Modulation in Vacuum Tubes Used as Amplifiers

By EUGENE PETERSON and HERBERT P. EVANS

SYNOPSIS: Recent developments in amplifier design tending toward more rigorous quality requirements have shown that the solutions of Van der Bijl and Carson are inadequate for certain purposes since they are based upon a convenient assumption which is not satisfied in fact. In particular, a detailed investigation of carrier current repeaters used for the simultaneous transmission of several channels, and upon which in consequence the modulation or crosstalk requirements are particularly severe, showed the modulation currents measured to be quite different from those specified by the theory, as was the law of variation of these currents with the circuit constants.

The cause of the discrepancy was found to reside in the neglect of the variation of the amplification factor (μ) with both plate and grid potentials. When the actual state of affairs was taken into account in the analysis by the application of a general method involving no assumptions, theory and experiment were found to be in good accord. The new expressions have been developed in terms of the amplification factor (μ), the internal output resistance of the tube (R_0), and their differential parameters, which are involved in the representation of the characteristic tube equation by a double power series. Expressions for the current components are developed in terms of the coefficients of the series, and modifications of Miller's method for greater convenience and precision in determinations of tube characteristics are described from which the series coefficients may be evaluated.

Conclusions are drawn from the solutions as to desirable tube characteristics by which, for example, a single tube may take the place of two tubes in push-pull connection. Finally, certain properties of different types of tubes under conditions of maximum output power are compared on the basis of μ constant and μ variable.

THE amount of modulation produced in vacuum tube amplifiers is in many cases a controlling factor in their application and it becomes of importance to determine how modulation products arise, so that the possibility of reducing them by tube and by circuit design may be studied.

In restricting discussion to amplifiers, and particularly to those used in communications, we are treating cases most amenable to analysis; in which, normally, the applied potential variation maintains the grid always negative so that conductive grid current does not flow, but in which, on the other hand, the greatest negative potential does not exceed the negative end of the plate current grid potential characteristic. In applying these two detailed restrictions we are incidentally insuring against prohibitive quantities of modulation; we know that, for example, the flow of conductive grid current may, under special conditions, produce an exceptionally efficient modulator of great service as such, but highly undesirable as an amplifier.

The necessity for suppressing modulation proceeds from the disturbing effects attendant on it, by which there may result reduction of quality in speech amplifiers, and crosstalk in the multi-channel amplifiers of carrier telephony, to take but two examples. The modulation level in the last case is restricted to much smaller values than are tolerated in the first; it is commonly required to reduce modulation products to the thousandth part, or even less, of the fundamentals which produce them. This last case is the one in which we are primarily interested; other cases of greater distortion referred to above may be treated by an extension of the methods used below in the case of grid current flow, and by Fourier series or expansions in terms of Bessel functions when the negative end of the tube characteristic is exceeded.

A thoroughgoing study of the amplifier problem would relate the static characteristics of a tube and the parameters of the circuit in which it works to its operating characteristics, and then would relate its static characteristics to the internal structure of the tube; it would in brief enable us to link the details of tube structure to the fundamental and modulation currents produced in the output wave of the amplifier. In the following, however, we shall treat only that part of the general problem which relates the operating characteristics and circuit parameters to the static characteristics.

A consideration of the usual plate current characteristics of a three electrode vacuum tube, as shown in Fig. 1, demonstrates the well-

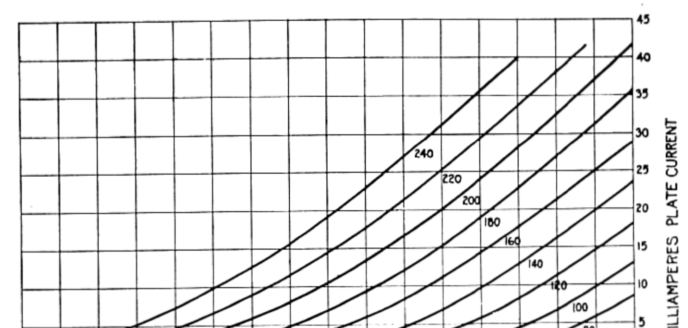

Fig. 1—Plate current as a function of grid potential with plate potential as a parameter. EL tube No. 109,150. I_f = 1.1 amperes

known dependence of the plate current upon the two variables, the grid and plate potentials. That is to say, the plate current varies with the grid potential when the plate potential is fixed, and it varies with the plate potential when the grid potential is fixed. It has been found of great convenience in the past to utilize an approximate relation between the grid and plate potentials as expressed in what is sometimes described as the fundamental theorem of the vacuum tube. The theorem states that a potential change in the grid circuit appears as a voltage generated in the plate circuit, the magnitude of which is equal to the grid potential change multiplied by the amplification factor μ. Solutions have been obtained for the output current components with the aid of this relation through the work of Van der Bijl and of Carson, which have been of great practical importance.

These solutions are approximate because of the simplifying assumption of the constancy of the amplification factor, which is certainly not accurate as the curves of Fig. 2 demonstrate. In this diagram

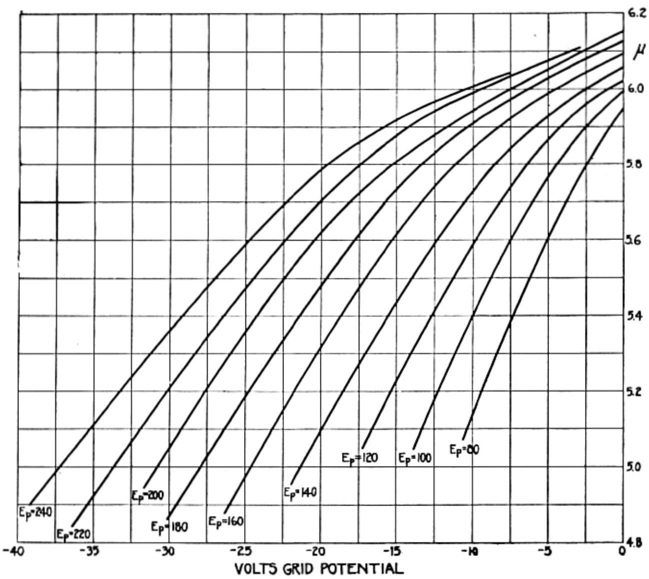

Fig. 2—μ as a function of the grid potential with plate potential as a parameter. EL tube No. 109,150. $I_f = 1.1$ amperes

the amplification factor for small applied potentials is plotted as ordinate with the grid potential as abscissa and the plate potential as parameter. The variation of μ is observed to be of the order of twenty per cent over the operating range. When we are interested in the distortion of the input wave, this variation cannot be ignored since to do so would in some cases yield results of another order of magnitude than those found experimentally. The treatment for our specific needs must therefore be modified to take account of the actual state of affairs.[1]

There are two ways open for a treatment involving the variation

[1] Early calculations to show the effect of a variable μ upon distortion were given by H. Nyquist of the American Telephone and Telegraph Company in an unpublished memorandum of April, 1921.

of μ; one is a modification of Carson's analysis while the other involves a reconsideration of the tube characteristic equation. The modification of Carson's treatment may be carried out with the aid of an expedient as follows: with a single input frequency the wave form of the generator voltage acting in the plate circuit is distorted by the variable amplification factor of the tube, and it is this distorted wave which acts in the plate circuit, instead of the pure sine wave operating with constant μ. The method of procedure is then evident; if we refer the actual distorted generator potential to the grid circuit—that is, if we divide it by the average value of μ—we have an input wave which, when treated by Carson's well-known method [2] in which μ is assumed constant, will yield correct results inasmuch as the μ variation has been taken into account—somewhat indirectly, it is true. When complex waves are applied to the grid circuit the effective grid potential is made up of numerous components and the treatment becomes very cumbersome.

Tube Characteristic Expressed by Double Power Series

A more direct method which has been used with some success consists in expressing the tube current-voltage relation by a double power series, without invoking any special relations regarding the connection between a grid potential change and the equivalent plate potential. If we use the symbols I_b, E_p, E_c to denote the plate current, plate potential, and grid potential, respectively, we may express the plate current as a double power series as follows:

$$\begin{aligned} I_b = f(E_p, E_c) &= \Sigma a_{mn} E_p{}^m E_c{}^n \\ &= a_{00} + a_{10}E_p + a_{01}E_c \\ &\quad + a_{20}E_p{}^2 + a_{11}E_pE_c + a_{02}E_c{}^2 \\ &\quad + a_{30}E_p{}^3 + a_{21}E_p{}^2E_c + a_{12}E_pE_c{}^2 + a_{03}E_c{}^3 \\ &\quad + \cdots \end{aligned} \tag{1}$$

where

$$a_{mn} = \frac{1}{m!n!}\,\frac{\partial^{m+n} f(0,0)}{\partial E_p{}^m \partial E_c{}^n}, \tag{2}$$

and in which it is understood that the development applies with the operating point on the characteristic. The derivatives, it will be noted, are evaluated at the point at which both E_p and E_c are zero. Some of the coefficients of Eq. (1) may of course be eliminated by reference to the evident properties of the tubes, but this need not concern us here since it is more convenient to formulate the tube equation in another way.

[2] *Proc. I. R. E.*, 1919.

Under normal conditions of amplifier operation E_p and E_c are fixed, and the alternating grid and plate potentials vary about these potentials. It then becomes convenient to express the coefficients as derivatives referred to the specific point E_{p_0}, E_{c_0}. If we indicate the variable components of the grid and plate potentials by e and v, respectively, the tube equation may be put in the form

$$\begin{aligned} I_b &= f(E_{p_0} + v,\ E_{c_0} + e) = \Sigma b_{mn} v^m e^n \\ &= f(E_{p_0},\ E_{c_0}) + b_{10}v + b_{01}e \\ &\qquad + b_{20}v^2 + b_{11}ve + b_{02}e^2 \\ &\qquad + b_{30}v^2 + b_{21}v^2e + b_{12}ve^2 + b_{03}e^3 \cdots, \end{aligned} \tag{3}$$

where

$$b_{mn} = \frac{1}{m!\,n!}\,\frac{\partial^{m+n} f(E_{p_0},\ E_{c_0})}{\partial^m E_p \partial^n E_c}.$$

The b coefficients are functions of the operating point—specified by E_{p_0}, E_{c_0}—and change with the operating point, in general. The a coefficients are definitely referred to the origin, however, so that each b coefficient may be expressed in terms of the a coefficients which correspond to it. It is possible, by determining this relation, to follow the variation of the b's in terms of E_{p_0} and E_{c_0}. To do this we substitute for E_c and E_p in Eq. (1) the expressions $E_{c_0} + e$, $E_{p_0} + e$, respectively, find the constant term, and obtain succeeding coefficients by differentiation. Thus

$$\begin{aligned} b_{00} &= a_{20}E_{p_0}{}^2 + a_{30}E_{p_0}{}^3 + a_{40}E_{p_0}{}^4 \\ &\quad + a_{21}E_{p_0}{}^2E_{c_0} + a_{31}E_{p_0}{}^3E_{c_0} + a_{22}E_{p_0}{}^2E_{c_0}{}^2, \end{aligned}$$

and further

$$b_{10} = \frac{\partial b_{00}}{\partial E_{p_0}}, \qquad b_{01} = \frac{\partial b_{00}}{\partial E_{c_0}}.$$

This concludes our consideration of the tube characteristics without reference to the circuit to which the tube may be connected. Eq. (3) rather than Eq. (1) will be used in the following.

It should be noted in terminating this part of the discussion that the treatment is capable of easy extension to characteristics depending upon a larger number of variables. Thus a four element (double grid) tube characteristic may be expressed by a triple power series, and so on. When the potential of the second grid is maintained constant it is evident that the tube characteristic is given by a double power series in which the coefficients depend in addition upon the potential of the second grid. To determine the dependence quantitatively, the triple series will serve.

Solutions for the Plate Circuit Components

We now pass on to a consideration of the operation of the tube working into a plate resistance. The more general case of a load impedance which is a function of frequency may be treated by application of the equations derived above, but it will serve our purpose here to deal with the case of a pure resistance load since the experimental work was done for that particular case which is of considerable practical importance.

If J is the alternating component of the plate current, we have from (3)

$$J = I_b - f(E_{p_0}, E_{c_0})$$

and J, it is seen, is a function of the two variables v and e. The quantity v depends on e of course, so that J may evidently be expressed as a function of e alone or

$$J = \sum_{k=1}^{k=\infty} C_k e^k. \tag{4}$$

A solution of the problem therefore consists in determining the C's in terms of the circuit and tube parameters.

The change in plate potential v may further be expressed as

$$v = -RJ = -R\sum_{k=1}^{k=\infty} C_k e^k. \tag{5}$$

The C's are then determined by putting (4) and (5) in (3) and identifying coefficients of similar powers of the variable. We have then

$$\begin{aligned} v &= -RC_1e - RC_2e^2 - RC_3e^3 \quad \cdots, \\ v^2 &= \qquad\qquad R^2C_1^2e^2 + 2R^2C_1C_2e^3 \quad \cdots, \\ v^3 &= \qquad\qquad\qquad\qquad - R^3C_1^3e^3 \quad \cdots, \end{aligned}$$

in which powers higher than the third are neglected for this, the first approximation. Carrying through the substitutions we obtain the solutions

$$\begin{aligned} C_1 &= b_{01}/(1 + b_{10}R), \\ C_2 &= (b_{02} + b_{20}R^2C_1^2 - b_{11}RC_1)/(1 + b_{10}R), \\ C_3 &= \frac{b_{03} - RC_1b_{12} + R^2C_1^2b_{21} - R^3C_1^3b_{30} - RC_2b_{11} + 2R^2C_1C_2b_{20}}{1 + b_{10}R}. \end{aligned} \tag{6}$$

The first equation, which leads to the first approximation to the fundamental current, is identical with that obtained on the basis of μ constant, but the higher orders are distinctly changed. When e is a pure sine wave C_2 contributes to the constant term which represents the change in direct current, C_1 and C_3 contribute to the fundamental component of the plate current, C_2 gives rise to the second harmonic and C_3 gives rise to the third harmonic current. When e is a complex wave the subscripts indicate the order of modulation [3] to which each coefficient applies.

The b coefficients may be readily converted into quantities dependent on μ, R_0, and their derivatives; we have

$$\mu = \frac{\partial I_b/\partial E_{c0}}{\partial I_b/\partial E_{p0}} = \frac{b_{01}}{b_{10}}$$

and

$$1/R_0 = \partial I_b/\partial E_{p0} = b_{10},$$

so that

$$\mu/R_0 = b_{01}.$$

Succeeding coefficients are obtained by differentiating with respect to E_p and to E_c. For example,

$$b_{20} = -\frac{1}{2R_0^2}\frac{\partial R_0}{\partial E_{p0}},$$

$$b_{11} = \frac{1}{R_0}\frac{\partial \mu}{\partial E_{p0}} - \frac{\mu}{R_0^2}\frac{\partial R_0}{\partial E_{p0}} = -\frac{1}{R_0^2}\frac{\partial R_0}{\partial E_{c0}},$$

$$b_{02} = \frac{1}{2R_0}\frac{\partial \mu}{\partial E_{c0}} + \frac{\mu}{2R_0}\frac{\partial \mu}{\partial E_{p0}} - \frac{\mu^2}{2R_0^2}\frac{\partial R_0}{\partial E_{p0}}.$$

The b coefficients may be obtained directly from the family of characteristic curves either graphically or analytically, when the operating point is specified. If we obtain our coefficients from the μ and R_0 curves, however, derivatives of a lower order are required than we need in dealing directly with the static characteristics. A family of μ-curves is shown in Fig. 2, and a family of R_0 curves is shown in Fig. 2*a*.

Results applying to the four element tube which are obtained by methods analogous to the above may be stated briefly. If we express the change in plate current by

$$J = C_{10}\epsilon + C_{01}e + C_{20}\epsilon^2 + C_{11}\epsilon e + C_{02}e^2 \cdots,$$

where ϵ and e represent the alternating potentials on the two grids,

[3] The new frequencies produced by modulation are given by the expression

$$F = |mf_1 \pm nf_2 \pm \cdots|,$$

where f_1, f_2 are impressed frequencies and m, n are integers or zero; the order is simply the sum of m, n, $\cdots$.

we find

$$C_{10} = b_{010}/(1 + Rb_{100}),$$

$$C_{01} = b_{001}/(1 + Rb_{100}),$$

$$C_{11} = \frac{b_{011} + 2R^2 C_{10} C_{01} b_{200} - RC_{01} b_{110} - RC_{10} b_{101}}{1 + Rb_{100}},$$

in which

$$b_{rst} = \frac{1}{r!\,s!\,t!} \frac{\partial^{r+s+t} f(E_{p_0}, E_{n_0}, E_{c_0})}{\partial E_p{}^r \partial E_n{}^s \partial E_c{}^t}$$

and E_{n_0} is the fixed potential of the second grid.

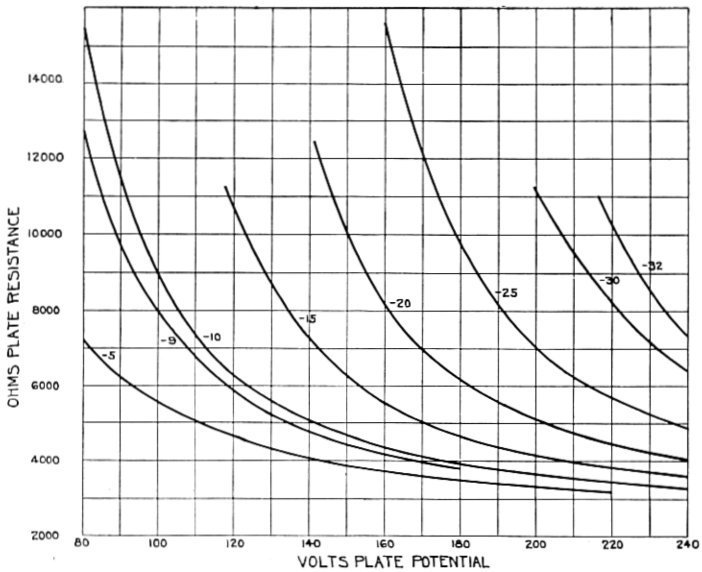

Fig. 2*a*—Plate resistance as a function of plate potential with grid potential as a parameter. EL tube No. 109,150. $I_f = 1.1$ amperes

We proceed to the methods used for the experimental determination of μ and R_0 for the three element tube.

Measurement of Tube Parameters

The amplification constant and plate impedance of a three element tube may be measured with precision by a well-known method due in principle to J. M. Miller [4] which requires no explanation here. The method as originally proposed is somewhat inconvenient in that the space current of the tube under test passes through a resistance common to the alternating measuring current, so that the operating

[4] *Proc. I. R. E.*, Vol. 6.

point of the tube is changed during manipulation for balance. Everitt's modification,[5] which consists in separating the direct and alternating current paths by a retard coil and condenser in the usual manner, is therefore preferable in this respect, but a complicating factor enters in the introduction of a reactive component due to the retard coil which cannot be balanced out by the variable resistances originally provided. This may be taken care of in a more or less obvious way by shunting a reactance around the grid resistance as shown in Fig. 3, the effect of which is to correct for the introduced

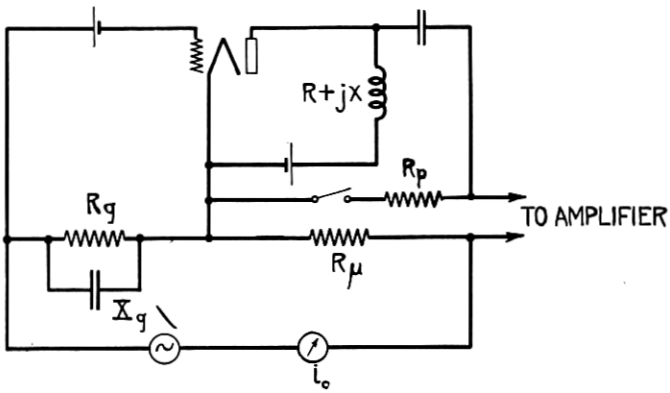

Fig. 3—Modification of Miller's method for determining μ and R_0

phase unbalance due to the retard coil and so lead to a precisely determinable null point instead of to a broad minimum, as is otherwise found.

The effect of the inserted reactance may be calculated by direct methods. Referring to the figure, there is no potential of fundamental frequency across the amplifier used to indicate balance at the null point, and if the grid-filament impedance of the tube is much greater than R_g (50 ohms), the total oscillator current passes through R_g and X_g in parallel, with R_μ in series. The potential impressed on the grid is then

$$e_g = jR_gX_gi_0/(R_g + jX_g),$$

which appears in the plate multiplied by the amplification factor of the tube and reversed in sign. The nomenclature is clearly indicated in the Figure. An alternating current flows in the plate circuit which is just balanced by the drop across R_μ so that we may write

$$\frac{j\mu R_gX_g}{R_g + jX_g} \cdot \frac{R + jX}{R_0 + R + jX} = R_\mu,$$

[5] See p. 201 of van der Bijl's "Thermionic Vacuum Tube."

which yields, after some reduction,

$$\mu = \frac{R_\mu}{R_g}\left(1 + \frac{R_0 R + R_0^2}{X^2 + R_0 R + R^2}\right).$$

As to the order of magnitude of the various quantities involved, R^2 is usually negligible before X^2, while R_0 and R may be of the same order of magnitude, so that we have

$$\mu = \frac{R_\mu}{R_g}\left(1 + \frac{2R^2}{X^2}\right).$$

In the specific case of a 101-D tube we had $X = 2.1 \times 10^5$, $R = 7 \times 10^3$ and

$$\mu = \frac{R_\mu}{R_g}(1 + 0.002).$$

The correction term, amounting to two parts in a thousand, drops out without the retard coil and we arrive at Miller's formula $\mu = R_\mu/R_g$. In measuring the output impedance of the tube after the settings for μ have been determined, R_g is doubled and R_p is connected in the plate circuit and varied until balance is again attained. It has been shown [6] by extension of the method used above that

$$R_0 = R_p\left[1 + \frac{3}{2}\left(\frac{R_p}{X}\right)^2\right]$$

and the correction term is of the same order of magnitude as that previously found for the amplification factor.

Balances may be obtained precise to one part in a thousand or better, but in much of our own work the observations are not ordinarily corrected for finite reactance. In order for the balancing action to take place the two reactances must be of opposite sign since amplification produces a 180° phase shift. If we balanced by a reactance shunted around R_μ instead of around R_g, the inserted reactance would, of course, be of the same sign as that of the plate retard coil, which was inductive at the frequency of 1,000 cycles at which the balances were made. The alternative scheme of shunting a variable condenser around R_g was adopted purely as a matter of convenience.

Applications of the Analysis

Second Order Modulation in Voltage Amplifiers

A striking illustration of the difference in the results of the two analyses, one based on the assumption of constant amplification

[6] By Mr. V. A. Schlenker.

factor and the other based on actual tube characteristics, is provided by a consideration of the second order modulation in the case of large external plate resistance.

The ratio of second harmonic to the fundamental, when μ is assumed invariable, comes out proportional to

$$(R + R_0)^{-2},$$

which shows that the ratio tends toward zero as R is made indefinitely great, a condition approximated in voltage amplifiers. According to this expression, the distortion would be eliminated by increasing the external plate resistance. That this is not really so is demonstrated by the analysis above which gives for the same ratio

$$\lim_{R \to \infty} \left(\frac{C_2}{C_1} \right) = (b_{02} + \mu^2 b_{20} - \mu b_{11})/b_{01}.$$

The ratio therefore approaches a constant value different from zero as R is indefinitely increased. The second harmonic level referred to the fundamental is about 40 T.U. down with a 101-D tube, which is prohibitively large distortion for certain classes of work such as multi-channel amplification used in carrier telephony; for a 104-D tube the level is about 32 T.U. down on the fundamental.

In order to bring out some important points involved in the theory, we shall discuss them in connection with experimental data on a standard type of tube (101-D) which are due to Mr. A. G. Landeen. The method used in measuring the current components is described in his paper on current analysis in the *Bell System Technical Journal* for April 1927.

Output Currents of a Representative Tube

Fig. 4 shows the calculated effect of varying the plate resistance on the fundamental, second, and third harmonic currents produced by a representative 101-D tube, which are indicated by circles, triangles, and crosses, respectively. In this drawing the values of the coefficients as calculated by Mr. J. G. Kreer are plotted as ordinates, and the external plate resistances are plotted as abscissæ. The agreement with the values obtained from experiment, and shown by the full lines, is seen to be rather close and within the limits of accuracy of the measurements except perhaps for the third harmonic at high load resistances. The coefficients used in the calculation of the quantities C_1, C_2, C_3 were obtained by graphical methods, which consisted in determining tangents to curves derived from μ and R_0 measurements. The precision obtained is sufficient for our present purposes, but for greater precision it may be desirable to use analytical methods for the determination of the b coefficients.

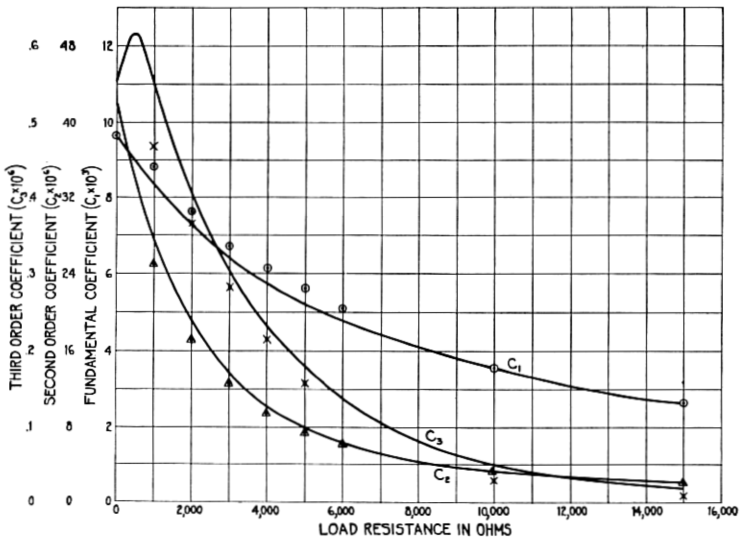

Fig. 4—Modulation coefficients for EL tube No. 109,150. $E_c = -9$, $E_p = 120$

Consideration of the expression for the second order coefficient, C_2, shows that the three terms of the numerator are all important, in general, except that the last term is negligible at very low resistances,

$$C_2 = \left[\frac{\partial\mu}{\partial E_{c_0}} - C_1(R - R_0)\frac{\partial\mu}{\partial E_{p_0}} - R_0 C_1^2 \frac{\partial R_0}{\partial E_{p_0}} \right] \frac{C_1}{2\mu}. \qquad (7)$$

Now in amplifiers, the condition for maximum power delivered to the load resistance at maximum gain demands equality of internal and external resistances, and this coincides with the requirement for minimum reflection coefficient [7] when the amplifier is connected to a line of definite characteristic impedance.

Under normal conditions, then, we have for the second order coefficient

$$C_2 = \left[\frac{\partial\mu}{\partial E_{c_0}} - \frac{\mu^2}{4R_0}\frac{\partial R_0}{\partial E_{p_0}} \right] \frac{1}{4R_0}, \qquad (8)$$

in which the variation of μ with respect to E_p does not enter, the only determining quantities being the variation of μ with respect to E_{c_0}, and the variation of R_0 with respect to E_{p_0}. The second order modula-

[7] The reflection coefficient is expressed as the quotient of the difference by the sum of the two connected impedances.

tion vanishes when

$$\frac{\partial \mu}{\partial E_{c_0}} \Big/ \frac{\partial R_0}{\partial E_{p_0}} = \mu^2/4R_0, \tag{9}$$

which is a valuable property of amplifier tubes when the equality can be secured.

A more general relation for which the second order vanishes occurs when we set Eq. (7) equal to zero. As a matter of experience these conditions are not satisfied with the usual type of tube; they are found to hold in tubes of rather special construction. The null points are, of course, independent of the character of the applied grid potential, provided that the restrictions on the original development for the tube characteristic are not exceeded and that contributions of higher to lower order terms are negligible.

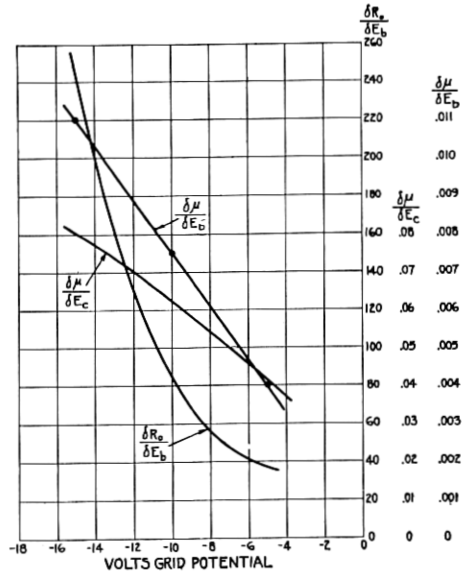

Fig. 5—Variation of tube parameters with grid potential. EL tube No. 109,150. $E_p = 120$

The expression for the third order coefficient contains six terms in the numerator, three of which are of opposite sign. If we consider the contribution of each of these terms as a function of the external plate resistance, we find that at very low resistances the single term b_{03} is predominant, while for resistances comparable to that of the internal plate resistance of the tube itself, no one of the six terms, of which three are negative and three are positive, may be neglected.

As a consequence of the subtraction of quantities of the same order of magnitude, the calculation for the third order coefficient is not capable of any great precision.

Fig. 5 shows how the fundamental coefficients $\partial\mu/\partial E_p$, $\partial\mu/\partial E_c$, $\partial R_0/\partial E_p$, which are involved in the second order term, vary as a function of the grid potential when the plate potential is maintained constant at 120 volts; $\partial R_0/\partial E_p$ is negative in sign.

Variation of C_1 and C_2 with Grid and Plate Potentials

To summarize our analysis up to this point, we have formulated an expression for the characteristic surface of a vacuum tube and have manipulated it to derive expressions for the fundamental and for the second and third order current coefficients. These theoretical

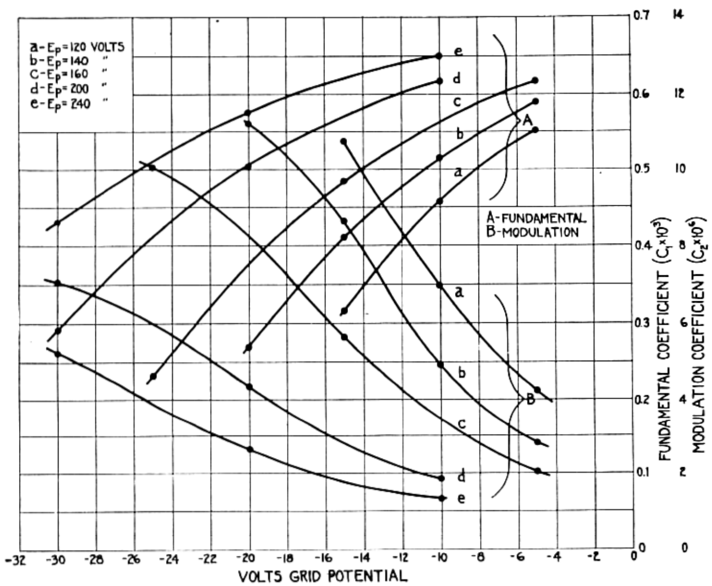

Fig. 6—EL tube No. 109,150. External resistance = 6,000 ohms. I_f = 1.1 amperes

relations have been compared with experimental determinations of the three quantities involved as a function of the external plate resistance, and a sufficiently good agreement has been obtained to indicate that the processes which we have treated are sufficient to account for experimental observations. We now present calculations of the coefficients C_1 and C_2 as a function of plate potentials and grid potentials for several values of the external plate resistance. It is seen from Figs. 6, 7, and 8 that these coefficients vary inversely with the plate and grid potentials.

As the plate resistance is increased all coefficients decrease, but C_2 decreases more rapidly than does C_1. The question then arises as to the conditions which would lead to the smallest amount of distortion while maintaining a definite fundamental power output at a definite

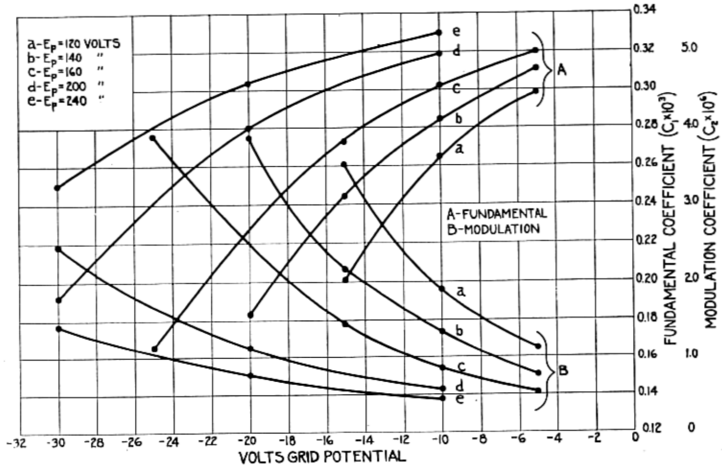

Fig. 7—EL tube No. 109,150. External resistance = 15,000 ohms. I_f = 1.1 amperes

plate potential. This depends evidently upon the desired power. The results in an illustrative case are depicted by Fig. 9, which represents the level of second harmonic current referred to the fundamental current (Δ) plotted as a function of the external plate resistance. At the point of minimum distortion—in which the second harmonic

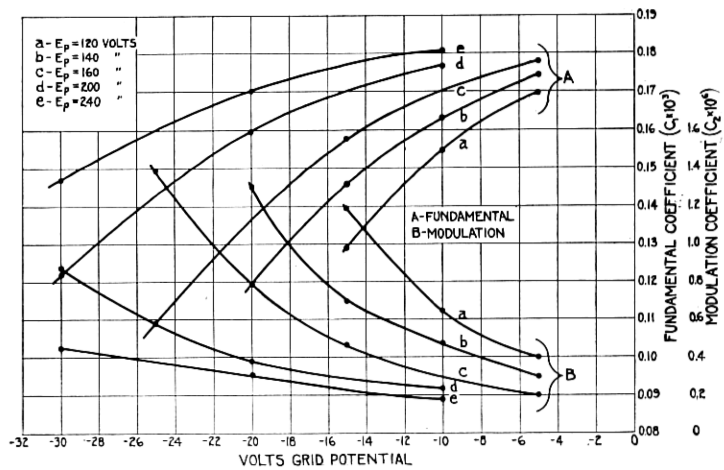

Fig. 8—EL tube No. 109,150. External resistance = 30,000 ohms. I_f = 1.1 amperes

level is 27.5 T.U. down on the fundamental—the external resistance is twice the internal, the grid potential is -10, and the improvement in the relative reduction of J_2 over customary conditions (for which $R = R_0$ and $E_c = -9$) is about 3.5 T.U. A similar survey made

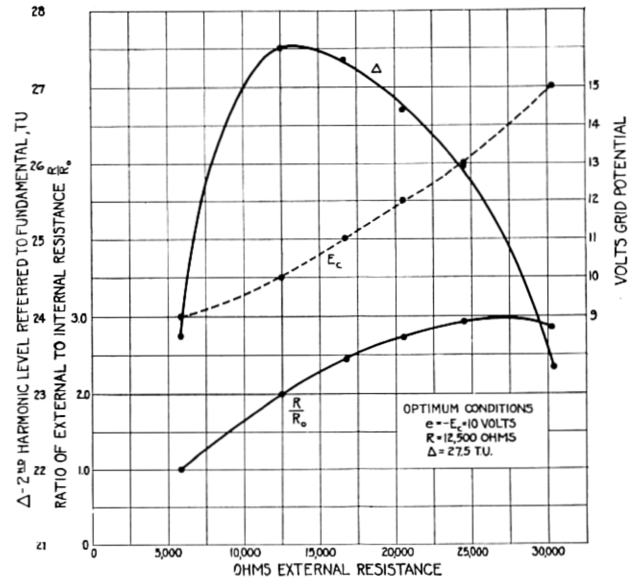

Fig. 9—Ratio of second harmonic to fundamental as a function of load resistance. EL tube No. 109,150. $E_p = 120$, $e = -E_c$ (variable). Power output constant $= .056$ watt.

with regard to the maximum fundamental power obtainable with constant plate potential, and with plate resistances matched, shows it to be had at -13.5 volts grid potential (Fig. 10), and to represent a gain of about 35 per cent in output power over that obtainable at the customary operating point. Other considerations such as stability with battery voltage variations operate in repeater practice to fix the grid potential at the customary values.

There has been considerable discussion recently as to the maximum undistorted power obtainable from a tube with fixed plate potential, and variable load resistance and grid potentials. The above analysis shows that in the strict sense of the word, distortion (C_2, C_3) always exists, while with some arbitrary criterion of distortion, results must depend upon the specific criterion adopted. Now as to the maximum fundamental power obtainable, it may be shown that with a parabolic tube characteristic the maximum is obtained for

$$R \doteq \frac{4}{3} R_0,$$

$$E_c \doteq -0.58 E_b/\mu.$$

These values are not very critical, however, since when we put $R = R_0$ and $E_c = -E_b/2\mu$ the power drops by about ten per cent. The last condition is that for maximum power at maximum gain,

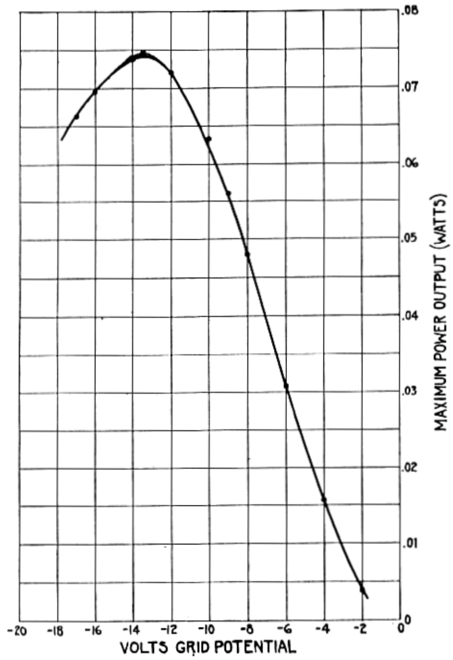

Fig. 10—Maximum power output as a function of the grid potential. EL tube No. 109,150. $E_p = 120$, $E = E_c$, $R = R_0$

in which the output power for a definite alternating grid potential is maximum. In view of the fact that it approximates optimum conditions and is much more convenient as a basis for calculation, we shall use the maximum power—maximum gain criterion of the performance of tubes.

Current and Power Relations.

The preceding analysis has shown that the maximum power obtainable at maximum gain without drawing grid current and without exceeding the negative end of the characteristic—at which grid potential variation produces substantially no plate current variation—is to be found at a grid voltage about midway between the two voltages specified by these conditions. It is again instructive to compare the predictions of the two theories as to the power dissipated in the tube, the a.c. power delivered to the load, the second order modulation, and the relations between these quantities at this operating point. It is a simple matter to calculate these quantities on the basis of Carson's and of van der Bijl's relations.

Thus the plate current is given by

$$i = \alpha(E_p + \mu E_c)^2,$$

the internal plate resistance to alternating currents is

$$R_0 = 1/2\alpha(E_p + \mu E_c),$$

and the internal plate resistance to direct current is

$$R_{dc} = E_b/\alpha(E_p + \mu E_c)^2.$$

At the operating point for maximum power we have $E_b = -2\mu E_c$, and when the alternating grid potential is equal in amplitude to the grid bias, the above expressions may be manipulated to give

$$\left.\begin{array}{lll} \text{1. The d.c. power dissipation} & P = \alpha E_p^3/4, \\ \text{2. The a.c. power delivered} & W = \alpha E_p^3/32, \\ \text{3. The 2d harmonic current} & J_2 = \alpha E_p^2/64, \\ \text{4. The fundamental current} & J_1 = \alpha E_p^2/4. \end{array}\right\} \qquad (10)$$

From these we have for the ratio of d.c. to a.c. powers

$$P/W = 8,$$

or the efficiency of power conversion at the maximum power condition is $12\frac{1}{2}$ per cent. We find also

$$W/J_2 = 2E_p,$$

or the relation of the fundamental power to the second harmonic current depends upon just one parameter, the plate potential. Other relations of interest are the two following:

$$R_{dc}/R_0 = 4,$$
$$J_1/J_2 = 16.$$

These four relations are independent of tube structure (μ, R_0) and we know that they cannot be accurate in view of the assumptions made in deriving them. In view, however, of the importance of general relations of this type in the design of amplifiers, it is of interest to compare these relations with the ones existing, as calculated by the more accurate theory in which μ variation enters.

Accordingly there are tabulated below the maximum power conditions for a number of tubes of different structure with plate potentials from 120 to 350 volts, operated under the conditions for maximum power output.

CONDITIONS FOR MAXIMUM POWER

Tube	E_b	$-E_c$	μ	R_0	P/W	W/J_2	R/R_0	J_1/J_2
101-D	120	13.5	5.36	8,800	4.8	132	4.55	7.17
101-D	240	28.5	5.43	5,900	4.1	270	4.68	6.95
104-D	120	34.0	2.13	2,930	5.4	132	4.10	7.50
104-D	240	72.0	2.13	2,200	4.3	211	4.54	5.48
205-D	350	32.0	6.95	5,780	3.8	350	5.40	6.53
Special A	250	16.9	9.35	5,800	3.9	267	5.07	6.76
" B	240	9.0	18.1	7,400	3.9	224	4.45	5.50
" B	120	4.4	17.6	9,500	4.5	116	4.14	5.96
" C	130	77.0	1.13	1,430	3.6	131	4.96	6.05

The last four columns represent computations by the double series method which are to be compared with the approximate relations of Eq. (10). Thus J_1/J_2 of the last column is given as 16 when μ-variation is neglected whereas it actually varies between 5.48 and 7.50; R/R_0 on the approximate theory is put at 4, whereas it varies actually between 4.1 and 5.4; W/J_2 is given as $2E_p$ which actually comes out close to half that, and finally the P/W is given as 8, while it really varies between 3.8 and 5.4. On the other hand, the approximate independence of tube structure is shown by these four ratios as given in the last four columns of figures.