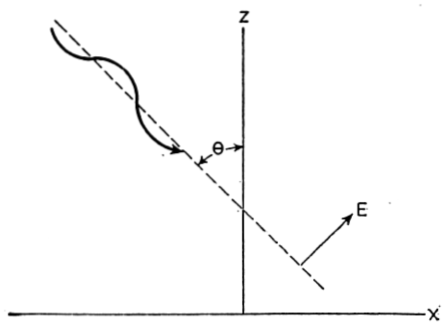

Fig. 1

Then the total field is

$$E_x = e^{i\omega t - ik_1 x \sin \theta} \cos \theta (e^{i\eta} - Re^{-i\eta}),$$
$$E_z = e^{i\omega t - ik_1 x \sin \theta} \sin \theta (e^{i\eta} + Re^{-i\eta}),$$

where $\eta = k_1 z \cos \theta$.

$$E_x = e^{i\omega t - ik_1 x \sin \theta}\, 2 \cos \theta \,\frac{i \cos \theta \sin \eta + \tau\sqrt{1 - \tau^2 \sin^2 \theta} \cos \eta}{\cos \theta + \tau\sqrt{1 - \tau^2 \sin^2 \theta}},$$

$$E_z = e^{i\omega t - ik_1 x \sin \theta}\, 2 \sin \theta \,\frac{\cos \theta \cos \eta + i\tau\sqrt{1 - \tau^2 \sin^2 \theta} \sin \eta}{\cos \theta + \tau\sqrt{1 - \tau^2 \sin^2 \theta}}.$$

In order to see better the significance of these formulas it is necessary to write $\theta = \pi/2 - \delta$ where δ is small, say less than 20°, and suppose that $k_1 z \cos \theta$ is small. Then we may use the expansions

$$\cos \theta = \sin \delta = \delta - \delta^3/3! + - \cdots,$$
$$\sin \theta = \cos \delta = 1 - \delta^2/2! + - \cdots,$$
$$\tan \eta = \cos \theta \cdot k_1 z(1 + k_1^2 z^2 \delta^2/3 + \cdots),$$
$$\cos \eta = 1 - k_1^2 z^2 \delta^2/2 + - \cdots.$$

If terms of third and higher order in δ are dropped we have left

$$E_x = e^{i\omega t - ik_1 x \sin\theta}\, 2\delta\left(1 - \frac{\delta^2}{6}\right)\left(1 - k_1^2 z^2 \frac{\delta^2}{2}\right)$$

$$\left(\tau\sqrt{1-\tau^2} + \frac{\tau^3\delta^2}{2\sqrt{1-\tau^2}} + ik_1 z\delta^2\right) \Big/ \left(\tau\sqrt{1-\tau^2} + \delta + \frac{\tau^3\delta^2}{2\sqrt{1-\tau^2}}\right),$$

$$= e^{i\omega t - ik_1 x \sin\theta} \frac{\tau\sqrt{1-\tau^2}}{\delta + \tau\sqrt{1-\tau^2} + \tau^3\delta^2/2\sqrt{1-\tau^2}}\, 2\delta.$$

$$[1 - \delta^2(\tfrac{1}{6} + k_1^2 z^2/2 - \tau^2/2(1-\tau^2) - ik_1 z/\tau\sqrt{1-\tau^2})],$$

$$E_z = e^{i\omega t - ik_1 x \sin\theta}\, 2\left(1 - \frac{\delta^2}{2}\right)\delta\left(1 - \frac{\delta^2}{6}\right)\left(1 - k_1^2 z^2 \frac{\delta^2}{2}\right)$$

$$\left[1 + \left(\tau\sqrt{1-\tau^2} + \frac{\tau^3\delta^2}{2\sqrt{1-\tau^2}}\right) ik_1 z(1 + k_1^2 z^2 \delta^2/3)\right] \Big/$$

$$\left(\tau\sqrt{1-\tau^2} + \delta + \frac{\tau^3\delta^2}{2\sqrt{1-\tau^2}}\right),$$

$$= e^{i\omega t - ik_1 x \sin\theta} \frac{1 + \tau\sqrt{1-\tau^2} k_1 z i}{\delta + \tau\sqrt{1-\tau^2} + \tau^3\delta^2/2\sqrt{1-\tau^2}}\, 2\delta$$

$$\left[1 - \delta^2\left(\frac{2}{3} + k_1^2 \frac{z^2}{2} - i\frac{\tau^3 k_1 z/2 + \tau(1-\tau^2)k_1^3 z^3/3}{(1 + \tau\sqrt{1-\tau^2} k_1 z)\sqrt{1-\tau^2}}\right)\right].$$

The wave tilt is then

$$\frac{E_x}{E_z} = \frac{\tau\sqrt{1-\tau^2}}{1 + \tau\sqrt{1-\tau^2} k_1 z i}\left\{1 + \delta^2\left[\frac{1}{2} + \frac{ik_1 z}{\tau\sqrt{1-\tau^2}} + \frac{\tau^2}{2(1-\tau^2)}\right.\right.$$

$$\left.\left. - i\frac{\tau^3 k_1 z/2 + \tau(1-\tau^2)k_1^3 z^3/3}{(1 + \tau\sqrt{1-\tau^2} ik_1 z)\sqrt{1-\tau^2}}\right]\right\}.$$

The wave tilt in the Zenneck wave is just τ.

As a particular and probably typical case we may take $\epsilon = 9$, $\sigma = 2 \times 10^{-14}$ and $f = 60{,}000$ and then $\tau = k_1/k_2 = 1/\sqrt{\epsilon - i2c\lambda\sigma}$ $= 1/\sqrt{9 - i600} = .04082\,\underline{|44.570°}$. If $z = 30$ ft. then

$$k_1 z = 2\pi\, 30 \times 30.48/5 \times 10^5 = .01149.$$

If $\delta = 10° = .1745$ radian then $\delta^2 = .03045$. The coefficient of τ in E_x/E_z then turns out to differ from unity by only about 1 per cent.

These figures show that if we retain only the principal terms in our formulæ we have

$$E_x = e^{i\omega t - ik_1x \sin\theta} \frac{\tau\sqrt{1-\tau^2}}{\delta + \tau\sqrt{1-\tau^2}} 2\delta \left[1 - \delta^2\left(\frac{1}{6} - \frac{ik_1z}{\tau}\right)\right],$$

$$E_z = e^{i\omega t - ik_1x \sin\theta} \frac{1 + \tau\sqrt{1-\tau^2} ik_1z}{\delta + \tau\sqrt{1-\tau^2}} 2\delta[1 - \tfrac{2}{3}\delta^2],$$

$$\frac{E_x}{E_z} = \frac{\tau\sqrt{1-\tau^2}}{1 + \tau\sqrt{1-\tau^2} ik_1z}[1 + \delta^2(\tfrac{1}{2} + ik_1z/\tau)].$$

As a rule the wave tilt is so nearly equal to the value τ predicted by Zenneck that present day wave tilt measurements do not distinguish between the two.

PART THREE—THE WAVE TILT OF THE Q_0-WAVE

It would be but natural for a reader to ask what wave tilt would be observed at the surface of a flat earth if there were no Heaviside layer. It was shown in Part One that the asymptotic expansion of the complete wave function is $Q_0 + Q_2$, of which Q_2 is negligible. The function Q_0 there considered is the surface value of a detached wave that carries energy to infinity in all directions. One would therefore expect that at the surface of the earth the Q_0-wave would act like the detached plane wave employed in Part Two. It will now be shown that it does.

It was shown in paper 8 that in the air

$$Q_0 \sim \frac{e^{-ikr}}{r} + \frac{e^{-ikr_2}}{r_2}\left\{ g_{01}(c) - \frac{g_{02}(c)}{ikr_2} + \frac{g_{03}(c)}{(ikr_2)^2} + \cdots \right\},$$

where

$$g_{01}(c) = \frac{c - \tau\sqrt{1-\tau^2+\tau^2c^2}}{c + \tau\sqrt{1-\tau^2+\tau^2c^2}},$$

$$g_{0(n+1)}(c) = \frac{n-1}{2} g_{0n}(c) - \frac{c}{n} g_{0n}'(c) + \frac{1-c^2}{2n} g_{0n}''(c),$$

$c = \cos\theta$ and r_2 and θ are shown in Fig. 2,

$$r_2 = \sqrt{\rho^2 + w^2}, \qquad c = w/r_2, \qquad w = z + a.$$

We need to compute (elm. units are employed, $\mu = 1$)

$$E_\rho = \frac{-\mu i\omega}{k^2}\frac{\partial^2 Q_0}{\partial\rho\partial z},$$

$$= \frac{-i\omega}{k^2}\left(\sqrt{1-c^2}\frac{\partial}{\partial r_2} - \frac{c\sqrt{1-c^2}}{r_2}\frac{\partial}{\partial c}\right)\left(c\frac{\partial}{\partial r_2} + \frac{1-c^2}{r_2}\frac{\partial}{\partial c}\right)Q_0$$

and

$$E_z = -\mu i\omega\left[1+\frac{1}{k^2}\frac{\partial^2}{\partial z^2}\right]Q_0,$$

$$= -\ i\omega\left[1+\frac{1}{k^2}\left(c\frac{\partial}{\partial r_2}+\frac{1-c^2}{r_2}\frac{\partial}{\partial c}\right)^2\right]Q_0$$

at a great distance near the interface; that is to say, retaining only the leading terms in c/r_2 and $1/r_2^2$.

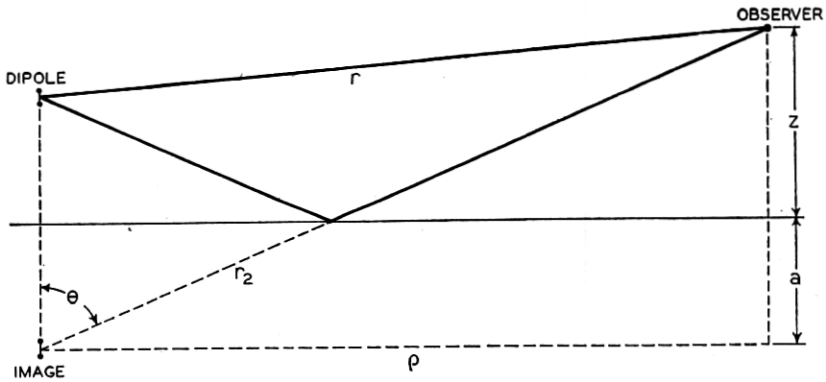

Fig. 2

The complete calculation of E_ρ and E_z is too long to be included. If $a = 0$ so that $r = r_2$

$$E_\rho = -\ i\omega\frac{e^{-ikr_2}}{r_2}\left\{-c\sqrt{1-c^2}(1+g_{01}(c))+\frac{\sqrt{1-c^2}}{ikr_2}[(1-2c^2)g_{01}'(c)\right.$$

$$-3c(1+g_{01}(c))+cg_{02}(c)]-\frac{\sqrt{1-c^2}}{(ikr_2)^2}[-(2-5c^2)g_{01}'(c)$$

$$-c(1-c^2)g_{01}''(c)+(1-2c^2)g_{02}'(c)+3c(1+g_{01}(c))$$

$$\left.-5cg_{02}(c)+cg_{03}(c)]+\cdots\right\},$$

$$E_z = -\ i\omega\frac{e^{-ikr_2}}{r_2}\left\{(1-c^2)(1+g_{01}(c))-\frac{1}{ikr_2}[(1-c^2)g_{02}(c)\right.$$

$$-(1-3c^2)(1+g_{01}(c))-2c(1-c^2)g_{01}'(c)]$$

$$+\frac{1}{(ikr_2)^2}[-(1-5c^2)g_{02}(c)+(1-c^2)g_{03}(c)-(1-c^2)^2g_{01}''(c)$$

$$\left.+(1-3c^2)(1+g_{01}(c))+c(1-c^2)(5g_{01}'(c)-2g_{02}'(c))]+\cdots\right\}.$$

Since c is to be very small it is best to expand $g_{01}(c)$ into an ascending power series in c.

$$g_{01}(c) = -1 + \frac{2c}{\tau\sqrt{1-\tau^2}} - \frac{2c^2}{\tau^2(1-\tau^2)} + \frac{(2-\tau^4)c^3}{\tau^3(1-\tau^2)^{3/2}}$$
$$- \frac{2(1+\tau^2)c^4}{\tau^4(1-\tau^2)} + \frac{(8-12\tau^4+3\tau^8)c^5}{4\tau^5(1-\tau^2)^{5/2}} - \frac{2(1+\tau^2)^2c^6}{\tau^6(1-\tau^2)} + - \cdots.$$

The recurrence relation then gives us

$$g_{02}(c) = \frac{-2}{\tau^2(1-\tau^2)} + \frac{(6-2\tau^2-\tau^4)c}{\tau^3(1-\tau^2)^{3/2}} - \frac{(12+6\tau^2)c^2}{\tau^4(1-\tau^2)}$$
$$+ \frac{(40-24\tau^2-36\tau^4+12\tau^6+3\tau^8)c^3}{2\tau^5(1-\tau^2)^{5/2}} + \cdots,$$

$$g_{03}(c) = \frac{-6-4\tau^2}{\tau^4(1-\tau^2)} + \frac{(120-72\tau^2-108\tau^4+36\tau^6+9\tau^8)c}{4\tau^5(1-\tau^2)^{5/2}} + \cdots.$$

After dropping all but the leading terms there is left

$$E_\rho = -i\omega\frac{e^{-ikr_2}}{r_2}\left\{\frac{1}{ikr_2}\left[\frac{2}{\tau\sqrt{1-\tau^2}}\right]\right\},$$
$$E_z = -i\omega\frac{e^{-ikr_2}}{r_2}\left\{\frac{2(1+\tau\sqrt{1-\tau^2}ikw)}{ikr_2\tau^2(1-\tau^2)}\right\}.$$

The wave tilt near the surface of the ground is then

$$\frac{E_\rho}{E_z} = \frac{\tau\sqrt{1-\tau^2}}{1+\tau\sqrt{1-\tau^2}ikz}.$$

This is the wave tilt in the asymptotic field of a quarter wave antenna or flat top antenna.

If a is not zero but c is small the final field expressions are

$$E_\rho = -i\omega\frac{e^{-ikr_2}}{r_2}\left\{\frac{2+\tau\sqrt{1-\tau^2}ik[w+(2a-w)e^{ik2az/r_2}]}{ikr_2\tau\sqrt{1-\tau^2}}\right.$$
$$-\frac{3}{ikr_2^2}[(w-2a)e^{ik2az/r_2}-w]$$
$$\left.-\frac{6-6\tau^2-3\tau^4+6\tau\sqrt{1-\tau^2}ikw+2\tau^2(1-\tau^2)(ikw)^2}{(ikr_2)^2\tau^3(1-\tau^2)^{3/2}}+\cdots\right\},$$

$$E_z = -i\omega\frac{e^{-ikr_2}}{r_2}\left\{(e^{ik2az/r_2}-1)\left(1+\frac{1}{ikr_2}\right)+\frac{2(1+\tau\sqrt{1-\tau^2}ikw)}{ikr_2\tau^2(1-\tau^2)}\right.$$
$$+\frac{(e^{ik2az/r_2}-1)(1+k^2w^2)-6k^2a(w-a)e^{ik2az/r_2}}{(ikr_2)^2}$$
$$\left.+\frac{2-6/\tau^2-(6-8\tau^2+5\tau^4)ikw/\tau\sqrt{1-\tau^2}-2(ikw)^2}{(ikr_2)^2\tau^2(1-\tau^2)}\right\}.$$

If $k2az/r_2 \ll 1$ the leading terms give

$$E_\rho = -i\omega\frac{e^{-ikr_2}}{r_2}\cdot\frac{2(1+\tau\sqrt{1-\tau^2}ika)}{ikr_2\tau\sqrt{1-\tau^2}},$$
$$E_z = -i\omega\frac{e^{-ikr_2}}{r_2}\cdot\frac{2(1+\tau\sqrt{1-\tau^2}ikz)(1+\tau\sqrt{1-\tau^2}ika)}{ikr_2\tau^2(1-\tau^2)}$$

and E_ρ/E_z is the same as obtained above with $a = 0$.

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