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Application of the Minimum-Weight Spanning-Tree Algorithm to Assignment of Communication Facilities

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COSMOS—the Computer System for Main Frame Operations—is an operational support system that inventories and assigns central office facilities to serve customer circuits. As part of this assignment responsibility, COSMOS must provide the central office personnel who will physically connect the circuit not only with information about the facilities to be connected, but also the order in which they will be connected (i.e., connection sequence or "connectivity"). Also, COSMOS must determine the circuit connectivity to permit automatic assignment of tie pairs—inter- and intra-frame cables that permit the connection of facilities that are widely separated physically. A new algorithm has been added to COSMOS to permit the determination of connectivity. This algorithm is based on the algorithm that determines the minimum-weight spanning tree of a connected graph. However, the algorithm is specialized for COSMOS by taking into account such factors as minimizing the maximum number of connections at any node and restricting certain nodes to a maximum number of connections.

I. INTRODUCTION

When a mechanized system assigns facilities to provide a telephone circuit (to fulfill a request for service, say), it must accomplish three things. It must

1. Determine which facility types are required to provide the service

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2. Select a particular unit which is available for each such facility type

3. Determine the circuit topology of the assigned facilities.

Steps 1 and 2 can be reduced to an algorithm using straightforward procedures. Step 3, however, has proved to be difficult and has been left for manual determination in all but the simplest cases. In this paper an algorithm is reported that has been successful in determining the circuit topology for most of the circuits encountered in telephony.

II. A PARTICULAR APPLICATION—COSMOS

COSMOS is the name of a minicomputer system (DEC PDP 11/70 and PDP 11/45) designed for use by telephone operating companies to assign and administer central office equipment.[1] A major impetus for its development and continued deployment is to increase the efficiency of central office personnel who must physically connect, rearrange, and disconnect facilities to provide service to customers. Accordingly, an important feature of COSMOS is its ability to produce a report (called the Frame Output Report or "FOR") for the central office personnel that clearly specifies what should be connected to what. Most circuits for which COSMOS must create a FOR are simple, i.e., only two facilities must be interconnected on the frame. Some circuits, however, can be quite complicated in that some facilities in the circuit must be interconnected in series while others must be connected in parallel. An example of such a case would be a circuit with a main line and an off-premises extension where the bridge point is in the central office. This example becomes more complex if signal conditioning equipment must be placed in series with each line.

Such an example is illustrated in Fig. 1. This circuit includes a main line (cable pair 4-980) and three off-premises extensions (cable pairs 4-981, 4-982, and 4-983). Each cable pair must be connected to the line equipment through a bridge lifter (BL 49, 50, 51, and 52). Since the bridge lifters are located on a different frame from the line equipment and the cable pairs, tie pairs (TP 107, 304, 305, 306, and 307) must be used to interconnect all the components of this circuit.

Since the FOR must unambiguously state how the connections are to be made, either the person establishing the order for service in COSMOS must provide the connection sequence ("connectivity"), or COSMOS itself has to be capable of determining the connectivity. All initial versions of COSMOS had to be connected manually. Starting about 1977 logic was added so COSMOS could automatically determine connectivity in certain situations. The current generic of COSMOS (generic 9.0) is being developed to incorporate connectivity determination in all cases but still allow the user to manually override the automatic connectivity logic if necessary. This paper presents the

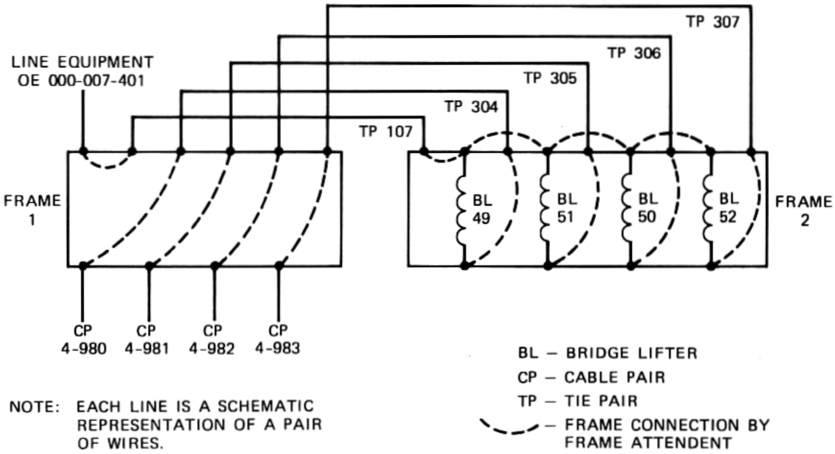

Fig. 1—Example of a main line with three off-premises extensions.

algorithm developed to achieve this capability and illustrates how it benefits the COSMOS user.

III. REVIEW OF COSMOS CAPABILITIES

As we already mentioned, COSMOS accepts a service order as input and creates the FOR as output. The service order is input to COSMOS by a clerk in the Loop Assignment Center (LAC). Certain information must be entered by the clerk so the order can be processed by COSMOS, while other information is optional, depending on the particular order. The required information is the order number and the order due date. If a switching equipment connection is to be assigned to the customer, then the switching equipment features and the customer class of service must be specified also. Specific facilities to be assigned to the customer can either be specified when entered or automatically assigned by COSMOS. Actually, the automatic assignment takes place in two levels, depending on the facilities: 1) COSMOS determines the need for the facility and then selects a particular facility for the circuit, or 2) the LAC clerk specifies the need for a facility on input and COSMOS selects a particular facility for the circuit. Table I lists the facilities administered by COSMOS and how they are selected for a particular circuit by COSMOS—i.e., manual specification of the particular facility, manual specification of the need for the facility, or complete automatic selection by COSMOS. Table I also specifies that some facilities are terminated on a Main Distributing Frame (MDF), while others are not. The facilities that are not terminated on an MDF either have no physical termination (for

Table I—COSMOS administered facilities

Facility	MDF Frame Termination	Manual	Assignment Mode	
			Need Specified	Full Auto
Telephone Number (TN)	No	Yes	Yes	No
Extra Number (XN)	No	Yes	No	No
Group (GP)	No	Yes	No	No
Terminal (TER)	No	Yes	No	No
Relay (RLY)	No	Yes	Yes	Yes
Message Register (MR)	No	Yes	Yes	Yes
Private Line Number (PL)	No	Yes	No	No
Special Equipment (SE)	No	Yes	No	No
Special Equipment (SE)	Yes	Yes	No	No
Cable Pair (CP)	Yes	Yes	No	No
Line Equipment (OE)	Yes	Yes	Yes	No
Concentrator (CON)	Yes	Yes	No	No
Tie Pair (TP)	Yes	Yes	Yes	Yes
Bridge Lifter (BL)	Yes	Yes	Yes	Yes
Trunk (TK)	Yes	Yes	No	No

example, telephone numbers, groups, and terminals on an electronic switching system are software variables) while others are terminated on an intermediate distributing frame [such as relays and message registers on a No. 5 crossbar switching system (5XB)].

Appendix A describes an overview of the input language for the Service Order Establishment (SOE) transaction. An appreciation of the language is helpful in understanding the example presented in Appendix B, which shows the effect of the connectivity algorithm on the user input. The example in this appendix shows the service order input, as well as excerpts from the FOR to connect the circuit shown in Fig. 1. The detailed functioning of the connectivity algorithm for a particular example is described in Appendix C.

So far, only orders resulting from customer requests for service have been described as input to COSMOS. Another major source of input to COSMOS are work orders; i.e., orders initiated by the telephone company personnel to change out defective equipment or to rearrange circuits to accommodate growth. These transactions also use the connectivity algorithm.

IV. THE CONNECTIVITY ALGORITHM

The connectivity algorithm, which was first proposed by H. L. York,[2] is based on the concept of a minimum-weight spanning tree of a connected graph. For each circuit whose connectivity is to be determined, a graph whose nodes correspond to each of the elements of the circuit is constructed. The edges of this graph are assigned weights such that the smaller the weight the more likely the two circuit elements (nodes) are to be connected directly to each other. The final connections between the circuit elements is determined by finding a spanning tree whose edges have a total weight less than or equal to all other spanning trees for this graph. Well-known methods are available for finding the minimum-weight spanning tree of a connected graph. One very straightforward algorithm is given by E. Horowitz and S. Sahni[3] (see especially Section 6.2). An apparently more efficient algorithm plus other extensions of the spanning-tree concept is given by R. C. Prim.[4] As this paper describes later, none of these algorithms could be applied directly to the COSMOS problem because of additional side-constraints that had to be imposed on real circuits. These in turn led to a more efficient algorithm than can be obtained for the general case. The problem of determining circuit connectivity is now reduced to obtaining pair-wise connection weights for all facility combinations and to specifying the particular algorithm for calculating the minimum-weight spanning tree. These will each be discussed in turn.

4.1 Determination of connective weights

In Table I, two types of special equipment (SE) are noted: those with a frame location and those without. Even among SE terminated on the frame there are subgroupings that must be treated differently by the connectivity algorithm. These will now be described.

The SE file in COSMOS contains "miscellaneous" equipment that is not explicitly recorded in any of the other COSMOS equipment files. The name of the SE is created during the order input and a record for the SE is allocated at that time. When an order to disconnect the circuit is established and completed, the record allocated to this SE is released to a list of free records. During input of the name of the SE, the frame location (if one exists) is input also. The SE receives special treatment by the connectivity algorithm, depending on the SE name and the presence or absence of a frame location. The various subcategories of SE are shown in Table II.

With these subdivisions of the equipment that can be represented in the SE field, plus the other facilities that have frame terminations as listed in Table I, the user can construct a complete list of facility

Table II—Subcategories of special equipment

Input	Action
If no frame location is input	Ignore SE for connectivity determination
First two characters are RE	Treat as a REG (repeater with gain)
First two characters are DL	Treat as a DLL (dial Long Lines) unit
First two characters are VR	Treat as a VR (voice repeater)—an example would be an E6 repeater
First four characters are DPP– or character string begins with "."	Treat as a trunk
Anything else	Treat as an SE

types that must be processed by the connectivity algorithm. The next step is to construct a matrix whose rows and columns represent each of these facilities and whose elements are the numerical weights associated with how likely the two facilities are to be connected directly to one another. This will be referred to as the generalized weight table. When the algorithm is presented an actual list of elements that must be connected together, the weights for the graph constructed for this circuit will be obtained from the generalized weight table.

The generalized weight table is constructed as follows:

1. All possible facility types are classified in four broad categories: switching equipment, conditioning equipment, metallic facilities, and tie pairs. In general, any circuit must be connected in the order: switching equipment–conditioning equipment–metallic facilities. Tie pairs are assigned as needed to facilitate these connections.

2. Table III shows this classification of facilities. When assigning weights, the user should note that some conditioning equipment is likely to be connected directly to another conditioning equipment of the same type while other types would be unlikely to be connected directly to each other. For example, if several bridge lifters (BLs) were in the same circuit, they would likely all be connected together. Facilities of this type are noted as "bunching" on Table III.

The range of weights is arbitrarily chosen to lie between zero and one hundred. With the considerations just described plus a review of many likely circuits, the generalized weight table shown in Table IV was developed.

As mentioned earlier, for a particular circuit a graph is established and the weights for the edges are taken from the generalized weight table. After that step the weights are further modified if any of the following additional information applies to the circuit.

1. If tie pairs are already present in the circuit (i.e., an existing circuit is being modified), then the two facilities connected by the tie pair are recorded in the tie pair record. The weight between these two facilities and the tie pair is reduced to a small value.

Table III—Grouping of facilities

I Switching equipment	Line equipment
II Conditioning equipment	Bridge lifter (bunching) Special equipment (RE) Special equipment (DL) Special equipment (VR) Special equipment (bunching)
III Metallic facilities	Cable pair Trunk Concentrator Special equipment (DPP–) Special equipment (.)

Table IV—Generalized weight table

	OE	BL	SE: DL	SE: RE	SE: VR	SE	CP	TK	SE: .	SE: DPP–	CO	TP
OE	90											
BL	35	5										
SE:DL	40	50	90									
SE:RE	45	55	90	90								
SE:VR	45	55	7	90	90							
SE	45	55	10	15	10	10						
CP	65	60	30	30	6	30	70					
TK	65	60	25	25	5	25	75	70				
SE:.	65	60	25	25	5	25	75	70	70			
SE:DPP–	65	60	25	25	5	25	75	70	70	70		
CO	65	60	70	70	73	70	75	75	75	75	90	
TP	90	90	90	90	90	90	90	90	90	90	90	90

2. If a party circuit is being processed, then the facilities associated with each party are identified by a party number. Those facilities that do not belong to the same party have their weights increased to a maximum value.

3. If a circuit with one or more off-premises extensions is being processed, then those facilities belonging to the same "leg" will have a Different Premise Address (DPA) value assigned to them. Consequently, the weights are increased to a maximum value for those edges connecting facilities in different "legs".

4. If the circuit contains tie pairs, then the weights between facilities terminated on different frames will be increased somewhat. This is done to avoid assigning tie pairs unnecessarily.

4.2 Fundamental considerations

There are two special conditions that apply to determining circuits for central office facilities that do not apply to circuit connectivity in general. Not only do these conditions enable COSMOS to determine the correct configuration, but their use speeds up the algorithm as well since some nodes can be eliminated from consideration after a certain point in the algorithm has been reached. These special conditions are:

1. When a user is choosing among several Minimum-Weight Spanning Trees (MWSTs) (they need not be unique for a given graph), the tree with the minimum number of branches at the node with the most branches is preferred. An example of this is shown in Fig. 2. The algorithm does not actually calculate all possible MWSTs and then choose the one with the minimum number of branches at the node with the maximum number of branches. Instead, several strategies are employed, depending on the circuit being processed. If the circuit contains office equipment (OE), then most nodes have a maximum number of connections to them which is calculated before the MWST processing begins (as described in item 2 of this listing). The only

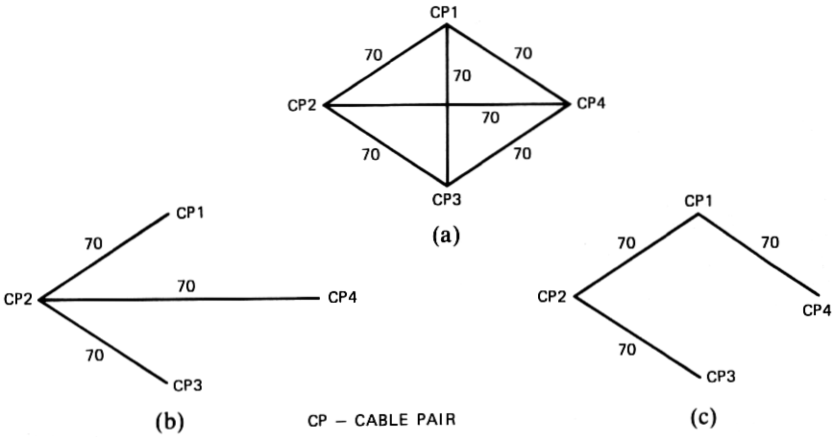

Fig. 2—The minimum-weight spanning tree showing (a) the graph; (b) a maximum of three branches; (c) a maximum of two branches. In this case (c) is the preferred tree.

exceptions are nodes that represent BLs. To prevent all BLs from connecting to one BL, the edge weights of all BL edges not yet selected for the MWST terminating on a BL just connected are incremented. For a circuit that contains no OE, the edge weights of all edges not yet selected for the MWST terminating on a node that has been selected for the MWST are incremented. These strategies direct the algorithm towards selecting the MWST with the required branch minimization. This requirement relates to how circuits are actually wired on the frame. If too many connections must be made at one terminal, the craftsperson may physically run out of room on the terminal and thus be unable to complete all the connections. Also, if an order is subsequently received to disconnect one of the legs of the circuit, proper "housekeeping" might require dismantling all connections at a terminal and then reconnecting the remaining legs. This process is much simplified if the number of connections at a terminal is minimized.

2. When an OE is present in the circuit, each facility is allowed a maximum number of "outward connections." An outward connection is defined as a connection away from the OE. When an OE is present, it will be the root of the tree and therefore a direction away from the OE (root) is always defined. The maximum number of outward connections is determined by the following rules:

(a) Metallic facilities (see Table III) have zero outward connections since they must always be at the outermost "tips" of the branches.

(b) Conditioning equipment (see Table III) is allowed one outward connection. In determining outward connections, a connection between two BLs is not counted. This is because a BL will usually be at a branch point and therefore will have additional outward connections.

(c) If no BLs are present in the circuit, the number of outward connections from the OE equals the number of metallic facilities. If BLs are present, the number of outward connections from the OE equals the number of metallic facilities minus the number of BLs plus one. This formula reflects the actual way in which such circuits are wired: If BLs are present, they are the bridge point instead of the OE. In fact, BLs are often hard-wired in parallel in anticipation of their use as bridge points.

This set of rules (a through c) constitutes the principal reason for a speed-up of this algorithm over the general case, since once the maximum number of outward connections is achieved, a particular node no longer needs to be considered.

3. When an edge is chosen for the MWST, its weight is increased to the maximum value (100). This was chosen as the most efficient method to signal the algorithm not to consider this edge for the MWST again.

4.3 Detailed description

This section describes determining the list of facilities to be connected, the actual algorithm, and how the output list of facilities in connectivity order is assembled from the internal tables populated by the connectivity algorithm. This breakdown parallels the construction of the actual software.

4.3.1 The list of facilities to be connected

Connectivity processing is initiated when another COSMOS module determines that connectivity must be established. If this is the case, the connectivity module is invoked and a list of facilities is presented to it. Before this list can be passed along to the connectivity algorithm, certain facilities must be "weeded out".

There are two types of facilities that must be excluded from connectivity considerations. The first type includes facilities that have no mainframe terminations. These facilities are telephone numbers (TNs), No. 1XB coded terminals (XNs), No. 5XB relays, electronic switching system groups and terminals (GP and TER), and special equipment (SEs) for which no frame termination has been entered.

The second type of facility that must be excluded is frame-terminated facilities that will not be in the circuit at the time that the order being processed will be worked. This situation can arise because COSMOS allows multiple orders to be established on the same circuit if they are logically consistent with one another. Thus order number

1 with due date X may be removing facilities from an established circuit while order number 2 (the one being processed, say) with due date Y is adding facilities to the same circuit. If due date X precedes due date Y, then in processing order 2 the facilities being removed by order 1 should not be considered. However, if due date X is later than due date Y, then all facilities must be considered in processing order 2.

With these two considerations, a list of facilities is prepared for processing by the connectivity algorithm. After connectivity is determined for these facilities, those facilities that were excluded are added to the end of the list of facilities that were placed in connectivity order.

4.3.2 The algorithm itself

The first step performed by the algorithm is to identify the equipment types that have been presented to it. It then proceeds to calculate the connection weights for all the edges of the graph describing the circuit using the considerations outlined in Section 4.1. These weights are stored in a weight table. Next the actual MWST processing begins. This is facilitated by updating a "working" table. Each row of the table contains the following information: facility, count of connections to the facility, available outward connections, lowest connection cost, and the facility connected by the "lowest connection cost" edge. Also, as the algorithm proceeds, a third table, the connection list, is created. The connection list table maintains a list of the edges selected for the MWST.

The "working" table is populated as follows: each input facility is placed into the table. Initially, the count of all connections to the facility is set to zero for each facility. The available outward connections for each facility are determined based on the considerations described in item 2 of Section 4.2. The lowest cost connection and the corresponding facility are determined by scanning the weight table for each facility. In case of a tie the first edge encountered in the weight table is chosen for inclusion in the working table.

Now the first facility to be placed in the circuit must be chosen. If there is an OE in the working table, it is chosen as the first facility; otherwise the first facility in the working table is chosen. The first facility and the facility it is connected to in the working table are placed in the connection list.

In the following, the first facility is taken as a facility appearing in the connection list. While there are still facilities that have not been connected to the circuit, the following instructions are repeated:

1. Choosing among the facilities already in the circuit (i.e., in the connection list), find the facility in the working table with the lowest cost connection. In case of a tie take the facility that appears first in

the working table. The facility connected by the lowest cost connection edge will be referred to as the "new facility"; the original facility will be called the "old facility".

2. If the number of connections to the new facility is not zero, this edge cannot be part of the MWST or else a cycle would be formed. Skip to instruction 8 below.

3. Add this connection to the connection list.

4. Increment the number of connections for the two facilities.

5. If the circuit contains an OE, decrement the number of available outward connections for the old facility unless both facilities are BLs. If both facilities are BLs, add one to the cost of all edges in the weight table that emanate from the old BL. This will reduce the maximum number of connections made at one bridge point, as explained in Section 4.2.

6. If the circuit contains an OE, and if either the old or the new facility (or both) have zero outward connections available, change the costs in the weight table for all edges emanating from such a node to a maximum value.

7. If the circuit does not contain an OE, and the old facility has two or more connections, add one to the cost of all edges in the weight table that emanate from the old facility node.

8. Change the cost of the edge in the weight table that connects the old and the new facility to a maximum value.

9. Reestablish the working table based on the new weight table costs.

We may now assume that all facilities have been placed in the connection list. (Note that if there are N facilities to be connected, there will be N-1 entries in the connection list so that the end of the algorithm is readily detected.) Now the connection list must be converted to a linear list. A tree will be described by a linear list that enumerates each branch, one after another. The beginning of a new branch is detected by the repetition of a facility that already appears higher up on the list (the branch point).

The algorithm for creating the linear list makes use of the working table left over from the MWST algorithm and the connection list. The algorithm

1. Searches the working table (in reverse order) until a facility is found with only one connection. This facility is one end of a branch. It places the facility in the linear list.

2. Searches the connection list (in reverse order) for the facility just placed in the linear list. It places the facility connected to it in the linear list.

3. Decrements the connection count for both the old and the new facilities. It removes their connection from the connection list.

4. If the connection count for the new facility is greater than zero, it repeats Steps 2 and 3. If the connection count for the new facility is zero, the end of the current branch has been reached. It will then go to the next step.

5. For each facility already on the linear list, it determines the number of remaining connections in the working table. If all connections are zero, the linear list is complete. Otherwise, it selects the first facility encountered with a nonzero connection count.

6. Enters this facility in the linear list and proceeds to Step 2.

Note that the lists in Steps 1 and 2 are searched in reverse order so that the frame instructions are in a more "pleasing" sequence: line equipment first, then the first "leg", then the second "leg", etc.

The connectivity algorithm is now complete. The facilities that were excluded from consideration at the start of the algorithm can be added to the end of the list.

The steps just described are applied to a particular example in Appendix C.

V. ACKNOWLEDGMENTS

As we mentioned earlier, the basic idea of applying the MWST algorithm to the problem of determining circuit connectivity in COSMOS is due to H. L. York, who also programmed the original version. The additional modifications for circuits containing an OE were conceived of and designed by J. B. Sharpless. The algorithm was rewritten by E. W. Merrill, who worked under Sharpless' direction.[5] The current "owner" of the program is D. P. Bates.

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APPENDIX A

COSMOS Service Order Language

When COSMOS is ready to accept a command, it will print a prompt (%). Immediately preceding the prompt character two alphanumeric characters are printed. These two characters represent the wire center with whose facilities the user wishes to work. The wire center is identified by the user at log-in time.

After the prompt letters have been printed the user can enter the transaction name. All service orders are initiated in COSMOS through the transaction SOE (Service Order Establishment). Particular inputs to SOE are established on separate lines, as many as are needed to specify the order.

The first character of the first input line must be an H (standing for header). The remainder of the line contains general data pertaining to the order. Typical data items that appear on this line are the order number (identified by the prefix ORD), the order type (OT), and the due date (DD). The prefix-data groupings are separated by a vergule (/). This applies to all line types, not just to the H line.

If all the data do not fit on the first H line, they may be continued on subsequent H lines. Once all the header data have been entered, facilities to be connected on the order are entered on a line (or lines) whose first character is I (standing for "in"). Facilities to be disconnected by the order are entered on a line (or lines) whose first character is O (standing for "out"). Typical data items that appear on I or O lines are Cable Pair (CP), Telephone Number (TN), Office Equipment (OE), Universal Service Order Code (US), features (FEA), Telephone Number Exchange code (NNX), and Resistance Zone (RZ).

In the case of facilities that are automatically assigned by COSMOS, the facility prefix may be followed by a question mark (?). This is a signal to COSMOS to assign the facility automatically. For example, if COSMOS is to select a telephone number somewhere on the I line the construction

```
I ····· /TN ?/ ····
```

should appear. However, some wire centers contain several different switching entities. To distinguish among them the user is instead required to specify the exchange code. In this case automatic telephone number selection is triggered by the input.

```
I ···· /NNX 351/ ·····
```

When all I and/or O lines have been input the user types a "." on a single line. At this point processing of the order commences. It should also be noted that as each line is entered, rudimentary checks are performed. When this processing is completed COSMOS prints an underscore (_) as a prompt to indicate that the next line can be processed.

APPENDIX B

An Example of Automatic Connectivity Determination

In this case COSMOS will be asked to process order number NAS0789. This is a new connect order (OT NC) and has a due date of August 1, 1981. The exchange code is 111 and COSMOS is to assign the line equipment with a Universal Service Order Code (US) of 1FR and features (FEA) consisting of *Touch-Tone** service (T), nonsleeve lead (N), nonessential (N), and loop start (L). Four cable pairs are to be assigned to the order—pairs 980, 981, 982, and 983 in cable 4. The resistance zones of these pairs are 22, 11, 12, and 13, respectively. A parameter is maintained in the database to indicate whether bridge lifters are needed. If any one of these resistance zones exceeds this parameter, then all pairs will be assigned bridge lifters. In this case the parameter is set to 18, a value exceeded by the resistance zone of the first pair.

The input and the SOE response is as follows:

```
90% SOE
H ORD NAS0789/OT NC/DD 8-1-81
_I NNX 111/OE ?/US 1FR/FEA TNNL
_I CP 4-980/RZ 22
_I CP 4-981/RZ 11
_I CP 4-982/RZ 12
_I CP 4-982/RZ 13

_.
SO000122
ORD NAS0789
IN:   CP   4-0980
IN:   CP   4-0981
IN:   CP   4-0982
IN:   CP   4-0983
IN:   OE   000-007-401
IN:   TN   111-1096
IN:   BL   49
IN:   BL   51
IN:   BL   50
IN:   BL   52
IN:   TP   CM11-0107
IN:   TP   CM11-0304
IN:   TP   CM11-0305
IN:   TP   CM11-0306
IN:   TP   CM11-0307
**TRANSACTION COMPLETED
90%
```

The string "SO000122" immediately following the period is the record number in the service order file selected by COSMOS to hold information about the order. This record number is useful in the event

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* Registered service mark of AT&T.

the order is not established properly and manual corrective action is required.

The rest of the SOEs output are COSMOS assignments. First the four cable pairs are echoed back. These are followed by eleven automatically assigned facilities: an office equipment, a telephone number, four bridge lifters, and five tie pairs. (The tie pairs are needed to interconnect the bridge lifters and the office equipment and cable pairs since the bridge lifters are terminated on a different frame.) The facilities are listed by SOE in the order in which they are assigned. This is not the connectivity order.

To show the connectivity order the frame output report must be executed. This is the report used by telephone company personnel to actually wire the circuit in the central office. The report itself is in a lengthy format for ease of reading. Instead of reproducing the entire report here, only excerpts that show connectivity are listed below:

```
                  ⋮
LINE EQP IN          000-007-401
                  ⋮
TIE PAIR IN          CM11-0107
                  ⋮
MISC EQP IN          BL 49
                  ⋮
TIE PAIR IN          CM11-0304
                  ⋮
CABLE PR IN          4-0980
                  ⋮
*MISC EQP IN         BL 49
                  ⋮
MISC EQP IN          BL 51
                  ⋮
TIE PAIR IN          CM11-0305
                  ⋮
CABLE PR IN          4-0981
                  ⋮
*MISC EQP IN         BL 51
                  ⋮
MISC EQP IN          BL 50
                  ⋮
TIE PAIR IN          CM11-0306
                  ⋮
CABLE PR IN          4-0982
                  ⋮
*MISC EQP IN         BL 50
                  ⋮
```

MISC EQP IN BL 52
⋮
TIE PAIR IN CM11-0307
⋮
CABLE PR IN 4-0983
⋮

Note the first leg of the circuit—extending from OE 000-007-401 to CP 4-0980. The beginning of the next leg is indicated by the asterisk (*) and the repetition of the facility BL 49. This is the first bridge point. This leg extends down to CP 4-0981. Now BL 51 is shown as the next bridge point. Notice that BL 49 is not the bridge point for all legs. This is the effect of the algorithm described in Section 4.2 to minimize the maximum number of legs emanating from a single bridge point. The remaining two legs extend from BL 51 to CP 4-0982 and BL 50 to CP 4-0983.

APPENDIX C

An Example of the Algorithm's Execution

The algorithm described in Section 4.3.2 will be followed in detail for a particular set of facilities: two bridge lifters (BL1 and BL2), two cable pairs (CP1 and CP2), and one line equipment (OE). The first step is to determine the connection weights for all the edges of the graph. These weights are determined from Table IV. Note that the diagonal terms are given a weight of 100, since a facility cannot be connected to itself.

Step 1—Weight table

	BL1	BL2	CP1	CP2	OE
BL1	100	5	60	60	35
BL2	5	100	60	60	35
CP1	60	60	100	70	65
CP2	60	60	70	100	65
OE	35	35	65	65	100

In this particular case CP1 and BL1 have been assigned a DPA value of " " (i.e., a blank) and CP2 and BL2 have been assigned a DPA value of "999" by a previously invoked load module of SOE. Thus those edges connecting facilities in different "legs" (i.e., BL1-CP2 and BL2-CP1) have their weights changed to a maximum value. (In the next and in all following tables entries that have changed from the previous table are enclosed in parentheses.)

Step 2—Weight table

	BL1	BL2	CP1	CP2	OE
BL1	100	5	60	(100)	35
BL2	5	100	(100)	60	35
CP1	60	(100)	100	70	65
CP2	(100)	60	70	100	65
OE	35	35	65	65	100

Now the working table is constructed. Each facility has an entry and the connection count is initially set to zero. The Available Outward Connections (AOC) equal zero for the two metallic facilities (CP1 and CP2), equals one for the two conditioning facilities (BL1 and BL2), and equals one for the OE based on the formula:

$$\begin{aligned} \text{AOC} &= \#\ \text{metallic facilities} - \#\ \text{BL's} + 1 \\ &= 2 - 2 + 1 = 1 \end{aligned}$$

The lowest connection cost and corresponding facility are obtained from the weight table.

Step 3—Working table

Facility	Connection Count	Available Outward Connections	Lowest Connection Cost	Corresponding Facility
BL1	0	1	5	BL2
BL2	0	1	5	BL1
CP1	0	0	60	BL1
CP2	0	0	60	BL2
OE	0	1	35	BL1

Since the circuit contains an OE, this facility is chosen first and placed on the connection list.

Step 4—Connection list

OE-BL1

The number of connections to the OE and BL1 are incremented (Step 6, working table). The number of AOC to the old facility (the OE) is decremented (Step 6, working table). The old facility now has zero AOC so the weight of all edges emanating from it is changed to a maximum value (Step 5, weight table). Finally, the working table is modified due to changes in the weight table.

Step 5—Weight table

	BL1	BL2	CP1	CP2	OE
BL1	100	5	60	100	(100)
BL2	5	100	100	60	(100)
CP1	60	100	100	70	(100)
CP2	100	60	70	100	(100)
OE	(100)	(100)	(100)	(100)	100

Step 6—Working table

Facility	Connection Count	Available Outward Connections	Lowest Connection Cost	Corresponding Facility
BL1	(1)	1	5	BL2
BL2	0	1	5	BL1
CP1	0	0	60	BL1
CP2	0	0	60	BL2
OE	(1)	(0)	(100)	BL1

If we choose among the facilities already in the connection list (OE and BL1), the one with the lowest connection cost in the weight table is the first entry. The edge BL1-BL2 is added to the connection list.

Step 7—Connection list
OE-BL1
BL1-BL2

Since the connection count to BL2 is zero, this is an acceptable choice. The number of connections to BL1 and BL2 are incremented (Step 9, working table). However, the number of AOC to the old facility (BL1) is not decremented, since both facilities are BLs. Instead, one is added to the cost of all edges that emanate from BL1 (Step 8, weight table). Since neither the old nor the new facility has zero AOC, the edges emanating from these nodes do not have their weights set to 100. However, the BL1-BL2 weights are set to the maximum value (Step 8, weight table). Finally, the working table is modified according to changes in the weight table.

Step 8—Weight table

	BL1	BL2	CP1	CP2	OE
BL1	(101)	(100)	(61)	(101)	(101)
BL2	(100)	100	100	60	100
CP1	(61)	100	100	70	100
CP2	(101)	60	70	100	100
OE	(101)	100	100	100	100

Step 9—Working table

Facility	Connection Count	Available Outward Connections	Lowest Connection Cost	Corresponding Facility
BL1	(2)	1	(61)	(CP1)
BL2	(1)	1	(60)	(CP2)
CP1	0	0	(61)	BL1
CP2	0	0	60	BL2
OE	1	0	100	(BL2)

If we choose among the facilities already on the connection list (OE, BL1, and BL2), the one with the lowest connection cost in the weight table is the BL2 entry. Therefore, BL2-CP2 is added to the connection list.

Step 10—Connection list

OE-BL1
BL1-BL2
BL2-CP2

Since the connection count to CP2 is zero, this is an acceptable choice. The number of connections to BL2 and CP2 are incremented (Step 12, working table). The number of AOC to the old facility (BL2) is decremented (Step 12, working table). Since both BL2 and CP2 now have zero AOC, the weights for all edges emanating from BL2 and CP2 are set to the maximum value (Step 11, weight table). Finally, the working table is modified according to changes in the weight table.

Step 11—Weight table

	BL1	BL2	CP1	CP2	OE
BL1	101	100	61	101	101
BL2	100	100	100	(100)	100
CP1	61	100	100	(100)	100
CP2	101	(100)	(100)	100	100
OE	101	100	100	100	100

Step 12—Working table

Facility	Connection Count	Available Outward Connections	Lowest Connection Cost	Corresponding Facility
BL1	2	1	61	CP1
BL2	(2)	(0)	(100)	(BL1)
CP1	0	0	61	BL1
CP2	(1)	0	(100)	BL2
OE	1	0	100	BL2

If we choose among the facilities already on the connection list (OE, BL1, BL2, CP2), the one with the lowest connection cost in the weight table is the BL1 entry. Therefore, BL1-CP1 is added to the connection list.

Step 13—Connection list

OE-BL1
BL1-BL2
BL2-CP2
BL1-CP1

Since the connection count to CP1 is zero, this is an acceptable choice. The number of connections to BL1 and CP1 are incremented (Step 15, working table). The number of AOC to the old facility (BL1) is decremented (Step 15, working table). Since both BL1 and CP1 now have zero AOC, the weight of all edges emanating from BL1 and CP1 are set to the maximum value (Step 14, weight table). Finally, the working table is modified according to changes in the weight table.

Step 14—Weight table

	BL1	BL2	CP1	CP2	OE
BL1	101	100	(100)	101	101
BL2	100	100	100	100	100
CP1	(100)	100	100	100	100
CP2	101	100	100	100	100
OE	101	100	100	100	100

Step 15–Working table

Facility	Connection Count	Available Outward Connections	Lowest Connection Cost	Corresponding Facility
BL1	(3)	(0)	(100)	(BL2)
BL2	2	0	100	BL1
CP1	(1)	0	(100)	BL1
CP2	1	0	100	BL2
OE	1	0	100	BL2

The algorithm is completed when the connection list contains N-1 entries, where N equals the number of facilities. In this case N-1 = 4 and so all connections have been obtained. The remainder of the algorithm transforms the connection list to a linear list.

Initially, the connection count for each facility, the connection list, and the linear list are as shown in Step 16. Search the connection count (from the bottom) to find a facility with a connection count of one. In this case the facility found is the OE. Next, search the connection list (from the bottom) to find a corresponding facility. In this case the facility is BL1. Place these two facilities on the linear list and decrement the connection count for each.

Step 16—Linear list

Facility	Connection Count	Connection List	Linear List
BL1	3	OE-BL1	
BL2	2	BL1-BL2	
CP1	1	BL2-CP2	
CP2	1	BL1-CP1	
OE	1		

Applying this algorithm results in the table shown in Step 17. Since the connection count for BL1 is greater than zero, search the connection list (from the bottom) to find another entry for BL1. The connection BL1-CP1 is found, so CP1 is added to the linear list, and the connection count for both BL1 and CP1 are decremented.

Step 17—Linear list

Facility	Connection Count	Connection List	Linear List
BL1	2	------	OE
BL2	2	BL1-BL2	BL1
CP1	1	BL2-CP2	
CP2	1	BL1-CP1	
OE	0		

The table now changes to what is shown in Step 18. Since the connection count for CP1 is zero, the connection count list is again searched (from the bottom) but only for facilities on the linear list (i.e., OE, BL1, CP1) for an entry with a nonzero connection count. The entry found is BL1. Searching the connection list for a corresponding facility results in the addition of the BL1-BL2 connection to the linear list. The connection count of each of these facilities is therefore decremented.

Step 18—Linear list

Facility	Connection Count	Connection List	Linear List
BL1	1	------	OE
BL2	2	BL1-BL2	BL1
CP1	0	BL2-CP2	CP1
CP2	1	------	
OE	0		

The table now changes to what is shown in Step 19. Since the connection count for BL2 is greater than zero, search the connection list to find another entry for BL2. The connection BL2-CP2 is found so CP2 is added to the linear list and the connection count for both BL2 and CP2 are decremented.

Step 19—Linear list

Facility	Connection Count	Connection List	Linear List
BL1	0	------	OE
BL2	1	------	BL1
CP1	0	BL2-CP2	CP1
CP2	1	------	BL1
OE	0		BL2

The table now changes as shown in Step 20. Since all connection counts are zero, the algorithm terminates.

Step 20—Linear list

Facility	Connection Count	Connection List	Linear List
BL1	0	------	OE
BL2	0	------	BL1
CP1	0	------	CP1
CP2	0	------	BL1
OE	0	------	BL2
			CP2

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