Some Contemporary Advances in Physics—VII
Waves and Quanta

By KARL K. DARROW

THE invaluable agent of our best knowledge of the environing world, and yet itself unknown except by inference; the intermediary between matter and the finest of our senses, and yet itself not material; intangible, and yet able to press, to strike blows, and to recoil; impalpable, and yet the vehicle of the energies that flow to the earth from the sun—light in all times has been a recognized and conspicuous feature of the physical world, a perpetual reminder that the material, the tangible, the palpable substances are not the only real ones. Yet its apparent importance, to our forerunners who knew only the rays to which the eye responds and suspected no others, was as nothing beside its real importance, which was realized very gradually during the nineteenth century, as new families of rays were discovered one after the other with new detecting instruments and with new sources. Radiation is not absent from the places where there is no eye-stimulating light; radiation is omnipresent; there is no region of space enclosed or boundless, vacuous or occupied by matter, which is not pervaded by rays; there is no substance which is not perpetually absorbing rays and giving others out, in a continual interchange of energy, which either is an equilibrium of equal and opposite exchanges, or is striving towards such an equilibrium. Radiation is one of the great general entities of the physical world; if we could still use the word "element," not to mean one of the eighty or ninety kinds of material atoms, but in a deeper sense and somewhat as the ancients used it, we might describe radiation and matter, or possibly radiation and electricity, as coequal elements. Also the problem of the nature and structure of radiation is of no lesser importance than the problem of the structure and nature of matter; and in fact neither can be treated separately; they are so inextricably intertwined that whoever sets out to expound the present condition of one soon finds himself outlining the other. One cannot write a discourse on the nature of radiation alone nor on the structure of the atom alone, one can but vary the relative emphasis laid upon these two subjects, or rather upon these two aspects of a single subject; and in this article I shall restate many things about the atom which were stated in former articles, but the emphasis will be laid upon light.

Speaking very generally and rather vaguely, light has been much more tractable to the theorists than most of the other objects of enquiry in physics or chemistry. Over a rather long period of years, it was indeed generally regarded as perfectly intelligible. The famous battle between the corpuscular theory adopted by Newton, and the wave-theory founded by Descartes and Huyghens, died out in the earlier years of the nineteenth century with the gradual extinction of the former. The history of optics in the nineteenth century, from Fresnel and Young to Michelson and Rayleigh, is the tale of a brilliant series of beautiful and striking demonstrations of the wave-theory, of experiments which were founded upon the wave-theory as their basis and would have failed if the basis had not been firm, of instruments which were designed and competent to make difficult and delicate measurements of all sorts—from the thickness of a sheet of molecules to the diameter of a star—and would have been useless had the theory been fallacious. The details of the bending of light around the sides of a slit or the edge of a screen, the intricate pattern of light and shade formed where subdivisions of a beam of light are reunited after separation, the complexities of refraction through a curved surface, are represented by the theory with all verifiable accuracy; and so are the incredibly complicated phenomena attending the progress of light through crystals, phenomena which have slipped out of common knowledge because few are willing to undertake the labour of mastering the theory. The wave-theory of light stands with Newton's inverse-square law of gravitation, in respect of the many extraordinarily precise tests which it has undergone with triumph; I know of no other which can rival either of them in this regard.

By the term "wave-theory of light" I have meant, in the foregoing paragraph, the conception that light is a wave-motion, an undulation, a periodic form advancing through space without distorting its shape; I have not meant to imply any particular answer to the question, *what is it of which light is a wave-motion?* It may seem surprising that one can make and defend the conception, without having answered the question beforehand; but as a matter of fact there are certain properties common to all undulations, and these are the properties which have been verified in the experiments on light. There are also certain properties which are not shared by such waves as those of sound, in which the vibration is confined to a single direction (that normal to the wavefront) and may not vary otherwise than in amplitude and phase, but are shared by transverse or distortional waves in elastic solids, in which the vibration may lie in any of an infinity of directions (any direction tangent to the wavefront). Light possesses these properties, and therefore the wave-motion which is radiation may not be compared with the wave-motion which is sound; but a wide range of comparisons still remains open.

Of course, very many have proposed images and models for "the thing of which the vibrations are light", and many have believed with an unshakable faith in the reality of their models. The fact that light-waves may be compared, detail by detail, with transverse vibrations in an elastic solid, led some to fill universal space with a solid elastic medium to which they gave the sonorous name of "luminiferous aether". It is not many years since men of science used to amaze the laity with the remarkable conception of a solid substance, millions of times more rigid than steel and billions of times rarer than air, through which men and planets serenely pass as if it were not there. Even now one finds this doctrine occasionally set forth.[1]

In that image of the elastic solid, the propagation of light was conceived to occur because, when one particle of the solid is drawn aside from its normal place, it pulls the next one aside, that one the next one to it, and so on indefinitely. Meanwhile, each particle which is drawn aside exerts a restoring force upon the particle of which the displacement preceded and caused its own. Set one of the particles into vibration, and the others enter consecutively into vibration. Maintain the first particle in regular oscillation, and each of the others oscillates regularly, with a phase which changes from one to the next; a wave-train travels across the medium. One particle influences the next, because of the attraction between them. But in the great and magnificent theory of light which Maxwell erected upon the base of Faraday's experiments, the propagation was explained in an altogether different manner. Vary the magnetic field across a loop of wire in a periodic manner, and you obtain a periodic electric force around the loop, as is known to everyone who has dabbled in electricity. Vary the electric field periodically, and you obtain a periodic magnetic field—this a fact not by any means so well known as the other, one which it was Maxwell's distinction to have anticipated, and which was verified after the event. In a traveling train of light-waves the electric field and the magnetic field stimulate one another alternately and reciprocally, and for this reason the wave-train travels. Since the periodic electric field may point in any one of the infinity of directions in the plane of the wave-front, the wave-motion possesses all the freedom and variability of

[1] Apparently the image of the elastic solid was never quite perfected; one recalls the question as to whether its vibrations were in or normal to the plane of polarization of the light, which required one answer in order to agree with the phenomena of reflection, and another in order to agree with those of double refraction. Probably a *modus vivendi* could have been arranged if the whole idea had not been superseded.

form which are required to account for the observed properties of light.

Maxwell's theory immediately achieved the stunning success of presenting a value for the speed of the imagined electromagnetic waves, determined exclusively from measurements upon the magnetic fields of electric currents, and agreeing precisely with the observed speed of light. Two supposedly distinct provinces of physics, each of which had been organized on its own particular basis of experience and in its own particular manner, were suddenly united by a stroke of synthesis to which few if any parallels can be found in the history of thought. And this is by no means the only achievement of the electromagnetic theory of light; there will shortly be occasion to mention some of the others.

Now that there was so much evidence that light travels as a wave-motion, and that its speed and other properties are those of electromagnetic waves, it became urgently desirable to inquire into the nature of the *sources* of light. Granted that light *en route* outwards from a luminous particle of matter is of the nature of a combination of wave-trains, what is taking place in the luminous particle? To this question all our experience and all our habits of thought suggest one sole obvious answer—that in the luminous particle there is a vibrating something, a *vibrator*, or more likely an enormous number of vibrators—one to each atom, possibly—and the oscillations of these vibrators are the sources of the waves of light, as the oscillations of a violin-string or a tuning-fork are the sources of waves of sound. This analogy drawn from acoustics, this picture of the vibrating violin-string and the vibrating tuning-fork, has been powerful—indeed, it begins to seem, too powerful—in guiding the formation of our ideas on light. It is profitable to reflect that the evolution of thought in acoustics must have traveled in the opposite sense from the evolution of thought in optics. Whoever it was who was the first to conceive that sound is a wave-motion in air, must certainly have arrived at the idea by noticing that sounding bodies vibrate. One feels the trembling of the tuning-fork or the bell, one sees the violin-string apparently spread out into a band by the amplitude of its motion; it is not difficult to build apparatus which, like a slowed-down cinema film, makes the vibrations separately visible, or, like the stroboscope, produces an equivalent and not misleading illusion. This was not possible in optics, and never will be. In acoustics, one may sometimes accept the vibrations of the sounding body as an independently-given fact of experience, and reason forward to the wave-motion spreading outwards into the environing air; in optics, this entrance to the path is closed, one must reason in the inverse sense from the wave-motion to the qualities of the shining body. Inevitably, it was assumed that when the path should at last be successfully retraced, the shining body would be found in the semblance of a vibrator.

For a few years at the end of the nineteenth century and the beginning of the twentieth, it seemed that the desired vibrator had been found. Apparently it was the electron, the little corpuscle of negative electricity, of which the charge and the mass were rather roughly estimated in the late nineties, although Millikan's definite measurements were not to come for a decade yet. Maxwell had not conceived of particles of electricity, his conception of the "electric fluid" was indeed so sublimated and highly formal that it gave point to the celebrated jest (I think a French one) about the man who read the whole of his "Electricity and Magnetism" and understood it all except that he was never able to find out what an electrified body was. H. A. Lorentz incorporated the electron into Maxwell's theory. Conceiving it as a spherule of negative electricity, and assuming that in an atom one or more of these spherules are held in equilibrium-positions, to which restoring-forces varying proportionally to displacement draw them back when they are displaced, Lorentz showed that these "bound" electrons are remarkably well adapted to serve as sources and as absorbents for electromagnetic radiation. Displaced from its position of equilibrium by some transitory impulse, and then left to itself, the bound electron would execute damped oscillations in one dimension or in two, emitting radiation of the desired kind at a calculable rate. Or, if a beam of radiation streamed over an atom containing a bound electron, there would be a "resonance" like an acoustic resonance—the bound electron would vibrate in tune with the radiation, absorbing energy from the beam and scattering it in all directions, or quite conceivably delivering it over in some way or other to its atom or the environing atoms. There were numerical agreements between this theory and experience, some of them very striking.[2] Apparently the one thing still needful was to produce a plausible theory of these binding-forces which control the response of the "bound" electron to disturbances of all kinds. Once these were properly described, the waves of light would be supplied with

[2] Notably, the trend of the dispersion-curves for certain transparent substances, recently extended by Bergen Davis and his collaborators to the range of X-ray frequencies; the normal Zeeman effect; Wien's observations on the exponential dying-down of the luminosity of a canal-ray beam, interpreted as the exponential decline in the vibration-amplitudes of the bound electrons in the flying atoms; the dependence of X-ray scattering on the number of electrons in the atom.

their vibrators, the electromagnetic theory would receive a most valuable supplement. And, much as a competent theory of the binding-forces was to be desired, a continuing failure to produce one would not impugn the electromagnetic theory, which in itself was a coherent system, self-sustaining and self-sufficient.

This was the state of affairs in the late nineties. The wave-conception of light had existed for more than two centuries, and it was seventy-five years since any noticeable opposition had been raised against it. The electromagnetic theory of light had existed for about thirty years, and now that the electron had been discovered to serve as a source for the waves which in their propagation through space had already been so abundantly explained, there was no effective opposition to it. Not all the facts of emission and absorption had been accounted for, but there was no reason to believe that any particular one of them was unaccountable. Authoritative people thought that the epoch of great discoveries in physics was ended. It was only beginning.

In the year 1900, Max Planck published the result of a long series of researches on the character of the radiation inside a completely-enclosed or nearly-enclosed cavity, surrounded by walls maintained at an even temperature. Every point within such a cavity is traversed by rays of a wide range of wave lengths, moving in all directions. By the "character" of the radiation, I mean the absolute intensities of the rays of all the various frequencies, traversing such a point. The character of the radiation, in this sense, is perfectly determinate; experiment shows that it depends only on the temperature of the walls of the cavity, not on its material. According to the electromagnetic theory of radiation, as completed by the adoption of the electron, the walls of the cavity are densely crowded with bound electrons; nor are these electrons all bound in the same manner, so that they would all have the same natural frequency of oscillation—they are bound in all sorts of different ways with all magnitudes of restoring-forces, so that every natural frequency of oscillation over a wide range is abundantly represented among them. Now the conclusion of Planck's long study was this:

If the bound electrons in the walls of the cavity (i.e., *in any solid body*) *did really radiate while and as they oscillate, in the fashion prescribed by the electromagnetic theory, then the character of the radiation in the cavity would be totally different from that which is observed.*[3]

[3] The belief that the character of radiation within a cavity could not be explained without doing some violence to the "classical mechanics" had already been gaining ground for some years, by reason of extremely recondite speculations of a statistical nature. It is very difficult to gauge the exact force and bearing of such considerations.

However, *if the bound electrons do not radiate energy while they oscillate, but accumulate it and save it up and finally discharge it in a single outburst when it attains some one of a certain series of values* $h\nu$, $2h\nu$, $3h\nu$, etc. (h stands for a constant factor, ν for the frequency of vibration of the electrons and the emitted radiation)—*then the character of the radiation will agree with that which is observed*, provided a suitable value be chosen for the constant h.

The value required [4] for h in C.G.S. units (erg seconds) is $6.53 \cdot 10^{-27}$.

Here, then, was a phenomenon which the electromagnetic theory seemed to be fundamentally incapable of explaining. For this notion of a bound electron, which oscillates and does not meanwhile radiate, is not merely foreign to the classical theory, but very dangerous to it; one does not see how to introduce it, and displace the opposed notion, without bringing down large portions of the structure (including the numerical agreements which I cited in a foregoing footnote). However, Planck had arrived at this conclusion by an intricate process of statistical and thermodynamical reasoning. Statistical reasoning is notoriously the most laborious and perplexing in all physics, and many will agree that thermodynamical reasoning is not much less so. Planck's inference made an immense impression on the most capable thinkers of the time; but in spite of the early adherence of such men as Einstein and Poincaré, I suspect that even to this day it might practically be confined to the pages of the more profound treatises on the philosophical aspects of physics, if certain experimenters had not been guided to seek and to discover phenomena so simple that none could fail to apprehend them, so extraordinary that none could fail to be amazed.

Honour for this guidance belongs chiefly to Einstein. Where Planck in 1900 had said simply that bound electrons emit and absorb energy in fixed finite quantities, and shortly afterwards had softened his novel idea as far as possible by making it apply only to the act of emission, Einstein in 1905 rushed boldly in and presented the idea that these fixed finite quantities of radiant energy retain their identity throughout their wanderings through space from the moment of emission to the moment of absorption. This idea he offered as a "heuristic" one—the word, if I grasp its connotation exactly, is an apologetic sort of a word, used to describe a theory which achieves successes though its author feels at heart that it really is too absurd to

[4] I take the numerical values of the constant h scattered through this article from Gerlach. The weighted mean of the experimental values, with due regard to the relative reliability of the various methods, is taken as 6.55 or $6.56 \cdot 10^{-27}$. None of the individual values cited in these pages is definitely known to differ from this average by more than the experimental error.

be presentable. The implication is, that the experimenters should proceed to verify the predictions based upon the idea, quite as if it were acceptable, while remembering always that it is absurd. If the successes continue to mount up, the absurdity may be confidently expected to fade gradually out of the public mind. Such was the destiny of this heuristic idea.

I will now describe some of these wonderfully simple phenomena—wonderfully simple indeed, for they stand out in full simplicity in domains where the classical electromagnetic theory would almost or quite certainly impose a serious complexity. If Planck's inference from the character of the radiation within a cavity had been deferred for another fifteen years, one or more of these phenomena would assuredly have been discovered independently. What would have happened in that case, what course the evolution of theoretical physics would have followed, it is interesting to conjecture.

The *photoelectric effect* is the outflowing of electrons from a metal, occurring when and because the metal is illuminated. It was discovered by Hertz in 1889, but several years elapsed before it was known to be an efflux of electrons, and several more before the electrons were proved to come forth with speeds which vary from one electron to another, upwards as far as a certain definite maximum value, and never beyond it.

Here is a rather delicate point of interpretation, which it is well to examine with some care; for all the controversies as to continuity versus discontinuity in Nature turn upon it, in the last analysis. What is meant, or what reasonable thing can be meant, when one says that the speeds of all the electrons of a certain group are confined within a certain range, extending up to a certain limiting topmost value? If one could detect each and every electron separately, and separately measure its speed, the meaning would be perfectly clear. For that matter, the statement would degenerate into a truism. The fact is otherwise. The instruments used in work such as this perceive electrons only in great multitudes. Suppose that one intercepts a stream of electrons with a metal plate connected by a wire to an electrometer. If a barrier is placed before the electrons in the form of a retarding potential-drop, which is raised higher and higher, the moment eventually comes when the current into the electrometer declines. This happens because the slower electrons are stopped and driven back before they reach the plate, the faster ones surmount the barrier. As the potential-drop is further magnified, the reading of the electrometer decreases steadily, and at last becomes inappreciable. Beyond a certain critical value of the retarding voltage, the electrometer reports no influx of electrons. Does this really mean that there are *no* electrons with more than just the speed necessary to overpass a retarding voltage of just that critical value? Or does it merely mean that the electrons flying with more than that critical speed are plentiful, but not quite plentiful enough to make an impression on the electrometer? Is there any topmost speed at

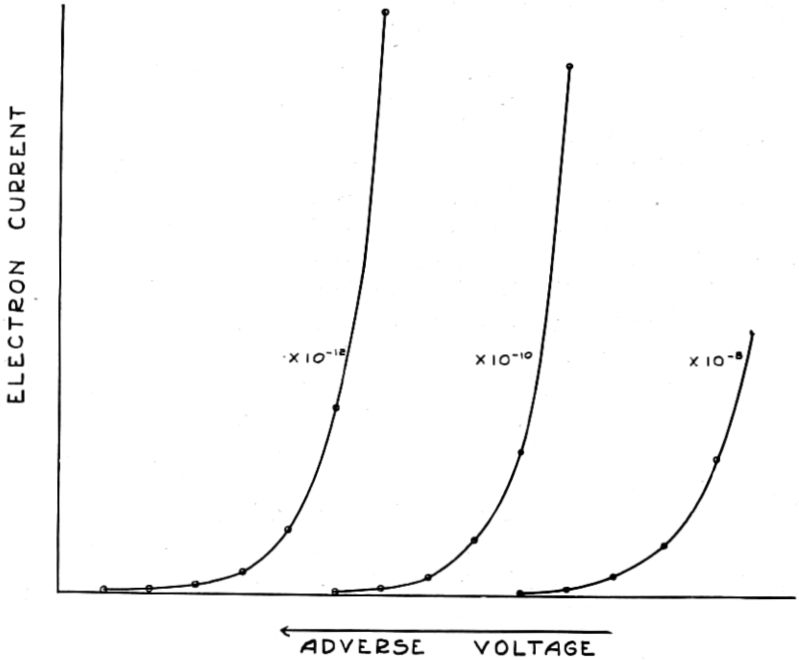

Fig. 1—Curves showing thermionic electron-current versus opposing voltage, demonstrating a distribution-in-speed extending over an unlimited range of speeds. Multiply the ordinates of the middle curve by 100, those of the right-hand curve by 10,000, to bring them to the same scale and make them merge into a single curve. (L. H. Germer)

all, or should we find, if we could replace the current-measuring device with other and progressively better ones *ad infinitum*, that the apparent maximum speed soared indefinitely upwards?

Absolute decisions cannot be rendered in a question of this kind; but it is possible, under the best of circumstances, to pile up indicatory evidence to such an extent that only an unusually strong will-to-disbelieve would refuse to be swayed by it. The judgment depends on the shape of the curve which is obtained by plotting the electrometer-reading *vs.* the retarding potential—in other words, the fraction y of the electrons of which the energy of motion surpasses the amount x, determined from the retarding-voltage by the relation $x = eV$. Look for example at the curves of Fig. 1, which refer to the electron-

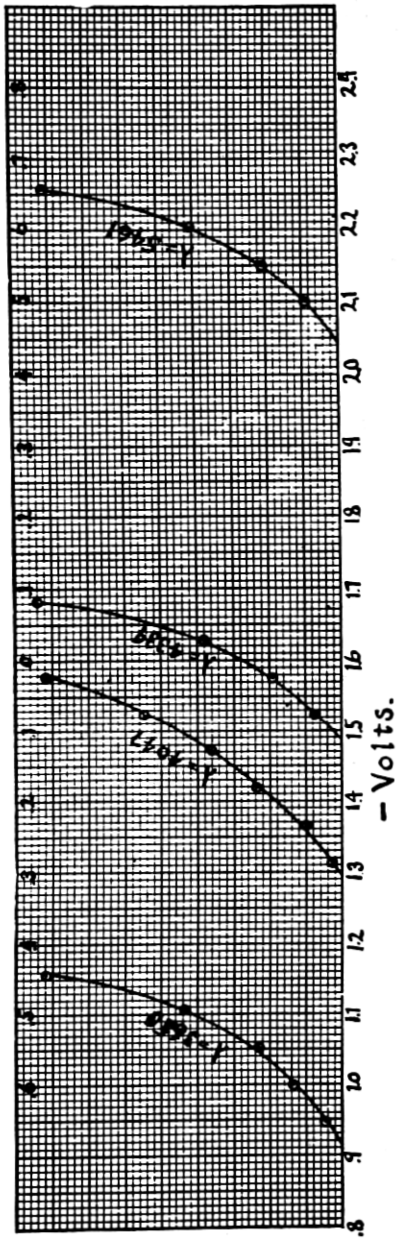

Fig. 2—Curves showing photoelectric electron-current versus opposing voltage, demonstrating a distribution-in-speed extending over a range limited at the top. (R. A. Millikan, *Physical Review*)

stream flowing spontaneously out of an incandescent wire; they are three segments of one single curve, plotted on different scales as the numerals show. This curve bends so gradually around towards tangency with the axis of abscissae, that one can hardly avoid the inference that it is really approaching that axis as if to an asymptote, and that if the electrometer at any point ceases to declare a current, it is because the electrometer is too insensitive to respond to the smaller currents, and not because there are no faster electrons. Look instead at the curves of Fig. 2, which refer to the electrons emerging from an illuminated surface of sodium. These curves slant so sharply towards the axis of abscissae, they bend so slightly in the portions of their courses where the data of experiment determine them, that the linear extrapolation over the little interval into the axis commends itself as natural and inevitable. Because the curves for the thermionic electrons approach the axis so gently, it is agreed that their velocities are distributed continuously over an unlimited range; because the curves for the photoelectrons cut into it so acutely, it is felt that their velocities are confined below a definite maximum value.

This therefore is the photoelectric effect: waves of light inundate the surface of a metal, and electrons pour out with various velocities, some nearly attaining and none exceeding a particular topmost value. I will designate this maximum speed, or rather the corresponding maximum kinetic energy, by E_{max}. Analyzing the process in the classical manner, one must imagine the waves entering into the metal and setting the indwelling electrons into forced oscillations; the oscillations grow steadily wider; the speed with which the electron dashes through its middle position grows larger and larger, and at last it is torn from its moorings and forces its way through the surface of the metal. Some of the energy it absorbed during the oscillations is spent (converted into potential energy) during the escape; the rest is the kinetic energy with which it flies away. Even if the electron were free within the metal and could oscillate in response to the waves, unrestrained by any restoring force, it would still have to spend some of its acquired energy in passing out through the boundary of the metal (the laws of thermionic emission furnish evidence enough for this). It is natural to infer that E_{max} is the energy absorbed by an electron originally free, minus this amount (let me call it P) which it must sacrifice in crossing the frontier; the electrons which emerge with energies lower than E_{max} may be supposed to have made the same sacrifice at the frontier and others in addition, whether in tearing themselves away from an additional restraint or in colliding with atoms during their emigration. This is not the only conceivable interpretation, but it seems unprofitable to enter into the others. It is therefore E_{max} which appears to merit the most attention.

Now the mere fact that there is a maximum velocity of the escaped electrons, that there is an E_{max}, is not in itself of a nature to suggest that the classical theory is inadequate. It is the peculiar dependence of this quantity on the two most important controllable qualities of the light—on its intensity and on its frequency—which awakens the first faint suspicions that something has at last been discovered, which the classical theory is ill adapted to explain. One would predict with a good deal of confidence that the greater the intensity of the light, the greater the energy acquired by the electron in each cycle of its forced oscillation would be, the greater the energy with which it would finally break away, the greater the residuum of energy which at the end would be left to it. But E_{max} is found to be independent of the intensity of the light. This is strange; it is as though the waves beating upon a beach were doubled in their height and the powerful new waves disturbed four times as many pebbles as before, but did not displace a single one of them any farther nor agitate it any more violently than the original gentle waves did to the pebbles that they washed about. As for the dependence of E_{max} on the frequency of the light, it would be necessary to make additional assumptions to calculate it from the classical theory; in any case it would probably not be very simple. But the actual relation between E_{max} and ν is the simplest of all relations, short of an absolute proportionality; this is it:

$$E_{max} = h\nu - P \tag{1}$$

Fig. 3 shows the relation for sodium, observed by Millikan.

The maximum energy of the photoelectrons increases linearly with the frequency of the light. P is a constant which varies from one metal to another. In the terms of the simple foregoing interpretation, P is the energy which an electron must spend (more precisely, the energy which it must invest or convert into potential energy) when it passes through the frontier of the metal on its way outward. Comparing the values of P for several metals with the contact potentials which they display relatively to one another, one finds powerful evidence confirming this theory. Having discussed this particular aspect of the question in the fifth article of this series, I will not enter further into it at this point.

The constant h is the same for all the metals which have been used in such experiments. The best determinations have been made upon two or three of the alkali metals, for these are the only metals

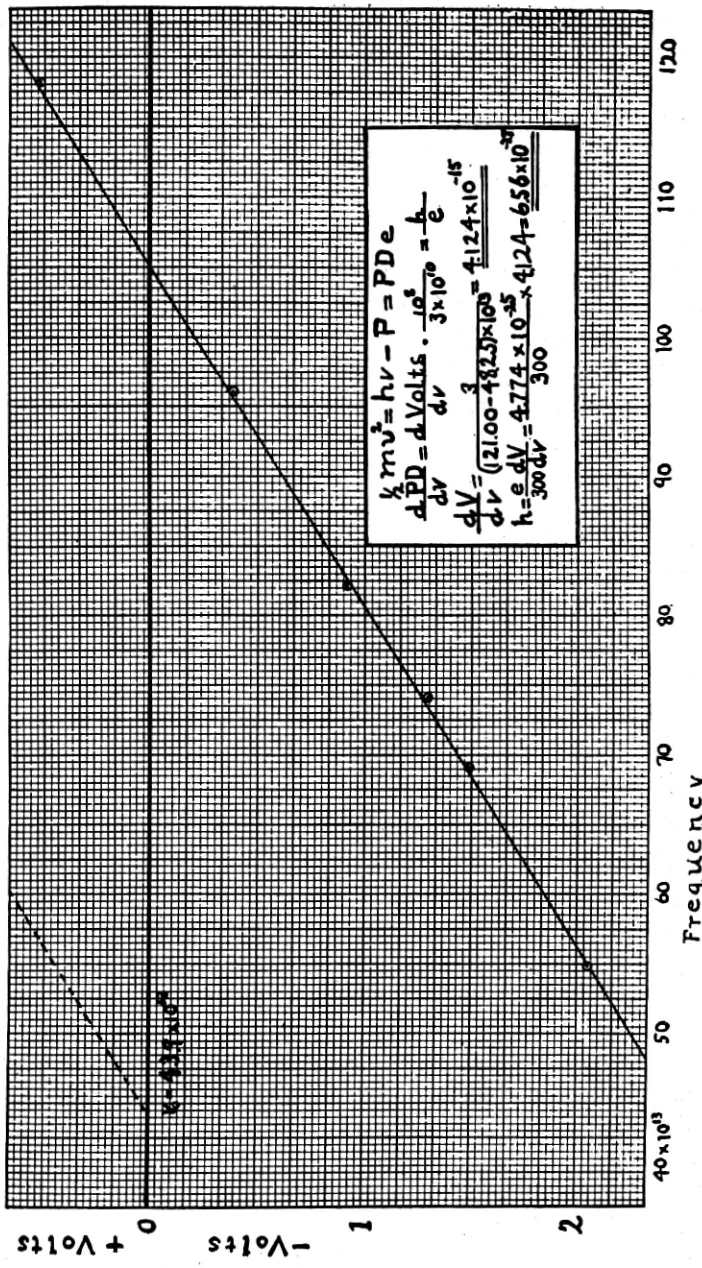

Fig. 3—Curve showing the linear relation between the maximum energy of photo-electrons and the frequency of the light which excites them. (Millikan, *Physical Review*)

which release electrons when illuminated with light of wide convenient ranges of frequency and color. Most metals must be irradiated with ultraviolet light, and the experiments become very difficult if they must be performed with light of frequencies far from the visible spectrum. The values which Millikan obtained for sodium and for lithium agree within the experimental error with one another and with the mean value

$$h = 6.57 . 10^{-27} \qquad (2)$$

The maximum energy of the electrons released by light of the frequency ν is therefore equal to a quantity $h\nu$ which is the same, whatever metal be illuminated by the light—a quantity which is characteristic of the light, not of the metal—minus a quantity P which, there is every reason to believe, is the quota of energy surrendered by each electron in passing out across the boundary-surface of the metal. It is *as if* each of the released electrons had received a quantity $h\nu$ of energy from the light. I will go one step further, and lay down this as a rule, with another cautiously-inserted *as if* to guard against too suddenly daring an innovation:

Photoelectric emission occurs as if *the energy in the light were concentrated in packets, or units, or corpuscles of amount $h\nu$, and one whole unit were delivered over to each electron.*

This is a perfectly legitimate phrasing of equation (1), but I doubt whether anyone would ever have employed it, even with the guarded and apologetic *as if*, but for the fact that the value of h given in (2) agreed admirably well with the value of that constant factor involved in Planck's theory, the constant to which he had given this very symbol and a somewhat similar role. Deferring for a few pages one other extremely relevant feature of the photoelectric effect (its "instantaneity") I will proceed to examine these other situations.

An effect which might well be, though it is not, called the *inverse photoelectric effect*, occurs when electrons strike violently against metal surfaces. Since radiation striking a metal may elicit electrons, it is not surprising that electrons bombarding a metal should excite radiation. Electrons moving as slowly as those which ultraviolet or blue light excites from sodium do not have this power; or possibly they do, but the radiation they excite is generally too feeble to be detected. Electrons moving with speeds corresponding to kinetic energies of hundreds of equivalent volts,[5] and especially electrons

[5] One equivalent volt of energy = the energy acquired by an electron in passing across a potential-rise of one volt $= e/300$ ergs $= 1.591 . 10^{-12}$ ergs. This unit is usually called simply a "volt of energy", or "volt", a bad usage but ineradicable. Also "speed" is used interchangeably with "energy" in speaking of electrons, and one finds (and, what is worse, cannot avoid) such deplorable phrases as "a speed of 4.9 volts" ! ! !

with energies amounting to tens of thousands of equivalent volts, do possess it. This is in fact the process of excitation of X-rays, which are radiated from a metal target exposed to an intense bombardment of fast electrons. The protagonists of the electromagnetic theory had an explanation ready for this effect, as soon as it was discovered. A fast electron, colliding with a metal plate, is brought to rest by a slowing-down process, which might be gradual or abrupt, uniform or *saccadé*, but in any case must be continuous. Slowing-down entails radiation; the radiation is not oscillatory, for the electron is not oscillating, but it is radiation none the less; it is an outward-spreading single pulsation or *pulse*, comparable to the narrow spherical shell of condensed air which diverges outward through the atmosphere from an electric spark and has been photographed so often, or to a transient in an electrical circuit.

One may object that the pulse is just a pulse and nothing more, while the X-rays are wave-trains, for otherwise the X-ray spectroscope (which is a diffraction apparatus) would not function. The objection is answered by pointing out the quite indubitable fact that any pulse, whatever its shape (by "shape" I mean the shape of the curve representing the electric field strength, or whatever other variable one chooses to take, as a function of time at a point traversed by the wave) can be accurately reproduced by superposing an infinity of wave-trains, of all frequencies and divers properly-adjusted amplitudes, which efface one another's periodic variations, and in fact efface one another altogether at all moments except during the time-interval while the pulse is passing over—during this interval they coalesce into the pulse. Thence, the argument leads to the contention that the actual pulse is made up of just such wave-trains, and the sapient diffracting crystal recognizes them all and diffracts each of them duly along its proper path. The problem is not new, nor the answer; white light has long been diagnosed as consisting of just such pulses, and the method of analyzing transient impulses in electrical circuits into their equivalent sums of wave-trains has been strikingly successful.

The application of the method to this case of X-ray excitation enjoyed one qualitative success. The spherical pulse diverging from the place where an electron was brought to rest should not be of equal thickness at all the points of its surface; it should be broader and flatter on the side towards the direction whence the electron came, thinner and sharper on the side towards the direction in which the electron was going when it was arrested. Analyzing the pulse, it is found that at the point where it is broad and low, the most intense of its equivalent wave-trains are on the whole of a lower frequency than the most intense of the wave-trains which constitute it where it is narrow and high. By examining and resolving the X-rays radiated from a target, at various inclinations to the direction of the bombarding electrons, this was verified—verified in part, not altogether. The X-rays radiated nearly towards the source of the electron-stream include a

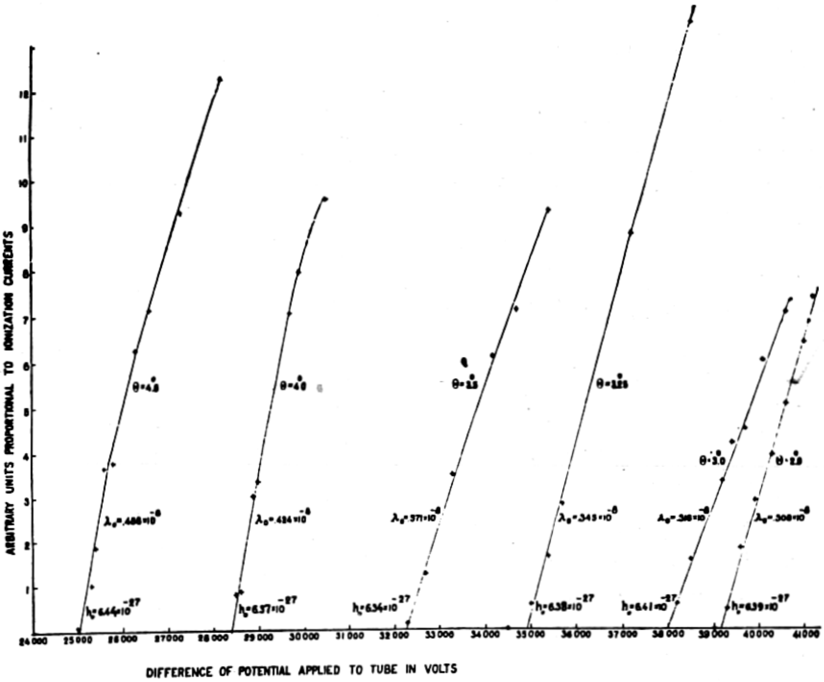

Fig. 4—Curves ("isochromatics") each representing the intensity of X-radiation of a very narrow range of frequencies, plotted versus the energy of the bombarding electrons. (Duane & Hunt, *Physical Review*)

lesser proportion of high-frequency wave-trains, they are *softer* as the phrase is, than the X-rays radiated nearly along the prolongation of the electron-stream. In the spectrum of each of these beams of X-rays, there is a wave length where the density of radiant energy attains a maximum, and this wave length is longer in the former beam than in the latter one. So much is implied in the classical theory.

But it is nowhere implied in the classical theory that the spectrum of an X-ray beam, produced when electrons of a constant energy rain down upon a metal, should extend upwards only to a certain maximum frequency, and then and there come to a sudden end; yet apparently it does. There is a *high-frequency limit* to each X-ray spectrum, and wave-trains of frequencies exceeding that limit are not detected; whereas the spectrum of the hypothetical pulses ought to include wave-trains of every frequency low or high, the amplitudes indeed declining to infinitely low values as one goes along the spectrum to infinitely high frequencies, but certainly declining smoothly and gradually. To demonstrate this high-frequency limit is a delicate experimental problem, quite like that other problem of demonstrating a sharply definite topmost value for the energies of photoelectrons. That question whether the curves of photoelectric current *vs.* retarding voltage, the curves of Fig. 2, cut straightly and sharply enough into the axis of abscissae to prove that there are no photoelectrons with velocities higher than the one corresponding to x_o, returns again in a slightly altered form.

The most reliable of the methods actually used to demonstrate the high-frequency limit depends on the fact that the high limiting frequency (which I will call ν_{max}) varies with the energy of the bombarding electrons, increasing as their velocity increases. Therefore, if the radiant energy belonging to rays of a certain fixed wave length or a certain fixed narrow range of wave lengths is separated out from the X-ray beam by a spectroscope, and measured for various velocities of the impinging electrons, passing from very high velocities step by step to very low ones; it will decrease from its first high value to zero at some intermediate velocity, and thereafter remain zero. But according to the classical theory also, it must decrease from its first high value to an imperceptibly low one; the descent however will be gradual and smooth. Thus the only question which can be settled by experiment is the question whether the descent from measurable intensities to immeasurably small ones resembles the gentle quasi-asymptotic decline of the curve of Fig. 1 or the precipitate slope of the curve of Fig. 2. The data assembled by Duane and Hunt are shown in Fig. 4 plotted in the manner I have described; there is little occasion for doubt as to which sort of curve these resemble most.[6]

Each of the curves in Fig. 4 represents that portion of the total intensity of an X-ray beam, which belongs to rays of wave lengths near the marked value of the frequency ν. This frequency is the high

[6] Three simple curves of the intensity-distribution in the X-ray spectrum are shown in Figure 5. The abscissa is neither frequency or wavelength, but a variable which varies continuously with either (it is actually *arc sin* of a quantity proportional to wavelength) so that the acute angle between each curve and the axis of abscissae, at the point where they meet, corresponds to and has much the same meaning as the acute angles in Figure 2—not so conspicuously.

limiting frequency ν_{max} for that value of the energy E of the bombarding electrons, which corresponds to the point on the axis of abscissae where the curve (extrapolated) intersects it. The relation between ν_{max} and E is the simplest of all relations:

$$E = \text{constant} \cdot \nu_{max} = h\ \nu_{max} \tag{3}$$

The constant h is the same for all the metals on which the experiment has been performed—a few of the least fusible ones, for metals of a low melting-point would be melted before E could be lifted far enough to give an adequate range for determining the relation between it and

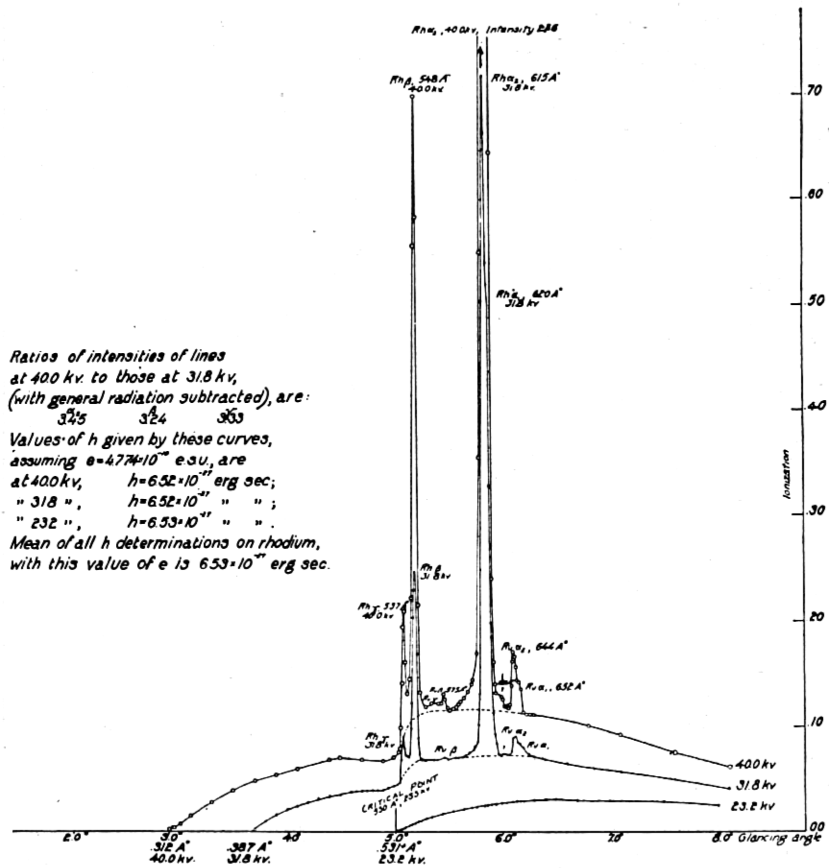

Fig. 5—The continuous X-ray spectrum for three values of the energy of the bombarding electrons, intensity being plotted versus a quantity varying uniformly with frequency. Ignore the peaks. (D. L. Webster, *Physical Review*.) See footnote 6

ν_{max}. The value[7] given for it by Gerlach, after a critical study of all the determinations, is

$$h = 6.53.10^{-27} \tag{4}$$

The highest frequency of radiation which electrons moving with the energy E are able to excite, when they are brought to rest by colliding with a metal target, is therefore equal to E divided by a constant independent of the kind of metal. So far as this high limiting frequency is concerned, it is perfectly legitimate to express equation (3) in these words,

Excitation of radiation by electrons stopped in their flight by collision with a metal occurs as if *the energy in the radiation were concentrated in units of amount $h\nu$, and one such unit were created out of the total energy which each electron surrenders when it is stopped.*

As for the radiation of frequencies inferior to the high limiting frequency, it is very easily explained by asserting that most of the electrons come to rest not in one operation, but in several successive ones, dividing their energy up among several units of frequencies inferior to ν_{max} or E/h; or possibly they lose energy in various sorts of impacts or various other ways before making the first impact of the sort which transforms their energy into energy of X-rays. Nothing about it contradicts the italicized rule. Still it is not likely that anyone would have formulated equation (3) in such language, if the value of the constant h which appears in it were not identical with the value which we have already once encountered in analyzing the photoelectric effect, and with the value at which Planck earlier arrived.

I think it is too early in this discourse to fuse these italicized Rules for the release of electrons by radiation and the excitation of radiation by electrons into a single Rule; but by contemplating the two Rules side by side one arrives without much labor at an inference which could be tested even though we had no way of measuring the frequency of a radiation, and in fact was verified before any such way existed. For if electrons of energy E can excite radiation of frequency E/h, and radiation of frequency E/h striking a piece of metal can elicit electrons of energy $h\ (E/h) - P$; then, if a target is bombarded with electrons, and another metal target is exposed to the radiation which emanates from the first one, the fastest of the electrons which escape from the second target will move with the same velocity and

[7] Gerlach regards this as the most accurate of all the methods for determining h, an opinion in which probably not all would concur. It has been maintained that the high-frequency limit, like the wavelength of maximum intensity in the X-ray spectrum, depends on the inclination of the X-ray beam to the exciting electron-stream. I do not know whether the experiments adduced in support of this claim have been adequately confuted.

the same energy as the electrons which strike the first one (minus the quantity P which, however, is immeasurably small and perfectly negligible in comparison with the energy of the electrons which excite ordinary X-rays). This fact emerged from a series of experiments which were performed by various people in the first decade of this century, the results of which were generally phrased somewhat in this way, "the energy of the secondary electrons depends only on the energy of the primary electrons, not on the nature of the material which the primary electrons strike or on that from which the secondary electrons issue, nor on the distance over which the X-rays travel." Upon these results Sir William Bragg based his corpuscular theory of X-rays; for (he argued) the most sensible interpretation of the facts is surely this, that some of the electrons striking the first target rebound with their full energy, and rebound again with their full energy from the second target, each of them carrying with it from the first to the second target a positive particle which neutralizes its charge over that part of its course, and so defeats all the methods devised to recognize a flying electron. Not many years later, Sir William cooperated in the slaying of his own theory, by developing the best of all methods for proving that X-rays are undulatory and measuring their wave-lengths; but it was only the imagery of the theory that perished, for its essence, the idea that the energy of the first electron travels as a unit or is carried as a parcel to the place where the second electron picks it up, had to be resurrected. All the mystery of the contrast between wave-theory and quantum-theory is implicit in this phenomenon, for which Sir William found an inimitable simile: "It is as if one dropped a plank into the sea from a height of 100 feet, and found that the spreading ripple was able, after travelling 1,000 miles and becoming infinitesimal in comparison with its original amount, to act upon a wooden ship in such a way that a plank of that ship flew out of its place to a height of 100 feet."

Among the radiations excited from a metal by electrons of a single energy E, there are many of which the frequencies differ from the interpreted frequency E/h, being lower. Among the electrons expelled from a metal by radiation of a single frequency ν, there are many of which the energies differ from the interpreted energy-value $h\nu$, being lower. These were accounted for by supposing that the electrons are troubled by repeated encounters with closely-crowded atoms. If then a metal vapor or a gas were bombarded with electrons or exposed to radiation, would all the excited radiation have a single frequency conforming to equation (3), would all the released electrons have a single energy conforming to equation (1)? One could not affirm this *a priori*, for a solid metal is not a collection of free atoms close together as a gas is an assemblage of free atoms far apart, but rather a structure of atoms which interfere with one another and are distorted, and there are many electrons in a solid of which the bonds and the constraints are very different from those by which the electrons of free atoms are controlled and vice versa. When a plate of sodium or a pool of mercury is exposed to a rain of electrons, not exceeding say 10 equivalent volts in energy, nothing apparent happens.[8] When the vapor of either metal is similarly exposed, the atoms respond in a manner from which they are inhibited, when they are bound together in the tight latticework of a solid or the promiscuous crowding of a liquid; and light is emitted.

The phenomena are clearest when the bombarded vapor is that of a volatile metal, such as mercury, sodium, or magnesium. The atoms in such vapors are not usually bound together two by two or in greater clusters, as they are in such gases as oxygen or hydrogen, of which the response to electron-impacts or to radiation is not quite understood to this day; and the first radiations which they emit are not in the almost inaccessible far ultra-violet, like those of the monatomic noble gases, but in the near ultra-violet or even in the visible spectrum. Dealing with such a vapor, I will say mercury for definiteness, one observes that so long as the energy of the bombarding electrons remains below a certain value, no perceptible light is emitted; but beyond, there is a certain range of energies, such that electrons possessing them are able to arouse one single frequency of radiation from the atoms. Ordinarily, as when a vapor is kept continuously excited by a self-sustaining electric discharge throughout it, the atoms emit a great multitude of different frequencies of radiation, forming a rich and complicated spectrum of many lines. But if the energy of the bombarding electrons is carefully adjusted to some value within the specified range, only one line of this spectrum makes its appearance; under the best of circumstances this single line may be exceedingly bright, so that the absence of its companions—some of which, in an ordinary arc-spectrum, are not much inferior to it in brightness—is decidedly striking. The one line which constitutes this *single-line spectrum* is the first line of the principal series in the complete arc-spectrum of the element; its wave length is (to take a few examples) 2536A for mercury, 5890 for sodium (for which it is a doublet), 4571 for magnesium.

[8] According to a very recent paper by C. H. Thomas, radiations from iron excited by electrons with as low an energy as some two or three equivalent volts have been detected.

Does this single line appear suddenly at a precise value of the energy of the impinging electrons? This question suggests itself, when one has already studied the excitation of X-rays from solids by electrons and the excitation of electrons from solids by light. Here again we meet that tiresome but ineluctable problem, as to what constitutes a *sudden* appearance, and how we should recognize it if it really occurred. The only consistent way to meet it (consistent, that is, with the ways already employed in the prior cases) would be to measure the intensity of the line for various values of the energy of the electrons, plot the curve, and decide whether or not it cuts the axis of abscissae at a sharp angle. This is in principle the same method as is used in determining whether a given X-ray frequency appears suddenly at a given value of the energy of the electrons bombarding a solid; the curves of Fig. 4 were so obtained. Attempting to apply this same method to such a radiation as 2,536 of mercury, one has the solitary advantage that the frequency of the light is sharp and definite (it is not necessary to cut an arbitrary band of radiations out of a continuous spectrum) and two great counteracting disadvantages: the intensity of the light cannot be measured accurately (one has to guess it from the effect upon a photographic plate) and the impinging electrons never all have the same energy. Owing probably to these two difficulties, there is no published curve (that I know of) which cuts down across the axis of abscissae with such a decisive trend as the curves of Figs. 2 and 4. Still it is generally accepted that the advent of the single line is really sudden. The common argument is, that one can detect it on a photographic film exposed for a few hours when the energy of the bombarding electrons is (say) 5 equivalent volts, and not at all on a plate exposed for hundreds of hours when the bombarding voltage is (say) 4.5 volts. In this manner the energy of the electrons just sufficient to excite 2536 of mercury has been located at 4.9 equivalent volts. Dividing this critical energy (expressed in ergs) by the frequency of the radiation, we get

$$(4.9e/300) \,/\, (c/.00002536) = 6.59 \cdot 10^{-27} \qquad (5)$$

It agrees with the values of the constant which I designated by h in the two prior cases, and the data obtained with other kinds of atoms are not discordant. Gerlach arrives at $6.56 \cdot 10^{-27}$ as the mean of all values from experiments of this type upon many vapours. The evidence is not quite so strong as in the prior cases, but fortunately it is supplemented and strengthened by testimony of a new kind.

When electrons strike solids and excite X-rays, it is impossible to follow their own later history, or the adventures of a beam of radiation after it sinks into a metal. We have inferred that the electrons which collide with a piece of tungsten and disappear into it transfer their energy to X-rays, but the inference lacked the final support which would have been afforded by a demonstration of these very electrons, still personally present after the collision but deprived of their energy. Now when electrons are fired against mercury atoms, this demonstration is possible, and the results are very gratifying. I have already several times had occasion to remark, in this series of articles, that when an electron strikes a free atom of mercury, the result of the encounter is very different, according as its energy of motion was initially less than some 4.9 equivalent volts, or greater. In the former case, it rebounds as from an elastic wall, having lost only a very minute fraction of its energy, and this fraction spent in communicating motion to the atom; but in the latter case, it may and often does lose 4.9 equivalent volts of its energy *en bloc*, in a single piece as it were, retaining only the excess of its original energy over and above this amount. Thus if electrons of an energy of 4.8 equivalent volts are shot into a thin stratum of mercury vapor, nothing but electrons of that energy arrives at the far side; but if electrons of an only slightly greater energy, say 5.0 equivalent volts, are fired into the stratum, those which arrive at the far side will be a mixture of electrons of that energy, and very slow ones. The very slow ones can be detected by appropriate means, and the particular value of the energy of the bombarding electrons, at which some of them are for the first time transformed into these very slow ones, can be determined. Once more we meet that question as to whether the transformation does make its first appearance *suddenly*, but in this case the indications that it does are rather precise and easy to read. Furthermore it is possible to measure the energy of the slow electrons, and one finds that it is equal to the initial energy of the electrons, minus the amount 4.9 equivalent volts. (These measurements are not so exact as is desirable, and it is to be hoped that somebody will take up the task of perfecting them.)

We, therefore, see both aspects of the transaction which occurs when an electron whereof the energy is 4.9 equivalent volts, or greater, strikes a mercury atom. It loses 4.9 equivalent volts of energy, and we measure the loss; the atom sends forth radiation of a certain frequency, and no other; the atom does not send forth even this frequency of radiation, if none of the electrons fired against it has at least so much energy. We have already compared the energy transferred with the frequency radiated, and as in the case of X-rays excited from a solid target by very fast electrons, it is legitimate to say for these radiations which form the single-line spectra of metallic atoms, that

Excitation of the ray forming a single-line spectrum, by the collision of an electron against an atom, occurs as if *the energy in the radiation were concentrated in units of amount $h\nu$, and one such unit were created out of the total energy which the electron surrenders.*

There are yet several phenomena which I might treat by the same inductive method, arriving after each exposition at a Rule which would resemble one or the other of those which I have thus far written in italics; but it is no longer expedient, I think, to pass in each instance through the same elaborate inductive detour. These three phenomena which I have discussed already combine into an impressive and rather formidable obstacle to the classical manner of thinking. Here is a mercury atom, which receives a definite quantity of energy U from an electron, and distributes it in radiation of a definite frequency U/h. Here again is a multitude of atoms locked together into a solid, and when an electron conveys its energy U to the solid, it redistributes that energy in radiation of a definite frequency U/h. (It is true that many other radiations issue from the solid, but they are all explicable if one assumes that the electron may deliver over its energy in stages, and there is no radiation of the sort which would controvert the theory by virtue of its frequency exceeding U/h.) And when that radiation of frequency U/h in its turn strikes a metal, it is liable and able to release an electron from within the metal, conferring upon it an energy which is apparently equal to U. Apparently there is some correlation between an energy U and a frequency U/h, between a frequency ν and an energy $h\nu$. Apparently a block of energy of the amount U tends to pass into a radiation of the frequency U/h; apparently a radiation of the frequency ν tends to deliver up energy in blocks of the amount $h\nu$. The three italicized Rules coalesce into this one:

Photoelectric emission, and the excitation of X-rays from solids by electrons, and the excitation of single-line spectra from free atoms, occur as if *radiant energy of the frequency ν were concentrated into packets, or units, or corpuscles, of energy amounting to $h\nu$, and each packet were created in a single process and were absorbed in a single process.*

If the neutralizing *as if* were omitted, this would be the corpuscular theory *rediviva*. It is good policy to leave the *as if* in place for awhile yet. But conservatism such as this need not and should not deter anyone from using the idea as basis for every prediction that can be founded upon it, and testing every one of the predictions that can be tested by any possible way. Just so were the three phenomena cited in these Rules discovered. All of them involve either the emission or the absorption of radiation, and so do all the others which I could have quoted in addition, if this account had been written three years ago. Reserving to the end the one new phenomenon that transcends this limitation, I must explain the relation between this problem and the contemporary Theory of Atomic Structure.

The classical notion of a source of radiation is a vibrating electron. The classical conception of an atom competent to emit radiations of many frequencies is this: a family or a system of electrons, each electron remaining in an equilibrium-position so long as the system is not disturbed, one or more of the electrons vibrating when the system is jarred or distorted. A system with these properties would have to contain other things than electrons, otherwise it would fly apart; it would have to contain other things than particles of positive and particles of negative electricity intermixed, otherwise it would collapse together. One would have to postulate some sort of a framework, some imaginary analogue to a skeleton of springs and rods and pivots, to hold the electrons together in an ensemble able to vibrate and not liable to coalesce or to explode. This would not be satisfying, for in making atom-models one wants to avoid the elaborate machinery and in particular the non-electrical components; it would be much more agreeable to build an atom out of positive and negative electricity associated with mass, omitting all masses or structures not electrified. Nevertheless, if anyone had succeeded in devising a framework having the same set of natural frequencies as (say) the hydrogen atom exhibits in its spectrum—if anyone expert in dynamics or acoustics had been able to demonstrate that some peculiar shape of drumhead or bell, if anyone versed in electricity had been able to show that some particular arrangement of condensers and induction-coils has such a series of natural vibrations as some one kind of atom displays—then, it is quite safe to say, that framework or that membrane or that circuit would today be either the accepted atom-model, or at least one of the chief candidates for acceptance. Nobody ever succeeded in doing this; it is the consensus of opinion today that the task is an impracticable one.[9]

[9] It is difficult to put this statement into a more precise form. Rayleigh was of the opinion that the hydrogen spectrum could not be regarded as the ensemble of natural frequencies of a mechanical system, because it is the general rule for such systems that the *second* power of the frequency conforms to simple algebraic formulae, while in the hydrogen spectrum it is the *first* power for which the algebraic expression is simple. He admitted, however, that it was possible to find "exceptional" mechanical systems for which the first power of the frequency is given by a simple formula; which goes far to vitiate the conclusion. Another aspect of the formula (6) for

This set of natural frequencies which baffled all the efforts to explain it, the set constituting the two simplest of all spectra (the spectrum of atomic hydrogen and the spectrum of ionized helium), is given by the formula

$$\nu = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{6}$$

the different lines being obtained by assigning different integral values to the parameters m and n; lines corresponding to values of m ranging from 1 to 5 inclusive, and to values of n ranging from 2 to 40 inclusive, have already been observed, and there is no reason to doubt that lines corresponding to much higher values of m and n actually are emitted, but are too faint to be detected with our apparatus. The constant R has one value for hydrogen, another almost exactly four times as great for ionized helium.

Here, then, is the problem in its simplest presentation: How can a model for a hydrogen atom be constructed, which shall emit rays of the frequencies given by the formula (6), only these and no others? The obvious answer "By constructing a mechanical framework having precisely these natural frequencies" is practically excluded; it seems infeasible. Something radically different must be done. The achievement of Niels Bohr consisted in doing a radically different thing, with such a degree of success that the extraordinary divergence of his ideas from all foregoing ones was all but universally condoned. I do not know how Bohr first approached his theory; but it will do no harm to pretend that the manner was this.

Look once more at the formula for the frequencies of the hydrogen spectrum. It expresses each frequency as a difference between two terms, and the algebraic form of each term is of an extreme sim-

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the hydrogen spectrum is this, that it specifies infinitely many frequencies within finite intervals enclosing certain critical values, such as R, $4R$, $9R$, and so forth. Poincaré is said to have proved that the natural frequencies of an elastic medium with a rigid boundary cannot display this feature, so long as the displacements are governed by the familiar equation $d^2\phi/dt^2 = k^2\nabla^2\phi$. For a membrane this equation is tantamount to the statement that the restoring-force acting upon an element of the membrane is proportional to the curvature of the membrane at that element. Ritz was able to show that the natural frequencies of a square membrane would conform to the formula (6), *if* the restoring-force upon each element of the membrane, instead of being proportional to the curvature of the membrane at that element, depended in an exceedingly involved and artificial manner upon the curvature of the membrane elsewhere. He apologized abundantly for the extraordinary character of the properties with which he had been obliged to endow this membrane, in order to arrive at the desired formula; but his procedure might have proved unsuspectedly fruitful, if Bohr's interpretation had not supplanted it.

plicity. Multiply now each member of the formula by h, that same constant h which we have encountered three times in the course of this article; and reverse the signs of the terms.[10] The formula becomes

$$h\nu = (-h\ R/n^2) - (-h\ R/m^2) \tag{7}$$

In the left-hand member there stands $h\nu$. The reader will have become more or less accustomed to the notion that, under certain conditions and circumstances of Nature, radiant energy of the frequency ν apparently goes about in packets or corpuscles of the amount $h\nu$; now and then, here and there, energy is absorbed from such radiation in such amounts, or energy is converted into such radiation in such amounts. *Suppose that this also happens when a hydrogen atom radiates*, whatever the cause which sets it to radiating. Then the left-hand member of the equation (5) represents the energy which the hydrogen atom radiates; so also does the right-hand member; but the right-hand member is obviously the difference between two terms; *these terms are respectively the energy of the atom before it begins to radiate, and the energy of the atom after it ceases from radiating.*

The problem of the hydrogen atom has now experienced a fundamental change. The proposal to make a mechanical framework, having the natural vibration-frequencies expressed by (6), has been laid aside. The new problem, or the new formulation of the old problem, is this: how can a model for a hydrogen atom be constructed, which shall be able to abide only in certain peculiar and distinctive states or shapes or configurations, in which various states the energy of the atom shall have the various values $-hR$, $-hR/4$, $-hR/9$, $-hR/16$, and so forth?

Bohr's own model has become one of the best-known and most-taught conceptions of the whole science of physics, in the twelve years of its public existence. He based it upon the conception, then rapidly gaining ground and now generally accepted, that the hydrogen atom is a microcosmic sun-and-planet system, a single electron revolving around a much more massive nucleus bearing an electric charge equal in magnitude and opposite in sign to its own. This is really a most unpromising conception, very ill adapted to the modification we need to make. We want an atom which shall be able to assume only those definite values of energy which were listed above: $-hR$, $-hR/4$, $-hR/9$ and the rest. Now the energy of this sun-and-planet atom depends on the orbit which the electron is describing.

[10] For the explanation of this rather confusing reversal, see my third article (page 278; or page 11 of the reprint).

If the energy may assume only those definite values, the electron may describe only certain definite orbits. But there is no obvious reason why the electron should not describe any of an infinity of other orbits, circular or elliptical. To consider only the circular orbits: if the atom may have no other values of energy than $-hR$, and $-hR/4$, and $-hR/9$, and the rest of the series, then it may not revolve in any other circular orbits than those of which the radii are $e^2/2hR$, and $e^2/2(hR/4)$, and $e^2/2(hR/9)$, and so forth; but why just these? What prevents it from revolving in a circular orbit of radius $e^2/2(hR/2)$, or any other value not in the series? And for that matter how can it revolve in a closed orbit at all, since according to the fundamental notions of the electromagnetic theory it must be radiating its energy as it revolves, and so must sink into the nucleus in a gradually narrowing spiral?

Bohr did not resolve these difficulties, and no one has ever resolved them except by ignoring them. The customary procedure is to select some common feature of these permitted orbits, and declare that it is this feature which makes these orbits permissible, and forbids the electron to follow any other. For example, there is the fact that the angular momentum of the electron in any one of the permitted circular orbits is an integer multiple of the constant quantity $h/2\pi$, h being the same constant as we have met hitherto, which is hardly an accidental coincidence. If one could only think of some plausible reason why an electron should want to revolve only in an orbit where it can have some integer multiple of $h/2\pi$ for its angular momentum, and should radiate no energy at all while so revolving, and should refuse to revolve in an orbit where it must have a fractional multiple of $h/2\pi$, the model would certainly be much fortified. Failing this it is necessary to put this assertion about the angular momentum as a downright assumption, in the hope that its value will be so great and its range of usefulness so widespread that it will commend itself as an ultimate basic principle such as no one thinks of questioning. So far this hope has not been thoroughly realized. On the one hand, Sommerfeld and W. Wilson did succeed in generalizing it into a somewhat wider form, and using it in this wider form they explained the fine structure of the lines of hydrogen and ionized helium, and Epstein explained the effect of an electric field upon these lines. These are truly astonishing successes, and no one, I think, can work through the details of these applications to the final triumphant comparisons of theory with experiment, and not experience an impression amounting almost or quite to conviction. Yet on the other hand this generalization does not account for the frequencies forming the spectra of other elements.[11] There is the spectrum of neutral helium, for example, and the spectrum of sodium, and the spectrum of mercury; in each of these there are series of lines, of which the frequencies are clearly best expressed each as the difference between a pair of terms, and these terms should be the energies of the atom before and after radiating. But we have not the shadow of an idea what the corresponding configurations of the atom are; it may be that the outermost electron has certain permissible orbits, but we do not know what these orbits are like nor what common feature they possess.

Is it then justifiable to write down a Rule such as this: *the frequencies of the rays which free atoms emit are such as to confirm the idea that radiant energy of the frequency ν is emitted in packets or corpuscles of the amount $h\nu$*? Very few men of science, I imagine, would hesitate to approve this. However one may fluctuate in his feelings about Bohr's model of the atom, there always remains that peculiar relation among the frequencies emitted by the hydrogen atom, which is so nearly copied by analogous relations in the spectra of other elements. When one has once looked at the general formula

$$h\nu = \left(-\frac{hR}{n^2}\right) - \left(-\frac{hR}{m^2}\right) \tag{7}$$

and has once interpreted the first term on the right as the energy of an atom before radiating, the second term on the right as the energy of the atom after radiating, and the quantity $h\nu$ as the amount of the packet of energy radiated, it is very difficult to admit that this way of thinking will ever be superseded; particularly when one remembers the auxiliary facts, such as that fact about the electrons transferring just 4.9 equivalent volts to the mercury atoms which they strike, no more and no less. Analyzing the mercury spectrum in the same way as the hydrogen spectrum was analyzed, we find the frequencies expressible as differences between terms; interpreting the terms as energy-values, we find that between the normal state of the mercury atom and the next adjacent state, there is a difference in energy of 4.9 equivalent volts, and between this and the next adjacent state there is a further difference of 1.8 volts. This then is the reason why an electron with less than 4.9 equivalent volts of

[11] The mathematical experts who have laboured over the theory of the helium atom (two electrons and a nucleus of charge $+2e$) seem to have convinced themselves that the features which distinguish the permitted orbits of the electrons in this atom, whatever they may be, are definitely not the same features as distinguish the permitted orbits of the electron in the hydrogen atom. This cannot be said with certainty for any other atom.

energy can communicate no energy at all to a mercury atom; and an electron with 5 or 6 equivalent volts of energy can transfer only 4.9 of them. It is conceivable that other conditions may be found to govern the orbits of the electrons, so that the atoms shall have only the prescribed energy-values and no others; it is even conceivable that the conception of electron-orbits may be discarded; but the interpretation of the terms in the formula (7) as energies will, in all human probability, be permanent.

The foregoing Rule is thus very strongly based; but let us nevertheless rephrase it in a somewhat milder form as follows: *The idea that radiant energy of frequency ν is emitted in packets of the amount $h\nu$, and the contemporary theory of atomic structure, between them give a attractive and appealing account of spectra in general, and a convincingly exact explanation of two spectra in particular.*

But what has happened meanwhile to the Vibrator, to the oscillating electron, to the postulated electrified particle of which the vibrations caused light-waves to spread out from around it like sound waves from a bell? It has disappeared from the picture; or rather, since the attempt to account for the frequencies of a spectrum as the natural frequencies of an elastic framework was abandoned, no one has tried to re-insert it. But there are some who will never be quite happy with any new conception, until the vibrator is established as a part of it.

Ionization, the total removal of an electron from an atom, affords another chance to see whether radiant energy behaves as though it could be absorbed only in complete packets of amount $h\nu$. That it requires a certain definite amount of energy to deprive an atom of its loosest electron, an amount characteristic of the atom, may now be regarded as an experimental result quite beyond question, and not requiring the support of any special theory. Thus, a free-flying electron may remove the loosest electron from a free mercury atom which it strikes, if its energy amounts to 10.4 equivalent volts, not less; or the loosest electron from a helium atom if its energy amounts to at least 24.6 equivalent volts. If radiant energy of frequency ν goes about in parcels of magnitude $h\nu$, the frequency of a parcel which amounts just exactly to 10.4 equivalent volts is $\nu_o = 2.53.10^{16}$, corresponding to a wave length of 1188A. Light of inferior frequency should be unable to ionize a mercury atom; light of just that frequency should just be able to ionize it; light of a higher frequency ν should be able to ionize the atom, and in addition confer upon the released electron an additional amount of kinetic energy equal to $h\ (\nu - \nu_o)$. The same could be said, with appropriate numerical changes, for every other kind of atom. Of all the phenomena which might serve to illuminate this difficult question of the relations between radiation and atoms, this is the one which has been least studied. The experimental material is scanty and dubious. There is no reason to suppose that light of a lower frequency than the one I have called ν_o is able to ionize; but it is not clear whether perceptible ionization commences just at the frequency ν_o, although it has been observed at frequencies not far beyond. The energy of the released electrons has not been measured.

The removal of deep-lying electrons, the electrons lying close to the nuclei of massive atoms, is much better known; and the data confirm in the fullest manner the idea that radiant energy of the frequency ν is absorbed in units amounting to $h\nu$. When a beam of X-rays of a sufficiently high frequency is directed against a group of massive atoms, various streams of electrons emanate from the atoms, and the electrons of each stream have a certain characteristic speed. The kinetic energy of each electron of any particular stream is equal to $h\nu$, minus the amount of energy which must be spent in extracting the electron from its position in the atom; for this amount of energy is independently known, being the energy which a free-flying electron must possess in order to drive the bound electron out of the atom, which is measurable and has been separately measured. Here again I touch upon a subject which has been treated in an earlier article of this series—the second—and to prevent this article from stretching out to an intolerable length, I refrain from further repetition of what was written there. The analogy of this with the photoelectric effect will escape no reader. Here as there, we observe electrons released with an energy which is admittedly not $h\nu$, but $h\nu$ minus a constant; the idea that this constant represents energy which the electrons have already spent in escaping, in one case through the surface of the metal and in the other case from their positions within atoms, is fortified by independent measurements of these energies which give values agreeing with these constants.

We have considered various items of evidence tending to show that radiant energy is born, so to speak, in units of the amount $h\nu$, and dies in units of the amount $h\nu$. Whether energy remains subdivided into these units during its incarnation as radiation remains unsettled; to settle this question absolutely, one would have to devise some way of testing the energy in a beam of radiation, otherwise than by absorbing it in matter; and such a way has not yet been discovered. There is, however, another quality which radiant energy possesses.

Conceive a stream of radiation in the form of an extremely long train of plane waves, flowing against a blackened plate facing normally against the direction in which they advance, which utterly absorbs them. This wave-train shall have an intensity I; by which it is meant, that an amount of energy I appears, in the form of heat, in unit area of the blackened plate in unit time. Furthermore, the radiation is found to exert a pressure p against the blackened plate; by which it is meant, that unit area of the plate (or the framework upholding it) acquires in unit time an amount of momentum p. According to the classical electromagnetic theory, verified by experience, p is equal to I/c. Unit area of the plate acquires, in unit time, energy to the amount I and momentum to the amount I/c.

Where is this energy, and where is this momentum, an instant before they appear in the plate? One might say that they did not exist, that they had vanished at the moment when the radiation left its source, not to reappear until it arrived at the plate; but such an answer would be contrary to the spirit of the electromagnetic theory, and we have long been accustomed to think of the energy as existing in the radiation, from the moment of its departure from the source to the moment of its arrival at the receiver; the term "radiant energy" implies this. Momentum has the same right to be conceived as existing in the radiation, during all the period of its passage from source to receiver. In the system of equations of the classical electromagnetic theory, the expression for the stream of energy through the electromagnetic field stands side by side with the expression for the stream of momentum flowing through the field. If the second expression is not so familiar as the first, and the phrase "radiant momentum" has not entered into the language of physics together with "radiant energy," the reason can only be that the pressure which light exerts upon a substance is very much less conspicuous than the heat which it communicates, and seems correspondingly less important,—which is no valid reason at all. Radiant energy and radiant momentum deserve the same standing; it is admitted that the energy I is the energy which is brought by the radiation in unit time to unit area of the plate which blocks the wave-train, and with it the radiation brings momentum I/c in unit time to unit area of the plate. The density of radiant energy in the wave-train is obviously I/c, the density of radiant momentum is I/c^2.

Now let that tentative idea, that radiant energy of the frequency ν is emitted and absorbed in packets of the amount $h\nu$, be completed by the idea that these packets travel as entities from the place of their birth to the place of their death. Let me now introduce the word "quantum" to replace the alternative words *packet*, or *unit*, or *corpuscle*; I have held to these alternative words quite long enough, I think, to bring out all of their connotations. Then the energy I is brought to unit area of the plate, in unit time, by $I/h\nu$ of the quanta; which also bring momentum amounting to I/c. Shall we not divide up the momentum equally among the quanta as the energy is divided, and say that *each is endowed with the inherent energy* $h\nu$ *and with the inherent momentum* $h\nu/c$?

The idea is a fascinating one, but not so easy to put to the trial as one might at first imagine. None of the phenomena I have described in the foregoing pages affords any means of testing it. In studying the photoelectric effect, we concluded that each of the electrons released from an illuminated sodium plate had received the entire energy of a packet of radiation; but this does not imply that each of them had received the momentum associated with that energy; the momentum passed to the plate, to the framework supporting it, eventually to the earth. The same statement holds true for the release of electrons from the deep levels of heavy atoms, such as de Broglie and Ellis observed. Even if the same experiments should be performed on free atoms, as for example on mercury vapor, no clear information could be expected; for the momentum of the absorbed radiation may divide itself between the released electron and the residuum of the atom, and this last is so massive that the speed it would thus acquire is too low to be noticed. Only one way seems to be open; this is, to bring about an encounter between a quantum of radiation and a free electron, so that whatever momentum and whatever energy are transferred to the electron must remain with it, and cannot be passed along to more massive objects where the momentum, so far as the possibility of observing it goes, is lost. *A priori* one could not be certain that even this way is open; radiation might ignore electrons which are not tightly bound to atoms.

Arthur H. Compton, then of Washington University, is the physicist whose experiments were the first that clearly and strikingly disclosed such encounters between quanta of radiation and sensibly free electrons. Others had observed the effect which reveals them, but his were the first measurements accurate enough for inference. Unaware at the moment of the meaning of his data, he realized it almost immediately afterward, and so established the fact and the explanation both—a twofold achievement of a very unusual magnitude, whence the phenomenon received the name of "Compton effect" by a universal acceptance, and deservedly.

What Compton observed was not the presence of electrons possessed of momentum acquired from radiation—these electrons were however to be discovered later, as I shall presently mention—but the presence of radiation of a new sort, come into being by virtue of the encounters between the original radiation and free electrons. We have not encountered anything of this sort heretofore. When a quantum of radiant energy releases an electron from an atom, it dies completely and confers its entire energy upon the electron. The disposal of its momentum gives no trouble, for as I have mentioned the atom takes care of that. When the electron is initially free, and there is no atom to swallow up the momentum of the radiation, it cannot be ignored in this simple fashion. For if the quantum did utterly disappear in an encounter with a free electron, the velocity which the electron acquired would have to be such that its kinetic energy and its momentum were separately equal to the energy and momentum of the quantum; but these distinct two conditions would generally be impossible for the electron to fulfil. Hence in general, a quantum possessed of momentum cannot disappear by the process of transferring its energy to a free electron, whatever may be the case with an electron bound to a massive atom. This reflection might easily have led to the conclusion that radiation and free electrons can have nothing to do one with the other.

What actually happens is this: the energy and the momentum of the quantum are partly conferred upon the electron, the residues of each go to form a new quantum, of lesser energy and of lesser and differently-directed momentum, hence lower in frequency and deflected obliquely from the direction in which the original quantum was moving. The encounter occurs much like an impact between two elastic balls; what prevents the analogy from being perfect is, that when a moving elastic ball strikes a stationary one, it loses some of its speed but remains the same ball, whereas the quantum retains its speed but changes over into a new and smaller size. It is as though a billiard-ball lost some of its weight when it touched another but rolled off sidewise with its original speed. I do not know what this innovation would do to the technique of billiards, but it would at all events not make technique impossible; the result of an impact would still be calculable, though the calculations would lead to a new result. The rules of this microcosmic billiard-game in which the struck balls are electrons and the striking balls are quanta of radiant energy are definite enough to control the consequences. The rules are these:

Conservation of energy requires that the energy of the impinging quantum, $h\nu$, be equal to the sum of the energy of the resulting quantum, $h\nu'$, and the kinetic energy K of the recoiling electron. For this last quantity the expression prescribed by the special relativity-theory [12] is used, viz.

$$K = mc^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)$$

in which m stands for the mass of the electron and $c\beta = v$ for its speed. The equation of conservation of energy is then

$$h\nu = h\nu' + mc^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right). \tag{8a}$$

Conservation of momentum requires that the momentum of the impinging quantum be equal to the sum of the momenta of the resulting quantum and the recoiling electron. Momentum being a vector quantity, this rule requires three scalar equations to express it, which three may be reduced to two if we choose the x-axis to coincide with the direction in which the impinging quantum travels, and the y-axis to lie in the plane common to the paths of the recoiling electron and the resulting quantum. Designate by ϕ the angle between the paths of the impinging quantum and the recoiling electron; by θ the angle between the paths of the two quanta. The magnitude of the momentum-vector is, by the special relativity-theory, $mv/\sqrt{1-\beta^2}$. Conservation of momentum then requires:

$$\begin{aligned} h\nu/c &= (h\nu'/c)\cos\theta + \frac{mv}{\sqrt{1-\beta^2}}\cos\phi, \\ O &= (h\nu'/c)\sin\theta + \frac{mv}{\sqrt{1-\beta^2}}\sin\phi. \end{aligned} \tag{8b}$$

Eliminating ϕ and v between these three equations, we arrive at this relation between ν and ν', the frequencies of the impinging quantum and the recoiling quantum—or, as I shall hereafter say, between the frequencies of the primary X-ray and the scattered X-ray—and the angle θ between the directions of the primary X-ray and the scattered X-ray:

$$\frac{\nu'}{\nu} = \frac{1}{1+\dfrac{h\nu}{mc^2}(1-\cos\theta)}. \tag{9}$$

[12] If the reader prefers to use the familiar expressions $\frac{1}{2}mv^2$ for the kinetic energy and mv for the magnitude of the momentum of the electron, he will arrive at a formula for ν^1 which, while apparently dissimilar to (9) and not so elegant, is approximately identical with it when v is not too large—or, which comes practically to the same thing, when $h\nu$ is small in comparison with mc^2; a condition which is realized for all X-rays now being produced.

The relation between λ' and λ, the wavelengths of the primary beam and of the scattered beam, is still simpler, being

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta). \tag{10}$$

The intrusion of this angle θ into the final equation may seem to contradict my earlier statement that the results of the impact are calculable; for it is true that there are not equations enough to eliminate θ, and yet I have offered no additional means of calculating it. In fact it cannot be calculated with the data at our command. All that we are able to say is that *if* the resulting quantum goes off in the direction θ, then its frequency is given by (9). What determines θ in any particular case? Reverting to the image of the billiard-balls, it is easy to see that the direction in which the rebounding ball rolls away depends on whether it gave a central blow, or a glancing blow, or something in between, to the initially stationary ball. If we knew just which sort of a blow was going to be given, we could calculate θ; otherwise we can only apply our conditions of conservation of energy and conservation of momentum to ascertain just how much of its energy the rebounding ball retains when θ has some particular value, and then produce—or, if we cannot produce at will, await—a collision which results in that value, and make our comparison of experiment with theory. So it is in this case of the rebounding quantum. When a beam of primary electrons is scattered by encountering a piece of matter, some quanta rebound in each direction, and all the values of θ are represented. We cannot know what determines the particular value of θ in any case; but we can at least select any direction we desire, measure the frequency of the quanta which have rebounded in that direction, and compare it with the formula. Fig. 6 is a diagram illustrating these relations.[13]

The comparison, which has now been made repeatedly by Compton, repeatedly by P. A. Ross, and once or oftener by each of several other physicists—notably de Broglie in Paris—is highly gratifying. The value of the frequency-difference between the primary X-rays and the scattered X-rays, that is to say, between the impinging quanta and the rebounding quanta, is in excellent accord with the formula, whether the measurements be made on the quanta recoiling at 45°, at 90° or at 135°, or at intermediate values of the angle θ. The method consists in receiving the beam of scattered X-rays into an X-ray spectroscope, whereby it is deflected against an ionization-chamber or a photographic plate at a particular point, of which the location is the measure of the wave-length. An image can be made on the same plate at the point where the beam would have struck it, if it had retained the frequency of the primary beam. The two images then stand sharply and widely apart. Indeed it is not necessary to make a special image to mark the place on the plate where a scattered beam of unmodified wave-length would fall, for there

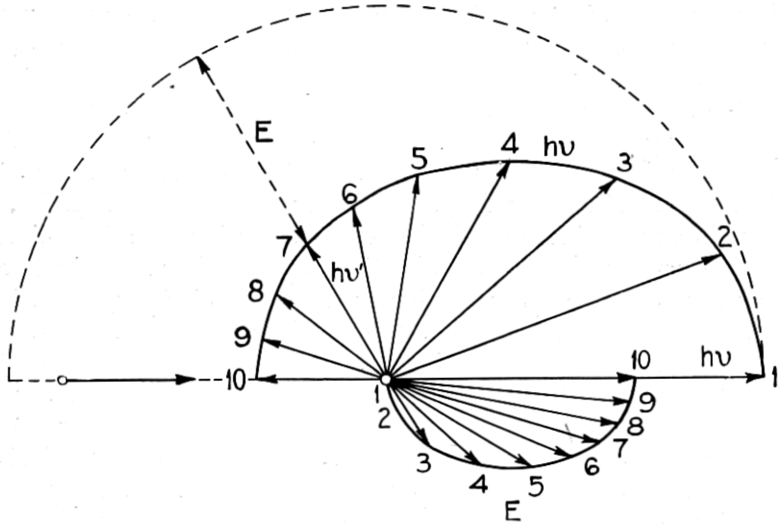

Fig. 6—Diagram showing the energy-relations ensuing upon an impact between a quantum and a free electron. (After Debye.) See footnote 13

nearly always is such a beam and such an image. A plausible explanation is easy to find; one has only to assume that the quanta composing this beam have rebounded from electrons so rigidly bound into atoms that they did not budge when the impinging quanta struck them, and these were reflected as from an immovable wall.[14]

[13] The diagram in Fig. 6 is designed to illustrate the relations between the energy of the primary quantum (radius of the dotted semicircle), the energy of the rebounding quantum (radius of the upper continuous curve), and the energy of the recoiling electron (radius of the lower continuous curve). Thus the two arrows marked with a 5 are proportional respectively to the energies of the secondary quantum and of the recoiling electron, when the encounter has taken place in such a fashion that the angle θ is equal to the angle between the arrow 10 and the upper arrow 5. In the same case, the angle between arrow 10 and lower arrow 5 is equal to ϕ of the equations (9).

[14] As a matter of fact we have no independent means of knowing that the recoiling electrons are initially free, or that the scattered beam with the modified frequency originates from collisions of primary quanta with initially free electrons; we know only that the frequency of the scattered quanta is such as would be expected if little or no energy is spent in freeing the electrons, and little or no momentum is transferred otherwise than to the electrons—which, of course, is not quite the same

In the photographs which I reproduce,[15] the imprints of these two beams stand side by side. In the first of them, Fig. 7, the spectrum of the primary rays is specially depicted on the upper half of the plate; one sees the α, β, and γ lines of the K-series of molybdenum, three lines (the first a doublet) of which the wavelengths are respec-

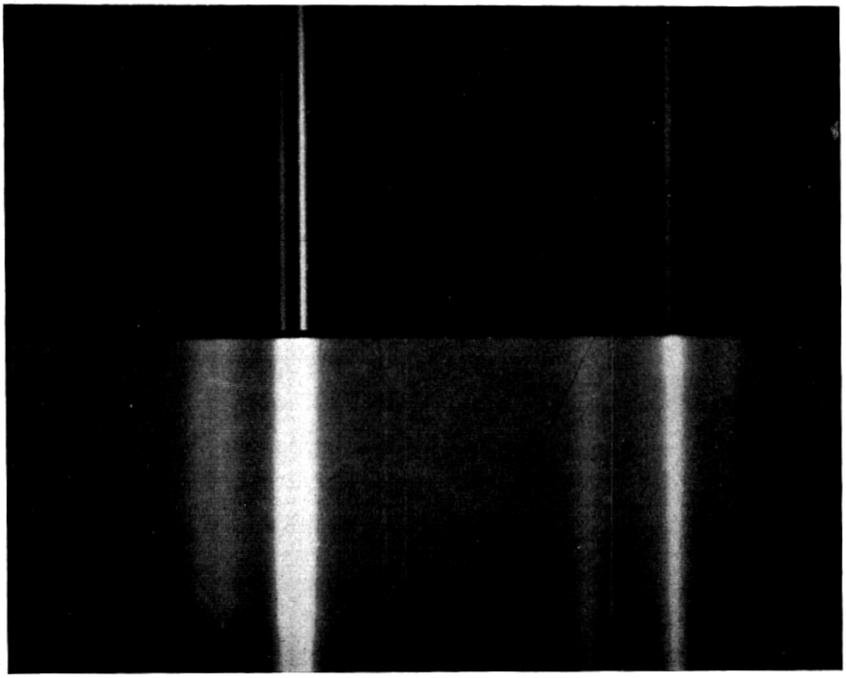

Fig. 7—Above, the K-spectrum of molybdenum (α-doublet, β-line, γ-line from left to right); below, the spectrum of this same radiation after scattering at 90° from aluminum (each line doubled). (P. A. Ross)

tively .710 − .714A, .633A, .618A. Below, the spectrum of the secondary rays scattered at the angle θ is spread out; to each of the primary rays there corresponds a scattered ray of the same wavelength, and beside it another ray of which the wavelength exceeds that of its companion by the required amount.

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thing. The Compton effect has been demonstrated only where there are electrons associated with atoms. It may be that the rebound occurs only from an electron which is connected to an atom by some peculiar liaison, weak so far as the energy required to break it is concerned, but able to control the response of the electron to an impact. Something of this sort may have to be assumed to explain why the effect is apparently not greater for conductive substances than for insulating ones and is certainly feebler for massive atoms with numerous loosely-bound electrons than for light atoms with few.

[15] I am indebted to Professor Ross for these photographs.

Another series of photographs, in Fig. 8, shows the two scattered rays produced when a beam of the $K\alpha$-radiation of molybdenum falls upon various scattering substances: carbon (the sixth element of the periodic table), aluminium (the thirteenth), copper (the twenty-ninth), and silver (the forty-seventh). The relative intensity of the two rays—that is to say, the proportion between the number of quanta which rebound as from free electrons, and the number of quanta which recoil as from immobile obstacles—varies in a curious manner

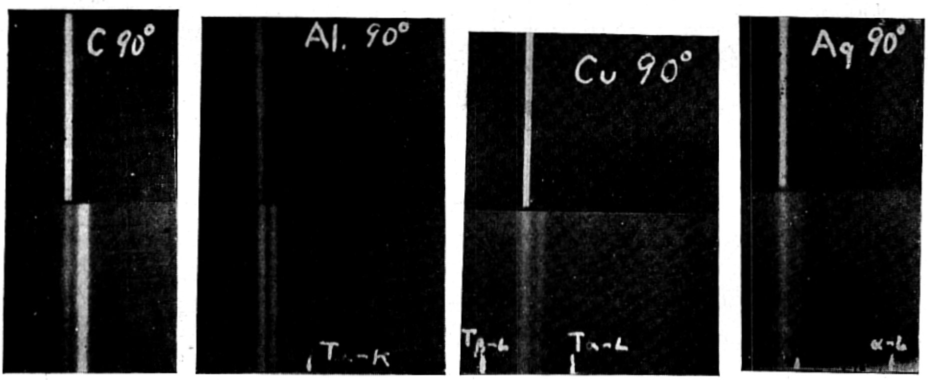

Fig. 8—Above, the K α-line of molybdenum; below, the same radiation after scattering at 90° from carbon, aluminium, copper and silver. (P. A. Ross)

from one of these elements to another. Most of the quanta scattered by lithium undergo the alteration in wavelength which we have calculated; nearly all of the quanta scattered by lead emerge with the same frequency as the incident quanta. Apparently, the heavier the atoms of a substance are, the less conspicuous does Compton's effect become. Further, the relative intensity of the two rays assumes different values for one and the same substance, depending on the direction of scattering. This is illustrated in Fig. 9, the curves of which may be interpreted as graphical representations of photographs like those of the foregoing Figure, the ordinate standing for the density of the image on the photographic plate. (Actually, the ordinate stands for a quantity which is much more nearly proportional to the true intensity of the rays—that is, the amount of ionization which they produce in a dense gas.) These curves show, in the first place, that the separation between the two scattered rays has the proper theoretical values at the angle 45°, at 90°, and at 135°; in the second place, among the quanta scattered at 45°, those that retain the primary wavelength are more abundant than the altered quanta, while among the quanta scattered at 135° the modified ones have the predominance. Why the relative commonness of these two kinds of scattering, of these two modes of interaction between quanta

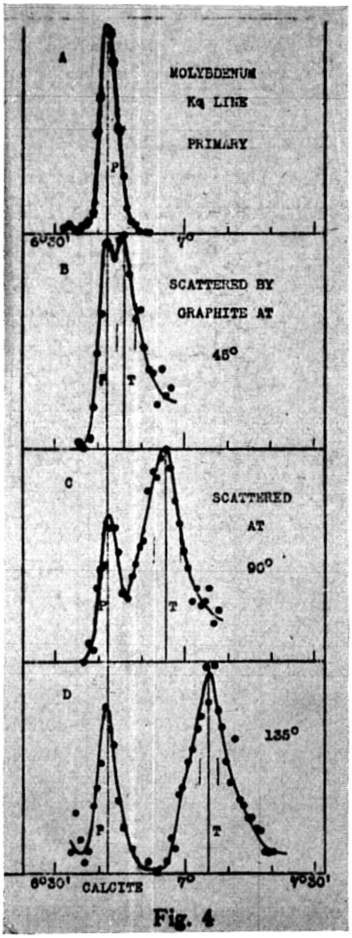

Fig. 9—The modified and unmodified scattered rays, at various inclinations, recorded by the ionization-chamber method. The vertical line T represents the position calculated from (9) for the modified ray. (A. H. Compton, *Physical Review*)

and matter, should depend on the substance and on the angle θ is a deeper question than any we have considered.

The recoiling electrons also have been detected; and Figs. 10 and 11, which are photographs of the trails left by flying electrons as they

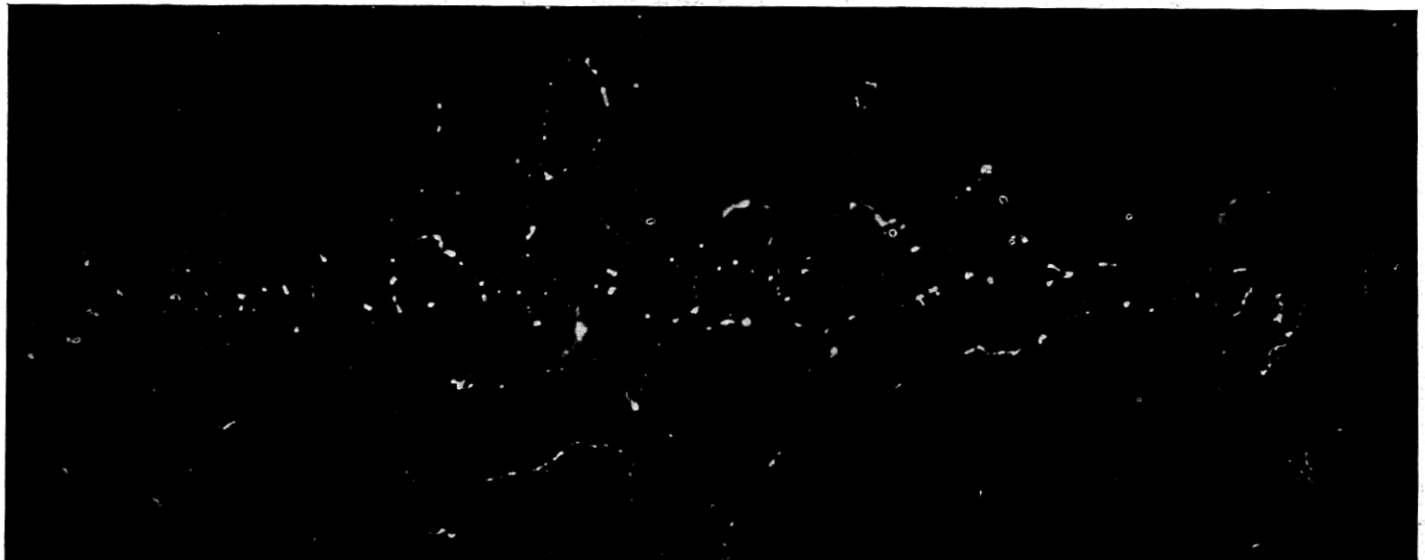

Fig. 10—Trails of recoiling electrons, mingled with long sinuous trails of electrons ejected from atoms by totally-absorbed quanta. (C. T. R. Wilson, *Proceedings of the Royal Society*)

proceed through air supersaturated with water vapor, shows evidence for these.[16] The long sinuous trails are those of fast electrons, which were liberated from their atoms by high-frequency quanta proceeding across the gas; each of these electrons possesses the entire energy of a vanished quantum (minus such part of it as was sacrificed when

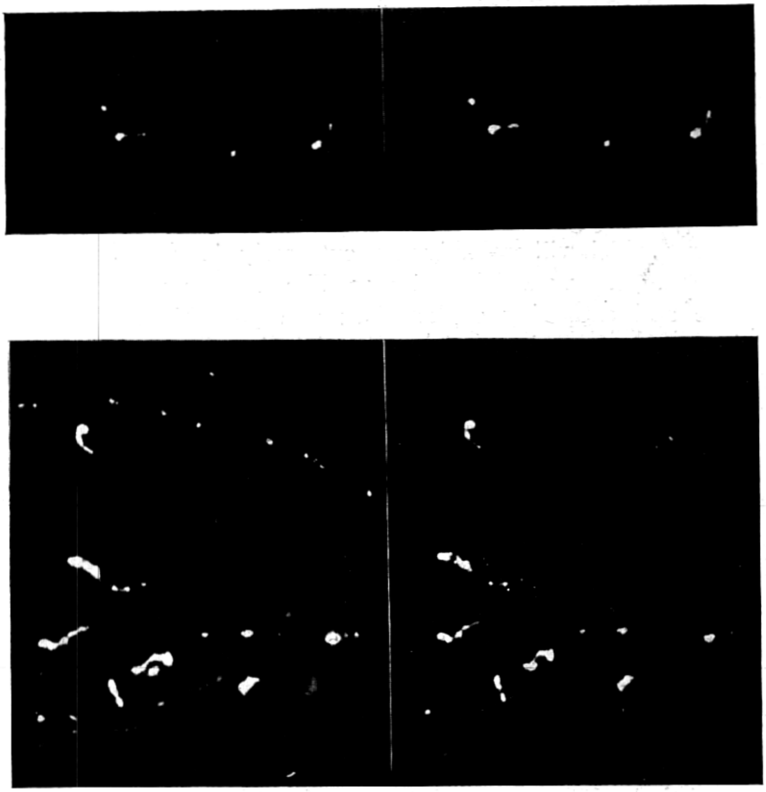

Fig. 11—Trails of recoiling electrons (C. T. R. Wilson, *Proceedings of the Royal Society*)

the electron emerged from its atom). The small slightly-elongated comma-like "blobs", the "fish tracks" as C. T. R. Wilson called them, are the trails of very slow electrons—these are the electrons from which quanta rebounded, transferring in the rebound a little of their energy and a little of their momentum. These appear only when the frequency of the X-ray quanta exceeds a certain minimum amount—a circumstance which, combined with others, shows that the com-

[16] I am indebted to Professor C. T. R. Wilson and to the Secretary of the Royal Society for permission to reproduce these photographs.

monness of the Compton effect depends not merely on the nature of the atoms and on the angle at which the scattering is observed, but also upon the frequency of the radiation. High-frequency quanta are liable to rebound in the manner prescribed by Compton's assumptions, but low-frequency quanta are not. Light of the visible spectrum suffers no change in wavelength when it is scattered.

Must we now concede that radiant energy travels about through space in the form of atom-like units, of corpuscles, of *quanta* every one of which, for a radiation of a specific frequency ν, possesses always the same energy $h\nu$ and always the same momentum $h\nu/c$? How indeed can we longer avoid admitting it? The phenomena which I have cited do certainly seem to close the case beyond any possibility of reopening it. Yet they might be interpreted in another way—a way which will probably seem extremely elaborate and artificial to the reader, a way which will seem like a mere excuse to avoid a simple and satisfying explanation; and yet this would not be sufficient to condemn it utterly. We might lay the whole blame and burden for all these "quantum" phenomena upon the atom. We might say that there is some mysterious mechanism inside every atom, which constrains it never to emit radiation of a frequency ν unless it has a quantity of energy $h\nu$ all packed up and ready to deliver, and never to absorb radiation of a frequency ν unless it has a special storeroom ready to receive just exactly the quantity of energy $h\nu$. This indeed is not a bad formulation of Bohr's theory of the atom. It would be necessary to go much further, and to say that not only every atom, but likewise every assemblage of atoms forming a liquid or a solid body, contains such a mechanism of its own; for the phenomena which I have called the "photoelectric effect" and the "inverse photoelectric effect" are qualities not of individual atoms, but of pieces of solid metal.[17] And it would be necessary to go much further yet, and make mechanisms to account for the transfer of momentum from radiation to electrons.

Yet even this would not be sufficient; for the most surprising and inexplicable fact of all is still to be presented. Here is the crux of the great dilemma. Imagine radiation of the frequency ν emerging from an atom, for a length of time determined by the condition that

[17] It was formerly contended that this explanation, while applicable to the behavior of free atoms which respond only to certain discrete frequencies, would not avail for a solid substance like sodium which delivers up electrons with energy $h\nu$, whatever the frequency ν may be. This contention, however, is probably not forcible, as it can be supposed that the solid has a very great number of natural frequencies very close together. This in fact was the inference from Epstein's theory of the photoelectric effect.

the total energy radiated shall be $h\nu$ exactly. According to the wave-theory, it emerges as a spherical wave-train, of which the wave-fronts are a series of expanding spheres, widening in all directions away from the atom at their common centre. Place another atom of the same kind some little distance away. Apparently it can absorb no radiant energy at all, unless it absorbs the whole amount $h\nu$ radiated from the first atom. But how can it do this, seeing that only a very small portion of each wavefront touched it or came anywhere near it, and much of the radiant energy went off from the first atom in a diametrically opposite direction? How can it reach and suck up all the energy from the entire wavefront, so little of which it actually intercepts? And the difficulty with the momentum is even greater.

But, of course, this experiment is unrealizable. In any laboratory experiment, there are always great multitudes of radiating atoms close together, and the atoms exposed to the radiation are bathed in myriads of wave-trains proceeding from myriads of sources. Does then the atom which absorbs the amount $h\nu$ of energy take it in little bits, one from this wavetrain and another from that, until the proper capital is laid up? But if so, it surely would require some appreciable time to gather up the separate amounts. According to the classical electromagnetic theory, a bound electron placed in a wavetrain of wavelength λ will gather up energy from an area of each wavefront, of the order of magnitude of the quantity λ^2. Hence we should not expect that the exposed atom would finish the task of assembling the amount of energy $h\nu$ from the various wavetrains which pass by it, until the lapse of a time-interval sufficient for so much energy to flow against a circle of the area λ^2, set up facing the rays at the point where the atom stands. Set up a mercury arc, or better still, an X-ray tube, and measure the intensity of the radiation from it at various distances. You will easily find a position sufficiently near to it for convenience, and yet sufficiently far from it, so that if a circular target of this area were set in that position, the radiant energy falling upon it would not mount up in one minute—nor in one day—nor in one year, to the amount $h\nu$. Yet cover the source of rays with a shutter, and then put a piece of matter in that position, and then lift the shutter; and you will not have to wait a year, nor a day, nor a minute, for the first electron which emerges from the matter with a whole quantum of energy; it will come out so quickly that no experimenter has, as yet, demonstrated a delay. What possible assumptions about the structure of the *atom* can account for this?

More and more the evidence is piled up to compel us to concede that radiation travels around the world in corpuscles of energy $h\nu$ and momentum $h\nu/c$, which never expand, or at all events always remain small enough to be swallowed up in one gulp by an atom, or to strike an electron with one single concentrated blow.

But it is unfair to close the case without pleading once more the cause of the undulatory theory—the more so because, in the usual fashion, I have understated the old and presumptively familiar arguments in its favor, and given all the advantages to the arguments of the opposition, which still have the force and charm of novelty. Furthermore, I may have produced the impression that the conception of the quantum actually unites the corpuscular theory with the wave-theory, mitigating discord instead of creating it. Why are we not really voicing a perfectly competent wave-theory of light, when we imagine wave-trains limited both in length and in breadth, so narrow that they can dive into an atom, but so long that they contain $h\nu$ of energy altogether? *filamentary* wave-trains, so to speak, like the tracing of a sine-wave in chalk upon a blackboard, or the familiar picture of a sea-serpent?

Well, the difficulty is that the phenomena of interference and of diffraction, which are the basis of the wave-theory, imply that the wave-trains are broad, that they have a considerable cross-sectional area; these phenomena should not occur, if the wave-trains were filaments no thicker than an atom, or even so wide that their cross-sectional area amounted to λ^2. Let me cite one or two of these phenomena, in tardy justice to the undulatory theory, as a sort of a makeweight to all the "quantum" phenomena I have described. Imagine an opaque screen with a slit in it; light flows against the screen from behind, some passes through the slit. The slit may be supposed to be half a millimetre wide, or even wider. If light consists of quanta only as thick as an atom, or even as thick as the wave-length of the light, they will shoot through the slit like raindrops or sand-grains through a wide open skylight. If they are all moving in parallel directions before they reach the slit, they will continue so to move after they pass through it—for how shall they know that the slit has any boundaries, since they are so small and the slit is so large? The beam of light which has passed through the slit will always retain the same cross-section as the slit. But we know that in truth the beam widens after it goes through the slit, and it develops a peculiar distribution of intensity which is accurately the same as we should expect, if the wavefront is *wider* than the slit—so much wider, that the slit cuts a piece out of it, which piece spreads outwards independently in its own fashion.[18]. Therefore the quantum must be wider than the widest slit which displays clear diffraction-phenomena—and this makes it at least a millimetre wide! But this is not the limit! Cut another slit in the screen, parallel to the first one, a distance *d* away from it. Where the widening diffracted light-beams from the two slits interpenetrate one another, they will produce interference-patterns of light and shade, accurately the same as we should expect if the wavefront is wider than the distance *d*. The quantum must therefore be wider than the greatest distance between two slits, the light-beams passing through which are able to interfere with one another. The slits may be put quite far apart, and the light-beams brought together by systems of prisms and mirrors. This is the principle of Michelson's famous method of determining the diameters of stars. He obtained interference fringes when the two beams of light were taken from portions of the wavefront *twenty feet apart!*[19]

Therefore the quantum is twenty feet wide! This is the object from which an atom one ten-millionth of a millimetre wide can suck up all its energy! this is what enters as a unit into collision with an electron ten thousandfold smaller yet!

The evidence is now before the reader; not the entire evidence for either of the two conceptions of radiation, but, I think, a fair sampling for both. If either view has been inequitably treated, it is the undulatory theory which has been underrated; for, as I have said already but cannot say too often, the evidence that light partakes of the nature of a wave-motion is tremendously extensive and tremendously compelling; it seems the less powerful only because it is so thoroughly familiar, and through much repetition has lost the force of novelty. Still, it is not necessary to hold all the relevant facts continually in mind. If one could reconcile a single typical fact of the one sort, such as the interference between beams of light brought together from parallel courses far apart, with a single outstanding fact of the other sort, such as the instantaneous emergence of electrons with great energy from atoms upon which a feeble beam of light has only just been directed—if one could unify two such phenomena as these, all of the others would probably fuse spontaneously into a harmonious system. But in thinking about these things, there is one more all-important

[18] One might, of course, inquire, why should a *piece* of the wavefront of a quantum, cut out of it by the edges of a slit, expand after passing through the slit when the quantum itself apparently rushes through space without expanding?

[19] It might be argued that these quanta from stars have come an enormously long way, and possibly have had a better chance to expand than the quanta passing across a laboratory room from an X-ray tube or a mercury arc to a metal plate. However, since the photoelectric cell is used to measure the brightness of a star, they evidently produce the same sort of photoelectric effect as newborn quanta.

fact that must never be forgotten: the quantum-theory involves the wave-theory in its root and basis, for *the quantum of a given radiation is defined in terms of the frequency of that radiation, and the frequency* is determined from the wavelength, and the wavelength *is determined by applying the wave-theory* to measurements on interference and diffraction patterns. Was there ever an instance in which two such apparently contradictory theories were woven so intimately the one with the other!

The fusion of the theories is not likely to result from new experimental evidence. Indeed there are already indications that further experiments will merely accentuate the strangeness, much as happened with the numerous experiments devised and performed three or four decades ago in the hope of settling whether the earth does or does not move relatively to the aether. More probably what is required is a modification, indeed a revolutionary extension in the art of thinking—such a revolution as took place among a few mathematicians when non-Euclidean geometry was established by the side of Euclidean, as is taking place today among the disciples of Einstein who are striving to unlearn the habitual distinctions between time and space—such a revolution, to go centuries back into the past, as occurred in the minds of men generally when they learned to realize that the earth is round, and yet at every place upon it the sky is above and the ground is below. Our descendants may think pityingly of us as we of our ancestors, who could not comprehend how a man can stand upright at the Antipodes.