Physical Measurements of Audition and Their Bearing on the Theory of Hearing*

By HARVEY FLETCHER

SYNOPSIS: The author states his purpose to be the presentation of certain facts of audition which have been determined recently with considerable accuracy and the discussion of the theory which best explains these facts.

Making use of data of Knudsen's as well as his own measurements of the auditory sensation area, the author estimates that the normal ear can perceive approximately 300,000 different pure tones. This is taking account of all possible variations in both pitch and intensity. Knudsen's data show that for considerable ranges the minimum perceptible difference in intensity bears a constant ratio to the intensity and the minimum perceptible difference in frequency bears a constant ratio to the frequency. These relations have been termed by psychologists "The Law of Weber and Fechner."

A loudness scale is proposed such that the difference in loudness between two tones is equal to ten times the common logarithm of their intensity ratio. A pitch scale is proposed such that the difference in pitch is equal to one hundred times the logarithm to the base two of the frequency ratio. A method for measuring the loudness of complex sounds is mentioned but is to be discussed in a later paper. A method is proposed for expressing quantitatively different degrees of deafness.

Reference is made to data obtained by the author on the masking of one pure tone by another. The minimum audible intensity of a pure tone depends upon the presence of another tone of different frequency. A low pitched note will, in general, exert a surprisingly large masking effect upon notes of higher frequency. The masking of a low note by a higher is not nearly as pronounced. From his observations, the author draws certain interesting conclusions. For example, given a complex tone consisting of three frequencies 400, 300 and 200 cycles with relative loudness values of 50, 10 and 10, respectively, the ear would hear only the 400 cycle tone and the 200 cycle tone. If the sound is now increased 30 loudness units, without distortion, the 400 cycle tone and the 300 cycle tone only, will be heard.

Binaural masking in which each ear receives one of the two sounds is considered and the conclusion reached that the masking effect noted results from conduction of the masking tone through the bones of the head to the ear receiving the masked tone.

It is stated on the basis of data obtained by Wegel and Lane that the oscillatory system of the ear, comprised by the membranes and little bones of the middle and inner ears, does not obey Hooke's Law regarding the proportionality of stress and strain. Consequently, the ear, when stimulated by a pure tone, introduces harmonics and the workers cited have observed harmonics as high as the 4th order. The non-linear transmission characteristic of the vibratory system of the ear is held to account for the greater masking of a high frequency by a lower.

A theory of hearing is advanced which pictures the basilar membrane as being caused to vibrate by incident sound waves. In the case of a pure tone, the membrane is supposed not to vibrate uniformly throughout its length but the region of maximum amplitude determines the pitch of the tone as interpreted by the ear and the maximum amplitude determines the intensity.—*Editor.*

THE question of how we hear has been a subject for discussion by scientists and philosophers for a long time. Practically every year during the past fifty years articles have appeared discussing the

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pros and cons of various theories of hearing. These discussions have been participated in by men from the various branches of science and particularly by the psychologists, physiologists, otologists, and physicists. During the past two or three years this discussion has been particularly acute. It is not uncommon to pick up an article and read in the beginning or concluding paragraphs statements such as the Helmholtz theory of audition seems to have sunk beyond recovery, [90, 65] † and at the same time an article written probably a month later will have the conclusion that the Helmholtz theory of audition is definitely established beyond all controversy.[70-75]

There is apparently a great deal of misunderstanding between various writers because of different points of view due to different training. To the physicist it seems that most of the discussions show a profound ignorance of the dynamics of the transmission of sound by the mechanism of the ear. Those discussions by the physicists are frequently open to criticism by the otologist and psychologist, due to his lack of knowledge of the structure of the ear or the mental reaction involved in the process of interpretation. I think it is fortunate that some of these scientists from the different branches are now cooperating in their research work as is evinced by the appearance of several joint papers. (Papers by Dean and Bunch, Minton and Wilson, Wegel and Fowler, Kranz and Pohlman, and others.)

It is not my purpose to discuss the merits of the various theories of hearing, but I desire to present some of the facts of audition which have been recently determined with considerable accuracy, and then discuss the theory of hearing which best explains these facts.

Hearing is one of the five senses. It is that sense that makes us aware of the presence of physical disturbances called sound waves. For my purpose, sounds may be classified into two groups, namely, pure tones and complex sounds. A pure tone is specified psychologically by two properties, namely, the pitch and the loudness. These sensory properties are directly related to the physical properties, frequency and intensity of vibration. Mixtures of pure tones of different loudness, but of the same pitch, fall under the first class, since such mixtures give rise to a pure tone. The complex sounds are varying mixtures of pure tones. It will be noticed that phase has not been taken into account. Except when using the two ears for locating the direction of sources of sound, phase differences are not ordinarily appreciated by the ear.*

† These numbers refer to the bibliography at the end of the paper.

These tones are usually transmitted by means of air waves through the outer ear canal to the drum of the ear. From here the vibrations are transmitted by means of the bones in the middle ear to the mechanism of the inner ear.

Those facts of audition which are familiar to almost everybody are as follows:

1. Pure tones are sensed by the ear and differentiated by means of the properties *pitch* and *loudness*.

2. When two notes, separated by a musical interval, are sounded together, they are sensed as two separate notes. They would never be taken for a tone having the intermediate pitch. In this respect, hearing is radically different from seeing. When a red and a green light are mixed together, the impression received by the eye is that of yellow, an intermediate color between the two.

3. There is a definite limiting difference in pitch that can just be sensed.[1-9]

4. There is a definite limiting difference in intensity that can just be sensed.[10-14]

5. There is a minimum intensity of sound below which there is no sensation.[15-31]

6. There is an upper limit on the pitch scale above which no auditory sensation is produced.[32-44]

7. There is a lower limit on the pitch scale below which there is no auditory sensation produced.[45-51]

8. The ear perceives tones separated by an octave as being very similar sensations.

Another quality of audition which is not so commonly known was pointed out by A. M. Mayer.[52] He stated that high tones can be completely masked by louder lower tones while intense higher tones cannot obliterate lower ones though the latter are very weak. Experiments to be described later in the paper show that this statement must be modified somewhat. Very intense low ones will produce a masking effect upon still lower tones, although the masking effect is very much more pronounced in the opposite case. Many of the opponents of the Helmholtz resonant theory of hearing claim that this fact is fatal to such a theory.[52]

* This statement may require modification when more experimental data are available. As shown later in the paper the middle ear has a non-linear response. Consequently it would be expected that phase differences, especially between tones which are harmonic, would produce spacial differences in nerve stimulation.

Limits of the Field of Audition

The new tools which have made possible more accurate measurements in audition are the vacuum tube, the thermal receiver and the condenser transmitter. When connected in a proper arrangement of circuits, the vacuum tube is capable of generating an oscillating electrical current of any desired frequency. This electrical vibration is translated into a sound vibration by means of the telephone receiver. Between the receiver and the oscillator, a wire network called an attenuator [28] is interposed which makes it possible to regulate the

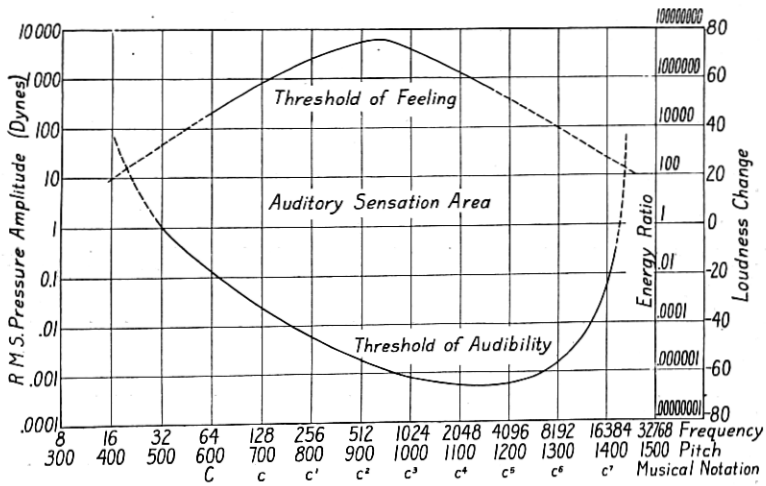


Fig. 1

volume of sound. The theory [54-55] of the thermal receiver has been worked out so that it is possible to calculate its acoustic output from the electrical energy it is absorbing. In this way, it is possible to calculate the pressure variation produced in the outer ear canal when a tone is being perceived. A detailed description of the apparatus and method used in such measurements was given in a paper presented before the National Academy of Science, November 14, 1921.[55] Such a combination of apparatus which has been calibrated is called an audiometer and is suitable for measuring abnormal as well as normal hearing. A receiver more rugged than the thermal may be substituted when its efficiency compared to the thermal receiver is known for all frequencies. By using such an audiometer the average absolute sensitivity for approximately 100 ears which were considered to be normal was determined. The lower curve in

Fig. 1, labelled the threshold of audibility, shows the results of such measurements. The ordinates give the amplitude of the pressure variation in dynes per square centimeter that is just sufficient to cause an auditory sensation and the abscissæ give the frequency of vibration of the tone being perceived. Both are plotted on a logarithmic scale. The experimental difficulties made it impossible to make a very accurate determination for those parts of the curve shown by dotted lines. More work needs to be done on these portions of the curve. In the important speech range, namely, from 500 to 5,000 cycles, it requires approximately .001 of a dyne pressure variation in the air to cause an auditory sensation. This corresponds to a fractional change of about one-billionth in the atmospheric pressure, which shows the extreme sensitiveness of the hearing mechanism.

In order to obtain an idea of the intensity range used in hearing, an attempt was also made to obtain an upper limit for audible intensities. When the intensity of a tone is continually increased, a value is reached where the ear experiences a tickling sensation. Experiments show that the intensity for this sensation is approximately the same for various individuals and the results can be duplicated as accurately as those for the minumum intensity value. It was found that if this same intensity of sound is impressed against the finger, it excites the tactile nerves, In other words, the sensation of feeling for the ear is practically the same as for other parts of the body. When the intensity goes slightly above this feeling point, pain is experienced. Consequently, this intensity for the threshold of feeling was considered to be the maximum intensity that could be used in any practical way for hearing. The two points where these two curves intersect have interesting interpretations. At these two points, the ear both hears and feels the tone. At frequencies above the upper intersecting point, the ear feels the sound before hearing it, and in general would experience pain before exciting the sensation of hearing. Consequently, the intersection point may be considered as the upper limit in pitch which can be sensed. In a similar way, the lower intersection point represents the lowest pitch than can be sensed.

There has been considerable work [32-51] in the past to determine the upper frequency and lower frequency limits of audibility, but it would appear that without the criterion just mentioned, such limiting points apply only to the particular intensity used in the determination. Not enough attention has been paid to the intensity of the tones for such determinations. It is quite evident from this figure that both the upper and lower limits of audibility which are found in any particular experimental investigation will very largely depend upon the intensity of the tones sounded. For example, if the intensity were along the .01 dyne line, the limits would be 200 and 12,600 cycles.

The area enclosed between the maximum and minimum audibility curves has been called the auditory-sensation area and each point in it represents a pure tone. The question then arises: How many such pure tones can be sensed by the normal ear?

The answer to this question has been made possible by the recent work of Mr. V. O. Knudsen.[14] In this work Knudsen made determinations of the sensibility of the ear for small differences in pitch and intensity. In Fig. 2, the average results of his measurements for

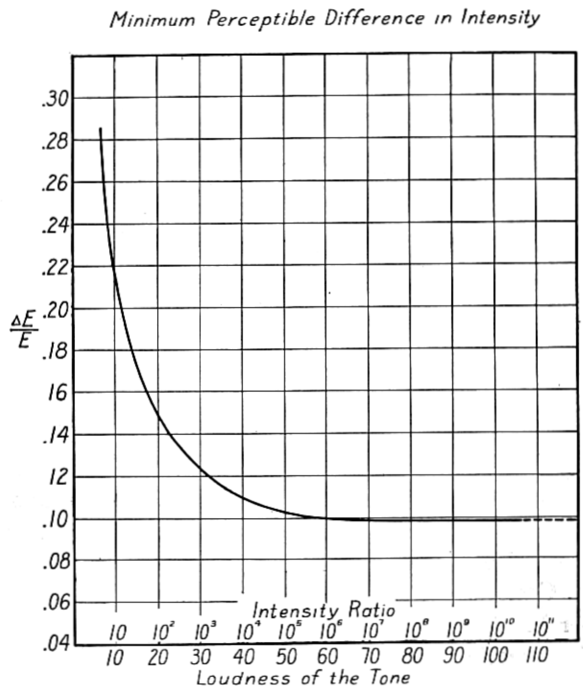


Fig. 2

changes in intensity are shown. Each ordinate gives the fractional change in the sound energy which is just perceptible, this fractional change being called the Fechner ratio. The abscissæ are equal to ten times the logarithm of the ratio of intensities, the zero corresponding to the intensity at the threshold of audibility. For intensities greater than 10^4 times the threshold of audibility, the Fechner ratio has the constant value of approximately one-tenth. It was found that this ratio is approximately the same for all frequencies. In Fig. 3 is shown the results taken from Knudsen's article on the

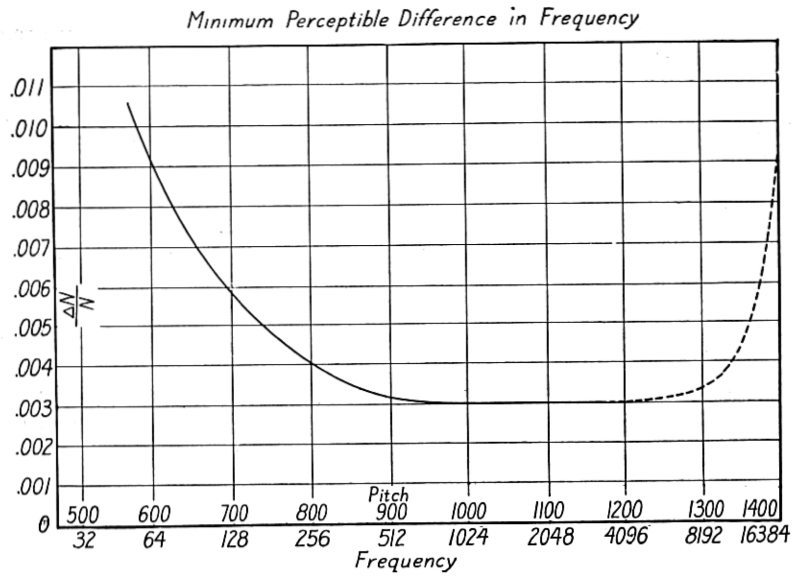


Fig. 3

pitch sensibility. The ordinates give the fractional change in the frequency which is just perceptible and the abscissæ give the frequency on a logarithmic scale. The meaning of the pitch scale at the bottom of this figure will be discussed later. For frequencies above 400 this fractional change is a constant equal to .003. This ratio probably becomes larger again for the very high frequencies. It was found that it varied with intensity in approximately the same way as that given for the energy ratio.

Using these values in connection with the auditory-sensation area, it is possible to calculate the number of pure tones which the ear can perceive as being different. For example, if, starting at the minimum audibility curve, ordinate increments are laid off along a constant pitch line, that are successively equal to the value of ΔE at the intensity position above the threshold, then the number of such increments between the upper and lower curves in Fig. 1 is equal to the number of pure tones of constant pitch that can be perceived as being different in volume. If the minimum and maximum audibility curves were plotted on an energy scale, the increment length ΔE near the maximum audibility curve would be a million million times longer than its length in the minimum audibility curve, whereas when they are plotted on a logarithmic scale, this increment length remains approximately constant, changing by less than a factor 2 for 90 per cent. of the distance across the auditory-sensation area. The calculation shows (see Appendix A) that the number of such increments on the 100-cycle frequency line is 270, that is, 270 tones having a frequency of vibration of 1,000 cycles can be perceived as being different in loudness.

What has been said of the intensity scale applies equally well to the frequency scale. The calculation (see Appendix A) indicates that the number of tones that are perceivable as being different in pitch along the 10-dyne pressure line is approximately 1,300.

If an ordinate increment corresponding to ΔE and an abscissa increment corresponding to ΔN be drawn, a small rectangle will be formed which may be considered as forming the boundary lines for a single pure tone. All tones which lie in this area sound alike to the ear. The number of such small rectangles in the auditory-sensation area corresponds to the number of pure tones which can be perceived as being different. The calculation (see Appendix A) of this number indicates that there are approximately 300,000 such tones.

One might well ask the question: How many complex sounds which are different can be sensed by the ear? At first thought, one might say that this number is represented by all the possible combinations of pure tones. Of course, such a number would be entirely too large, for some of these would sound alike to the ear, since the louder tones would necessarily mask the feebler ones. It is evident, however, that the number of such complex sounds will be very much larger than the number of pure tones.

Scales of Loudness and Pitch

It is seen that the use of the logarithmic scale in Fig. 1 is much more convenient not only on account of the large range of values necessary to represent the auditory-sensation area, but also because of its scientific basis. Psychologists have recognized this since Weber and Fechner formulated the relation between the sensation and the stimulus. Although logarithmic units have been used by various authors in measuring the amount of sensation, the numerical values have been quite different. It seems inevitable that there will be a greater cooperation in the future between men in the various branches of science working on this subject, so, in order to avoid misunderstanding, it would be very advantageous for all to use, as far as possible, the same units. With this in mind, I am taking the liberty of suggesting for discussion units for both loudness and pitch.

In the telephone business, the commodity being delivered to the customers is reproduced speech. One of the most important qualities of this speech is its loudness, so it is very reasonable to use a sensation scale to define the volume of the speech delivered. At the present time, an endeavor is being made to obtain an agreement of all the telephone companies, both in the United States and abroad, to adopt a standard logarithmic unit for defining the efficiency of telephone circuits and the electrical speech levels at various points along the transmission lines. The *chief interest* in changes in efficiency of transmission apparatus is their effects upon the loudness of the speech delivered by the receiver at the end of the telephone circuit. So it would be very advantageous to use this same logarithmic scale for measuring differences in loudness.

This scale is chosen so that the loudness difference is ten times the common logarithm of the intensity ratio. This means that if the intensity is multiplied by a factor 10, the loudness is increased by ten; if the intensity is multiplied by 100, the loudness is increased by 20; if the intensity is multiplied by 1,000, the loudness is increased by 30, etc. It was seen above that under the most favorable circumstances a change in loudness equal to 1/2 on this scale could just be detected. Knudsen's data indicate, however, that when a silent interval of only two seconds intervenes between the two tones being compared, a loudness change greater than unity on this scale is required before it is noticeable. So the smallest loudness change that is ordinarily appreciated is equivalent to one unit on this scale. It is also convenient because of the decimal relation between loudness change and intensity ratio. This relation is expressed by the formula:

$$\Delta L = L_1 - L_2 = 10 \log_{10} \frac{I_1}{I_2} \text{ or } \frac{I_1}{I_2} = 10^{\frac{\Delta L}{10}}$$

where L_1 and L_2 are the two loudness values corresponding to the intensities I_1 and I_2. Since intensities of sound are proportional to the square of pressure amplitudes this may also be written:

$$\Delta L = 20 \log \frac{p_1}{p_2}$$

The most convenient choice of the intensity or pressure used as a standard for comparison depends upon the problem under consideration. In the sensation area chart of Fig. 1, the intensity line corresponding to one dyne was used as the zero level, that is, p_2 was chosen equal to 1 so that

$$\Delta L = 20 \log p$$

The choice of the base of logarithms for the pitch scale is dictated by the fact mentioned before, that the ear perceives octaves as being very similar sensations. Consequently the base 2 is the most logical choice for expressing pitch changes. If the logarithm of the frequency to the base 2 were used, perceptible changes in pitch would correspond to inconveniently small values of the logarithm. It is better to use the logarithm to the base $\sqrt[100]{2}$ which is 100 times as large. On this scale the smallest perceptible difference in pitch is approximately unity—somewhat more for frequencies greater than 100 cycles or somewhat less for lower frequencies, according to Knudsen's data. The scale on the charts is chosen so that the change in pitch is given by

$$\Delta P = 100 \log_2 N$$

where N is the frequency of vibration.

It is now evident why such pitch and loudness scales were used in Fig. 1. With these scales, the number of units in any area gives approximately the number of tones that can be ordinarily appreciated in that area. For example, there are approximately 2,000 distinguishable tones in each square, there being more near the centre and fewer near the boundary lines than this number.

Experiments have shown that pure tones of different frequencies which are an equal number of units above the threshold value sound equally loud. This statement may require modification when very loud tones are compared, but the data indicated that throughout the most practical range this was true. Consequently, the absolute loudness of any tone can be taken as the number of units above the threshold value.

Loudness of Complex Sounds

In the measurement of the loudness of complex tones, the situation is not so simple. It has been found that if two complex tones are judged equally loud at one intensity level and then each is magnified equal amounts in intensity, they then may or may not sound equally loud. The curves shown in Figs. 4, 5 and 6 will illustrate this. The first (Fig. 4) shows the comparisons at different intensity levels of two sounds whose pressure spectra are shown in the two figures at the top. The x-axis gives units above threshold for sound A and the y-axis gives the units above threshold of the sound B when the two sound equally loud. In this case, the spectra are somewhat similar

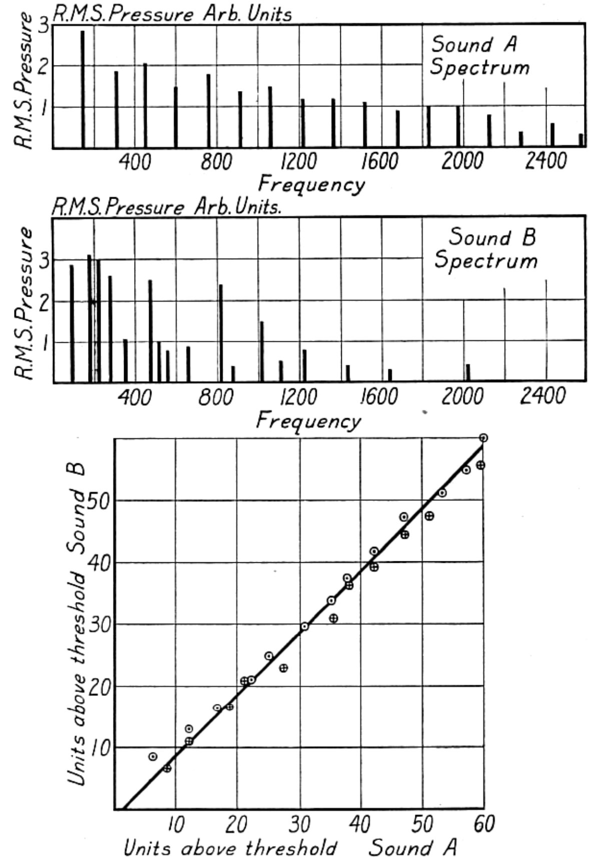


Fig. 4

and we have a straight line of slope 45° passing nearly through the origin. The two sounds are thus of practically equal loudness when they are the same number of units above threshold. In Fig. 5 we have similar data for two sounds which have quite different spectra as is indicated by the two charts at the top. The curve for C means that it was a practically continuous spectrum. It was produced by a device for making the "swishing" type of noises which are usually so prominent in office rooms. The curve representing the relation is not straight, since for values of intensity near the threshold, the loudness increases faster for the C sound for increments in the intensity than for the A sound. For example, it is seen that when the sound C is 30 units above the threshold, the sound A is 45 units

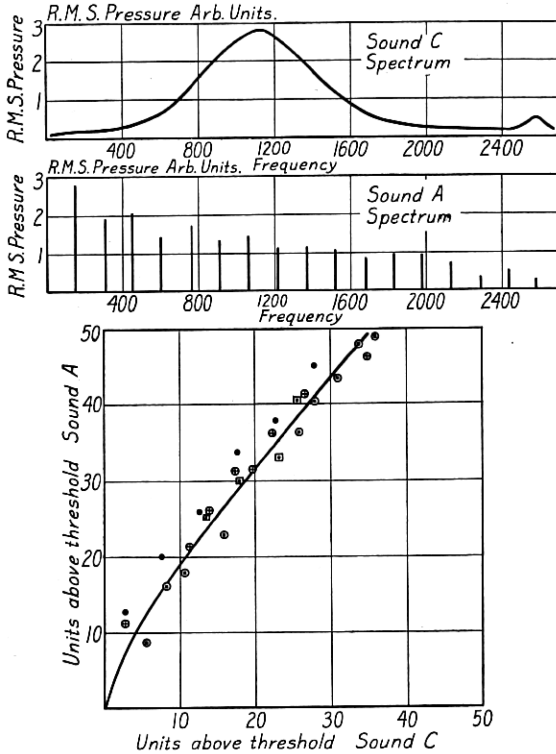


Fig. 5

above the threshold when the two sound equally loud. In Fig. 6 a comparison is given between the loudness of a pure tone of 700 cycles and a complex sound designated by A in the last figure. In this case again the relation is expressed by a curve. The technique of making such loudness measurements is rather difficult and requires a large number of observations before the values are reliable. A paper on this subject which will soon be published will give a detailed account of this work on loudness.

Enough data have been given to show that in order to give loudness a definite meaning for complex sounds, a more precise definition is necessary. It has been found convenient to define the loudness of any complex or pure tone in terms of the loudness of a sound standard.

This standard is a pure tone having a vibration frequency of 700 cycles per second. Its absolute loudness is defined as the change in loudness measured on the scale defined above, from the loudness value corresponding to the threshold pressure for normal ears which for 700 cycles is exactly 0.001 dyne. This frequency was arbitrarily chosen as a standard for measuring loudness because of this particular value of its threshold pressure, and because it is close to the frequency

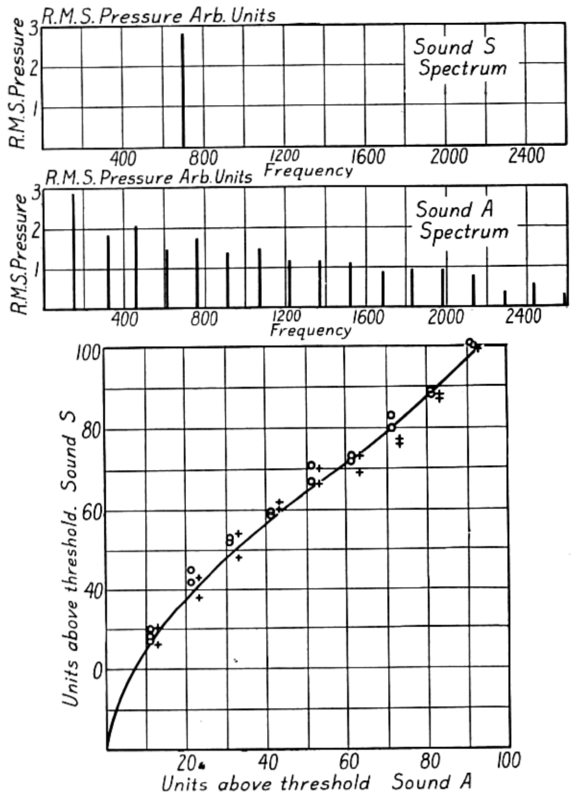


Fig. 6

at which the loudest tones used in conversational speech occur. By this definition, the loudness of a tone of frequency 700, for which p is the pressure variation, expressed as a root mean square value,

$$L = 60 + 20 \log p$$

and the loudness of any other sound, pure or complex, is defined as being equal to that of a tone of frequency 700, seeming equally loud. Such a definition implies that experimental measurements can be made to determine when any complex sound is equally loud to a 700-cycle tone. Such measurements can be made although the observational error is rather large and the judgment of various individuals is sometimes quite different, which means only that loudness as measured by various individuals is different. For use in engineering work, however, the average of a large number of individuals can be taken and this loudness will have a definite determinable value. For example in Fig. 6, the loudness of the A sound when it is 60 units above the threshold is 72, since it sounds as loud as a 700-cycle tone which is 72 units above its threshold. The loudness of complex sounds usually increases faster with increases in intensity than that of pure tones. This would be expected since the threshold is determined principally by the loudest frequency in the complex sound and as the intensity is increased the other frequencies begin to add to the total loudness.

Since pure tones of different pitches which are the same number of units above the threshold sound equally loud their loudness L can be represented by the formula

$$L = L_0 + 20 \log p$$

where p is the root mean square value of the pressure amplitude produced in the ear by the tone and L_0 is the number of units from the 1-dyne line to the minimum audibility curve. The values of L_0 can be read directly from the chart in Fig. 1.

Measurement of Degree of Deafness

The choice of the loudness and pitch units used above leads to a *rational definition of the degree of deafness.*

The number of possible pure tones that can be sensed by a deaf person is considerably smaller than that mentioned above obtained from the normal auditory-sensation area. A logical way of defining the amount of hearing is: *To give the per cent. of the total number of distinguishable pure tones audible to a person with normal hearing, that can be sensed by the deaf person.*

Some definition of this sort will be very helpful in clearing up the confusion that now exists in court cases involving the degree of deafness. It is well known that there are a number of laws which prevent people who have more than a defined amount of deafness from doing certain classes of work. For example, one cannot operate an automobile if he has a certain per cent. of deafness. At the present time, there is a large variation between the standards set up by the various doctors in different parts of the country.

From the discussion above it was seen that the number of tones corresponding to any region was approximately proportional to the area of that region when the logarithmic units were used. Consequently the per cent.* of hearing can be taken as the fractional part

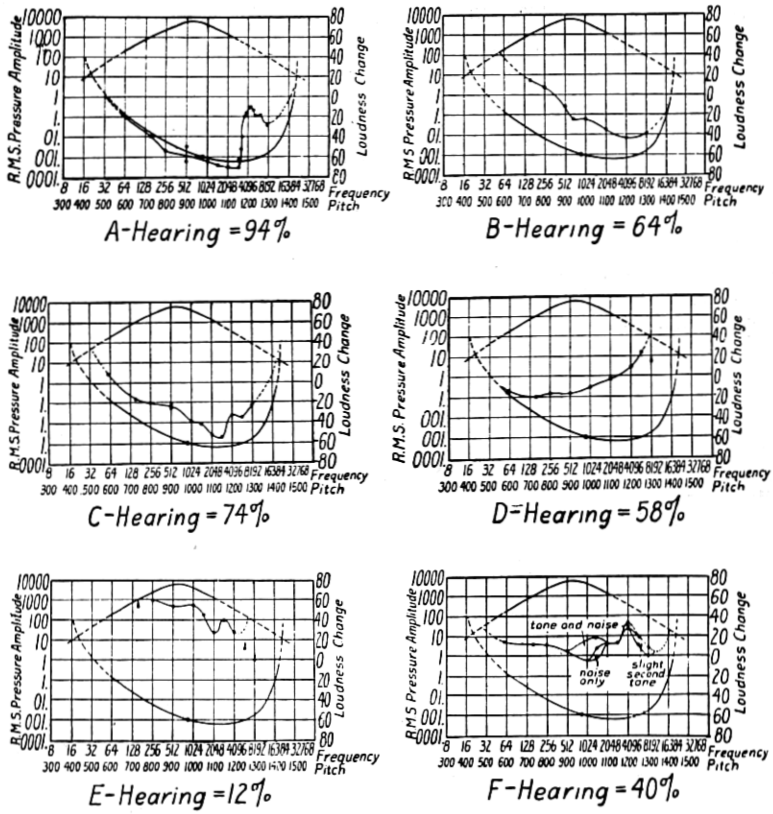


Fig. 7

Audiograms for Typical Cases of Deafness

of the normal auditory-sensation area in which tones can be properly sensed. The per cent. of deafness is, of course, 100 minus the per cent. of hearing.

To emphasize the meaning of this definition, some audiograms, that is minimum audible intensity curves, for some typical cases of deafness will be given. These are shown in Fig. 7. The first chart

* This assumes that the Fechner ratio for pitch and loudness is approximately the same for one having abnormal as for one having normal hearing.

shows a common type of deafness in which the sensitivity to the high frequencies suddenly decreases, as is indicated by the rise in the minimum audible intensity curve when the frequency exceeds 3,000 cycles per second. The sensation area for this person is 94 per cent. of that for the average. Consequently, his per cent. of hearing is 94 per cent. It is also convenient to speak of the per cent. of hearing for each pitch. It is evident that the logical definition for this is the ratio of the widths of the sensation area for the person tested and normal person, measured along the ordinate drawn at the frequency in question.[56] For example, in this audiogram the person had more than 100 per cent. hearing for most of the pitch range. At 4,000 cycles, however, the per cent. hearing was only 60 per cent. This means that for this pitch, the person when compared with one having normal hearing could sense only 60 per cent. as many gradations in tonal volume before reaching the threshold of feeling.

The second chart corresponds to a type of deafness that is not so common. It shows relatively large losses at the lower frequencies. The per cent. hearing in this case is seen to be 64 per cent.

The third type is very common and corresponds to a general lowering of the frequencies throughout the entire pitch range. In these first three cases, the deaf persons could carry on a conversation without any difficulty whatever. In the last two of these, difficulty was experienced in understanding a speaker at any considerable distance. In the first case, the person could not hear the steam issuing from a jet or any other high hissing sound. However, he could hear and understand speech practically as well as anyone with normal hearing.

The fourth case shows a falling off at the high frequencies, but this loss in hearing proceeds gradually as the pitch increases rather than abruptly as in the first case. As indicated in the figure the per cent. of hearing is 58 per cent.

The fifth case is one of extreme deafness and is typical of such cases. The per cent. of hearing is only 12 per cent. The last case shows not only the minimum audibility curve, but the quality of the sensation perceived. As indicated on the chart, in certain regions noises are heard when the stimulus is a pure tone. When computing the per cent. of hearing in such cases, it seems reasonable to take only the area where sensation of good quality is perceived. In some cases, this poor quality extends through practically the whole area and although the person hears sounds, he is unable to properly interpret them. Consequently, from a practical point of view, his per cent. of hearing is very low. For such cases, deaf sets or other aids to the hearing do not give any satisfactory help.

Masking of One Pure Tone by Another

We are now in a position to discuss another set of facts concerning the perception of tones, namely, the ability of the ear to perceive certain sounds in the presence of other sounds. Such data for pure tones have been obtained in our laboratories and will soon be published in some detail. The apparatus used consisted simply of two vacuum tube oscillators generating the two tones used and two attenuators which made it possible to introduce the tones into a single receiver with any desired intensities. In other words, it consists of two audiometers with a common receiver for generating the two tones. The curves shown in Fig. 8 give the general character of the results of this work.

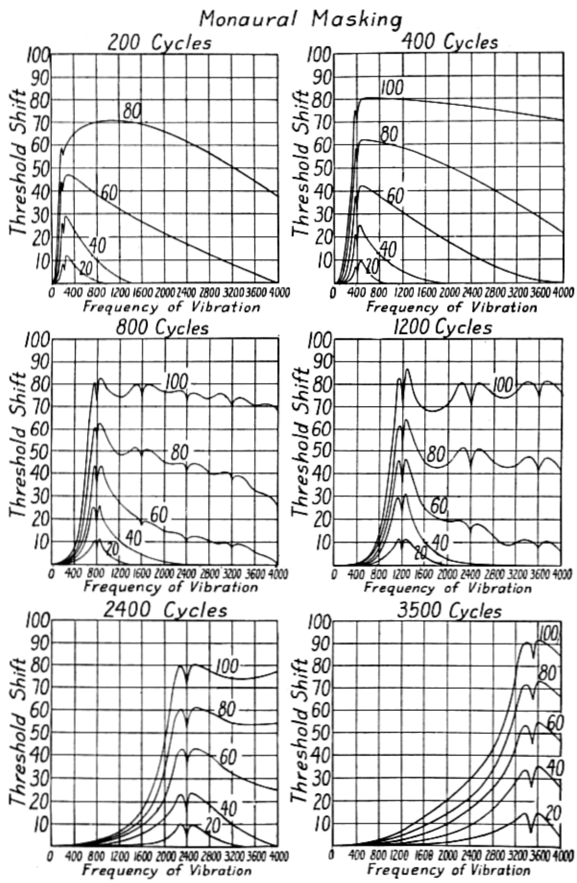


Fig. 8

The ordinates show the amounts in loudness units that the threshold value of a tone of any frequency called the "masked tone" is shifted due to the presence of another tone called the "masking tone." The frequency of the masking tone is given at the top of each set of curves.

The experimental procedure was as follows: The threshold values for the two tones were first determined. The intensity of the masking tone (the frequency of which is given above each graph) was then increased beyond its threshold value by the number of units indicated just above the curve. The masked tone was then increased in intensity until its presence was just perceived. The amount of this latter increase, measured on the loudness scale is called the *threshold shift* and is plotted as ordinate in Figs. 8, 9 and 10. The frequencies of the masked tones are given by the abscissæ.

For example, in the fourth chart, the masking effects of the tone having a frequency of 1,200 cycles are shown. It is seen that the greatest masking effect is near 1,200 cycles, which is the frequency of the masking tone. A tone of 1,250 cycles must be raised to 46 units above the threshold to be perceived in the presence of a 1,200-cycle tone which is 60 units above its threshold, or it must be raised to within 14 units of the masking tone before it is perceived. This corresponds to an intensity ratio between the tones of only 25. A tone of 3,000 cycles, however, can be perceived in the presence of a 1,200-cycle tone which is 60 units loud when it is only 8 units above its threshold. This means that the intensity ratio between these two tones, under such circumstances, corresponds to 52 units or to a ratio of approximately 160,000 in intensity. However, as the loudness of the masking tone is increased, all of the high tones must be increased to fairly large values before they can be heard. For example, the high frequencies must be raised 75 units above the threshold to be heard in the presence of a 1,200-cycle tone having a loudness of 100 units. But even for such large intensities for the masking tone, those frequencies below 300 are perceived by raising their loudness only slightly above the threshold value. It should be noticed that in all cases, those tones having frequencies near the masking frequency, whether they are higher or lower, are easily masked.

It is thus seen that Mayer's conclusion, that a low pitch sound completely obliterates higher pitched tones of considerable intensity and that higher pitched frequencies will never obliterate lower pitched tones, is true only under certain circumstances. A low tone will not obliterate to any degree a high tone far removed in frequency, except when the former is raised to very high intensities. Also a tone of higher frequency can easily obliterate a tone of lower frequency if the frequencies of the two tones are near together. When the two tones are very close together in pitch the presence of the masked tone is perceived by the beats it produces. This accounts for the sharp drop in the curves at these frequencies. A similar thing happens for those regions corresponding to harmonics of the masking frequency. In the charts for the 200- and 400-cycle masking tones these drops are not shown inasmuch as they were small, but in an accurate picture they should be shown.

Monaural Masking

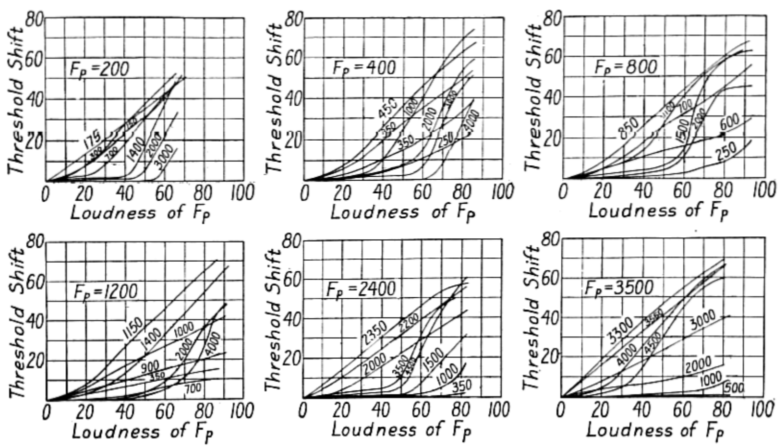


Fig. 9

In Fig. 9, these results are shown plotted in a different way. The abscissæ represent the loudness of the primary tones whose frequency is indicated at the top of each of the charts. The amounts that the threshold is shifted are plotted as ordinates as in the previous figure. For example, in Chart 1, the results are shown for a masking tone of 200 cycles. The curve marked 3,000 indicates the masking effect of a 200-cycle upon a 3,000-cycle tone. It is seen that the loudness of the low pitched tone can be raised to 55 units before it has any interfering effect upon the high pitched tone. For louder values than this it has a very marked effect. It will be noticed that in nearly all of the charts the curves for different frequencies intersect. This leads to some rather interesting conclusions, regarding the perception of a complex tone. For example, consider the curves for a masking tone having a frequency of 400 cycles. Assume we have a complex tone having three frequencies of 400, 300 and 200 cycles with relative loudness values of 50, 10 and 10, respectively. The ear will hear only the 400-cycle tone and the 200-cycle tone as is evident from the curves. It would be necessary to raise the 300-cycle tone above 16 units for it to be heard in the presence of 400 cycles of loudness 50. However, if the sound is magnified without distortion 30 loudness units, so that these three frequencies have loudness values of 80, 40

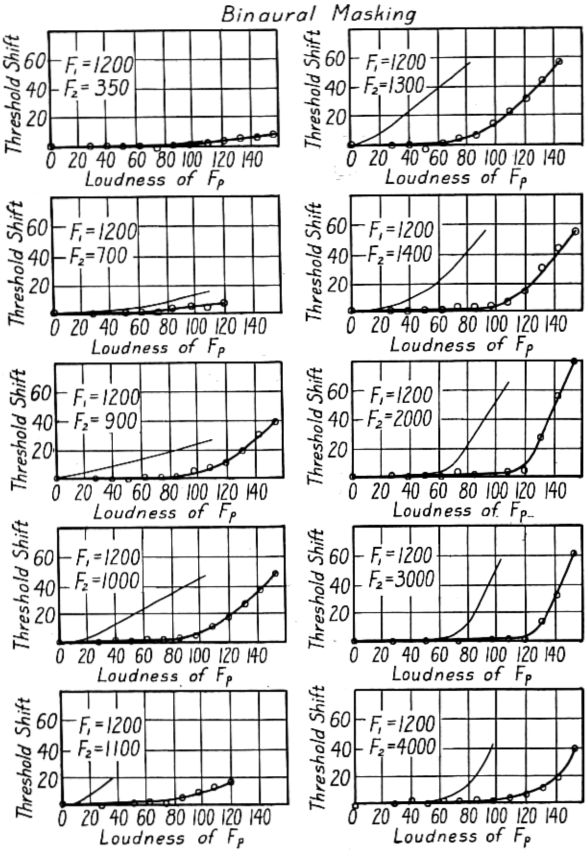


Fig. 10

and 40, respectively, then the 400-cycle tone and 300-cycle tone only will be heard. Under such conditions, the 300-cycle tone could be attenuated approximately 15 units before it would disappear. This means that the sensation produced by a complex sound is different in character as well as intensity when the sound is increased or decreased in intensity without distortion. In general, as the tone becomes more intense the low tones become more prominent because the high tones are masked. It is a common experience of one working with complex sounds to have the low frequencies always gain in prominence as the sound is amplified.

The question naturally arises, Does the same interfering effect exist when the two tones are introduced into opposite ears instead of both being introduced into the same ear? The answer is No. Curves showing the results in such tests are shown in Fig. 10. For comparison the results for the case when in tones are both in the same ear are given by the light lines. Take the case of 1,200 and 1,300 cycles. It is rather remarkable that a tone in one ear can be raised to 60 units, that is, increased in intensity one million times, before the threshold value for the tone in the other ear is noticeably affected. If the 1,300-cycle tone were introduced into the same ear as the 1,200-cycle tone, its loudness would need to be shifted 40 units, corresponding to a 10,000-fold magnification in intensity above its threshold intensity in the free ear before it can be heard. It is seen that if one set of curves is shifted about 50 units it will coincide with the second set. This strongly suggests that the interference in this case is due to the loud tone being transmitted by bone conduction through the head with sufficient energy to cause masking. The vibration is probably picked up by the base of the incus and transmitted from there to the cochlea in the usual way. There is other evidence * which I shall not have space here to discuss, which indicates that the effective attenuation from one ear to the other is approximately 50 units.

Theories of Audition

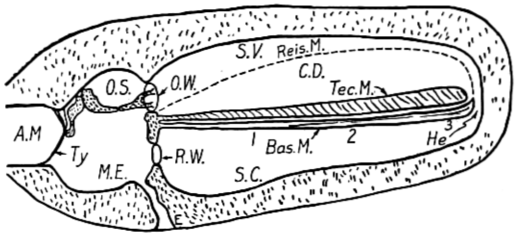
With these facts in mind, we are now ready to discuss the theory of hearing which will best account for them. I will refer briefly to just a few of the principal theories of hearing which have been proposed. The sketch shown in Fig. 11 gives a diagrammatic picture of the internal ear. In the Helmholtz theory, as first formulated, it is stated that the organ of Corti located between the basilar membrane and the tectorial membrane act like a set of resonators which are sharply tuned. Each tone stimulates a single organ depending upon its pitch. Later this theory was somewhat modified as it was thought that the resonant property might reside in one of the membranes in the cochlea.

* See paper by Wegel and Lane soon to be published in the *Physical Review* entitled "The Auditory Masking of One Pure Tone by Another and its Relation to the Dynamics of the Inner Ear."

In the "telephone" theory, as expounded by Voltalini, Rutherford, Waller and others, it is assumed that the basilar membrane vibrates as a whole like the diaphragm of a telephone receiver, and consequently responds to all frequencies with varying degress of amplitude. The discrimination of pitch takes place in the brain.

Meyers in his theory states that various lengths of the basilar membrane are set in motion depending upon the intensity of the stimulating tone. As in the previous theory, the pitch discrimination is accomplished in some way in the brain.

In the "non-resonant" theory of Emile ter Kuile it is assumed that the sound disturbance penetrates different distances into the



Diagrammatic representation of auditory function

A.M.	Auditory meatus	O. W.	Oval window
Bas. M.	Bas. mem. including organ of Corti	Reis. M.	Reissner's mem.
C. D.	Cochlear duct	R. W.	Rd. window
E.	Eustachian tube	S. C.	Scala cochlea
He.	Helicotrema	S. V.	Scala vestibuli
M. E.	Middle ear	Tec. M.	Tectorial membrane
O. S.	Ossicles (malleus, incus, stapes)	Ty.	Tympanic membrane

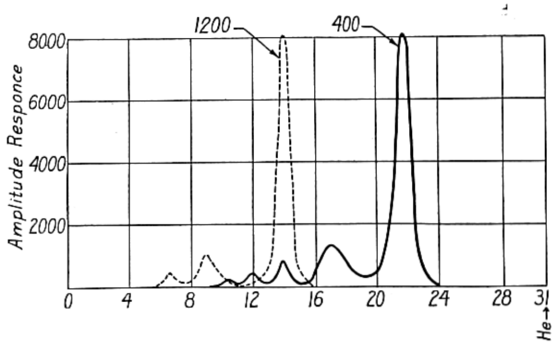


Fig. 11

cochlea depending upon the frequency of the stimulating tone. The further along the membrane the disturbance reaches, the lower will be the pitch sensation. A low pitch tone then stimulates all of the nerve fibres that would be stimulated by tones of higher pitch plus some additional nerve fibres.

The theory of maximum amplitudes was first put into definite form by Gray in 1899.[69] It assumes that the position of maximum amplitude of the basilar membrane varies with the pitch of the stimulating tone. Although a considerable portion of the membrane vibrates when stimulated by a pure tone, the ear judges the pitch by the position of maximum response of the basilar membrane. Roaf has shown that some action of this sort must take place due to the dynamical constants involved.[61] It is an amplification of this theory that I desire to propose as the one which most satisfactorily accounts for the facts.

When a sound wave impinges upon an ear-drum, its vibrational motion is communicated through the middle ear (Fig. 11) by means of the chain of small ossicles (malleus, incus and stapes) to the oval window. Here the vibration is communicated to the fluid contained in the cochlea. If the pitch of the tone is low, say below 20 vibrations, the fluid is moved bodily back and forth around the basilar membrane through the helicotrema, the motion of the membrane at the round window and the oval window being just opposite in phase, the former moving inward while the latter moves outward. For very high frequencies, the mass reactions of the ossicles and the fluid are so great that very little energy can be transmitted to the cochlea. For example, when the elastic forces are negligible it requires a force 10,000 times as large as produce the same amplitude of vibration at 10,000 cycles as that required at 100 cycles. For intermediate frequencies the mass reactions, the elastic restoring forces and the frictional resistances which are brought into play are such that the wave is transmitted through the basilar membrane causing the nerves to be excited.

It is thus seen that the upper and lower limits of audibility are easily explained. When the forces upon the drum of the ear or walls of the ear canal are large enough to excite the sensation of feeling and the pitch of the tone is either too low or too high to cause any perceptible vibration of the basilar membrane, we are beyond the lower or upper limit of audibility respectively. At frequencies between these limits, the vibrational energy is first communicated to the fluid in the scala vestibuli and then transmitted through the two membranes into the fluid of the scala cochlea. As the basilar membrane transmits the sound wave it takes up a vibration amplitude which stimulates the nerve fibres located in it. The entire membrane vibrates for every incident tone, but for each frequency there is a corresponding spot on the membrane where the amplitude of the vibration is greater than anywhere else. Our postulate is that only those nerves are stimulated which are at the particular parts of the membrane vibrating with more than a certain critical amplitude; and that we judge the pitch from the part of the membrane where the nerves are stimulated. According to this conception, the variation with frequency of the minimum audible intensity is due principally to the variation with frequency of the transmission efficiency of the mechanical system between the auditory meatus and the basilar membrane. Pure tones of equal loudness correspond either to equal amplitudes or to equal velocities of vibration of the basilar membrane or to some function of the two. Whatever is assumed, the dependence of the minimum audible intensity upon frequency for the ear can be explained entirely by the vibrational characteristics of the ear mechanism. For the sake of clearness it will be assumed that equal amplitudes of vibration of the basilar membrane correspond to equal sensations. For loud pure tones, there are several regions of maximum amplitude on the membrane, corresponding to the tone and to the harmonic introduced by the non-linear response of the middle ear, the latter maxima increasing very rapidly as the stimulation increases.

It is a strange thing that the phenomenon of the masking of tones which, as stated in the beginning, has been considered by some to be so fatal to any resonator theory, is the very thing that has furnished experimental data which makes it possible to calculate the vibration characteristics of the inner ear. Such a calculation must be based upon assumptions which will be uncertain, but will seem reasonable. It is not my purpose to discuss those here, but I shall give only the final result of such a calculation made by Mr. Wegel and Mr. Lane of our laboratories. At the bottom of Fig. 11, the two curves show the amplitude of vibration of different portions of the basilar membrane for the two frequencies 400 and 1,200 cycles. For purposes here these curves may be considered to be simply illustrative. This membrane has a length of 31 mm. and a width of .2 mm. at the base and .36 mm. at the helicotrema end. The x-axis in this figure gives the distance in millimeters from the oval window and the y-axis gives the amplitudes of vibration in terms of the amplitude corresponding to the threshold of audibility. The loudness of the stimulating tones in both cases is 80 units. It will be seen that the maximum response for the high frequencies is near the base of the cochlea, while that for the low frequencies is near the helicoptrema. It will be noticed that the amplitude of the membrane has several maxima corresponding to the subjective harmonics.

With this picture in mind, it is clear why the perception of one tone is interfered with by the presence of a second tone when their frequencies are close together, since the nerves necessary to perceive the first tone are already stimulated by the second tone. Also when their frequencies are widely separated, entirely different sets of nerves carry the impulses to the brain, and consequently there is no interference between the tones except that which occurs in the brain. Although this brain interference may not be entirely negligible, especially for very loud sounds, it is certainly very much smaller than that existing in the ear for tones close together in pitch.

It is also seen that the reason why the low tones mask the high tones very much more easily that the reverse is due to the harmonics introduced by the transmission mechanism of the ear. Inasmuch as these harmonics are due to the second order modulations, they are proportional to the square of the amplitude and, therefore, become much more prominent for the large amplitudes. When two tones are introduced, summation and difference tones as well as the harmonics will necessarily be present (see Appendix B). With the proper apparatus for generating continuously sounding tones, these subjective tones are easily heard. Their frequency can be quite accurately located by introducing from an external source a frequency which can be varied until it produces beats with the subjective tone.

Messrs. Wegel and Lane who are working in this field have observed modulation frequencies created in the ear as high as the fourth order. They will soon publish* an account of this work on the vibrational characteristics of the basilar membrane. It is seen that the quality as well as the intensity of the sensation produced by a pure tone should change as the intensity of stimulus is increased due to the increasing prominence of the harmonics. This is in accordance with one's experience while listening to pure tones of varying intensity. The non-linear character of the hearing mechanism is also sufficient to account for the falling off in the ability of one to interpret speech when it becomes louder than about 75 units. The introduction of the summation and difference tones and the harmonics makes the interpretation by the brain more difficult. Its action in this respect is very similar to the carbon transmitter used in commercial telephone work or to an overloaded vacuum tub. This characteristic of the ear also explains why we should expect departures from non-linearity when making loudness balances for complex tones. It also suggests that a similar thing might be ex-

* Wegel and Lane, see paper already cited.

pected when comparing the loudness of pure tones if the balances are made at very high intensities. No such balances have yet been made.

What happens to the ear when one becomes deaf? This question, of course, is one for the medical profession to answer, but let us take one or two simple cases and see if they fit into this theory. First assume that the nerve endings are diseased for a short distance away from the base of the cochlea so that they send no impulses to the brain. Under certain assumptions the kind of an audiogram one should obtain can be calculated from the vibrational characteristics determined as mentioned above. Such a calculation shows that an audiogram similar to that shown in Fig. 4, which has a rapid falling off in sensitiveness, can be accounted for, both quantitatively as well as qualitatively. On a pure resonant theory corresponding to that first proposed by Helmholtz, a tone island would exist corresponding to the affected region for such a case. Although we have tested a large number of cases, no such islands have ever been found. When the intensity of the tone is raised sufficiently to bring the amplitude of the area containing the healthy nerve cells which are adjacent to the diseased portion to a value above that corresponding to the threshold, the tone will then be perceived.

Again assume that due to some pathological condition, the tissue around the oval window where the stapes join the cochlea has become hardened. Its elasticity will then be greatly increased so that vibrational energy at low frequencies will be greatly discriminated against. For such a case, an audiogram similar to that shown in Fig. 7-B would be obtained.

A number of things can cause a general lowering of the ear sensitivity, such as wax in the ear canal, affections of the ear-drum, fixation of any of the ossicles, thickening of the basilar membrane, affections of the nerve endings or loss in nervous energy being supplied to the membrane, etc. However, one would expect that each type of trouble would discriminate, at least to some extent, against certain frequency regions so as to produce some characteristic in the audiogram. Ear specialists are beginning to realize the possibility of obtaining considerable aid in the diagnosis of abnormal hearing from such accurate audiograms.

There are a large number of facts obtained from medical research which necessarily have a bearing upon the theory of hearing, but as far as I know none of them is contrary to the theory of hearing given above. It was seen that there are approximately 300,000 tone units in the auditory-sensation area. According to the anatomists, there are only 4,000 nerve cells in the basilar membrane with four or five fibre hairs for each cell. Assuming that each hair fibre acts as a unit there are still insufficient units for each perceivable tone and according to the theory given above, a large number of these units must act at one time. Consequently the ear must be able to interpret differences in the intensity of excitation of each nerve cell as well as determine the position of each nerve cell excited.

Most modern neurologists believe in the "none or all" excitation theory of nerve impulses.[59-60] They also claim that nerve impulses can never be much more rapid than about 50 per second and cannot therefore follow frequencies as high as those found in sound waves. The second statement only emphasizes the necessity of assuming that the intensity position as well as place position is necessary to

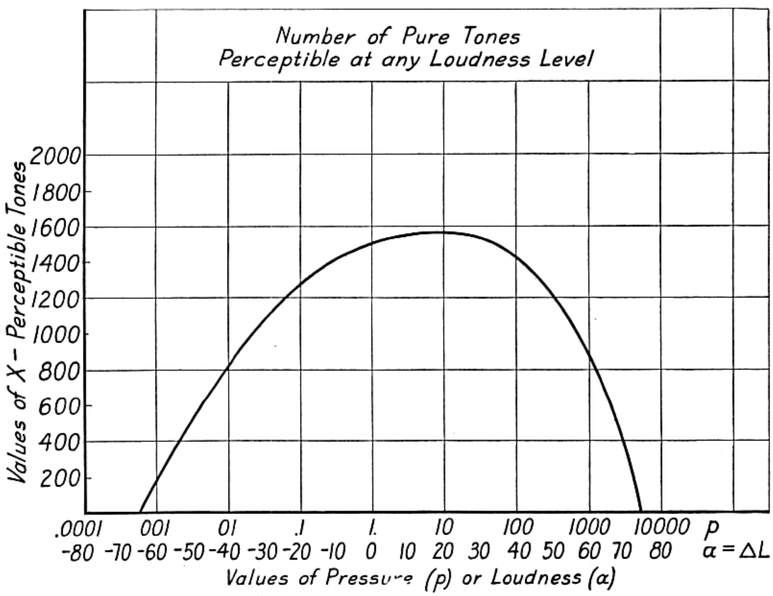


Fig. 12

account for the differentiation of pure tones. The first statement is not necessarily in conflict with such an idea since anatomists are not agreed upon the number of nerve fibres radiating from each nerve cell. Since each nerve fibre can serve to give a unit nerve impulse, the intensity of stimulation sent from a single nerve cell can increase with stimulation depending upon the number of nerve fibres brought into action. The intensity of the sensation produced is then directly related to the total number of nerve fibres giving off impulses. It seems to me that the spacial and intensity configurations which are possible, according to this theory, are sufficient for an educated brain to interpret all the complex sounds which are common to our experience.

In conclusion then, it is seen that the pitch of pure tones is determined by the position of maximum response of the basilar membrane, the high tones stimulating regions near the base and the low tones regions near the apex of the cochlea.

A person can sense two mixed tones as being distinctly two tones while he cannot sense two mixed colors, since in the ear mechanism there is a spacial frequency selectivity while in the eye mechanism there is no such selectivity.

The limiting frequencies which can be perceived are due entirely to the dynamical constants of the inner ear as is also the dependence of minimum audible intensity on frequency.

The so-called subjective harmonics, summation and difference tones are probably due to the non-linear transmission characteristics of the middle and inner ear.

These subjective harmonics account for the greater masking effect of low tones on high tones than high tones on low tones. Due to this non-linear characteristic, the quality as well as the intensity of the sensation produced, especially by complex tones, change as the intensity of the stimulus increases.

The facts obtained from audiograms of abnormal hearing are consistent with the theory of hearing which has been outlined.

Although this theory of hearing involves the principle of resonance, it is very different from the Helmholtz theory as usually understood. In the latter it as assumed that there are four or five thousand small resonators in the ear, each responding only to a single tone; while in the former it is assumed that a single vibrating membrane which vibrates for every impressed sound is sufficient to differentiate the various recognizable sounds by its various configurations of vibration form.

A loudness scale has been chosen such that the loudness change is equal to ten times the common logarithm of the intensity ratio. A pitch scale has been chosen such that the pitch change is equal to 100 times the logarithm to the base two of the frequency ratio. The loudness of complex or simple tones is measured in terms of the number of loudness units a tone of 700 cycles must be raised above its average threshold value before it sounds equally loud to the sound measured.

The degree of deafness is measured by the fractional part of the normal area of audition in which the sensation is either lacking or false.

Appendix A

The calculations of the number of pure tones perceivable as being different in pitch at a given intensity or being different in loudness at a given pitch involves a line integral. The calculation of the number of pure tones perceivable as being different either in loudness or pitch involves a surface integral.

Let the coordinates used in Fig. 1 corresponding to ΔL and ΔP be designated α and β, respectively. Then the relations shown in Figs. 2 and 3 can be expressed by the equations

$$\frac{\Delta E}{E} = f(\alpha - \alpha_0) \text{ and} \tag{1}$$

$$\frac{\Delta N}{N} = \varphi(\beta) \tag{2}$$

where α_0 is the value of α along the normal minimum audibility curve shown in Fig. 1. Knudsen's data indicated that the curve shown in Fig. 3 held only for values of $\alpha - \alpha_0$ corresponding to the flat part of the curve in Fig. 2. For lower intensities the pitch discrimination fell off in about the same way as that shown for the intensity discrimination. To represent this mathematically, $\varphi(\beta)$ can be multiplied by a factor which is unity for the loud tones and which increases similarly to $f(\alpha - \alpha_0)$ for the weaker tones. Such a factor is $10\, f(\alpha - \alpha_0)$ since $f(\alpha - \alpha_0)$ is approximately $\frac{1}{10}$ for the louder tones. So the corrected formula for $\frac{\Delta N}{N}$ is

$$\frac{\Delta N}{N} = 10\varphi(\beta) \cdot f(\alpha - \alpha_0). \tag{3}$$

Let dx be the number of perceivable tones of constant intensity corresponding to α in the pitch region between β and $\beta + d\beta$ and let dy be the number of perceivable tones of constant pitch corresponding to β in the region between α and $\alpha + d\alpha$. Then

$$dx = \frac{dN}{\Delta N} \tag{4}$$

$$dy = \frac{dE}{\Delta E}. \tag{5}$$

But the values of β and α are given by

$$\beta = 100 \log_2 N \tag{6}$$

$$\alpha = 10(\log_{10} E - \log_{10} E_1) \tag{7}$$

where E_1 is the value of intensity corresponding to a pressure amplitude of 1 dyne.

Substituting values of dN and dE in terms of α and β we have

$$dx = \frac{1}{100}\frac{N}{\Delta N}\log_e 2\, d\beta = \frac{\log_e 2}{1000}\frac{d\beta}{\varphi(\beta)\cdot f(\alpha-\alpha_0)} \tag{4'}$$

$$dy = \frac{1}{10}\frac{E}{\Delta E}\log_e 10\, d\alpha = \frac{\log_e 10}{10}\frac{d\alpha}{f(\alpha-\alpha_0)}. \tag{5'}$$

The number of tones of constant intensity which are perceivable as different in pitch is then

$$x = \frac{\log_e 2}{1000}\int_{\beta_1}^{\beta_2}\frac{d\beta}{\varphi(\beta)\cdot f(\alpha-\alpha_0)}$$

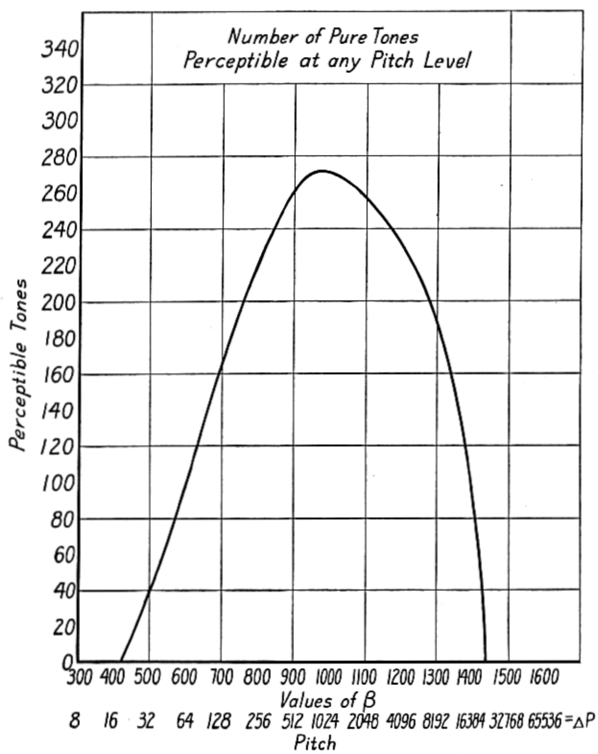


Fig. 13

where β_1 and β_2 are the points where the particular intensity line cuts the boundary lines of the auditory-sensation area. For example, the limits for the line corresponding to 1-dyne pressure ampli-

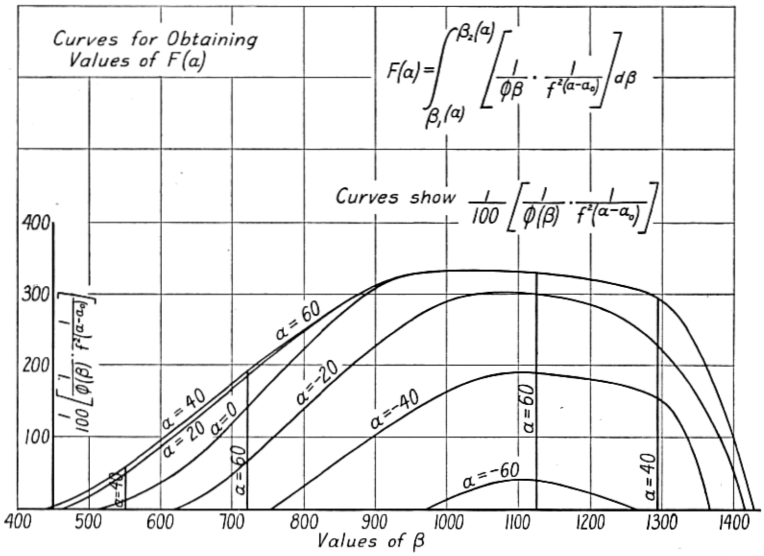


Fig. 14

tude are 500 and 1420. Similarly the number of tones of constant pitch which are perceivable as being different in loudness is given by

$$y=\frac{\log_2 10}{10}\int_{\alpha_1}^{\alpha_2}\frac{d\alpha}{f(\alpha-\alpha_1)}$$

where α_1 and α_2 are determined by the intersection of the particular pitch line with the boundary lines of the auditory-sensation area.

The values of these integrals were computed graphically. Figs. 12 and 13 show the results of these calculations. It is seen that the maximum number of tones perceivable as different in loudness is in the frequency range 700 to 1,500 which is also the important speech range. The number in this range is approximately 270.

In the pressure range from 1 to 100 there are approxmiately 1,500 tones which can be perceived as being different in pitch.

The number of tones ΔT in a small area $d\beta\ d\alpha$ situated with one corner at the point (α, β) is given by $dx\ dy$ or

$$\Delta T=dx\ dy=\frac{\log_e 2\log_e 10}{10{,}000}\ \frac{d\alpha\ d\beta}{\varphi(\beta)f^2(\alpha-\alpha_0)},$$

$$T=\frac{\log_e 2\log_e 10}{10{,}000}\int\int\frac{d\beta\ d\alpha}{\varphi(\beta)f^2(\alpha-\alpha_0)}.$$

The function $\frac{1}{\varphi(\beta)\cdot f^2(\alpha-\alpha_0)}$ must be integrated throughout the auditory-sensation area. This was done by graphical methods as shown in Figs. 14 and 15 with the result that $T=324{,}000$.

Appendix B

Let the pressure variation of the air in front of the drum of the ear be designated by δp. Since the pressure of the air in the middle

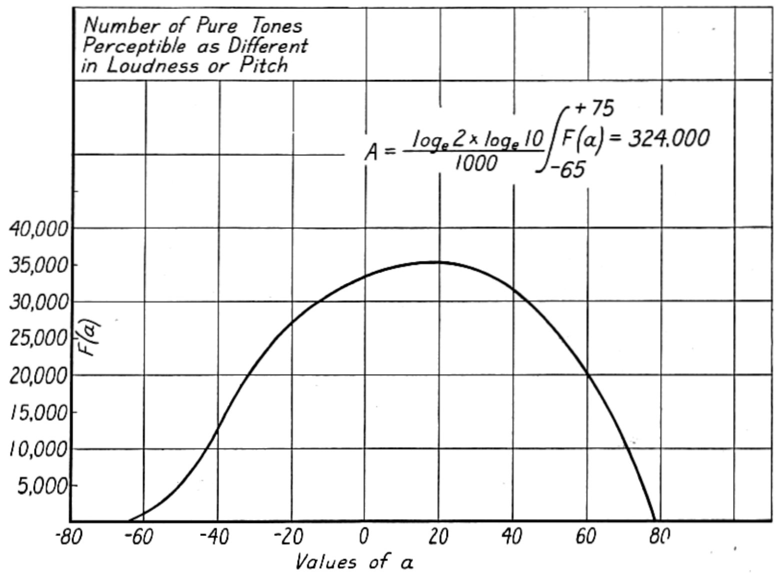


Fig. 15

ear balances the undisturbed outside air pressure this change in pressure multiplied by the effective area of the ear-drum is the only effective force that produces displacements. Let the displacement of the fluid of the cochlea near the oval window be designated by X. If Hookes law held for all the elastic members taking part in the transmission of sound to the inner ear then

$$X=k\delta p \tag{1}$$

where k is a constant.

It would be expected from the anatomy of the ear that Hookes law would start to break down even for small displacements. So in general the relation between the force δp and the displacement X can be represented by

$$X=f(\delta p)=a_0+a_1\delta p+a_2(\delta p)^2+a_3(\delta p)^3+\ldots. \tag{2}$$

where the coefficients α_0, α_1, α_2 . . . belong to the expansion of the function into a power series. Now if δp is a sinusoidal variation then

$$\delta p = p_0 \cos \omega t \tag{3}$$

where $\frac{\omega}{2\pi}$ is the frequency of vibration. Substituting this value in (2), terms containing the cosine raised to integral powers are obtained. These can be expanded into multiple angle functions. For example, for the first four powers

$$\cos^2 \omega t = \tfrac{1}{2} \cos 2\,\omega t + \tfrac{1}{2}, \tag{4}$$

$$\cos^3 \omega t = \tfrac{1}{4} \cos 3\,\omega t + \tfrac{3}{4} \cos \omega t, \tag{5}$$

$$\cos^4 \omega t = \tfrac{1}{8} \cos 4\,\omega t + \tfrac{1}{2} \cos 2\,\omega t + \tfrac{3}{8}. \tag{6}$$

It is evident then that the displacement X will be represented by a formula

$$X = b_0 + b_1 \cos \omega t + b_2 \cos 2\,\omega t + b_3 \cos 3\,\omega t + \ldots .$$

In other words when a periodic force of only one frequency is impressed upon the ear-drum this same frequency and in addition all its harmonic frequencies are impressed upon the fluid of the inner ear.

If two pure tones are impressed upon the ear then δp is given by

$$\delta p = p_1 \cos \omega_1 t + p_2 \cos \omega_2 t.$$

If this value is substituted in equation (2), terms of the form $\cos^n \omega_1 t$ and $\cos^m \omega_2 t$ and $\cos^n \omega_1 t \cos^m \omega_2 t$ are obtained. The first two forms give rise to all the harmonics and the third form gives rise to the summation and difference tones. For example, the first four terms are

$$a_0 = a_0$$

$$a_1 \delta p = a_1 (p_1 \cos \omega_1 t + p_2 \cos \omega_2 t)$$

$$a_2 (\delta p)^2 = a_2 [\tfrac{1}{2}\, p_1^2 \cos 2\,\omega_1 t + \tfrac{1}{2}\, p_2^2 \cos 2\,\omega_2 t + p_1 p_2 \{\cos(\omega_1 - \omega_2) t + \cos(\omega_1 + \omega_2) t\} + \tfrac{1}{2}(p_1^2 + p_2^2)]$$

$$a_3 (\delta p)^3 = a_3 \Big[\Big(\frac{3}{4}\, p_1^3 + \frac{3}{2}\, p_1 p_2^2\Big) \cos \omega_1 t + \frac{1}{4}\, p_1^3 \cos 3\,\omega_1 t + \Big(\frac{3}{4}\, p_2^3 + \frac{3}{2}\, p_1^2 p_2\Big) \cos \omega_2 t + \frac{1}{4}\, p_2^3 \cos 3\,\omega_2 t + \frac{3}{4}\, p_1^2 p_2 \cos(\omega_2 t + 2\,\omega_1 t) + \frac{3}{4}\, p_1^2 p_2 \cos(\omega_2 t - 2\,\omega_1 t) + \frac{3}{4}\, p_1 p_2^2 \cos(\omega_1 t + 2\,\omega_2 t) + \frac{3}{4}\, p_1 p_2^2 \cos(\omega_1 t - 2\omega_2 t) \Big].$$

Therefore unless there is a linear relation between a force acting on the ear-drum and the displacement at the oval window, that is unless all the coefficients in equation (2) are zero except a_1, all the harmonics and the summation and difference tones will be impressed upon the fluid in the cochlea of the inner ear.

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