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Practical Design Considerations for Coupled-Single-Amplifier-Biquad Active Bandpass Filters

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This paper presents a synthesis method and practical design considerations for the Coupled-Single-Amplifier-Biquad (CSAB) realization of all-pole symmetrical bandpass (BP) filters. The CSAB topology consists of a cascade of second-order SAB bandpass sections, together with negative feedback around adjacent sections. A straightforward procedure that leads to the block diagram representation of the CSAB is shown. Explicit design formulas are given for the optimum element values of the Deliyannis-Friend SAB bandpass section, as well as for the feedback resistors. This CSAB design offers improved performance over the cascade SAB approach without using additional operational amplifiers. Also described are the effects on the filter response due to finite amplifier gain, capacitor dissipations, noninfinite pole-Q sections, and their compensation techniques. These are followed by discussions on maximizing the filter dynamic range and tuning.

I. INTRODUCTION

It is commonly known that in the realization of high-order active filters, properly designed multiple-loop-feedback topologies offer far superior sensitivity performance than the approach of cascading biquadratic filter blocks.[1–6] The multiple-loop-feedback structure is particularly useful in the design of bandpass filters, where reduced sensitivity design is most often needed. Compared with the cascade biquad approach, two drawbacks are usually attributed to these topologies,

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namely, the more complicated design procedures and the use of more op amps. However, for the class of all-pole geometrically symmetrical Bandpass (BP) filters, these drawbacks do not exist for a particular multiple-loop-feedback structure commonly referred to as leap-frog, active-ladder, or coupled-biquad.[7–13] The coupled-biquad topology consists of a cascade of second-order sections together with negative feedback around adjacent biquad sections.

Most of the existing coupled-biquad descriptions assumed multiple op-amp biquads, but details on the use of Single-Amplifier Biquads (SABs) are scarce. Our discussion here focuses on the use of SAB and is further restricted to voice-frequency applications. Note that the Coupled-Single-Amplifier-Biquad (CSAB) topology described here requires n op amps for a $2n$-order BP filter.

The first part of this paper presents a general design method for the CSAB realization of all-pole symmetrical BP filters. A straightforward procedure is given that leads to the block diagram representation of the CSAB. Each of the second-order sections is then implemented by the Deliyannis-Friend[14] SAB configuration. Optimum element values for these SABs are computed according to Fleischer's results.[15] The second part of the paper discusses the effects on the filter response due to finite op-amp gain, capacitor dissipations, noninfinite pole-Q sections, and their compensation techniques. These are followed by discussions on maximizing the filter dynamic range and tuning.

II. CSAB DESIGN PROCEDURE

This section presents a straightforward design procedure for the CSAB realization of all-pole symmetrical BP filters. Optimum design equations are given for the Deliyannis-Friend SAB bandpass section and the feedback resistors.

2.1 Block diagram representation of CSAB configuration

The starting point for the CSAB realization of an all-pole symmetrical BP filter, e.g., a Bessel-, Butterworth-, or Chebychev-type BP filter, is its normalized low-pass (LP) prototype ladder configuration. The ladder configuration is readily available from many existing handbooks and is shown in Fig. 1.

Let ω_0 = center frequency of the BP filter (in rad/s)

B = passband bandwidth of the BP filter (in rad/s)

A block diagram representation of the CSAB topology for the BP filter is given in Fig. 2, where the coupled biquadratic transfer functions,

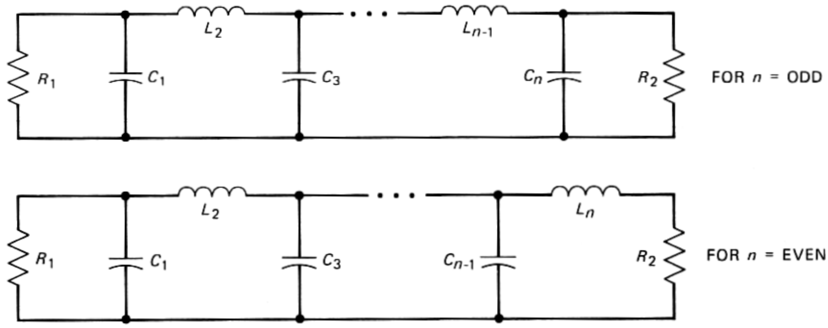

Fig. 1—LP prototype ladder configuration.

$T_i(s)$ and $T_i'(s)$, are related to the element values of the LP ladder by the following equations:

$$T_1(s) = K \cdot \frac{\frac{R_1 + R_2}{R_2} \cdot \frac{B}{R_1 C_1} s}{s^2 + \frac{B}{R_1 C_1} s + \omega_0^2} \tag{1}$$

$$T_1'(s) = \frac{\frac{B}{C_1} s}{s^2 + \frac{B}{R_1 C_1} s + \omega_0^2} \tag{2}$$

$$T_i(s) = T_i'(s) = \frac{\frac{B}{X_i} s}{s^2 + \omega_0^2} \qquad i = 2, 3, \cdots, n-1 \tag{3}$$

$$\text{and} \quad X_i = \begin{cases} L_i \text{ for } i \text{ even} \\ C_i \text{ for } i \text{ odd} \end{cases}$$

$$T_n(s) = \begin{cases} \dfrac{\frac{B}{C_n} s}{s^2 + \frac{B}{R_2 C_n} s + \omega_0^2} & \text{for} \quad n \text{ odd} \\[2ex] \dfrac{\frac{B}{L_n} s}{s^2 + \frac{B R_2}{L_n} s + \omega_0^2} & \text{for} \quad n \text{ even,} \end{cases} \tag{4}$$

where K determines the overall gain of the filter. For unity (0 dB) voltage gain, set $K = 1$.

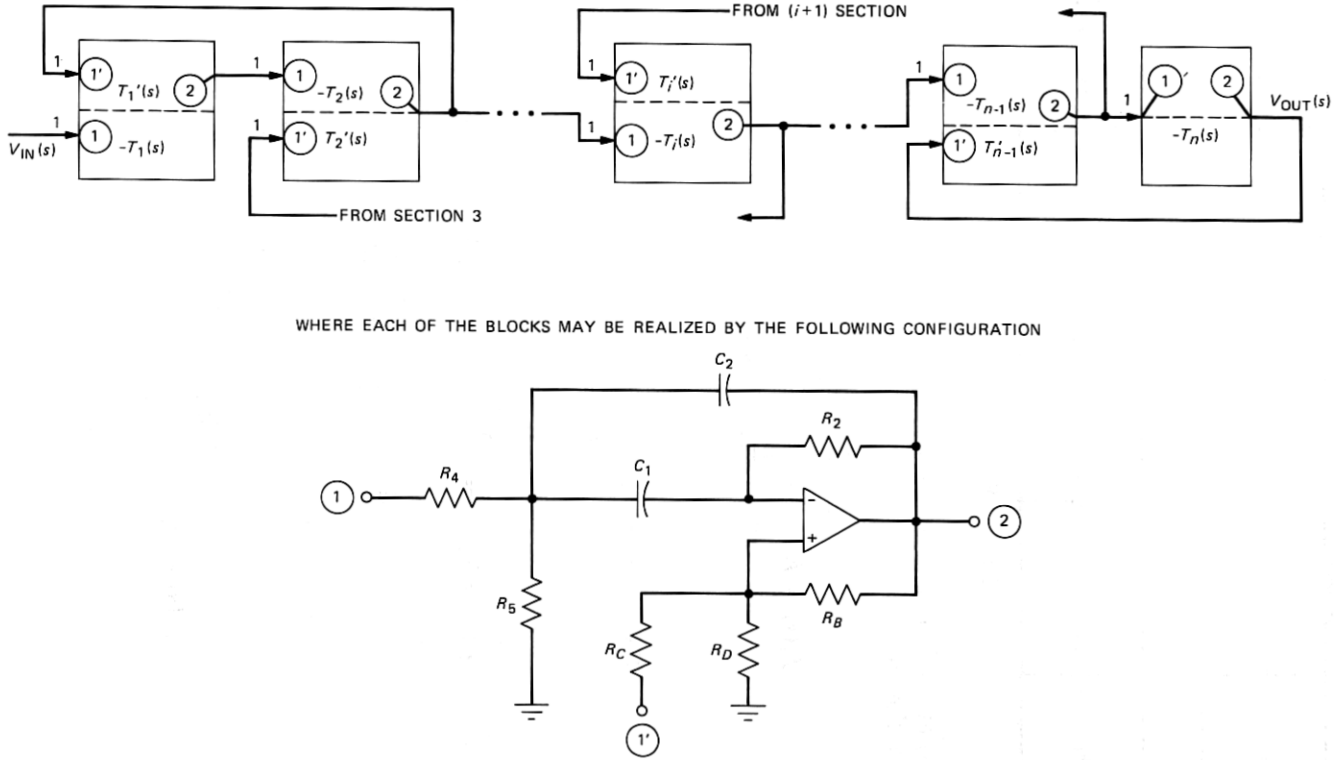

Fig. 2—Coupled SAB configuration for all-pole symmetrical BP filters.

The above derivation is straightforward and is omitted here. For the three-section case, discussions can be found in pages 726 to 729 of Ref. 3 and pages 348 to 351 of Ref. 6. Note that in Fig. 2, the Deliyannis-Friend SAB configuration is also shown. Besides its good sensitivity properties, this particular SAB can simultaneously realize both the forward and the feedback paths as required by Fig. 2.

One final step in the CSAB realization is to obtain the element values for the individual SAB blocks. The optimum formulas are given in the next section.

2.2 Optimum CSAB element values

For each of the second-order blocks shown in Fig. 2, let the forward and feedback voltage transfer functions be represented by:

$$\frac{V_2(s)}{V_1(s)} = -T_i(s) = \frac{-n_1 s}{s^2 + d_1 s + d_0} \tag{5}$$

and

$$\frac{V_2(s)}{V_1'(s)} = T_i'(s) = \frac{n_1' s}{s^2 + d_1 s + d_0}. \tag{6}$$

As shown in Fig. 2 and eqs. (1) to (3), the internal sections, i.e., $i = 2, 3, \cdots, n-1$, have an infinite pole-Q value or $d_1 = 0$. For improved filter performances, a later section suggests the use of a very high-Q value, say several hundreds, instead of the infinite value. This corresponds to the use of a small value for d_1 in eqs. (5) and (6). The high but fixed Q-value can be designed into the CSAB configuration by the familiar predistortion technique.

Element values are first obtained for the forward transfer function, $T_i(s)$, by the formulas given in Ref. 14, where the conductance value of a fictitious resistor, R_1, is first computed and the values of C_1, C_2, R_A, and R_B can be conveniently chosen such that

$$G_1 = \frac{C_2 R_B}{2R_A}\left\{-d_1 + \left[d_1^2 + 4d_0\left(1 + \frac{C_1}{C_2}\right)\frac{R_A}{R_B}\right]^{1/2}\right\} \tag{7}$$

$$R_2 = \frac{G_1}{C_1 C_2 d_0} \tag{8}$$

$$R_4 = \frac{1 + \dfrac{R_A}{R_B}}{n_1 C_2} \tag{9}$$

$$R_5 = \frac{1}{G_1 - G_4} \tag{10}$$

$$R_B = \text{arbitrary [can be chosen according to eq. (15)]} \quad (11)$$

$$R_D = R_A \quad (12)$$

$$R_C = \text{infinite.} \quad (13)$$

As shown by Fleischer,[15] the value of R_B or the ratio of R_A/R_B should be chosen so as to minimize the overall SAB variability (sensitivity) due to the combined active and passive elements variations. His results are given below:*

$$Q_0 = \left[|A(s_0)|^2 \frac{8\sigma_R^2 + \sigma_c^2}{8\sigma_{A(s0)}^2} \right]^{1/4} \quad (14)$$

$$\frac{R_A}{R_B} = \frac{C_2}{C_1 + C_2} \frac{1}{Q_0} \left(\frac{1}{Q_0} - \frac{d_1}{\sqrt{d_0}} \right), \quad (15)$$

where $A(s)$ represents the gain of the op amp, σ_R^2, σ_C^2, and $\sigma_{A(s)}^2$ correspond to the variances of $\Delta R/R$, $\Delta C/C$, and $\Delta A(s)/A(s)$, respectively, and s_0 is the pole location of the particular SAB transfer function.

The values of R_B and R_C are next modified according to the following formulas to implement the feedback transfer function as well [see Fig. 2 and eq. (6)]:†

$$K_F = \frac{n_1'}{\left(\frac{G_1 + G_2}{C_2} + \frac{G_2}{C_1} \right)} \quad (16)$$

$$\text{(new) } R_C = \frac{1 - K_F}{K_F} R_D \quad (17)$$

$$\text{(new) } R_B = \frac{R_B R_C}{R_C + R_D} \quad (18)$$

$$R_D = R_A \text{ (as before).} \quad (19)$$

In summary, the optimum element values can be computed from eqs. (14), (15), (7) to (12), and (16) to (18).

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* Equations (57), (61), (13b), (2) and (7) of Ref. 15 were used. For the case of $C1 = C2$ and single-pole op-amp characteristics, simpler formulas are given in page 323 of Ref. 2.

† The derivation is given in the appendix. Note that, as shown in pages 362 to 364 of Ref. 6, the feedback transfer function realized is not exactly a BP function but has negative and real transmission zeros. However, in the vicinity of the pole frequency, where the effect of feedback is of interest, the function behaves like a BP function. In any event, the error introduced is negligible.

III. PRACTICAL DESIGN CONSIDERATIONS

The preceding section describes one practical design consideration, namely, that component statistics are used to obtain an optimum design parameter, R_A/R_B, for the individual SAB blocks. This section further discusses some important factors that cause the SAB or CSAB response to deviate from its ideal characteristic. Compensations in the form of predistortion techniques are described. These compensations can be achieved by modifying the pole locations of the individual second-order blocks as given by eqs. (5) and (6). In a good design, they must be considered as part of the original design and optimum element values are to be computed from the modified transfer functions.

3.1 Nonideal op amp characteristics

For a given SAB realizing the transfer functions (5) and (6), the major effect of finite op-amp gain is to shift the desired pole location, or the roots of

$$s^2 + d_1 s + d_0 = s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = (s - s_0)(s - s_0^*) \tag{20}$$

from s_0 to $s_0 + \Delta s_0$. Fleischer[15] has shown that this deviation is approximately given by

$$\Delta s_0 = -\left(1 + \frac{R_A}{R_B}\right) \frac{1}{A(s_0)} \frac{D_2(s_0)}{s - s_0^*}, \tag{21}$$

where

$$D_2(s) = s^2 + \left(\frac{C_1 + C_2}{C_1 C_2} \frac{1}{R_2} + \frac{G_1}{C_2}\right) s + \frac{G_1}{R_2 C_1 C_2}. \tag{22}$$

In the actual design, one can apply the negative of this shift to the nominal transfer function. Equivalently, as shown in pages 415 to 416 of Ref. 2, the shift in Δs_0 can be represented by:

$$\frac{\Delta\omega_0}{\omega_0} = Re\left(\frac{\Delta s_0}{s_0}\right) \tag{23}$$

$$\frac{\Delta Q}{Q} = -2Q Im\left(\frac{\Delta s_0}{s_0}\right). \tag{24}$$

Hence, instead of using the design parameters ω_0 and Q in eq. (20), one can use the modified parameters $\omega_0' = \omega_0 - \Delta\omega_0$ and $Q' = Q - \Delta Q$.

As pointed out by Fleischer, eq. (21) shows that the fractional change in the complex pole location is inversely proportional to the magnitude of the amplifier gain at the pole frequency. Hence, the use of a two-pole, one-zero compensated op amp would provide at least an order of magnitude smaller pole shift magnitude than that obtained from using a single-pole compensated op amp in most of the audio frequency band.* For many practical voice-frequency-band applications, this pole shift is quite small for the former compensation and its effect can often be ignored.

3.2 Nonideal capacitor characteristics

A nonideal capacitor is usually associated with a finite dissipation factor or tan δ. It is commonly represented by introducing a resistor (with conductance G_i) in parallel with the capacitor C_i, where

$$G_i = \omega_0 C_i \tan \delta_i$$

and ω_0 corresponds to the pole frequency of the SAB block, since we are particularly interested in the variations of the transfer function for frequencies near ω_0.

As with the op-amp finite gain, the major effect of the capacitor dissipation factor is to cause a shift in the desired pole locations, which again can be compensated for. Weyten's approach[16] is described here. For a two-capacitor biquad section realizing eq. (5), Weyten has shown that

$$\frac{\Delta d_1}{d_1} = Q(\tan \delta_1 + \tan \delta_2) \qquad (25)$$

$$\frac{\Delta d_0}{d_0} = \tan \delta_1 \tan \delta_2 - \frac{1}{Q}\,(S^{d_1}_{C_1} \tan \delta_2 + S^{d_1}_{C_2} \tan \delta_1), \qquad (26)$$

where

$$-S^{d_1}_{C_i} = 1/2 + S^{Q}_{C_i} \qquad (27)$$

and the coefficients in eq. (20) move from d_i to $d_i + \Delta d_i$. For the SAB under consideration (Fig. 2), Ref. 15 gives

$$S^{Q}_{C_1} = -S^{Q}_{C_2} = \frac{C_2}{C_1 + C_2}\frac{Q}{Q_0} - \frac{1}{2}, \qquad (28)$$

where Q_0 is as given before.

Hence, instead of using the design parameters d_1 and d_0 in eqs. (5) and (6), the modified parameters $d_1' = d_1 - \Delta d_1$ and $d_0' = d_0 - \Delta d_0$ can be used. Note that the fractional change of d_0 is usually very small and can practically be ignored. Alternately, eqs. (25) and (26) can be rewritten as

$$\frac{\Delta Q}{Q} \simeq -\frac{\Delta d_1}{d_1} = -Q(\tan \delta_1 + \tan \delta_2) \qquad (29)$$

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* See for example, pages 84 to 85 of Ref. 6.

$$\frac{\Delta\omega_0}{\omega_0} \simeq \frac{1}{2}\tan\delta_1\tan\delta_2 - \frac{1}{2Q}\,(S^{d_1}_{C_1}\tan\delta_2 + S^{d_1}_{C_2}\tan\delta_1) \qquad (30)$$

since the variations of ω_0 are usually very small. The decrease in pole-Q value due to capacitor dissipation factors is usually appreciable and should be compensated for. As an illustration, for a medium-Q BP section, say $Q = 20$, and for a capacitor dissipation factor of 0.0015, such as that encountered with thin-film capacitors, this fractional change in the pole-Q value is 6 percent.

3.3 Infinite pole-Q sections

As we see in Fig. 2, all of the second-order sections except the first and the last have an infinite pole-Q value, i.e., $d_1 = 0$ in eq. (5). Note that the negative feedbacks in the coupled-biquad configuration move these poles away from the $j\omega$-axis and into the desired pole locations. Except for very high-Q BP filters, realization of the infinite pole-Q sections in the CSAB configuration is not very critical as long as the value of these pole-Qs is high, say several hundreds. The effects of using a high but finite pole-Q value in these sections are a lower overall gain of the filter and a reduction of the effective passband bandwidth.[17] A decrease in gain is easily compensated for in active filter design, while the reduced passband bandwidth may usually be absorbed in the original design margin.

On the other hand, a closer approximation to the desired response is obtained if each of the internal sections is a priori designed to have a high but finite pole-Q value. This value can be chosen to be the highest and practically realizable pole-Q value, say Q_M. For the SAB, this is in the order of a few hundreds. Note that in the actual realization, these pole-Q values will deviate a great deal (higher or lower than Q_M). Computer simulations have shown that the overall circuit variability is smaller for the finite internal pole-Q design than the design with infinite pole-Qs.

Having presented the virtues of noninfinite internal pole-Q sections, we show now how this can be achieved with the classical predistortion technique. If we refer back to Section 2.1, instead of starting with the ladder configuration, we see that the overall transfer function for the normalized LP prototype is used (again this is available from many handbooks). A shift α is introduced to the complex frequency variable s, $s = p - \alpha$, where p represents the new complex frequency variable, and

$$\alpha = \frac{\omega_0}{BQ_M}. \qquad (31)$$

In eq. (31) all variables are as defined before. The new transfer function in p is used to realize the ladder configuration shown in Fig. 1. With the following modifications to eqs. (1) through (4), where R_1' and R_2' are used in place of R_1 and R_2, respectively, the CSAB configuration is again as shown in Fig. 2:

$$R_1' = \frac{1}{\dfrac{1}{R_1} + \alpha C_1} \tag{32}$$

$$R_2' = \begin{cases} \dfrac{1}{\dfrac{1}{R_2} + \alpha C_n} & \text{for} \quad n \text{ odd} \\[2ex] R_2 + \alpha L_n & \text{for} \quad n \text{ even} \end{cases} \tag{33}$$

$$T_i(s) = T_i'(s) = \frac{\dfrac{B}{X_i} s}{s^2 + \dfrac{\omega_0}{Q_M} s + \omega_0^2} \qquad i = 2, 3, \cdots, n-1. \tag{34}$$

In addition, the value of K in eq. (1) must be modified to obtain the desired overall gain of the filter. This value is easily determined by computing the gain of the ladder in Fig. 1 after replacing each inductor L_i with a resistor of value $\alpha^* L_i$ and each capacitor C_i with a resistor of conductance $\alpha^* C_i$.

The ladder realization of the shifted LP prototype transfer function can be obtained by the classical synthesis technique, or with a filter synthesis program.[18] For best sensitivity performance, this ladder must correspond to the maximum power transfer design.[19]

3.4 Maximizing the filter dynamic range

A rule-of-thumb design procedure for maximizing the dynamic range of a high-order filter is to make the maximum voltage output level at the various amplifiers equal. Variations among the various amplifier outputs in the CSAB are generally much smaller than in the corresponding cascade SAB design. For many applications, one may find the CSAB design as obtained before to be satisfactory.

The maximum voltage output levels at the amplifiers usually occur at frequencies within the filter passband or the transition bands. A practical procedure to maximize the filter dynamic range is to evaluate the filter frequency responses (via a computer program) at each of the amplifier outputs and at a set of discrete frequencies chosen from the passband and transition bands. The maximum output values thus obtained are used to rescale the gain of each of the transfer functions given by eqs. (1) through (4). More formally, let

$$G_i = \text{Max}(V_i) - \text{Max}(V_{out}) \text{ in dB},$$

where V_i (in dB) is the voltage level at the output of the ith amplifier and Max(V_i) is the maximum value over the chosen set of frequencies.

Compute

$$H_i = 10^{-\frac{G_i}{20}} \qquad i = 1, 2, \cdots, n-1$$

with $H_0 = H_n = 1$.

Multiply each of the forward transfer functions, $T_i(s)$ by K_i, where

$$K_i = H_i/H_{i-1} \qquad i = 1, 2, \cdots, n,$$

and multiply each of the feedback transfer functions, $T_i'(s)$, by K_i', where

$$K_i' = H_i/H_{i+1} \qquad i = 1, 2, \cdots, n-1.$$

3.5 CSAB tuning

When the STAR realization[14] is used to implement the SAB, the manufacturing tuning procedure is to measure values of the two capacitors on the substrate and then compute the resistor values, based on these measurements, from the predistorted transfer function using the equations given in Section 2.2. The resistors are laser-trimmed to these values. The individual SAB blocks are then connected as shown in Fig. 2. In general, this procedure is sufficient for all practical purposes.

For extremely high-precision filter applications where functional tuning may be desirable, the SAB, like any other single-amplifier-biquad configuration, does not exhibit an orthogonal set of tuning parameters. However, when the desired tuning range is small, e.g., during final mop-up trimming, the following tuning sequence is suggested: For the forward BP transfer function, with R_C connected to ground, use R_4 to adjust for the gain at the pole frequency, R_5 for the pole-Q, and then R_2 for the pole frequency. The adjustments for the pole-Q and pole frequency can be monitored by the 45-degree phase-shift points and the 180-degree phase-shift point, respectively. Finally, with R_4 connected to ground, the feedback factor can be adjusted by R_C. As in any coupled-biquad configuration, the CSAB overall response is relatively insensitive to these feedback resistors.

IV. BP FILTERS WITH FINITE TRANSMISSION ZEROS

Extensions of the CSAB design to BP filters with finite transmission zeros, e.g., elliptic-type BP filters, are available. There are two approaches here. The first is to realize these transmission zeros within the individual SAB blocks;[11,12] however, their design procedures are rather complicated. A second approach is to form these transmission zeros by a weighted sum of the individual SAB BP sections (Fig. 2) with an additional summing amplifier (in a manner analogous to the feedforward technique described in Ref. 9). Simple design formulas exist for this purpose. The second approach is particularly useful and exhibits excellent sensitivity properties for low-order filters, say fewer than five sections.

V. CONCLUSIONS

A straightforward design procedure is given for the coupled-single-amplified-biquad realization of high-order, all-pole, symmetrical BP filters. Practical limitations and their compensation techniques are discussed. Many of these considerations, i.e., optimum choice of element values, nonideal op-amp and capacitor characteristics, are the same for the cascade or coupled designs. With this in mind, the CSAB design procedure is seen to be not more (if not less) complicated than the cascade design, since the individual second-order transfer functions are more readily computed.

The CSAB approach uses the same number of op amps as that of the cascade SAB approach. This number is equal to n for a $2n$-order BP filter and is approximately half the number required by the many inductance simulation techniques, e.g., the two-op-amp Generalized Impedance Converter (GIC) designs. Sensitivity performances of the coupled-biquad may be considered as the best among all the various multiple-loop-feedback topologies[20] and are far superior to the cascade biquad. These observations, together with the fact that the Deliyannis-Friend SAB, from a sensitivity point of view, is as good as any other circuit in the audio frequency band,[15] suggest that the CSAB described here should be the choice for the design of low to medium-high Q (say, $Q < 60$) BP filters in the audio frequency band.*

VI. ACKNOWLEDGMENT

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* Practical element values constraints and the lack of an orthogonal tuning sequence for the SAB usually limit its maximum realizable stable pole-Q to 30. Note that in the CSAB design, the maximum pole-Q value for the two end sections is usually less than or equal to one-half the maximum pole-Q of the overall design.

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APPENDIX

Derivation of the Element Values for the Feedback Transfer Function

With ideal op amp, the voltage transfer functions for the SAB circuit of Fig. 2 are given by:

$$\frac{V_2(s)}{V_1(s)} = -\frac{n_1 s}{s^2 + d_1 s + d_0} \tag{35}$$

and

$$\frac{V_2(s)}{V_1'(s)} = \frac{K_F(s^2 + Fs + d_0)}{s^2 + d_1 s + d_0}, \tag{36}$$

where

$$d_1 = \left(\frac{C_1 + C_2}{C_1 C_2}\right) G_2 - \frac{(G_4 + G_5)}{(G_C + G_D)} \frac{G_B}{C_2} \tag{37}$$

$$d_0 = \frac{(G_4 + G_5)G_2}{C_1C_2} \tag{38}$$

$$n_1 = \left(\frac{G_4}{C_2}\right)\left(1 + \frac{G_B}{G_C + G_D}\right) \tag{39}$$

$$K_F = \frac{G_C}{G_C + G_D} \tag{40}$$

$$F = \left(\frac{G_4 + G_5 + G_2}{C_2} + \frac{G_2}{C_1}\right). \tag{41}$$

The element values as given by eqs. (7) through (13) satisfy the forward transfer function, eqs. (35) and (37) through (39). Note that $G_C = 0$.

In the vicinity of the pole frequency, the feedback transfer function, eq. (36), is closely approximated by

$$\frac{V_2(s)}{V_1'(s)} \simeq \frac{n_1's}{s^2 + d_1s + d_0}, \tag{42}$$

where

$$n_1' = K_F \cdot F. \tag{43}$$

To realize the feedback transfer function, the value of G_C must be finite, say G_C'. If we let the value of G_B take on a new value, G_B', and, furthermore, let all the remaining elements take on the values as given before, then the forward and feedback transfer functions, Eqs. (35) and (37) through (43), can be simultaneously satisfied if the following two conditions are met:

$$\frac{G_B}{G_D} = \frac{G_B'}{G_C' + G_D} \tag{44}$$

and

$$K_F = \frac{n_1'}{F} = \frac{G_C'}{G_C' + G_D}. \tag{45}$$

Note that in eq. (45) the coefficient n_1' corresponds to the like coefficient of eq. (6).

The new value of G_C is obtained from eq. (45) and is given by:

$$G_C' = \frac{K_FG_D}{1 - K_F}. \tag{46}$$

Equations (44) and (46) yield the new value of R_B, which is given by:

$$G_B' = \frac{G_B(G_C' + G_D)}{G_D}. \tag{47}$$

Equations (46) and (47) correspond to eqs. (17) and (18), respectively.

AUTHOR

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