Spontaneous Resistance Fluctuations in Carbon Microphones and Other Granular Resistances

By C. J. CHRISTENSEN and G. L. PEARSON

Voltage fluctuations which occur in resistance elements of the granular type when a direct current is flowing have been measured in the granular carbon microphone, commercial grid leaks, and sputtered or evaporated metal films. The results can be experessed by the formula

$$\overline{V_c^2} = KV^{\alpha}R^{\beta} \log (F_2/F_1),$$

where $\overline{V_c^2}$ is the mean square fluctuation voltage, V is the d.-c. voltage across the resistance R, α and β are constants having values of about 1.85 and 1.25, respectively, and F_2 and F_1 are the limits of the frequency range over which the fluctuation voltage is measured. The constant K depends, among other things on the temperature, the surrounding medium, and the dimensions and material of the resistance element; for a commonly used carbon transmitter at ordinary operating conditions its value is about 1.3×10^{-11}.

The spontaneous voltage fluctuations and the signal due to acoustic modulation are affected in almost an equivalent manner by the applied d.-c. voltage which suggests that the two effects arise from the same type of mechanism, namely a fluctuating resistance at the points of contact between granules. Experiment shows that although the acoustic signal produces a resistance modulation which is in phase at all contacts the spontaneous resistance fluctuations are completely random.

On the assumption that a region of secondary conduction, wherein the resistance fluctuation lies, surrounds each area of primary conduction as postulated in recent contact theory a value of β consistent with experiment has been deduced. On the further assumptions that thermal energy produces the mechanical fluctuations and that the equipartition law governs the distribution of energy between oscillators the observed frequency distribution follows.

Introduction

When a direct current is passed through certain types of resistance elements a small potential fluctuation between the terminals of the resistance can be observed in addition to that caused by the thermal agitation of electric charge. The resistances in which this effect is particularly noted are granular carbon microphones and commercial grid leaks which are granular in nature, such as sputtered or evaporated metal films, and any of a number of composite materials containing carbon in a finely divided state. If such a resistance element is in a current-carrying circuit associated with a telephone receiver or loud speaker, particularly when amplification is present, a steady hissing noise which sounds like that due to shot effect or thermal agitation of electric charge is heard. It is this noise which sets a practical limit to the use of the carbon microphone in sound fields of low intensity, and of commercial grid leaks in circuits carrying direct current and working at low signal levels.

The resistance of a granular conductor has been shown experimentally to lie almost entirely within very small volume elements in the regions of the contact areas.[1] It is our hypothesis that there exist minute fluctuations of resistance in the region of contact, and when such an element carries direct current a potential fluctuation between the terminals can be observed. Accordingly we propose for this phenomenon the term "contact noise."

In a study of the electrical disturbances in a carbon transmitter Kawamoto [2] found that in addition to "carbon burning," which is a sharp crackling noise sometimes present in the carbon transmitter when the voltage across individual contacts is of the order of 0.5 volt or greater,[3] there is a continuous rushing sound which is always present no matter how well the transmitter is shielded from external disturbances. Kawamoto applied the term "carbon roar" to this phenomenon. Frederick [4] in discussing the disturbances in the carbon transmitter states that the noise power is proportional to the square of the direct current passing through the transmitter. More recently Otto [5]—who has been working on this subject contemporaneously with ourselves—has reported the results of an extended investigation of this phenomenon. The present report parallels to some extent the study of Otto but in addition new aspects of the phenomenon have been investigated, more accurate data have been obtained, and the conclusions drawn from these experimental results are fundamentally different from those of Otto.

Electrical disturbance in grid leaks, which becomes evident with the passage of current, was first reported by Hull and Williams [6] who observed the phenomenon in resistances formed by an India ink line. Preliminary reports have since been published concerning such noise in thin metallic films on glass.[7] The observations of Otto [5] were also extended to fine carbon wires and copper-oxide resistances. More recently Meyer and Thiede [8] have investigated the noise in resistances consisting of thin films of carbon on a refractory base.

We have performed noise measurements on each of the types of resistance elements mentioned above and the experimental results

[1] F. S. Goucher, *Jour. Franklin Inst.* **217**, 407 (1934); *Bell Sys. Tech. Jour.* **13**, 163 (1934).
[2] T. S. Kawamoto, Unpublished Report, Engineering Division, Western Electric Company, April, 1919.
[3] This disturbance undoubtedly has its origin in the heat generated at the carbon contact by the passage of current.
[4] H. A. Frederick, *Bell Telephone Quarterly* **10**, 164, July, 1931.
[5] R. Otto, *Hochfrequenztechnik und Elektroakustik* **45**, 187 (1935).
[6] A. W. Hull and N. H. Williams, *Phys. Rev.* **25**, 173 (1925).
[7] G. W. Barnes, *Jour. Franklin Inst.* **219**, 100 (1935).
[8] Erwin Meyer and Heinz Thiede, *E.N.T.* **12**, 237 (1935).

which are presented in this paper indicate that the noise observed in each case is of the same nature and is traceable to the existence of contacts between granules or perhaps granular boundaries.

Apparatus

The experimental arrangement used in the measurements to be described here is given in schematic form in Fig. 1. The system includes the input circuit, a high gain amplifier, appropriate filters, attenuator and output measuring device.

The input circuit consists of the resistance under test, a battery for supplying the direct current, a potentiometer for measuring resistance and voltage, a standard signal oscillator for calibration purposes and appropriate resistances and condensers for coupling to the amplifier. In some cases an input transformer having a high-turns ratio was also required in order to raise the signal level above the amplifier noise level. The granular resistance element was shielded from acoustical, mechanical and electrical shocks by suspending it with rubber bands

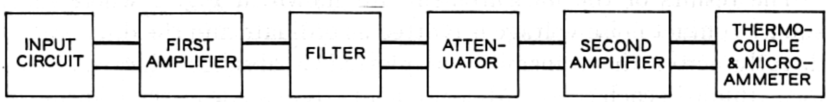

Fig. 1—Schematic amplifier circuit for measuring contact noise in granular resistance elements.

inside a tightly sealed iron box which was lined with alternate layers of hair felt and ¼-inch sheet lead. The remaining parts of the input circuit were also carefully shielded.

The high-gain amplifier consists of two separate resistance coupled units, each containing three stages. Each unit is so designed and shielded that the effect of external disturbances is eliminated. The total gain obtainable is about 165 db, with the frequency response uniform to within 2 db from 10 cycles to 15,000 cycles. In most of the measurements described here, however, a filter which transmitted only those frequencies above 100 cycles was inserted between the first amplifier unit and the attenuating network.

The gain of the amplifying system could be varied in steps of 20 db by means of interstage potentiometers. In addition, a 600-ohm attenuator having a range of 63 db in steps of 1 db was placed between the filter circuit and the second amplifier unit. The output measuring instrument was a 600-ohm vacuum thermocouple and microammeter. The deflection of the meter was closely proportional to the mean square voltage applied to the couple. Individual noise measurements were made by adjusting the attenuation so as to bring the deflection of the microammeter as near mid-scale as possible. Fractions of a db were estimated by means of the deviation from the standard mid-scale reading. Thus what was measured in each case is the insertion loss necessary to produce a standard electrical output.

Noise as a Function of Applied d.-c. Voltage

The contact noise in several different types of granular resistance elements was measured as a function of the applied d.-c. voltage, all other variables such as resistance, frequency range, temperature, etc., being held constant. The first of these measurements to be described is that obtained by using a standard handset telephone transmitter. The circuit used for coupling to the high-gain amplifier is shown in the insert of Fig. 2, the essential parts being an input transformer having a high-turns ratio, a d.-c. voltage supply, and a standard a.-c. signal generator. The resistance of the carbon transmitter was about 50 ohms.

The results of the measurement are shown in Fig. 2 where mean square contact noise voltage is plotted as ordinate and the d.-c. voltage directly across the transmitter is plotted as abscissa, the scale being logarithmic in each case. Measurements were made as the transmitter voltage was varied from 0.00145 to 4.5 volts. This is the greatest possible voltage range in which contact noise can be observed in this instrument since the contact noise is masked at the higher voltages by carbon burning and at the lower voltages by the thermal noise of the transmitter resistance. Thus the total noise at 0.00145 volt is only slightly above thermal noise and the measured value consists of thermal plus contact noise. The two effects have been calculated separately and the latter plotted as a cross. Using this method of plotting it is seen that there is a straight line relationship between contact noise and voltage over the entire lower range. These experimental data can be accurately represented by the equation

$$\overline{V_c^2} = \text{Const. } V^{\alpha}, \tag{1}$$

where $\overline{V_c^2}$ is the mean square contact noise voltage, V is the d.-c. voltage across the transmitter, and α is a numerical constant having in this case the value 1.85.

By this procedure the contact noise in a number of types of carbon transmitters, filled with carbons of various origins, was measured as a function of voltage. In each case the relationship given by Eq. (1) was followed very closely over a wide range of voltages. The value of

α varied slightly from cell to cell, the extreme values being 1.75 and 1.97 with an average of about 1.85.

Figure 3 gives the results of alternate measurements of contact noise and acoustic modulation performed on a particular telephone trans-

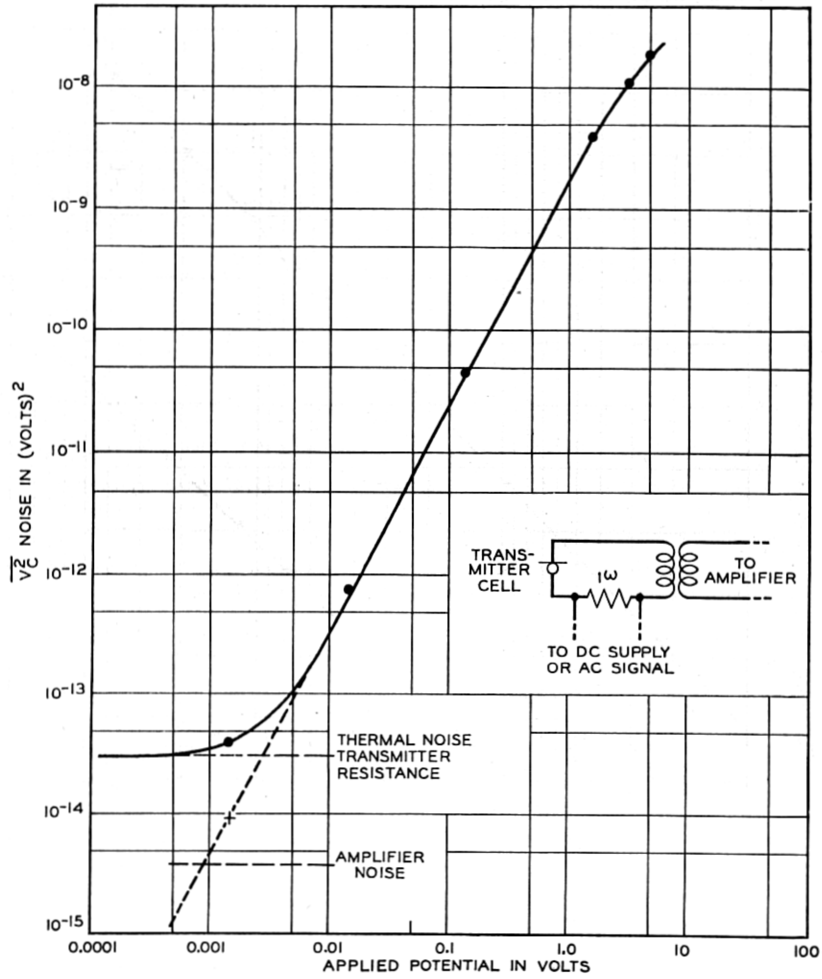

Fig. 2—The mean square contact noise voltage in a standard carbon transmitter as a function of the d.-c. voltage on the instrument. The noise level of the amplifier and the thermal noise level of the transmitter are indicated.

mitter. The acoustic field was of constant frequency and was supplied by an accurately controlled oscillator and "artificial mouth." The sound field was of such intensity that when the transmitter output was measured the background noise gave only an inappreciable part of the whole energy. In this figure the abscissæ represent the d.-c. voltage directly across the transmitter, the scale being logarithmic, and the ordinates represent mean square contact noise or acoustic signal voltage plotted in db above an arbitrary zero level. The experimental plots for both signal and noise are straight lines but of slightly different slope.

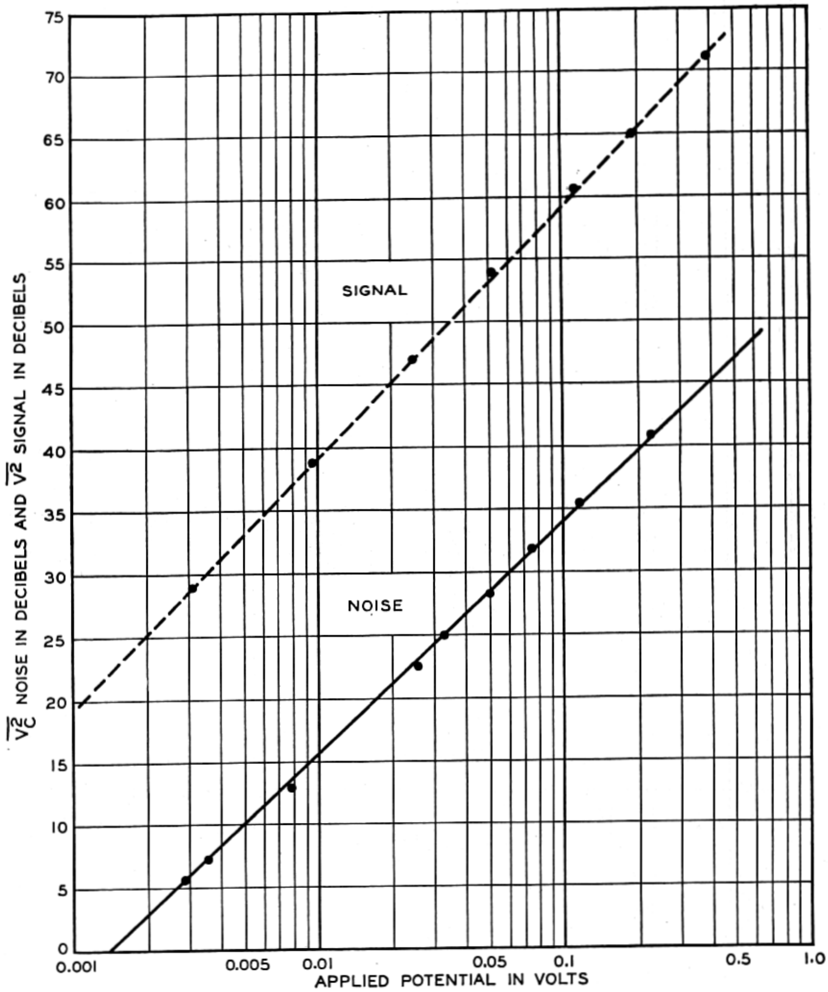

Fig. 3—The mean square contact noise and signal voltages in a standard carbon transmitter as a function of the d.-c. voltage on the instrument. The signal was obtained from a constant intensity sound field.

Analysis of the data shows that the mean square signal voltage due to acoustic modulation is accurately proportional to d.-c. transmitter voltage squared while the noise curve fixes the value of α at about 1.85. In the case of the signal the square relationship is to be expected since the sound field produces a constant resistance modulation at the points of contact between carbon granules and the granular aggregate obeys ohms law over the entire voltage range. The fact that the noise and the signal follow so nearly the same relationship indicates that the noise also arises from resistance modulation at the points of contact between carbon granules. Since α is slightly less than 2, however, the noise mechanism is not entirely independent of the applied voltage.

Noise as a function of applied voltage was also measured in single contacts between carbon particles. For these observations a cantilever bar device was used in which the contact can be rigidly fixed and manipulated at will. This apparatus, shown in Fig. 4, consists of a

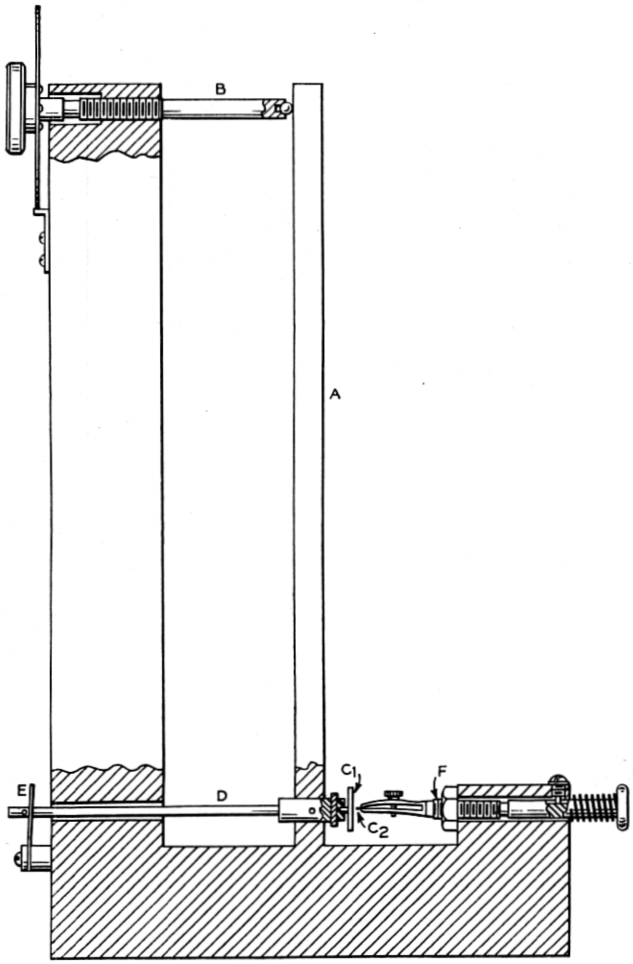

Fig. 4—Diagram of cantilever bar device for producing small contact displacements.

cantilever bar A integral with a massive L-shaped base, the entire device having been milled from a single piece of steel. A given displacement of the graduated screw B produces a greatly diminished displacement of the movable electrode C_1. The dimensions of the bar were so chosen that contact displacements of the order of 1×10^{-7} cm. could be produced. The motion of C_1 is made strictly linear by means of the pivoted rod D and any slack motion is eliminated by the

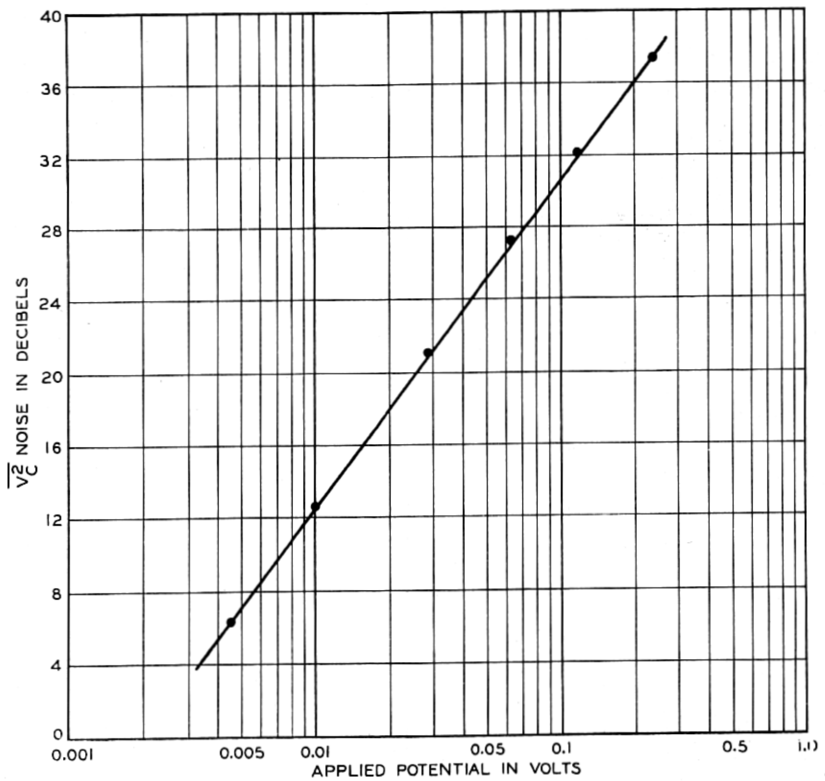

Fig. 5—The mean square contact noise voltage in a single carbon contact as a function of the applied voltage. The contact resistance was held constant at 76 ohms.

spring E. The contact is initially adjusted by the graduated screw F on which the stationary electrode C_2 is mounted. C_1 consisted of a flat polished disk of carbon like that used in the desk set transmitter, while C_2 was a composite carbon spheroid clamped securely between two gold plated jaws. Both the carbon plate and the spheroid were coated with a pyrolitic deposit of hard carbon.

The noise in a large number of single carbon contacts, connected in place of the transmitter in the input circuit shown in Fig. 2, was measured as a function of the voltage on the contact, the resistance being held fixed. In every case the general law given by Eq. (1) was found valid, α varying between the limits 1.75 and 1.95 for different contacts. The results of a typical measurement on a contact having a resistance of 76 ohms are shown in Fig. 5. The experimental points fall on a straight line having a slope corresponding to a value of α equal to 1.83.

Figure 6 gives the results of contact noise measurements performed on a commercial grid leak which was made by coating a thin layer of

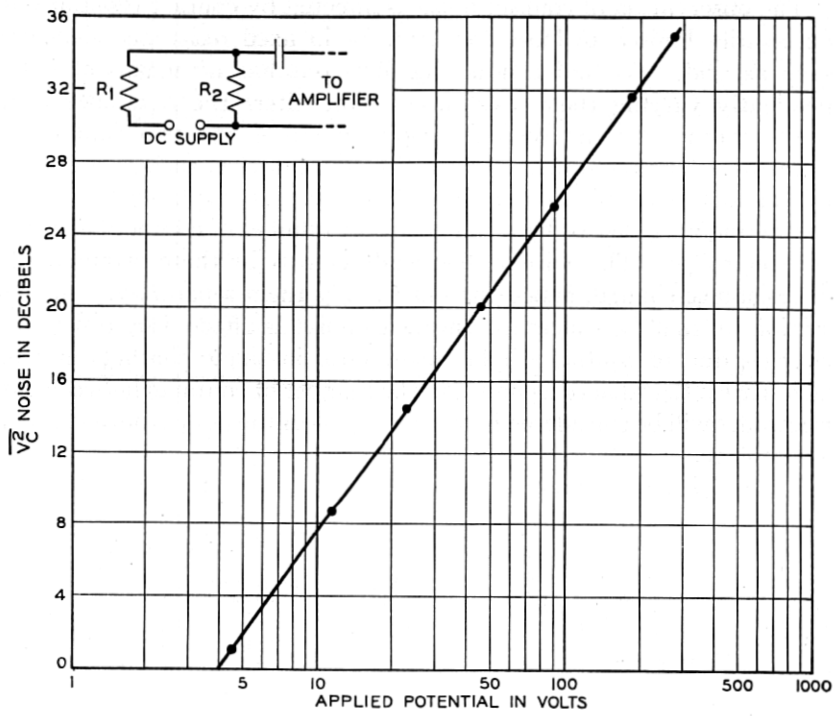

Fig. 6—The mean square contact noise voltage in a 50,000-ohm carbon grid leak resistor as a function of the applied voltage.

finely divided carbon and binder on glass. The input circuit shown in the insert, consists of R_1, the sample under test; R_2, a metal wire resistor which produces no contact noise; and a suitable source of d.-c. voltage. The resistance of both R_1 and R_2 was 50,000 ohms. The experimental points lie on a straight line the slope of which fixes the value of α at 1.90. Individual points could be reproduced within an accuracy of 0.1 db. Similar measurements of noise in thin metallic films deposited by either the cathode sputtering or evaporation process gave results in agreement with Eq. (1), the value of α lying between the limits mentioned above.

Thus it is seen that all types of granular resistance elements which we have tested, namely, carbon transmitters, single carbon contacts and those consisting of thin films of carbon or metal follow the same relationship for noise as a function of applied d.-c. voltage.

Noise as a Function of Contact Resistance

The observation of contact noise as affected by contact resistance is necessarily limited to loose contacts, for in fixed resistance elements such as grid leaks and conducting films one has no means of independently varying their resistances. One alters the resistance of a single contact by the relative displacement of the two contacting elements. The resistance of a multi-contact device, such as a carbon microphone, may be altered either by a relative displacement of the contacting particles, or by a change in the number of contacts between the electrodes. The noise is affected differently by these two methods of resistance change; hence one must study them separately. In this section we shall be concerned only with noise as affected by resistance changes due to contact displacement both in single contacts and in aggregates; that due to a change in the number of contacts between the electrodes will be considered in our discussion of the noise from a contact assemblage.

Figure 7 is a diagram of the input circuit used to study the relation-

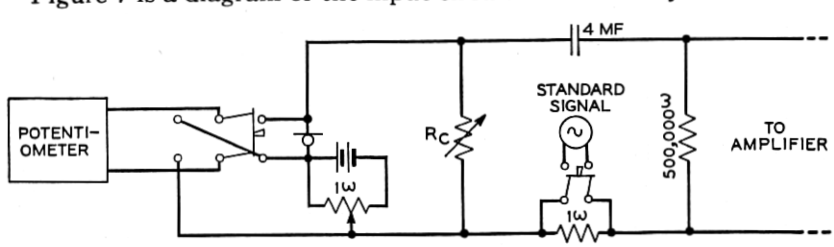

Fig. 7—Diagram of the circuit used in measuring contact noise as a function of resistance in granular resistance elements.

ship between noise and resistance. By means of the potentiometer we could measure both the voltage supplied to the contact circuit and that across the contact or transmitter. The resistance R_c is so adjusted for each noise observation that one half the voltage supplied to the circuit is across the contacts. The contact resistance for this condition is given by R_c. Also, in every case, one half the generated noise voltage is impressed on the input tube of the amplifier.

The single contacts studied were mounted in the cantilever bar device as described in the preceding section. We found it important to wait after the mounting of a contact long enough for the whole bar to come to thermal equilibrium before a measurement was attempted, otherwise very erratic results were obtained. Figure 8 is a typical curve obtained when the mean square noise voltage is plotted in db against the contact resistance on a logarithmic scale. For this

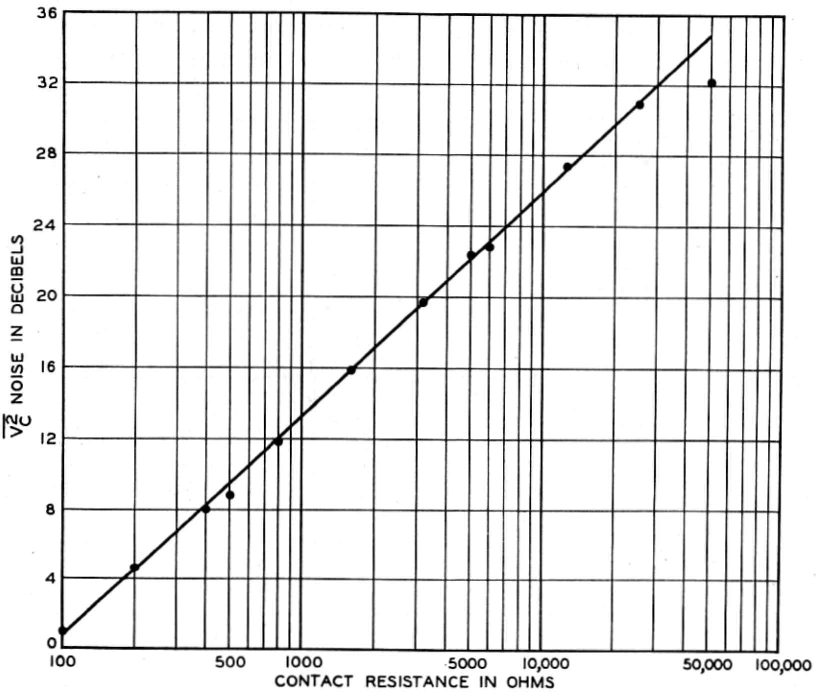

Fig. 8—The mean square contact noise voltage in a single carbon contact as a function of the contact resistance. The resistance was varied by changing the contact displacement while the applied voltage was held at 0.1 volt.

measurement the d.-c. voltage on the contact was 0.1 volt. The experimental relationship between noise and resistance is given by

$$\overline{V_c^2} = \text{Const. } R^{\beta}. \tag{2}$$

The data plotted in Fig. 8 give the value $\beta = 1.25$, which is the average value found for all the contacts studied. For individual contacts β varied between the extremes of 1.1 and 1.42. The studies on other properties of single contacts also exhibit a rather wide variability from contact to contact, hence the above result is not surprising.

A departure from the straight line relationship, such as is plotted in Fig. 8, occurs only when the contact is in a relatively high-resistance state due to a very slight compression of the contacting particles. When in this condition contacts are quite unstable and observations upon them erratic, but, in general, the noise originating in them is less than that expected if Eq. (2) held over the entire range of contact resistance values.

For the study of an aggregate of contacts the carbon cell from a barrier type transmitter was chosen. The structure of this cell (see insert Fig. 9) is such that one would expect the major portion of the

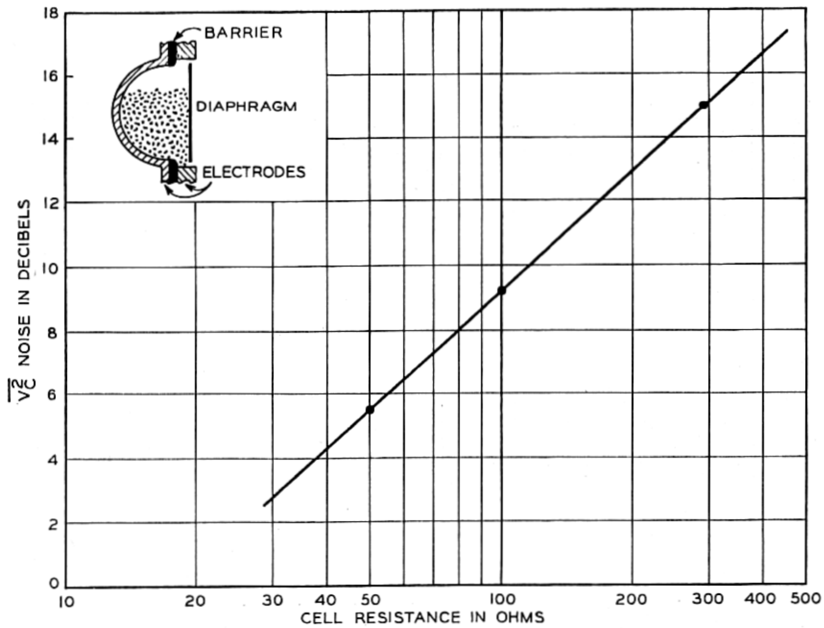

Fig. 9—The mean square contact noise voltage in a standard carbon transmitter as a function of the cell resistance. The resistance was varied by changing the amount of carbon filling while the transmitter voltage was held constant at 1.0 volt. Each experimental point represents the average of nine different readings.

current to be conducted through a small part of the total mass of granules near the bottom of the cell, and carbon added to the top of the cell to act mainly to increase the contact compressions of the conducting contacts. The electrodes of this cell are heavily gold plated, which assures that the resistance observed is almost entirely that of the contacts within the aggregate, a condition which is not fulfilled when carbon electrodes are used.

Figure 9 summarizes the results obtained in a typical set of measurements on the noise generated in the transmitter cell before mentioned. The resistance was varied by changing the height of the carbon layer above the carbon which was in the conducting path. Each of the plotted points is the average of nine observations, all of which occur in a range of 1 db. The relationship plotted in Fig. 9 can also be expressed by Eq. (2) and we again find $\beta = 1.25$. We shall see later—Eq. (8)—that when the resistance of an aggregate is altered by changing the number of conducting contacts between the electrodes quite another relationship between noise and resistance is obtained. Hence our assumption regarding the nature of the resistance change in this cell is consistent with the data of Fig. 9, and we believe that in this experiment we have measured the average value of β for all the contacts in the conducting path and have found it to be in agreement with the average value deduced from our single-contact measurements.

Noise as a Function of Frequency

For measuring the frequency distribution of the noise the filter shown in Fig. 1 was replaced by a frequency analyzer [9] having a constant band width of 20 cycles, the midpoint of which could be set at any point between 50 and 10,000 cycles per second. The calibration of the apparatus was checked by measuring the frequency distribution of thermal noise which was constant over this entire range, in accordance with theory.

The results of the measurements on a standard carbon transmitter, maintained at constant resistance and applied voltage, are shown in Fig. 10 where ordinates represent the mean square noise voltage over the 20-cycle band and abscissæ represent the mid-frequency of the band. It is seen that the experimental points fall on a straight line having a negative slope of about 1.0. This relationship may be represented by the equation

$$\Delta \overline{V_c^2} = \text{Const. } \Delta F/F, \tag{3}$$

where $\Delta \overline{V_c^2}$ is the mean square noise voltage for the frequency band ΔF. Integrating Eq. (3) between fixed limits we obtain:

$$\overline{V_c^2} = \text{Const. } \log (F_2/F_1), \tag{4}$$

which gives the total noise over the frequency range F_1 to F_2.

Figure 11 gives the results of similar measurements on a high-

[9] T. G. Castner, *Bell Laboratories Record* **13**, 267 (1935).

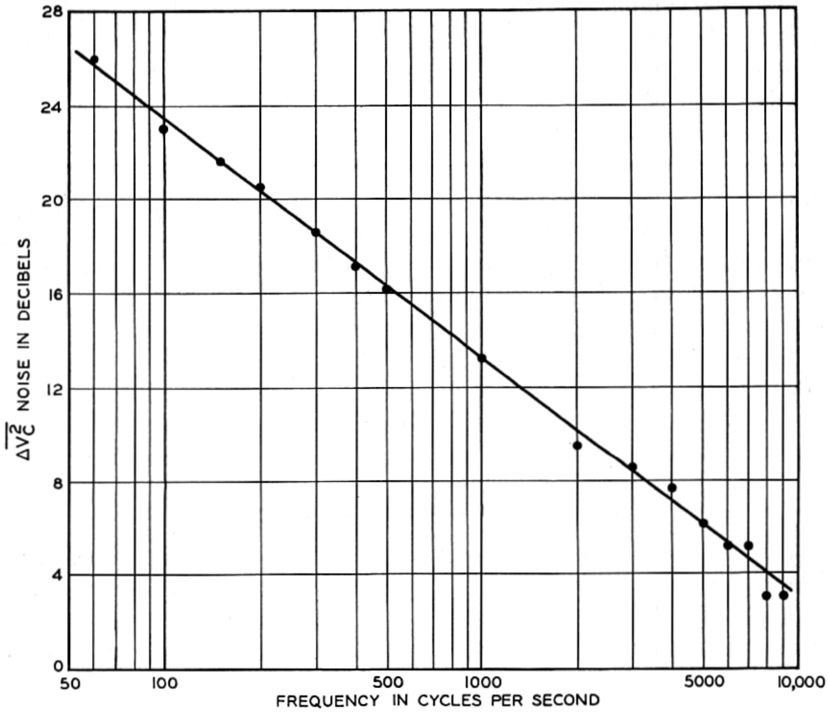

Fig. 10—The frequency distribution of contact noise in a standard carbon transmitter for normal operating conditions. The ordinates give the mean square contact noise voltage in a twenty-cycle band the midpoints of which are given by the abscissæ.

resistance carbon grid leak. It is seen that the noise has precisely the same frequency distribution in both the carbon transmitter and in the grid leak, which further supports our belief that the noise mechanism is the same in each case.

Otto [5] reported similar measurements of noise as a function of frequency in carbon transmitters, single contacts of carbon, carbon grid leaks and copper oxide resistances. Whereas we find an almost exact inverse relationship between noise and frequency for all types of elements tested he shows curves with negative slopes ranging from 1.0 to 1.4. Meyer and Thiede [8] in measurements on thin carbon films obtained negative slopes having values between 1.0 and 2.0.

Contact Noise as a Function of Temperature and Surrounding Medium

For the complete elucidation of contact noise the knowledge of its dependence upon temperature is important. However, the difficulties involved in such a measurement are so great that we have been unable,

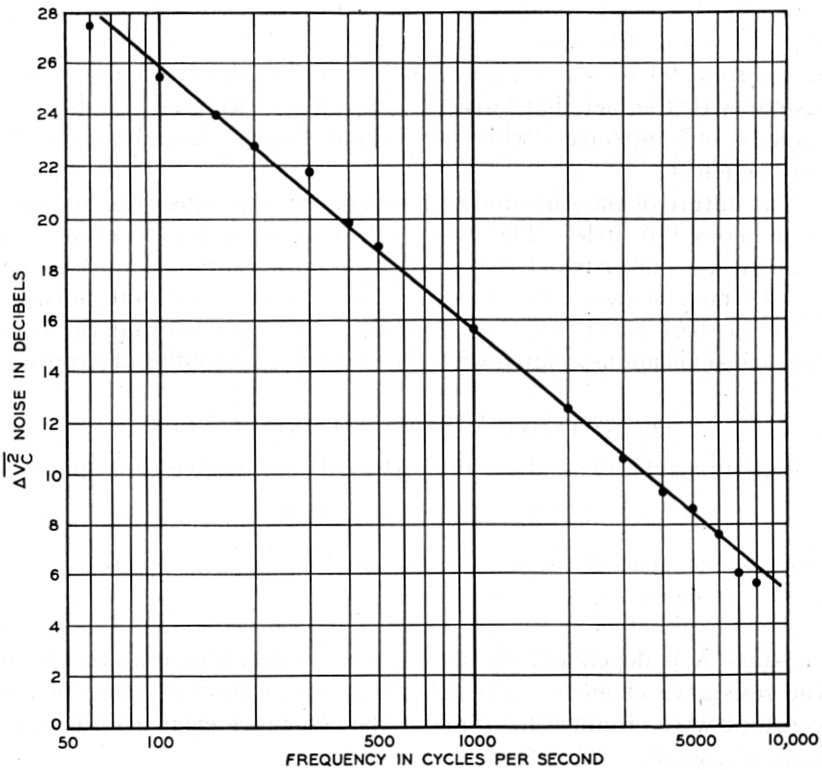

Fig. 11—The frequency distribution of the contact noise in a 50,000-ohm carbon grid leak resistor. The method of plotting the experimental points is the same as that shown in Fig. 10.

as yet, to obtain dependable results. For a satisfactory observation of the effect of temperature on the noise of contacts one must be certain that the conducting area in a contact remains constant and independent of temperature. Temperature variations can alter the conducting area in at least two ways; the contacting particles may be relatively displaced due to differential thermal expansions of the apparatus, and the change of the quantity of adsorbed gas on the contacting surfaces can alter the contacting areas without any relative displacement of the contacting particles. Both of these conditions are very difficult to control in any measurement involving temperature changes. However, our measurements of the relationship between contact noise and temperature, performed under the most carefully controlled conditions which we have been able to apply, indicate that contact noise may change either positively or negatively as a function of temperature depending upon the conditions of the experiment, but that the total change is not more than 3 or 4 db in the temperature range from 90 to 300 degrees Kelvin. These observations are consistent with the fact that Otto [5] has reported a decrease of noise with increase of temperature while Meyer and Thiede [8] have reported the reverse effect.

The nature of the surrounding medium seems to affect the intensity of the noise but little. The noise of the contact under oil seems to be slightly less, one or two decibels, than when the contact is in a vacuum of 10^{-5} mm. of mercury. The noise in air seems to be intermediate between these two extremes. This leads us to believe that the noise mechanism is not associated with the medium surrounding the contact.

Quantitative Values of Contact Noise

Equations (1), (2) and (4) may be combined to give the expression

$$\overline{V_c^2} = K\, V^{\alpha} R^{\beta} \log (F_2/F_1). \tag{5}$$

This is the general empirical equation found for noise in granular resistance elements as a function of voltage, resistance, and frequency. The average values for α and β are respectively 1.85 and 1.25. The constant K is dependent on the material, shape, temperature, etc. of the resistance element. The following representative values of this constant were obtained for some of the resistance elements which we have measured:

Single carbon contact	1.2×10^{-10}
Western Electric No. 395-B telephone transmitter	1.3×10^{-11}
100,000 ohm carbon grid leak	1.1×10^{-21}

For different single carbon contacts this constant did not vary more than 20 per cent as long as a given type of carbon was used, and a change in the type of microphonic carbon produced a variation by not more than a factor of two.

The contact noise in a Western Electric No. 395–B telephone transmitter under actual working conditions ($R = 45$ ohms, $V = 2.5$ volts, $F_1 = 200$ c.p.s. and $F_2 = 3000$ c.p.s.) is given by Eq. (5) as 9.8×10^{-5} volts. The output signal of this transmitter for standard voice operation is about 0.1 volt. The spread between signal and contact noise is so great that this noise is not a disturbing factor in the standard carbon transmitter as used in telephone service. This is not true, however, in the case of high quality carbon transmitters used for studio work and public address systems. The sound fields under the conditions wherein such instruments are apt to be used are much less intense than for the telephone transmitter under its normal condition of use, and hence, the contact noise becomes a limiting factor when the carbon microphone is used in weak sound fields.

Equation (2) is not suited for representing contact noise as a function of resistance in grid leaks since a change in resistance is brought about by variations in the dimensions and materials of the conducting film rather than by a change in contact displacement as is done in the case of loose contacts. For this reason the constant given above applies only to 100,000-ohm resistances of a given type.

It is of interest to note that the constant for the carbon grid leak resistor is smaller than that for the single contact by a factor of 10^9. This is due, in part, to the fact that the total voltage V across the grid leak resistor is divided among a network of contacts each of which produces noise independently of the others. The total noise from such an assemblage, as will be shown in the following section, is less than that arising from a single contact. This suggests that the contact noise in a solid carbon filament, if it exists at all, should be still smaller than that in the grid leak. We have made measurements on such a filament having a diameter of 0.0025 cm. and a resistance of 75,000 ohms. After taking great precautions to eliminate all the noise at the terminal connections we were unable to detect any noise in addition to that of thermal agitation for d.-c. loads as great as the filament would carry without being destroyed (a current density of 3×10^3 amperes per square cm.).

Noise From a Contact Assemblage

A transmitter cell contains an assemblage of contacts and we have shown that the noise from such an assemblage follows the empirical law set forth in Eq. (5), which also holds for single contacts. Several important deductions are possible when we study the noise from an assemblage as a function of the number and arrangement of the contacts within it.

Assemblage With Contacts in Parallel

Consider n contacts, $R_1, R_2 \cdots R_n$, placed in parallel across a direct current supply and inductance as shown in Fig. 12*A*. The inductance is large enough so that it offers an effectively infinite impedance to the fluctuation voltage we expect to study. Due to the fluctuation of resistance in the contacts and the passage of direct current they will act as a.-c. generators. Figure 12*B* is the equivalent a.-c. circuit, where $e_1, e_2 \cdots e_n$ are the instantaneous a.-c. voltages generated because of the fluctuating resistance in the respective contacts. The instantane-

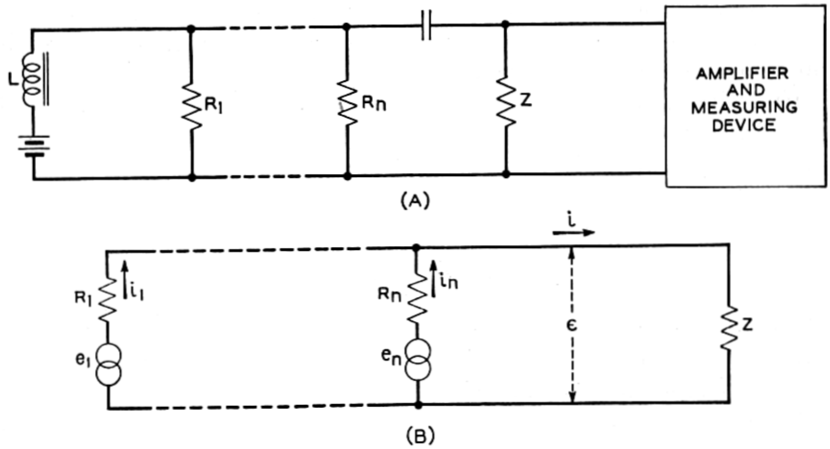

Fig. 12—(*A*) Circuit for measuring the contact noise of a parallel assemblage of resistance elements. (*B*) The equivalent a.-c. circuit of (*A*).

ous value of a.-c. voltage experienced across the impedance Z—which is the input impedance of a measuring circuit—is

$$\epsilon = e_1 - i_1 R_1 = \cdots e_n - i_n R_n = iZ,$$

where $i_1, i_2 \cdots i_n$ are the respective fluctuating currents flowing because of the generator action of the fluctuating contact resistances. Also we have

$$i = i_1 + i_2 \cdots i_n.$$

From these two expressions we get

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} + \cdots \frac{e_n}{R_n} = \epsilon \left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n} + \frac{1}{Z} \right] = \frac{\epsilon}{Y},$$

where

$$\frac{1}{Y} = \frac{1}{R_1} + \cdots \frac{1}{R_n} + \frac{1}{Z}.$$

Hence we obtain

$$\epsilon = e_1 \frac{Y}{R_1} + \cdots e_n \frac{Y}{R_n}.$$

The noise power dissipated in Z, and thus measured by the measuring device, is defined in the usual way as $\overline{\epsilon^2}/Z$, where $\overline{\epsilon^2}$ is the mean square voltage across the impedance Z and is defined as

$$\overline{\epsilon^2} = \underset{t \to \infty}{\text{limit}}\ (1/t) \int_0^t \epsilon^2 dt.$$

If the value of ϵ as given above is substituted in this equation, and the relative phases of the individual contact voltages $e_1 \cdots e_n$ are considered random, one derives the relationship

$$\overline{\epsilon^2}_{\text{parallel}} = \frac{\dfrac{\overline{e_1^2}}{R_1^2} + \dfrac{\overline{e_2^2}}{R_2^2} + \cdots \dfrac{\overline{e_n^2}}{R_n^2}}{\left[\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots \dfrac{1}{R_1} + \dfrac{1}{Z}\right]^2}. \tag{6}$$

The experimental test of this derived relationship will be given later.

Assemblage with Contacts in Series

Consider n contacts placed in series and supplied with current from a battery through an inductance as shown in Fig. 13*A*. The equivalent

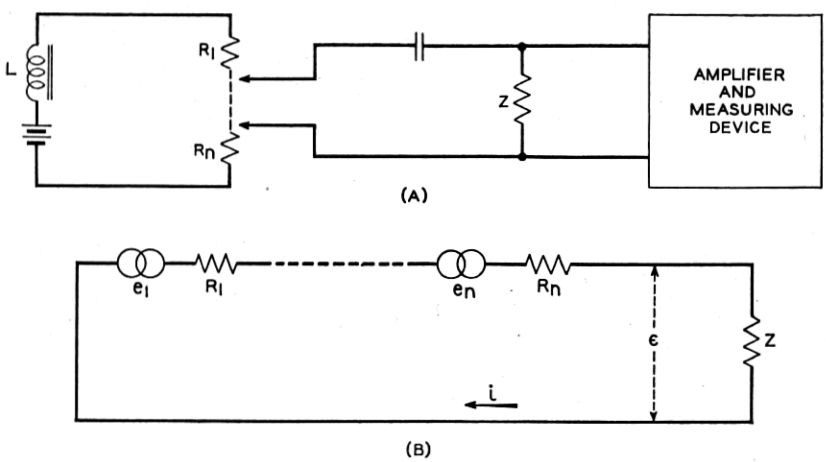

Fig. 13—(*A*) Circuit for measuring the contact noise of a series assemblage of resistance elements. (*B*) The equivalent a.-c. circuit of (*A*).

a.-c. circuit is shown in Fig. 13*B*. If the value of $\overline{\epsilon^2}$ impressed on the impedance Z of a measuring device is calculated, in a way similar to that outlined for the case of contacts in parallel, one gets

$$\overline{\epsilon^2}_{\text{series}} = \frac{[\overline{e_1^2} + \cdots \overline{e_n^2}]Z^2}{[R_1 + R_2 + \cdots R_n + Z]^2}. \tag{7}$$

Equations (6) and (7) are derived by considering the aggregate as made up of single contacts, but the same would be true if the aggregate were considered as made up of unit cells of arbitrary size, the values of e and R applying to the unit cells.

It is experimentally impracticable, if not impossible, to determine the noise and resistance of each contact in an aggregate; hence in the experimental test of the foregoing equations we have, in effect, divided the aggregate into unit cells. The aggregate of carbon granules was placed in four parallel grooves cut in a single phenol fibre block. Each groove was provided with five gold electrodes evenly spaced along the surface of the groove so that there were, effectively, four contiguous carbon cells in each groove. This permitted the measurement of the noise from each cell separately and also from the cells in various assemblages. Applying Eq. (6) or (7), as the case demanded, to the values of the noise from the single cells we calculated the expected noise of the assemblages and compared this with the measured values. Table I gives typical results for parallel assemblages and Table II typical results for series assemblages.

TABLE I

Cell	Cell Resistance Ohms	Noise in db	
		Measured	Calculated
a	3523	15.7	
b	4315	17.2	
c	4515	19.6	
d	3585	15.2	
$a + d$ parallel	1760	12.6	12.5
$b + c$ parallel	2215	15.7	15.5
$a + b + c + d$ parallel	980	11.2	10.9

TABLE II

Cell	Cell Resistance	Noise in db	
		Measured	Calculated
1	590	31.5	
2	720	34.0	
3	610	33.0	
4	510	30.0	
5	490	30.5	
6	545	31.0	
7	685	33.5	
8	427	26.5	
9	405	25.0	
10	226	19.0	
11	152	16.5	
Series 1 to 4	2510	38.5	38.2
5 to 8	2211	37.5	36.9
9 to 11	775	26.0	26.4
1 to 8	4720	41.0	40.4
5 to 11	2980	37.5	37.2

Considering the difficulty in holding a fixed granular configuration in a cell of loose contacts we feel that the experimental results justify the conclusion that the assumptions underlying the derivation of Eqs. (6) and (7) are essentially correct, which require that the phase of the noise voltage from each unit cell is entirely independent of that of any other cell in the aggregate. From this we conclude that the mechanism causing the noise is a small-scale effect capable of independently existing within a volume element much smaller than the size of the unit cell in any of the experiments we have performed on aggregates. In fact, as will be assumed later, we believe the noise mechanism to be located in a volume element smaller than that concerned with the properties of a contact between two particles.

If resistance elements are so chosen that each has the same resistance and noise, and these are placed in a circuit where the impedance Z is large compared to the resistance of the elements, then Eqs. (6) and (7) can be written respectively as follows:

$$\overline{V_c^2}_{\text{parallel}} = \frac{\overline{e^2}}{n}, \tag{6a}$$

and

$$\overline{V_c^2}_{\text{series}} = n\overline{e^2}. \tag{7a}$$

The resistance R of a parallel assemblage of like contact elements, each having the same resistance R_k, is obtained from $1/R = n/R_k$, or $n = R_k/R$.

Substituting this in Eq. (6a) we get

$$\overline{V_c^2}_{\text{parallel}} = \frac{R\overline{e^2}}{R_k}. \tag{6b}$$

For like contact elements in series we get $n = R/R_k$, hence Eq. (7a) becomes

$$\overline{V_c^2}_{\text{series}} = \frac{R\overline{e^2}}{R_k}. \tag{7b}$$

If now we have the further condition that the battery voltage is so adjusted for each new assemblage that $\overline{e^2}$ is always constant, then Eqs. (6b) and (7b) are equivalent and we have

$$\overline{V_c^2} = \text{Const. } R. \tag{8}$$

An equivalent relationship was derived and experimentally tested by Otto,[5] but it is clear from our derivation and measurements that it applies only to a change in the assemblage of like contacts, such as is realized when the dimensions of a granular aggregate or conducting film are altered, and is not valid for cases where the resistance is altered by changing the contact compressions. In this latter case Eq. (2) applies.

Another interesting property of an assemblage is obtained if we express $\overline{e^2}$ by means of Eq. (1) in terms of the battery voltage V. Thus for the parallel assemblage, $\overline{e^2} = \text{Const. } V^\alpha$, and for the series assemblage, $\overline{e^2} = \text{Const.} \left(\frac{V}{n}\right)^\alpha$. Thus Eqs. (6a) and (7a) can be written, respectively, as follows:

$$\overline{V_c^2}_{\text{parallel}} = \frac{\text{Const. } V^\alpha}{n} \tag{6c}$$

and

$$\overline{V_c^2}_{\text{series}} = \frac{\text{Const. } V^\alpha}{n^{\alpha-1}}. \tag{7c}$$

If we now accept as an approximation $\alpha = 2$ then Eqs. (6c) and (7c) are equivalent, and we can say that for any assemblage of contacts, where the value of $\overline{e^2}$ for each contact element is equal to that of every other contact element in the assemblage, the contact noise of the assemblage is inversely proportional to the number of contact elements in the assemblage. This principle we have established experimentally by building "square" assemblages—$\sqrt{n}$ parallel paths with $\sqrt{n}$ elements in series in each path—and measuring the noise as a function of n. The "square" assemblage is particularly interesting for it allows a control of the noise of an assemblage without altering its overall resistance. This suggests a principle which may be followed in designing grid leaks and carbon transmitters with low contact noise characteristics.

Discussion

It seems to us that the most logical hypothesis consistent with the foregoing experimental data is, as before indicated, that the noise mechanism lies in a fluctuating contact or boundary resistance. Assuming this we are led to the following considerations concerning the nature of the noise mechanism.

Careful measurement has established that the conduction through a carbon contact, as near as can be observed, is entirely ohmic. We have shown that when a carbon contact through which direct current is flowing is cyclically compressed, as in the acoustic modulation of a carbon transmitter, the generated a.-c. power is proportional to the square of the d.-c. voltage. This leads to the conclusion that the resistance modulation due to the cyclical compression is independent of the applied voltage. It is evident from Eq. (1) that the fluctuating resistance responsible for the noise cannot be equivalent to the resistance modulation introduced by a cyclical compression where the contacting granules move relatively as a whole, for the fluctuating resistance responsible for noise is somewhat voltage sensitive as indicated by the departure of α from the value 2. This means either that the conductance responsible for the noise is specifically non-ohmic or that the extent of the conduction wherein the noise mechanism lies is diminished as the applied d.-c. voltage is increased. Non-ohmic conductance is usually such that conductance increases with applied voltage, thereby demanding a value of α in Eq. (1) which is greater than 2; accordingly we are inclined to believe that applied voltage acts to diminish the area over which the noise mechanism operates.

If the noise mechanism were intimately associated with the total conductance of a contact one would expect the noise to be proportional to some simple integral power of the current in a contact, but this is denied by the observed value of β in Eq. (2).

These facts and deductions lead us to the hypothesis that there exist two mechanisms of conduction between particles in contact, a primary conduction which accounts for the major portion of the current, and a secondary conduction wherein a relatively small portion of the total current is transferred and in which the noise mechanism is found. Goucher [1] has given evidence that the primary conduction between contacting carbon particles is of the same nature as that in solid carbon, and since we have been unable to measure any noise in solid carbon we assume that the secondary conduction does not take place through the same region of the contact as the primary conduction.

Recent investigation of the elastic nature of carbon contacts [1] has led to the conclusion that the surface of each particle can be considered as covered with a layer of hemispherical hills of heights distributed according to the function $N_x = \text{Const. } x^n$, where N_x is defined as the quantity which when multiplied by dx gives the number of hills coming into coincidence with a plane as it moves from the position x to $x + dx$, and n is a constant whose experimentally determined value is about 0.6. The establishment of a contact consists in bringing into coincidence a number of these hills and enlarging the coincidence areas to the extent demanded by the displacement of the contacting elements after their initial coincidences. Let us accept this picture of a contact and inquire as to how it applies in the explanation of our empirical noise equation.

We assume that through each area of coincidence the primary conduction takes place and that secondary conduction, in which the noise mechanism lies, can take place between the surfaces which are not in primary contact and are not separated by more than a certain increment *dx*. Figure 14 is an attempt to picture a portion of the hypothetical plane of contact between two carbon granules.

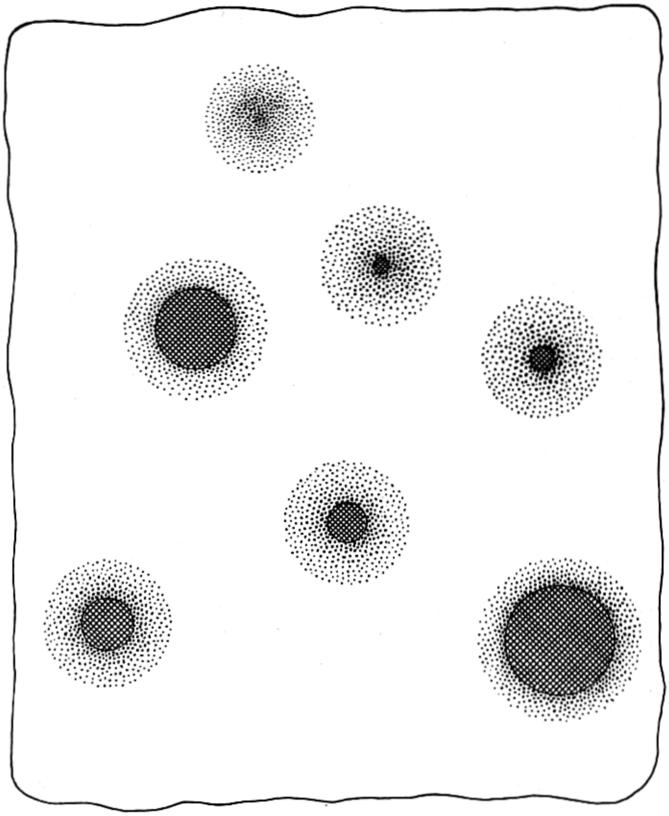

Fig. 14—A portion of the hypothetical plane of contact between two carbon granules. The cross hatched circular areas are the coincidence areas through which primary conduction takes place. The shaded areas, through which the secondary electrical conduction takes place, are regions where the granule surfaces are not separated by an interval larger than Δx. The area of each of these is independent of the coincidence area it surrounds.

The total number of hills in coincidence in a contact is given by

$$N_c = \int_0^D N_x \, dx = \text{Const.} \int_0^D x^n \, dx = \text{Const. } D^{n+1},$$

where D is the total displacement from the first coincidence. This number can also be expressed in terms of the contact resistance R by using Goucher's [1] derived equation: $1/R = \text{Const. } D^{n+3/2}$. Thus the above expression can be written

$$N_c = \text{Const. } R^{-(2n+2)/(2n+3)}. \tag{9}$$

The area of secondary electrical conduction surrounding each area of coincidence is precisely that area which would be added were the contact compressed by an increment of compression Δx. From the theory of Hertz [10] one can show that for smooth spherical hills in contact, $\Delta A/\Delta x$ is independent of the total hill compression and hence that the total area of secondary electrical conduction in a contact A_c is proportional to N_c, giving

$$A_c = \text{Const. } N_c = \text{Const. } R^{-(2n+2)/(2n+3)}. \tag{9a}$$

For purposes of analysis let us think of the secondary conduction area surrounding each primary area of contact as divided up into small elements of like nature, and further that the secondary conduction through each of these elements of area is independent of that in every other element. If such a contact is connected in a circuit as shown in Fig. 12*A* then the equivalent a.-c. circuit can be thought of as that shown in Fig. 15, where R is the mean resistance of the entire contact,

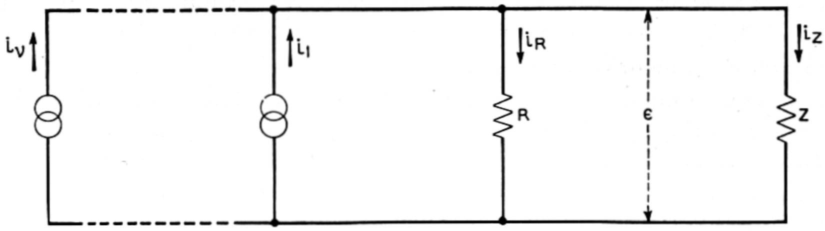

Fig. 15—The equivalent a.-c. circuit of a contact with ν elements of secondary conduction area, through each of which there is an independently fluctuating current, when such a contact is connected in a circuit as shown in Fig. 12*A*.

$i_1, \cdots i_2$ are the instantaneous deviations of the current from the mean current in each of the ν elements of secondary conduction area, and ϵ is the instantaneous value of the fluctuation voltage across the measuring device with impedance Z. Taking into account the random nature of the fluctuation currents through each element of secondary

[10] A. E. H. Love, "Mathematical Theory of Elasticity," 2nd ed., p. 192. This has been experimentally confirmed by J. P. Andrews, *Phys. Soc. Proc.* **43,** 1 (1931).

conduction area one can derive, in much the same manner as for Eq. (6), the relationship

$$\overline{\epsilon^2} = \nu \cdot \overline{i^2} \cdot (RZ)^2/(R + Z)^2, \tag{10}$$

where $\overline{i^2} = \overline{i_1^2} = \cdots \overline{i_\nu^2}$. Since ν is proportional to A_c we have by Eq. (9a) the relationship $\nu = \text{Const. } R^{-(2n+2)/(2n+3)}$. Substituting this into Eq. (10) we get

$$\overline{\epsilon^2} = \text{Const.} \cdot R^{(2n+4)/(2n+3)} Z^2/(R + Z)^2,$$

and if we make Z large compared with R this becomes

$$\overline{\epsilon^2} = \text{Const. } R^{(2n+4)/(2n+3)}. \tag{11}$$

Comparing this with Eq. (2), the experimentally derived relationship between noise and contact resistance, we get $n = 0.5$. Goucher [1] found by elastic measurements the value of $n = 0.6$.[11] In view of the fact that a slight change in the experimental value of the exponent β in Eq. (2) causes a rather large change in the value of n thereby determined from Eq. (11), we can say that the agreement between the results obtained from elastic measurements and noise measurements is surprisingly good, and that this agreement supports the hypothesis regarding the nature of a contact.[12]

In discussing the hypothesis that there exists a region of secondary conduction which is responsible for the noise, we have not made any assumption as to the nature of the secondary conduction. Several possibilities, however, have occurred to us, one of which it seems desirable to mention at this time.

If one assumes that the thermo-mechanical vibrations of a solid extend to the outside surface, then it is possible that the wave crests may be able to make periodic electrical contact across the secondary conduction area assumed in our hypothesis. This would permit a pulsating current to flow, the frequency of which, it is supposed, is determined by the frequency of the oscillator. For oscillators of audible frequencies the law of energy equipartition applies and each oscillator will have the usual $1/2\ kT$ of energy per degree of freedom. The energy of an elastic oscillator is also proportional to $(B \cdot F)^2$, where

[11] Goucher found a discrepancy between the measured resistance-displacement, resistance-force, and force-displacement relationships. But for reasons stated in his paper we are inclined to accept the distribution function found from the force-displacement measurements.

[12] If one assumes the existence of a film in the contact, which some older theories of microphonic action do, and that the number of independent elements of area through which current is conducted is proportional to the area of this film then one is led to the very unsatisfactory conclusion that $n = -3.5$.

B is the amplitude and F the frequency. From these two expressions of the energy we get $B \sim F^{-1} T^{1/2}$. The area of secondary electrical conduction surrounding each hill in coincidence is proportional to B, and since this area determines the number of elements of area through which secondary conduction takes place, as assumed in the derivation of Eq. (11), we arrive at the conclusion that

$$\overline{\epsilon^2} = \text{Const. } F^{-1} T^{1/2}. \quad (12)$$

While the hypothesis leading to Eq. (12) is only intended as a suggestion it does explain the inverse frequency relationship, and the temperature relationship is not an impossible one judging from the past unsatisfactory measurements. It may be possible, also, to explain the departure of α in Eq. (1) from the value 2, for one would expect electrostatic forces to distort the contacting surfaces so that the secondary conduction area would become smaller as the voltage increases. A satisfactory experiment on the effect of temperature on noise will do much to establish or disprove the tenability of this hypothesis.

Brillouin [13] has recently derived an expression for the noise in a conductor carrying a current by using the statistical method to deduce the most probable distribution of the electrons in such a system when it is in equilibrium. This method of calculation gives a noise energy, in addition to thermal noise, which is proportional to the square of the current and inversely proportional to the volume of the conducting material. We have made a calculation of the relative magnitudes of the two terms in Brillouin's equation which correspond to our experimental conditions. Assuming reasonable dimensions for a carbon contact and a current density as high as any we used it turns out that the magnitude of the term for contact noise is far below that for thermal noise. Furthermore it seems to us that Brillouin's mechanism would require a flat frequency distribution of noise rather than the distribution which we have observed. For these reasons we do not believe that the noise which we have studied is produced by the mechanism postulated by Brillouin.

In conclusion we wish to acknowledge our indebtedness to Dr. J. B. Johnson and Dr. F. S. Goucher for the helpful criticism they have given us during the course of this work.

[13] L. Brillouin, *Helv. Phys. Acta, Supt.* 2, **7**, 47 (1934). This theory is intended to explain the "Fluctuations de résistance dans un conducteur métallique de faible volume," reported by M. J. Bernamont, Comptes Rendus **198**, 1755 (1934); ibid. **198**, 2144 (1934).