A Shielded Bridge for Inductive Impedance Measurements at Speech and Carrier Frequencies [1]

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SYNOPSIS: A shielded, a-c., inductance bridge adapted to the measurement of inductive impedances at frequencies up to 50,000 cycles is described. The bridge comprises a balancing unit and associated standards of inductance and resistance. The balancing unit has resistance ratio arms specially constructed to meet the requirements imposed by the above frequency range. The reference standard makes use of inductance coils of a new type, their cores being of magnetic instead of non-magnetic material as is usually the case. The use of such cores results in coils that are smaller and hence better adapted to assembly in a multiple shielded standard.

The bridge is completely shielded so as to eliminate, to a high degree, errors due to parasitic capacitance currents. The shielding is also arranged so as to permit the correct measurement of either "grounded" or "balanced-to-ground" impedances. A series of diagrams is shown for the purpose of indicating the function of each part of the shielding system.

Equations expressing the errors resulting from any small residual capacitance unbalances in the resultant bridge network are given and calculations made of the balances required for the desired degree of measurement precision. Test data are presented illustrating a method of experimentally checking the residual shunt and series balances from which it is concluded that the bridge is capable of comparing two equal inductive impedances of large phase angle with an accuracy at the maximum frequency of 0.02 per cent for inductance and 1.0 per cent for resistance.

INTRODUCTION

THE limitations of the ordinary unshielded bridge network as a means of making precise a-c. measurements at speech frequencies were early recognized by telephone engineers. The solution of cross-talk problems arising in connection with the use of cable circuits was found to require an exact knowledge of the capacitive balances existing between such circuits at speech frequencies. For the ready and accurate determination of the capacitances defining these balances, together with their associated conductance values, G. A. Campbell devised the "shielded balance." [2] This is a bridge network having its parts individually and collectively shielded so as to define exactly the mutual electrostatic reaction of each with respect to all other parts of the electrical system affecting the balance condition.

As a means of more completely treating the cross-talk problems of cable circuits, Campbell conceived also the very valuable idea of "direct capacity" as distinguished from the "ground" and "mutual capacities" in use up to that time.[3] The shielded balance was found

[1] Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 11–12, 1926.

[2] G. A. Campbell: "The Shielded Balance," *Electrical World* and *Engineer*, April 2, 1904, p. 647.

[3] G. A. Campbell: "Measurement of Direct Capacities," BELL SYSTEM TECHNICAL JOURNAL, July, 1922, p. 18.

to be especially adapted to the precise measurement of direct capacities employing the substitution method devised by E. H. Colpitts.[4] Shortly thereafter, with the advent of loading for telephone lines, the same principles of shielding were extended to apply to bridge networks specially arranged for the measurement of the speech-frequency inductance and effective resistance of loading coils. As the successful commercial application of loading required the manufacture of these coils in large numbers to precise requirements, it was quite essential that testing means be available permitting a relatively unskilled tester to determine quickly whether the proper adjustment of the coils had been made. For this purpose the shielded balance has proved to be extremely valuable. More recently the employment of frequencies up to 50,000 cycles for carrier telephone and telegraph purposes has led to the need for correspondingly precise measurements at these higher frequencies. In this field the advantages of the shielded bridge are so great as to make it almost indispensable.

While the fundamental principles of the shielded balance are essentially the same for all impedance measurements, the practical application of shielding to any concrete bridge problem may vary according to the kind and range of impedances to be tested, the frequency range to be covered, and the precision required. It also presents special problems in the design and construction of several of the circuit elements. This paper describes a particular form of shielded bridge which has been developed to meet the conditions commonly encountered in the measurement of inductance at speech and carrier frequencies. The facts leading to the detailed construction are discussed and some experimental data given to illustrate the performance of the bridge.

General Features

A simple schematic diagram of the bridge circuit is shown in Fig. 1. To avoid confusion, no shielding is shown in this diagram. As will be noted, there are provided two equal non-inductive resistance ratio arms, an adjustable standard of self-inductance, an adjustable resistance standard, a thermocouple milliammeter, two transformers and two adjustable air condensers. Physically, this apparatus is grouped into three separate units, one comprising the standards of inductance, one the resistance standard, and the third, the remaining parts of the circuit. The last assembly constitutes what may be considered the balance element of the system, by means of which the unknown and standard impedances are compared. Figs. 2 and 3 show the arrange-

[4] See Note 3.

ment of the parts in this unit. Fig. 4 illustrates the appearance of the standard inductance unit and Fig. 5 shows how the units are associated when a test is being made. The thermocouple milliammeter indicates the total effective test current applied to the bridge and forms a

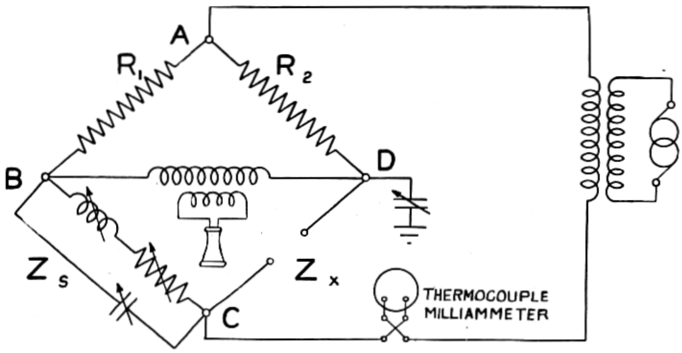

Fig. 1—Schematic diagram of bridge circuit

means of determining when this current has been adjusted to the desired value.

In operation, the air condensers are first adjusted to produce an initial or zero balance of the residual electrostatic capacitances of the apparatus. Aside from the initial balancing, the operation of the bridge follows the usual practise; that is, the standards of inductance

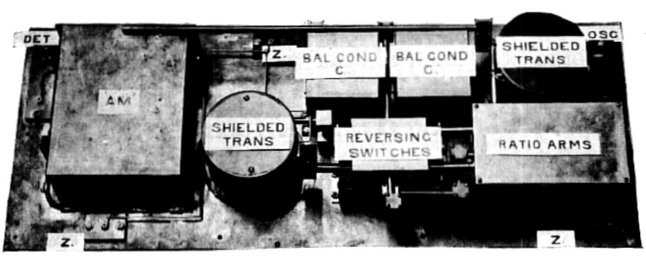

Fig. 2—Balance element of shielded bridge. Rear view of panel removed from case

and resistance are alternately adjusted until the balance detector indicates a condition of zero potential difference at every instant between the bridge points to which it is connected. The inductance and resistance values as indicated in the standard arm are then equal (within the precision limits of the bridge) to the corresponding constants of the unknown impedance.

Purpose of Shielding

The principal difficulties in attaining a satisfactory degree of precision in inductance measurements at relatively high frequencies by means of unshielded bridges are those due to the presence of residual or stray admittances existing between the bridge parts or from them to ground. All these parts have quite appreciable surface dimensions

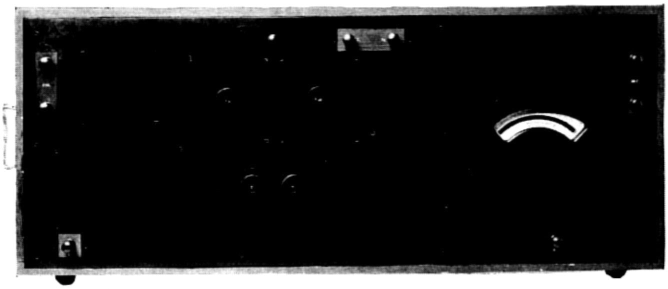

Fig. 3—Balance element of shielded bridge. Front view of panel and case

and when exposed at the usual separations to each other or to ground, have corresponding direct and grounded admittances. Leads to the source of testing current and to the balance detector also introduce rather large admittances. In a bridge intended for rapid operation, the parts subject to manipulation must be arranged compactly and

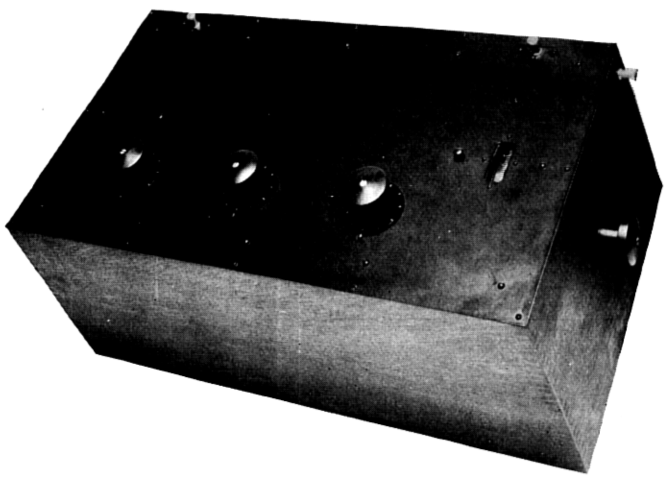

Fig. 4—Inductance standard

conveniently to the operator. This makes it impracticable to isolate them sufficiently to make the admittance values between these parts and between them and ground (the operator being considered to be at ground potential) negligibly small.

To make the matter more concrete, there is shown in Fig. 6 a

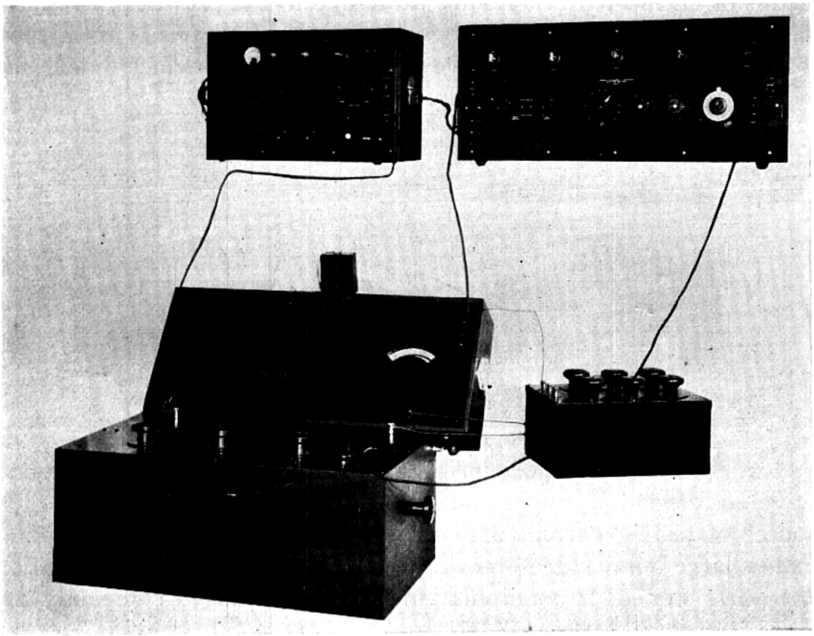

Fig. 5—Shielded bridge connected to vacuum tube oscillator and heterodyne detector

schematic diagram with possible positions of some of the more important of these admittances indicated as at C_1, C_2, etc. (With some exceptions, the capacitance components of these stray admittances substantially determine their full effect. In the diagrams and discussion, therefore, the conductance component will be neglected except where its effect is significantly large.) The capacitances between the two ratio arm coils, R_1 and R_2, and from each to ground, are shown as being uniformly distributed along the length of the coils symmetrically with respect to each other. If this symmetry is perfect these capacitances do not affect the bridge balance. In practise, however, they will only be approximately so, with the result that the two arms will be somewhat unbalanced to alternating currents, the effect of the unbalance increasing with the frequency. While the ratio arm capacitances can be made fairly small, others such as those indicated at C_1, C_2, C_3, and C_4 will commonly be much larger and hence of greater effect. Capacitances C_1 and C_2 are frequently comparatively large due to the use of long distributing wires, encased in grounded conduit, for supplying the testing current. C_3 may consist chiefly of the ground capacitance of the outer layer of the detector coil winding and C_4 that of dead-end coils of the reference standard, Z_S.

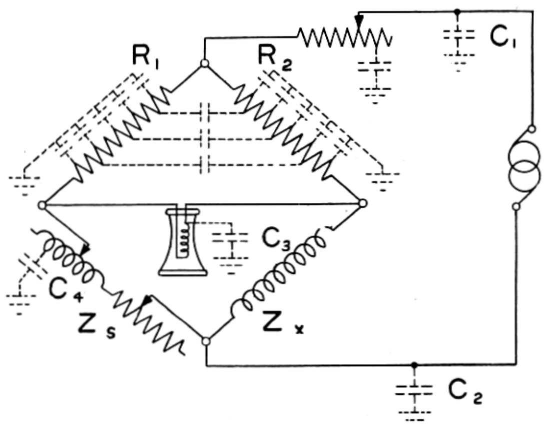

Fig. 6—Bridge circuit with stray admittances

Some of the currents flowing along the paths provided by these capacitances will complete their circuits external to the bridge network proper and will not affect the balance; for example, that through capacitances C_1 and C_2 in series. Other currents, however, will flow unsymmetrically through parts of the bridge circuit; for instance, that through C_1 and C_3 in series and the arm Z_X; also, that through C_2 and C_4 in series, returning through the ratio arm R_1. These latter currents and others of the same sort affect the potential distribution of the bridge and hence the values of the impedances required for balance. Certain of these capacitance currents in the bridge network tend to neutralize or balance the effects of others; for example, that through the arm Z_X due to the series action of capacitances C_1 and C_3 has a balancing effect with respect to that through C_1 and C_4 and the arm Z_S and would be without reaction on the bridge balance if capacitances C_3 and C_4 were exactly symmetrical with respect to the two detector terminals. Such balancing, however, is accidental in nature, seldom satisfactorily complete and, in part, not constant. Even were it made approximately complete for a particular arrangement, the substitution of another detector or the use of another source of testing current would probably destroy the balance. Variable effects would always be present; for example, those due to the changing position of the operator relative to the parts of the circuit or the effects of parallel loads on the supply generator. The distribution and value of the ratio arm ground capacitances described above are functions of the bridge surroundings; hence they are also subject to change if the bridge is moved from place to place. In the bridge being described, however, the shielding used affords a means of definitely fixing and controlling the various inter-circuit capacitances. Consequently, such variations cannot take place, balances between the resultant capacitance currents can be made as desired, and the bridge measurements are satisfactorily precise.

Shielding System Used

It is felt that the merits of the particular shielding system adopted for this bridge can best be brought out by showing, step by step, the reasons for using each of its elements.

The first step is to simplify, for further treatment, the initial residual capacitance network of the unshielded circuit. This is done by providing individual shields for each part of the circuit that it is desired to have function as an independent unit. Such shields can be connected to one of the terminals of the part enclosed and thus there is substituted, from the standpoint of terminal-to-terminal characteristics, a definite and invariable condition in place of that which was previously a function of the relation of the part to its surroundings. For example, as shown in Fig. 7, shields would be placed

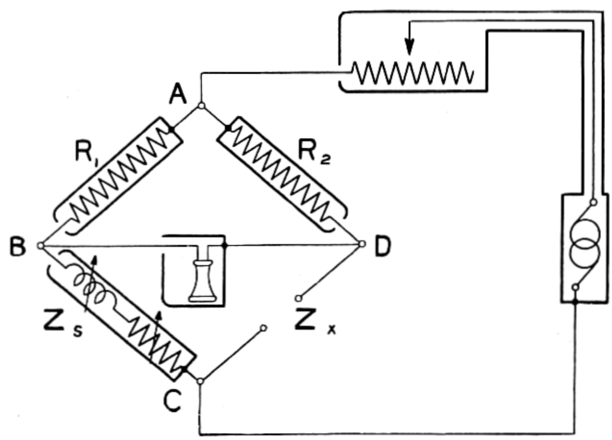

Fig. 7—Bridge circuit with local shields

around the resistance coils forming the ratio arms R_1 and R_2 and connected to the junction point A of the system, one enclosing the elements of the standard impedance Z_S and another around the source of testing current and connected at *C*; likewise, one around the detector is connected at *D*. It will readily be seen that these shields localize the effects of the various capacitance currents. Those circulating within the shields have, of course, no effect exterior to the shields, while those flowing between the various shields directly or by way of ground enter and leave the bridge system at definite points. By themselves, these shields do little good but they are necessary in order to make the next step, the balancing of the capacitances, practicable.

Generally it will not be found convenient to shield the current supply apparatus, especially if this is a power-driven generator. Also, to promote greater flexibility in respect to testing with a wide range of frequencies, it will often be desirable to substitute one source of current for another and likewise one detector for another. The shielding of this apparatus should therefore be reduced to a minimum. This is readily effected by making both the supply and detector branches of the bridge one of the windings of a transformer. This winding can be electrostatically shielded without affecting its transformer action and then any desirable source of current supply or any type of detector can be magnetically coupled with it.[5] Introducing this change the circuit becomes as shown in Fig. 8. The

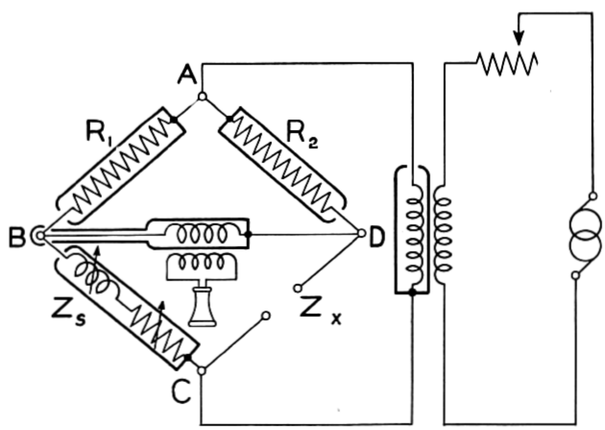

Fig. 8—Bridge circuit with shielded transformers and local shields

capacitances of the various shields to ground being still variable, the next step to correct this condition would simply be to add a ground shield around each. At this point, however, it becomes necessary to consider the ground admittance relations of the impedances to be tested.

[5] U. S. Patent No. 792248, June 13, 1905.

In general, the unknown impedance will have capacitances to ground and the effect of these will be properly included in the measurement only when certain conditions as determined by the nature of the apparatus are fulfilled. From this standpoint the impedances usually encountered are of three general classes: (1) Those having ground admittances negligibly small in comparison with the direct terminal-to-terminal admittance; (2) those having appreciably large admittances to ground approximately balanced with respect to the two test terminals; (3) those having one terminal directly grounded, the other having an appreciably large ground admittance.

In measuring apparatus of the first type it is evident that since in connecting it to the bridge circuit no additional ground admittances are introduced, the balance between those previously existing can be made without reference to the test impedance. The connection of an impedance of either of the other types will, however, introduce additional ground admittances into the bridge system, which, unless precautions have been taken, may cause the result to be something other than that which is wanted. In general, the desired test is that which gives the effective impedance applying to the apparatus as it is used. In the case of impedances having balanced admittances to ground, this is the effective value of the direct, terminal-to-terminal impedance as modified by the effect of the two ground admittances acting simply in series with each other. This condition is obtained when equal currents flow in each of these admittances, or, what is equivalent, when the electrical potentials of the terminals are balanced with respect to ground potential. To obtain this condition, when the impedance is being tested, the bridge terminals to which it is connected must likewise be balanced with respect to ground potential; that is, ground potential must be at the midpoint of the unknown impedance arm. If the only admittances to ground of the bridge system are those of the junction points (as is the case in Fig. 8), the potentials of these points with respect to ground are entirely determined by these admittances. To make any two points, such as the terminals of the unknown arm, have equal potentials to ground, it is sufficient to concentrate all of the ground admittances to these or other equipotential points and then balance the admittances from each. Referring to Fig. 9, if the testing current is applied at the points A and C, this condition is realized as shown by concentrating all ground capacitances at junction points B, C and D, and making the sum of the capacitances of junction points B and D equal to that of junction point C. This follows from the fact that when the bridge is balanced the junctions B and D are equipotential points. The mid-point of arm CD is now at ground potential. If, however, the testing current is applied at the points *B* and *D*, the equipotential points are the junctions *A* and *C*, the sum of whose ground admittances would then be made equal

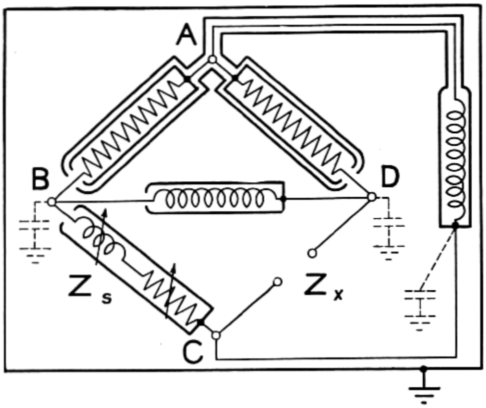

Fig. 9—Shielded bridge circuit showing location of ground admittances

to that of *D* and the arm *CD* again balanced with respect to ground potential. In this case there must be no ground admittance from junction *B*. To permit of testing under both conditions, point *A* and all connected conductors are protected with a shield which is then connected to the point *C*. Point *B* is likewise enclosed by a shield connected to point *D*. These two main shields then represent the junction points *C* and *D* of the bridge and are fixed with respect to capacitance to ground by a ground shield which may be common to the two.

There now exist, external to the local shields, direct capacitances only between points *A* and *C* and between *B* and *D* (which do not affect the bridge balance), and from points *C* and *D* to ground. These latter do, of course, affect the balance. Two courses are open. Their effective resultant value shunting the arm *CD* can be determined and allowed for by calculation. Such calculations would involve a considerable amount of labor, however, and can be avoided very simply by providing in the opposite arm an exactly equal shunt capacitance. To permit adjusting the ground capacitances of points *C* and *D*, an adjustable condenser is connected to ground from the point having the lower value. With the apparatus connected as shown this is usually point *D*. The shielded system then becomes as shown in Fig. 10.

When impedances, which in actual service are grounded at one terminal, are to be tested, the matter is much simpler. Then it is necessary only to definitely ground one of the bridge terminals to which the impedance is connected and establish the proper initial capacitance balance of the bridge for this condition. This is readily done by grounding junction point C and adjusting the capacitance from B to C to equal the ground capacitance of D. The shielding system may remain the same as in Fig. 10.

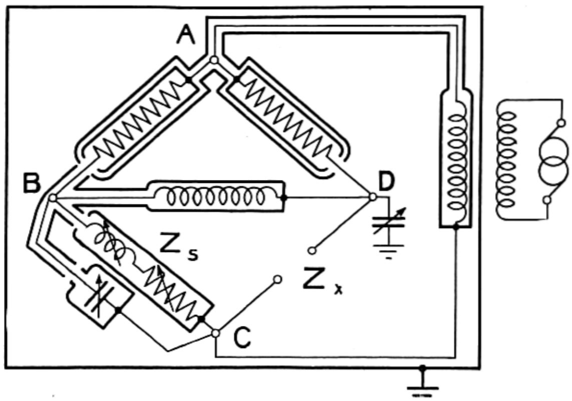

Fig. 10—Shielded bridge circuit with balancing condensers

In the case of the bridge being described it was desired to have a means of verifying by reversal the degree of balance of the ratio arms and also that of the impedance arms. The bridge is therefore equipped with reversing switches for this purpose. Due to the appreciable effect produced by a relatively small capacitance unbalance arising from factors present only when the arms are in circuit, it is quite important to be able to do this when a high degree of accuracy is desired. To effect the proper reversal, however, certain conditions must be definitely maintained. In reversing the impedance arms none of the inherent bridge admittances should be disturbed; that is, only the unknown impedance and the standard as read should be transferred. On the other hand, in reversing the ratio arms not only should the resistance element of these arms be transferred but also all associated shunt admittances. Moreover, in transferring these admittances they must be absolutely unchanged. A further requirement is that the ratio arm reversal must not occasion the shifting of any capacitances shunting the impedance arms. To accomplish these objects a suitable arrangement of shielded switches was worked out and added to the circuit of Fig. 10, the result being as shown in Fig. 11. In this arrangement all capacitances between the various parts of the switches which are subject to change due to physical movement of the switch parts are either short-circuited or connected across opposite bridge points and hence do not affect the bridge balance. The small capacitance C_R between the switch shield and that of the ratio coil R_2 shunts this coil and is not carried with it on reversal. For this reason a corresponding capacitance C_R', shunting the coil R_1, is pro-

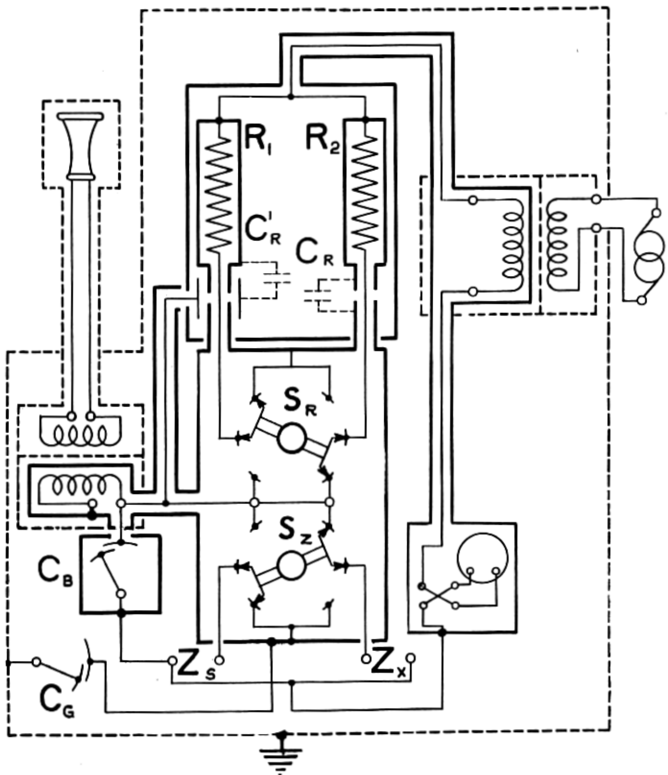

Fig. 11—Complete circuit diagram of balance unit with shielding

vided and connected to the opposite point of the switch. This is adjusted by test to equal the value of C_R. The diagram of Fig. 11 represents completely the circuit and shielding used for the balance unit of the bridge.

While, from the standpoint of the bridge balance alone, the parts comprising the standard impedance can be shielded with a local and ground shield as shown in Fig. 9, unless the standard has a very limited range, the resulting calibration is exceedingly laborious to make and use. To reduce calibration difficulties, additional shields can be used; this, of course, increasing the cost of construction. In arriving at the proper compromise between these conflicting factors the size and impedance value of the part to be shielded must be considered. This question will therefore be taken up in more detail in the following section.

Construction

The circuit and shielding features discussed so far are of general application to impedance measurements without restriction as to the particular range of values to be tested or frequencies to be used. The physical construction is, however, dependent upon these factors. As initially stated, the bridge is intended for the measurement of audio and carrier frequency inductances. By this is meant all apparatus having reactance values nearly equal to the respective impedance values. For the purpose of the present discussion, such inductances will be more exactly defined as those having ratios of reactance to resistance of not less than 10 (minimum phase angle of 84 deg., 20 min.). The difference between the reactance and the impedance of any such inductance does not exceed ½ per cent. The impedance values range from about 100 to 10,000 ohms and testing frequencies from 500 to 50,000 cycles.

On the basis of these conditions, the following construction was developed and is used for this bridge.

Ratio Arms. It is desirable from the standpoint of sensitivity of balance to have the ratio arm impedances of approximately the same value as those of the other two arms. Considering the range of impedances to be covered and giving due weight to the values which are of most importance in telephone circuits, a ratio arm resistance of 1000 ohms was selected. The problem then was to construct two 1000-ohm resistances, balanced both as to effective resistance and effective inductance for a frequency range from 500 to 50,000 cycles when subjected to the usual temperature and humidity variations.

Curtis and Grover have discussed the factors affecting the characteristics of a-c. resistances and have suggested forms suitable for general use at frequencies up to 3000 cycles.[6] A 1000-ohm resistance, constructed according to their specifications, is made by winding with a 1/10-mm. diameter, double-silk-covered manganin resistance wire, five 200-ohm bifilar sections on a 1-in. spool of insulating material. These sections are spaced about three mm. apart on the spool and are connected in series to form the 1000-ohm coil. Such a coil, when shellacked, baked and coated with paraffin, was found to be substantially constant in resistance and to have constant phase-angle

[6] H. L. Curtis and F. W. Grover: "Resistance Coils for Alternating Current Work," *Bulletin of the Bureau of Standards*, Vol. 8, No. 3.

effects equivalent to shunting capacitances of the order of 10 to 15 mmf. for all frequencies up to 3000 cycles. Since individual coils made according to this method may differ in their effective capacitances by as much as five mmf., some adjustment of these capacitances (as well as of the resistance) is required in order to make them suitable for use as the required ratio arms. Assuming that this is done by adding to the coil having the lower value a small capacitance of suitable constancy, it may be concluded that two coils so balanced would be suitable for use at frequencies up to 3000 cycles.

In arriving at the requirements for the more extended frequency range of this bridge, the necessary phase-angle balance was first considered. Designating by L_X and R_X the inductance and effective resistance of the impedance being tested, and by L_S and R_S, the corresponding components of the standard impedance required to balance it in a bridge circuit having ratio arms of exactly equal resistances R but shunted by slightly different capacitances, C_1 and C_2, and assuming that the quantities are such that $\omega^2R^2C_1^2$ and $\omega^2R^2C_2^2$ are small in comparison with unity, the equation for balance is

$$(R_X + jwL_X)(R - jwC_1R^2) = (R_S + jwL_S)(R - jwC_2R^2),$$

which reduces to

$$R_X = R_S + w^2R(C_2L_S - C_1L_X) \tag{1}$$

and

$$L_X = L_S - R(C_2R_S - C_1R_X). \tag{2}$$

Neglecting second order effects, these can be written

$$R_X = R_S + w^2RL_X(C_2 - C_1) \tag{3}$$

and

$$L_X = L_S - RR_X(C_2 - C_1). \tag{4}$$

If the readings R_S and L_S are taken as the values of the unknown resistance and inductance, respectively, it is evident that errors as given by the last terms of these equations will be present. The percentage errors in the two cases are as follows:

$$\Delta R_X\,(\%) = 100\omega^2R(C_2 - C_1)\frac{L_X}{R_X}$$

$$= 100\omega R(C_2 - C_1)\tan\theta, \tag{5}$$

$$\Delta L_X\,(\%) = 100R(C_2 - C_1)\frac{R_X}{L_X}\,. \tag{6}$$

For a given capacitance unbalance of the ratio arms, it is seen that the error in inductance is inversely proportional to the time constant L/R of the impedance arm and is independent of the frequency, while the error in resistance is proportional to the frequency and to the ratio of reactance to resistance, that is, to the tangent of the phase angle. The inductance error is, therefore, maximum for the minimum time constant apparatus to be tested. Within the range previously mentioned this occurs when an impedance having the minimum reactance to resistance ratio of 10 is being measured at the minimum frequency of 500 cycles. Under this condition R/L has a value of $(2\pi \times 500)/10$ or approximately 300. The corresponding percentage error in inductance per micro-microfarad of capacitance unbalance is then $300 \times 1000 \times 10^{-10} = 3 \times 10^{-5}$ or 0.00003 per cent. Evidently a very considerable unbalance can be tolerated. In the case of the resistance component, the error is maximum when an unknown impedance having the maximum reactance to resistance ratio is being tested at the maximum frequency. A reactance to resistance ratio of 300 is very rarely exceeded. For this value, the error per micro-microfarad unbalance at a frequency of 50,000 cycles amounts to about 9.5 per cent. Hence, to limit the error from this source to the order of 1 per cent requires a balance of about 0.1 micro-microfarad. It will be appreciated that this is an extremely close balance, the maintenance of which, under the different conditions of temperature and humidity to which the bridge may be subjected, requires careful consideration of the effects of these factors.

The effective phase-angle balance, though discussed above in terms of capacitance only, is, of course, the resultant of the inherent residual inductances and capacitances of the coil windings plus the additional capacitance effects due to the coil shields. The component due to residual magnetic induction is not appreciably affected by temperature or frequency changes. The capacitance component of the winding, however, tends to vary with temperature in accordance with the temperature coefficient of capacitance of the dielectric used for insulating the wire and with frequency to the extent that the capacitance is affected by absorption.

It is common practise to employ silk-insulated wire treated with varnish or wax for purposes of protection against moisture in such coils. In order to obtain data covering the temperature and absorption effects and also the phase-angle characteristics of silk insulation, both untreated and when treated with a number of the more common materials, various samples were constructed and tested as indicated in Table I.

TABLE I

Capacitance and Phase-angle Tests between Two Double-silk-covered No. 38 A. W. G. Wires, Each 88 in. (224 cm.) Long, Wound in Bifilar Fashion on a Glass Tube One In. (2.54 cm.) in Diameter and then Treated with Various Materials. Samples Dried before Testing.

Test sample number	Material used for treatment	Capacitance, micro-microfarads				Phase-angle tangent			
		Temp.—20 deg. cent.		Temp.—45 deg. cent.		Temp.—20 deg. cent.		Temp.—45 deg. cent.	
		1000 ∾	50,000 ∾	1000 ∾	50,000 ∾	1000 ∾	50,000 ∾	1000 ∾	50,000 ∾
1	None	193.3	188.2	188.7	184.1	0.0083	0.013	0.0076	0.011
2	Paraffin	251.1	243.2	232.1	244.3	0.0095	0.014	0.010	0.011
3	Collodion	238.9	228.0	231.4	222.5	0.016	0.021	0.015	0.018
4	Beeswax compound	258.2	248.7	273.2	265.8	0.013	0.014	0.010	0.013
5	Pyralin	253.1	241.1	258.5	246.6	0.016	0.021	0.018	0.022
6	Insulating varnish	346.4	320.1	361.5	337.0	0.027	0.036	0.030	0.033
7	Shellac	296.9	284.0	314.3	302.4	0.012	0.021	0.015	0.020

From the standpoint of percentage capacitance change (reckoning from the minimum temperature and frequency conditions as being those at which initial adjustments would be made), untreated and paraffin-treated silk insulation were found to be appreciably superior to any of the other materials. The change due to absorption effect (about three per cent for these two) was considered the more important, as normally variations in temperature would not be very large. As would be expected, the untreated silk had the lowest phase-angle effect and also the smallest capacitance. Assuming that a method of excluding moisture could be devised, it was concluded that an untreated silk-insulated winding would be the best to use, although the paraffin treatment was also considered promising. Discounting the fact that the two ratio coils would change in the same direction though not necessarily by the same amount, it was decided that a satisfactory factor of safety would be provided if it were assumed that the coils might become unbalanced by one half the observed change in one coil; that is, by about 1½ per cent. In order that such unbalance should not exceed 0.1 mmf., the capacitance of each coil would need to be not more than about 6.0 mmf. It should be noted that this limit applies to the true inherent capacitance and not to the resultant of the coil capacitance and inductance.

Considering now the variation in resistance over the frequency range of the bridge, it can be shown, following the methods of Curtis and Grover, that the effect of a capacitance of the value noted above on the resistance will not exceed one part in 100,000, which is quite satisfactory. The change in resistance (from the d-c. value) due to energy dissipation in the insulation is, however, somewhat larger than that due to the pure capacitance effect. This change is given to a close approximation by the expression

$$\Delta R = \frac{C\omega R^2 \tan \phi}{3},$$

where C is the total distributed capacitance between the wires of a bifilar winding and ϕ is the phase angle of the capacitance.[7] Clearly both the capacitance and its phase angle should be kept as small as practicable. An obvious and simple way of attaining the first object would be by using the very finest wire available. To do this, however, would in many cases result in excessive heating of the resistance. For bridge tests on telephone apparatus the ratio arm current will rarely exceed 25 milliamperes. The energy to be dissipated in a 1000-ohm coil is then about 0.5 watt, requiring a radiating surface of about 25 sq. cm. for a maximum temperature rise of 10 deg., which is a desirable limit. Since only the outer surface of such a coil is effective in radiating the generated heat the question of the number of layers requires consideration. Other factors being constant, it has been found that of the various possible arrangements that are easily constructed and mounted, a sectionalized, two-layer winding gives minimum capacitance. Hence one half of the winding is required to have an exposed surface of 25 sq. cm. The gauge of wire is then determined as a function of its specific resistance. A resistance alloy having a suitably low temperature coefficient (such as manganin, advance, etc.) will, on this basis, require that a wire no smaller than No. 38 A. W. G. be used. This is the size of wire used by Curtis and Grover and in the experiment covered by Table I. Using the data of this table, it was calculated that a Curtis and Grover type of coil, except for treatment, would have a change in resistance of not over one part in 50,000. The lower capacitance coils required from the phase-angle standpoint would have even smaller changes.

Summing up, then, the ratio arm coils were to be of approximately 1000-ohm resistance, wound with No. 38 A. W. G. double-silk-insulated manganin, or advance resistance wire, dried, but not impregnated with any moisture-resisting compound; the winding was to be arranged so that the true capacitances would not exceed 6.0 mmf. Besides being balanced for d-c. resistance, the resultants of their capacitance and inductance values were to be balanced to within 0.1 mmf. To

[7] See Note 6.

meet these requirements, the bridge coils were constructed as follows. The spool used is a glass cylinder $\frac{3}{4}$ in. in diameter. The winding is applied as follows: Starting at one end of the spool, a single strand of the wire is wound on until 14 inductive turns have been applied giving a resistance of approximately 50 ohms. Then the wire is tied, the direction of winding reversed, and an exactly equal number of turns wound over the first 14, but in an opposite direction. This brings the wire to the beginning. It is again tied, carried parallel to the axis of the spool over this first section and a second section wound. This is continued until ten sections have been applied. A thin sheet of mica is tied in place around the winding and the projecting ends of the wire bared of insulation. The whole is then baked to anneal the wire and dry the insulation. While hot, it is dipped several times in molten asphalt compound until a continuous coating of this moisture-proof material has been formed over the winding and surrounding mica wrapping. Adjustment for resistance balance is made by varying the length of the two wire ends.

The effective reactances of coils made as above are positive before assembly in their shields. The effect of the shield is to increase the capacitance. Table II gives data obtained on the two coils made for

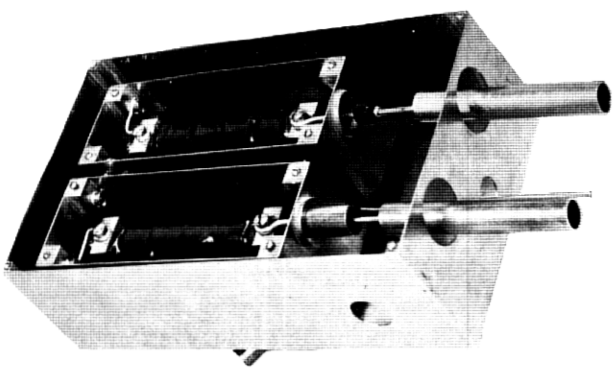

Fig. 12—Ratio arms

the bridge and shows the uniformity of phase-angle difference maintained by these coils over the operating frequency range. Final adjustment for reactance balance is made with the coils in the bridge circuit, a small amount of inductive coiling of the terminal leads sufficing for this purpose. In establishing this balance, use is made of the reversing switch described in the following section. Fig. 12 shows these coils assembled in their shields.

TABLE II

Effective Inductance of Ratio Arm Coils

Test Frequency	Microhenrys					
	Before Coating		After Coating		Assembled in Shield	
	Coil *A*	Coil *B*	Coil *A*	Coil *B*	Coil *A*	Coil *B*
1,000 cycles................	7.4	6.9	6.7	6.3	− 1.3	− 1.0
50,000 cycles................	7.4	6.9	6.7	6.2	− 1.2	− 0.8

Resistance of each coil = 1051.2 ohms

Reversing Switches. As will be noted from the diagram showing the circuit arrangement of the reversing switches, these are required to be completely enclosed in a shield which is connected to the junction point *D* of the bridge. They must also, of course, be subject to manipulation.

Obviously, then, this shield must be supported in some fashion from the outer enclosing ground shield of the bridge. The admittance between these two shields is a direct shunt on either one half or all of the impedance arm *CD*. While the capacitance component of this admittance can be readily balanced, it is more difficult to balance the conductance component, since the latter varies irregularly with frequency. Consequently, it is desired to make this factor so low that it can ordinarily be neglected. The construction adopted for this purpose is shown in the illustration, Fig. 13, which is a partially assembled view of the two reversing switches, their shield and its supporting brackets. It will be noted that the shield is supported by the brackets by means of small glass rods (four of which are shown). The low phase-angle characteristics of glass make it a favorable material to use, from this standpoint, but from the standpoint of machining into shape suitable for insulating supports, it is not so good. The construction shown, however, adapts it very well to this purpose. One other feature is worthy of special note; that is, the small change in position of any of the switch parts which occurs in effecting a reversal, the only metallic part moving being the small metal segments of the rotating disk.

Transformers. The transformers used for isolating the bridge circuit electrostatically from the source of testing current and from the detector system should have substantially zero external electromagnetic fields. This is to prevent inductive coupling to other parts of the bridge circuit. For this purpose the transformer core is made in toroidal or ring form and the windings, both primary and secondary, are uniformly distributed about its circumference. The wound toroid is also completely enclosed in a sheet iron case.

The winding which is connected to the bridge has an electrostatic shield completely surrounding it for the purpose of concentrating all

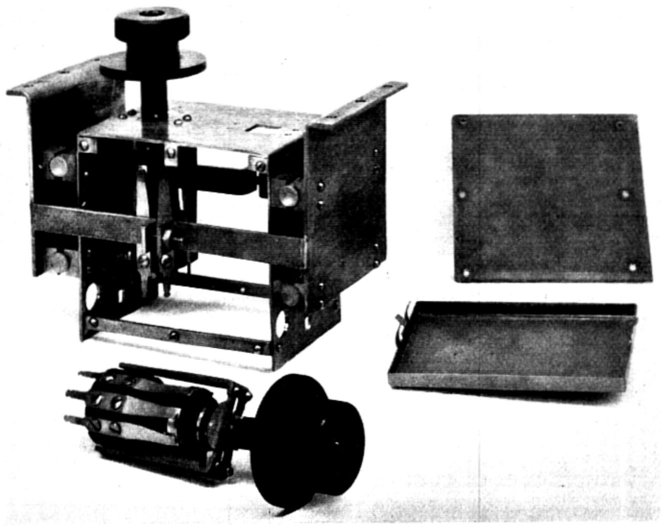

Fig. 13—Reversing switches

capacitance currents at one point. Around this localizing shield there is a second or ground shield. These two shields are made of sheet copper approximately No. 30 gauge (0.010 in.) in thickness. The inner winding terminal leads are brought out through a small brass tube leading into a terminal chamber which is an extension of the localizing shield. Since the admittance between the localizing and ground shields forms a major part of one of the balanced admittances shunting the impedance arms, it is desirable that the capacitance component be of low value and essential that it be constant. The conductance component should be negligibly small. To attain this end, the shields are separated at definite distances by means of hard rubber rings turned to fit the outer corners of the inner shield and the corresponding inner corners of the enclosing shield. These rings are made of the smallest cross-section consistent with mechanical strength requirements so as to introduce the minimum amount of solid material into the space between the shields. This minimizes the capacitance and conductance values. These shields must not, of course, be allowed to act as short-circuited secondaries on the transformer which would be the case if they linked conductively with the windings. Each is therefore made in two parts similar to toroidal channels which upon assembly have their overlapping inner circumferences insulated from each other by means of thin mica laminations. Further details of the construction will be evident from a study of Fig. 14.

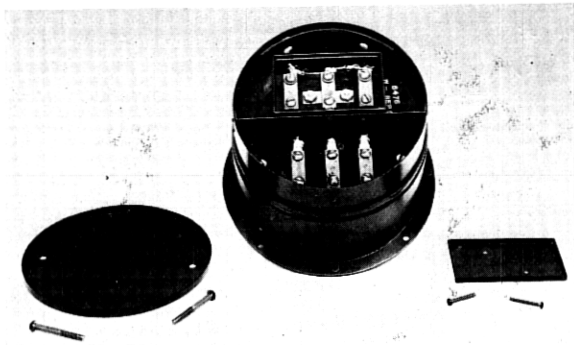

Fig. 14—Shielded transformer

The windings are, of course, proportioned so as to connect with a reasonable degree of efficiency the associated impedances. For best results two sets of transformers are used to cover the complete frequency range, one from 500 to 5000 cycles and the other from 5000 to 50,000 cycles.

Balancing Condensers and Impedance Arm Balance. It has been brought out previously that two adjustable capacitances are required, one to effect the proper adjustment of the bridge capacitances to ground and the other to balance the residual capacitances shunting one of the impedance arms. Such capacitances are provided in the form of adjustable air condensers each having a maximum value of about 500 mmf. The construction used is such as to give a high degree of stability of capacitance combined with low conductance characteristics. The arrangement of these condensers in relation to the other apparatus is shown in Fig. 3.

The effect on the accuracy of the bridge of the degree of balance of the impedance arms obtained by means of the balancing condenser C_b is determined as follows:

Capacitance Shunting the Impedance Arms. The equations giving the equivalent series inductance L' and resistance R' of a reactance of inductance L and resistance R paralleled by a capacitance C are

$$L' = \frac{L - CR^2 - \omega^2 CL^2}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2},$$

$$R' = \frac{R}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}.$$

When the bridge is balanced the equivalent series values of each component of the two impedance arms must be equal respectively to each other. If, however, the two arms have different shunting capacitances, it is evident that this equality will be obtained only by making the values of the two inductive branches of the parallel circuit somewhat different from each other. This difference represents the error introduced by the capacitance unbalance. When the values of the shunting capacitances are small these errors for the purpose of indicating their order are sufficiently closely given by the expressions

$$\Delta L_X = \omega^2 L_X^2 (C_X - C_S) \tag{7}$$

and

$$\Delta R_X = 2\omega^2 L_X R_X (C_X - C_S) \tag{8}$$

where C_X and C_S are the capacitances shunting the unknown and standard impedance arms, respectively. Reduced to percentages, these expressions become

$$\Delta L_X\ (\%) = 100\omega^2 L_X (C_X - C_S)$$

and

$$\Delta R_X\ (\%) = 200\omega^2 L_X (C_X - C_S)$$

and may also be written

$$\Delta L_X\ (\%) = 100 \frac{C_X - C_S}{C_R} \tag{9}$$

and

$$\Delta R_X\ (\%) = 200 \frac{C_X - C_S}{C_R}, \tag{10}$$

where C_R is the value of capacitance that would be required for resonance with inductance L_X at the test frequency.

These errors are thus proportional to the ratio of the capacitance unbalance to the resonating capacitance of the inductance under test. Ordinarily, values of the latter factor do not go below about 500 mmf. so that in the worst case a difference in capacitance of 0.1 mmf. corresponds to errors of 0.02 per cent and 0.04 per cent in inductance and resistance respectively.

Shields and Wiring. The shields have sufficient rigidity and are supported so as to maintain a definite and constant space relation to the part shielded and to the other shields. They are also of sufficiently high conductivity to maintain a common definite electrical potential at all points with respect to the part shielded.

The supports of shields or of bridge elements within the shields are as nearly as possible of constant specific inductive capacity, have low dissipative and leakage losses and are restricted to the minimum in number and size consistent with meeting the required rigidity of support.

Interconnecting conductors are shielded within brass tubes of approximately ½-in. diameter, the conductor which is of No. 10 gauge copper being supported at the axis of the tube by means of glass beads fitting snugly within the tubes and having holes through which the conductor passes. These beads are located longitudinally on the conductor by means of a small lump of solder placed on each side.

Standards. The impedance standards consist of adjustable self-inductance elements used in series with an adjustable non-inductive resistance. Each self-inductance element consists of a series of inductance coils and a low range inductometer of the Brooks type,[8] arranged in three decade formation and connected to dial switches by means of which any series combination of the coils can be selected. The inductometer is always in circuit and permits of balancing inductance values that fall between consecutive steps on the dials. Fig. 15 shows, schematically, the connections used for these standards and also the way in which they are shielded. It will be noted that the parts comprising each decade have a shield enclosing them and also all preceding decades of higher value. This makes a rather complicated mechanical arrangement but results in very important advantages from the standpoint of electrical performance. Due to the individual decade shields, each decade has effective values that are entirely independent of the settings of either of the other decades. Hence, once each individual setting of each dial has been calibrated, the value for the standard as a whole for any possible combination is obtained by simple addition of the separate dial values. This saves an immense amount of work in calibrating and also simplifies the reading of the standard. Without these shields the inter-coil and coil-to-ground admittances, at the higher frequencies, are sufficiently large to make the effective impedance of each decade setting depend to an appreciable extent upon the settings of the other decades. Under such conditions a calibration of

[8] H. B. Brooks and F. C. Weaver: "A Variable Self and Mutual Inductor," *Scientific Paper of the Bureau of Standards*, No. 290.

every combination would be required, and as this calibration would vary with frequency, a correction would be needed for each frequency value used.

In a standard of inductance to be used at high frequencies it is, of course, always desirable in order to minimize capacitance effects to have the inductance coils as small as possible. In the case of a completely shielded decade standard this is even more important on

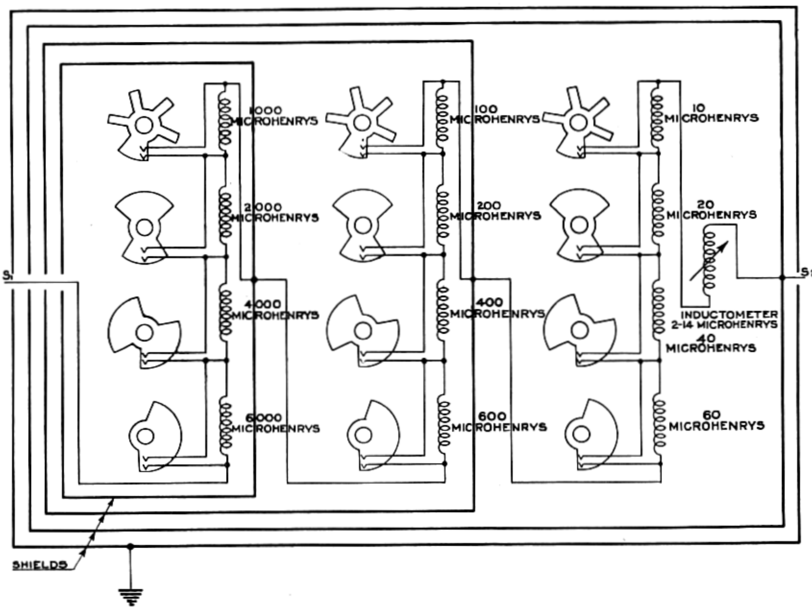

Fig. 15—Circuit diagram of shielded inductance standard

account of the capacitances added by the internesting shields. On the other hand the coil resistances should be quite small in comparison with their reactances, a requirement which tends to increase the coil dimensions. In addition to the above such coils should be highly stable in their inductance and effective resistance values with respect to the residual effects of direct and alternating currents and of temperature and humidity changes. Their values should also be of a satisfactory degree of constancy with respect to frequency and value of the testing current.

To meet these varied requirements the coils used in this bridge depart from the air core type ordinarily employed, in that they have a magnetic core of high stability and efficiency. Thus the desired inductance is obtained with a much smaller number of turns in the winding giving a satisfactorily low resistance even in a coil only a fraction of the size of the equivalent air core coil. An adaptation of the new magnetic material, permalloy, has made this type of inductance standard possible.[9] Their cores consist of finely laminated, high

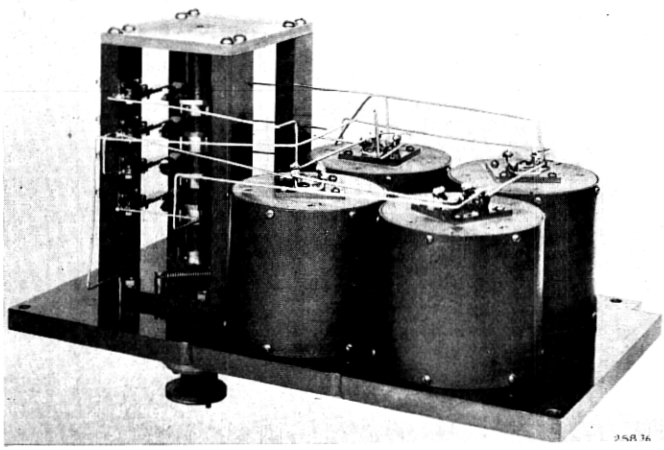

Fig. 16—Coil and dial switch assembly of typical inductance standard decade

specific resistance permalloy punchings, carefully annealed and assembled to form a toroidal structure whose effective permeability is about forty. On this is wound a sectionalized winding of insulated stranded conductor, the individual strands also being insulated from each other. The wound coils, after adjustment to the value desired, are sealed with moisture-proof compounds in phenol fiber cases. Fig. 16 shows an assembly of the four coils and switch which comprise one decade of the standard. In Table III are given data for typical coils illustrating their performance in respect to the above points.

The adjustable, non-inductive resistance is a commercial dial resistance box to which a shield has been added. It has five dials providing a range of 1000 ohms in steps of 0.01 ohm. Its shield is grounded in use, the resistance itself being connected usually between the *C* corner of the bridge and the inductance standard but in the case of an unknown impedance having a lower resistance than the standard from the *C* corner to the coil under test.

[9] H. D. Arnold and G. W. Elmen, *Franklin Institute Journal*, 195, 1923.

TABLE III

DATA ON COILS FOR INDUCTANCE STANDARDS

	Frequency Range	
	500 ∾ — 5,000 ∾	5,000 ∾ — 50,000 ∾
Overall Dimensions		
Diameter of case........	6½ in.	3½ in.
Length of case..........	4 in.	3⅛ in.
Inductance Characteristics		
Nominal value.........	0.100 henry	0.010 henry
Change with frequency..	+0.5% (500 ∾ — 5,000 ∾)	+2.5% (5,000 ∾ — 50,000 ∾)
Change with current....	+0.01% per milliampere	+0.007% per milliampere
Temperature coefficient..	−0.013% per deg. Fahr.	−0.005% per deg. Fahr.
Residual magnetization effect of one ampere d–c................	Less than 0.01%	Less than 0.01%
Resistance Characteristics		
At an alternating current (effective value) of		
2.0 milliamperes...	1,000 ∾ — 5.2 ohms 3,000 ∾ — 8.5 " 5,000 ∾ — 13.0 "	10,000 ∾ — 3.6 ohms 30,000 ∾ — 13.6 " 50,000 ∾ — 34.0 "
10.0 milliamperes...	1,000 ∾ — 5.5 " 3,000 ∾ — 9.1 " 5,000 ∾ — 13.9 "	10,000 ∾ — 3.7 " 30,000 ∾ — 13.9 " 50,000 ∾ — 34.7 "
Temperature coefficient...........	−0.017% per deg. Fahr. at 3,000 ∾	− < 0.01% per deg. Fahr. at 30,000 ∾

PERFORMANCE

As was stated earlier, the operation of the bridge involves an initial balancing of its capacitances. It is then ready for impedance testing which is done by suitably connecting the unknown and standard impedances to the proper terminals and adjusting the latter until a balance of the bridge is obtained. The corresponding constants of the two impedance arms are then taken as being equal. Those of the standards being known, by calibration, it follows that those of the impedance under test can be simply derived. The degree of precision obtained depends upon two major factors, the accuracy of the calibration of the standards and the accuracy of the bridge comparison. The matter of calibration is beyond the scope of this paper and it will be assumed that a suitable calibration of the standards is available.

Due to the construction used the factors determining the accuracy of the bridge comparison of impedances are reduced to the following:

1. The resistance balance of arms AB and AD.
2. The effective shunt capacitance balance of these arms.

3. The direct capacitance balance of arms BC and CD.
4. The direct conductance balance of arms BC and CD.
5. The series inductance balance of the interior wiring to the impedance terminals of arms BC and CD.

As was explained in the foregoing two switches are provided for independently reversing the ratio arms (AB and AD) and also the outside connected impedances. These, therefore, afford a very convenient means of checking the above balances of the bridge network. By a suitable choice of the test condition under which the reversals are made, a fairly good approximation of the effect of the separate items can be made. The following series of tests indicate how this was done on one of these bridges.

The junction point C was first grounded. Then, with a telephone receiver as the detector and with a test current having a frequency of 1600 cycles, the setting of the condenser C_b was varied until a balance was obtained. The arms Z_S and Z_X were both open-circuited in this test; hence the capacitances shunting these arms alone determined the balance point. This balance was very sharp indicating that the shunting conductances were either very small or else accidentally well balanced. Leaving the condenser set at its balance point, there was then connected into one of the impedance arms a toroidal self inductance standard having a nominal inductance of 0.200 henry and an effective resistance of about 50 ohms. In the other arm there was connected a similar standard of the same nominal but of slightly lower actual value in series with a small adjustable inductance and adjustable resistance, each of sufficient range to effect a balance of the corresponding constants. The extension inductance was graduated in steps of one microhenry and the resistance in steps of 0.001 ohm. Balances for the four combination settings of the reversing switches, S_R and S_Z, were then made, only the extension elements being varied in getting these balances. Readings as given in Table IV were obtained.

TABLE IV

Switch Position		Extension Inductance	Extension Resistance
S_R	S_Z		
Right	Right	124 ± 2 microhenrys	4.00 ± 0.01 ohm
Left	Right	120 " " "	4.00 " " "
Right	Left	124 " " "	3.87 " " "
Left	Left	128 " " "	3.87 " " "

Consideration of these figures led to the following conclusions:

1. The change in inductance balance due to reversal of the ratio arms is not more than eight parts in 200,000 or 0.004 per cent. It has been shown previously that the phase-angle balance of the ratio arms is not critical with respect to inductance readings. Hence, the change in inductance may be considered to be closely indicative of the resistance unbalance of the ratio arms. From the above data it is seen that this does not exceed 0.01 per cent.

2. The change in resistance due to reversal of the ratio arms being within the limits of observational error, the phase angles of the coils themselves are, as nearly as can be determined by this test, exactly balanced.

3. Since the change of inductance balance due to reversal of the impedance arms is no more than that due to the ratio arm reversal, the capacitance balance of the impedance arms is apparently satisfactory. It should be noted, however, that this balance is not critical under these test conditions. (See eq. 9.)

4. Since the resistance balance was appreciably affected by reversal of the impedance arms, it appeared that there was an unbalancing factor present which, if affecting the ratio arms, was not reversed with them but which if present in the impedance arms was reversed. The latter might have been an unbalance of the impedance arm shunt conductances but it was assumed that this unbalance was quite small. On the other hand, as was mentioned earlier in the paper, there are two small inter-shield capacitances shunting the ratio arms which are not reversed by the ratio arm switch and it seemed likely that an unbalance of these capacitances was causing the change in resistance reading. This proved to be the case, as adjustment of the balance of these capacitances for which, as previously noted, provision had been made, resulted in identical resistance readings being obtained for both positions of the impedance arm switch. After this adjustment had been made the previous tests were repeated, resulting in readings as given in Table V.

TABLE V

Switch Position		Extension Inductance	Extension Resistance
S_R	S_Z		
Right	Right	124 ± 2 microhenrys	3.93 ± 0.01 ohm
Left	Right	120 " " "	3.93 " " "
Right	Left	124 " " "	3.93 " " "
Left	Left	126 " " "	3.93 " " "

As a further check on the performance of this unit, two inductances, each of about 0.01 henry inductance, were compared at two frequencies, 25,000 and 50,000 cycles. Table VI gives the readings obtained in these tests.

TABLE VI

Frequency Cycles	Switch Position		Extension Inductance	Extension Resistance
	S_R	S_Z		
25,000	Right	Right	123 ± 1 microhenry	5.1 ± 0.1 ohm
	Left	Right	122 " " "	5.1 " " "
	Right	Left	129 " " "	5.1 " " "
	Left	Left	128 " " "	5.1 " " "
50,000	Right	Right	38 " " "	24.2 " " "
	Left	Right	36 " " "	24.2 " " "
	Right	Left	57 " " "	23.8 " " "
	Left	Left	57 " " "	23.8 " " "

At the 25,000-cycle frequency the maximum difference in inductance from the probable correct balance does not exceed ± 5 microhenrys or 0.05 per cent. The resistance balances check to within 0.1 ohm. At 50,000 cycles, the inductance change due to ratio arm reversal is still within ± 0.01 per cent while the resistance change is within 0.1 ohm which would be just under one per cent for a coil of this reactance and a reactance to resistance ratio of 300. This is a critical test of the ratio arm phase-angle balance. Hence it may be concluded that over the entire frequency range the ratio coils meet all balance requirements. The changes in inductance occurring at the higher frequencies when the impedance arms were reversed indicated that the residual capacitance unbalance of these arms was too large. Readjustment of the balancing condenser reduced the changes to less than 0.02 per cent. The difference in resistance balance at the 50,000-cycle frequency indicates that the conductances shunting the impedance arms are not negligible at this frequency. For more accurate results these conductances would require balancing. This would be quite practicable by means of a variable high resistance shunt.

In making each series of tests outlined above, the testing potential applied to the bridge was varied by means of a resistance potentiometer from the lowest value at which a balance could be made to the maximum of the supply oscillator. This was to check the completeness of the shielding and to detect the presence of any coupling with the supply circuit. In no case was there any discernible change in balance produced.

Conclusion

A system of electrostatic shielding for a direct reading bridge for the measurement of inductive impedances at frequencies up to 50,000 cycles has been described.

The general considerations defining the balances of the various capacitances which this shielding controls have been discussed and specific requirements derived for a typical range of impedances. The physical construction of a bridge designed to meet these requirements has been described and test data given illustrating its performance. These have shown it to be capable of comparing impedances over the above frequency range with a precision which approximates that ordinarily found in routine direct current resistance measurements.