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A Method to Characterize the Mechanical Properties of Undersea Cables

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A method has been developed to evaluate the mechanical properties of cables containing stranded-strength members in both linear elastic and nonlinear plastic regions. The method extends Cannon and Santana's general system of equations describing the cable mechanical characteristics. In the formulation of the method, the fundamental assumptions made by Cannon and Santana are first examined and justified. Next, instead of using elastic constants for the constituent cable materials in the system of equations, the regression analysis is applied to the tensile and torsional test data of dominating high-strength cable components to obtain least-squares-fit polynominals approximating the stress versus strain and shearing-stress versus shearing-strain curves. By differentiating the polynominals, the tensile and torsional moduli of these cable components as functions of their axial strain and twist are derived. The relations describing the mechanical properties of the cable in both elastic and plastic regions are obtained by substituting the tensile and torsional moduli of the high-strength cable components and the constant moduli for the low-strength cable components into the system of equations in differential form and integrating them. Application of the method to the present experimental undersea-lightguide cable yields excellent agreement with the tensile test results of the cable.

I. INTRODUCTION

Because of its long suspended length in deep ocean, undersea cable normally experiences high tension and strain during its installation and recovery.[1] High-strength stranded members are employed in the undersea cable design to support the tension and keep the cable strain below a prescribed level. The problems of high tension and strain cause even greater concerns for the undersea lightguide cable because of the static and dynamic fatigue of optical fibers.[2] Adequate protection of the fibers is essential to system reliability and requires accurate evaluation of the mechanical properties of the cable in development.

Mechanical characterization of cable containing helically wrapped or stranded members has been done by Cannon and Santana[3] using linear elasticity theory, which assumes that the tensile modulas E and Poisson's ratio ν are constant and independent of the cable load. Using two fundamental assumptions relating the external applied force and moment at the cable ends to the tension developed in individual helically stranded-strength members, a system of equations was developed that describes the mechanical characteristics of a cable having one or more layers of helically stranded members. Application of this theory to the present undersea lightguide cable yields excellent agreement with the experimental data up to a strain level of approximately 0.5 percent. Beyond this strain level, the theory predicts lower strain than the experimental data. This is not surprising because, at a strain level higher than 0.5 percent, the cable has been stretched beyond its elastic region to the plastic region, where the linear elasticity theory does not apply. Since undersea cable normally experiences high tension and strain, an understanding of the mechanical properties in both regions is essential to the design of undersea lightguide cable.

In this paper, a method is presented to accurately predict the mechanical properties of the cable in both elastic and plastic regions. Since the present method uses the system of equations developed in Ref. 3, the fundamental assumptions on which the system of governing equations is based will be examined and justified in Section II. The formulation of the new method is presented in Section III. Then the method is applied to the experimental undersea lightguide cable in Section IV. The results of the theory are compared with experimental data in Section V.

II. FUNDAMENTAL ASSUMPTIONS

The mechanical properties of cables containing helically stranded members have been analyzed in Ref. 3 using a model consisting of n identical wires of radius b parallel to one another in a circular array and twisted into helices with a common helix angle α and radius r, as shown in Fig. 1. The fundamental equations relating the applied force T_t and moment M_t along the cable axis to the tension T developed in individual wire were assumed to be

$$T_t = nT \sin \alpha \tag{1}$$

and

$$M_t = nTr \cos \alpha. \tag{2}$$

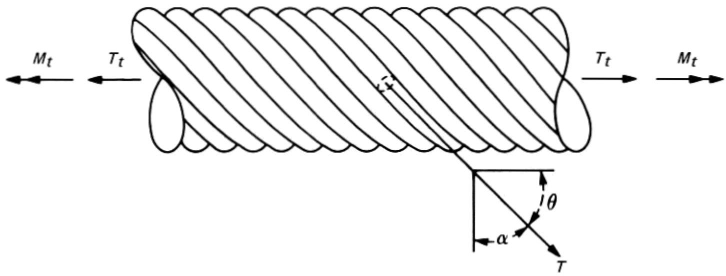

Fig. 1—Cable loads and wire tension.

From these assumptions, a system of equations was developed, which describe the mechanical properties of a cable containing helically wrapped elements.

However, from the equations of equilibrium of a bent and twisted thin rod, Costello and Phillips[4] have obtained a complete solution for the same problem illustrated in Fig. 1. The solution relates the applied tension and torque to the individual wire tension as

$$T_t = nT \sin \alpha + n \frac{\cos^3\alpha}{r} \left| C\tau - A \frac{\sin \alpha \cos \alpha}{r} \right| \tag{3}$$

and

$$M_t = n \left| T\tau \cos \alpha - \cos^2\alpha \sin \alpha \left| C\tau - A \frac{\sin \alpha \cos \alpha}{r} \right| + C\tau \sin \alpha + A \frac{\cos^3\alpha}{r} \right|, \tag{4}$$

where A and C are constants, depending on the elastic properties of the material and the shape and dimensions of the rod cross-sectional area. E is the tensile modulus, ν is Poisson's ratio, and, for a circular cross section of radius b,

$$A = \frac{\pi E b^4}{4} \quad \text{and} \quad C = \frac{\pi E b^4}{4(1 + \nu)}.$$

Notice that eqs. (3) and (4) can be reduced to (1) and (2) if the first term on the right of the equations is retained while neglecting the other terms. To neglect the second and third terms on the right-hand side of eq. (3) requires that

$$\frac{b^2\tau \cos^3\alpha}{4(1 + \nu) r \epsilon_s \sin \alpha} \ll 1 \tag{5}$$

and

$$\left| \frac{b^2 \cos^2 \alpha}{2r\sqrt{\epsilon_s}} \right|^2 \ll 1, \tag{6}$$

where ϵ_s is the strain of helical wire along its own axis. Similarly, to neglect the four terms on the right-hand side of eq. (4) requires that

$$\frac{\tau b^2 \cos \alpha \sin \alpha}{4(1+\nu) r E_s} \ll 1, \tag{7}$$

$$\left| \frac{b \cos \alpha \sin \alpha}{2r\sqrt{\epsilon_s}} \right|^2 \ll 1, \tag{8}$$

$$\frac{\tau b^2 \sin \alpha}{4(1+\nu) r \epsilon_s \cos \alpha} \ll 1, \tag{9}$$

and

$$\left| \frac{b \cos \alpha}{2r\sqrt{\epsilon_s}} \right|^2 \ll 1. \tag{10}$$

Inequalities (5), (6), (8), and (10) can easily be satisfied if $\alpha \simeq \pi/2$, i.e., for the large helix angle or small lay angle. However, to satisfy inequalities (7) and (9), the additional condition $\tau \simeq 0$ or $b^2/r \simeq 0$ is required.

In undersea cable design and manufacture, the helix angle α of the stranded members is usually close to $\pi/2$, and the members are actually made twist-free as they are formed into helices by a planetary-stranding machine. The condition $\tau \simeq 0$ can usually be satisfied. Therefore, the approximate relations (1) and (2) assumed in Ref. 3 and the resulting system of equations are justified.

III. FORMULATION OF THE NEW METHOD

For a cable containing multiple layers of helically stranded strength members plus the cable core, the total axial load carried by the cable is the sum of the core load and the strand load, according to the following equation[3]

$$T_t = T_c + \sum_{i=1}^{m} n_i T_{si} \sin \alpha_i, \tag{11}$$

and the total cable moment equals the sum of the moment contributions from each of the strand layers plus that of the core, as shown by

$$M_t = -\psi J_c G_c + \sum_{i=1}^{m} | n_i T_{si} r_i \cos \alpha_i - \psi (n_i J_{si} G_{si} \sin^2 \alpha_i) |, \tag{12}$$

where n_i is the number of stranded wires in the ith layer; T_c is the load experienced by the cable core; T_{si} is the tension supported by the ith strand layer; m is the number of strand layers in the cable; α_i is the common helix angle of the wires in the ith layer; ψ is the untwist turns experienced per unit length of the cable under applied tension; J_c and G_c are the cable core's moment of inertia and torsional rigidity; r_i is the radial location of the ith layer; J_{si} and G_{si} are the ith strand's moment of inertia and moment of rigidity. Note that eqs. (11) and (12) use the approximate relations (1) and (2) for the stranded wires.

Equations (11) and (12) can be transformed to the abbreviated forms

$$T_t = C_3\epsilon_c - C_4\psi \tag{13}$$

and

$$M_t = C_1\epsilon_c - C_2\psi, \tag{14}$$

where the coefficients C_i are defined by

$$C_1 = \sum_{i=1}^{m} |\sin^2\alpha_i - \nu_i^*\cos^2\alpha_i \| n_i E_{si} A_{si} r_i \cos\alpha_i|, \tag{15}$$

$$C_2 = J_c G_c + \sum_{i=1}^{m} |n_i J_{si} G_{si} \sin^2\alpha_i + \pi r_i^2 \sin 2\alpha_i | n_i E_{si} A_{si} \cos\alpha_i \||, \tag{16}$$

$$C_3 = E_c A_c + \sum_{i=1}^{m} |\sin^2\alpha_i - \nu_i^*\cos^2\alpha_i \| n_i E_{si} A_{si} \sin\alpha_i|, \tag{17}$$

and

$$C_4 = \sum_{i=1}^{m} |\pi r_i \sin 2\alpha_i | n_i E_{si} A_{si} \sin\alpha_i. \tag{18}$$

Here, ν_i^* is a pseudo-Poisson's ratio for the ith layer and is a measure of its diametric contraction; E_{si} and A_{si} are the tensile modulus and cross-sectional area of the stranded member in the ith layer. For a cable with its ends twist-restrained, $\psi = 0$, and eqs. (13) and (14) become

$$\epsilon_c|_{\psi=0} = \frac{T_t}{C_3} \tag{19}$$

and

$$M_t|_{\psi=0} = C_1\epsilon_c. \tag{20}$$

For a cable with its ends free to rotate, $M_t = 0$, and eqs. (13) and (14) become

$$\epsilon_c|_{M_t=0} = \frac{T_t}{C_3 - \dfrac{C_1 C_4}{C_2}} \tag{21}$$

and

$$\psi|_{M_t=0} = \frac{C_1}{C_2}\epsilon_c. \tag{22}$$

If the cable characterization parameters C_1, C_2, C_3, and C_4 are known, the cable strain ϵ_c, cable moment M_t, and cable twist can be evaluated from eqs. (19) through (22) for a given cable tension, T_t. If the elastic properties of cable components are all constant and the twist ψ is zero or very small, the cable characterization parameters C_1, C_2, C_3, and C_4 are all constant and the relations among the cable strain, moment, twist, and tension from eqs. (19) to (22) are all linear. This is the case illustrated in Ref. 3.

In reality, however, the tensile moduli E's and moduli of rigidity G's of the cable components, either stranded or unstranded, are nonlinear functions of the cable strain ϵ_c and twist ψ, i.e., $E = E(\epsilon_c, \psi)$, $G = G(\epsilon_c, \psi)$, and thus the cable characterization parameters C_1, C_2, C_3, and C_4 are also functions of the cable strain ϵ_c and twist ψ.

Thus, eqs. (13) and (14) can only be applied in differential forms

$$dT_t = C_3 d\epsilon_c - C_4 d\psi \tag{23}$$

and

$$dM_t = C_1 d\epsilon_c - C_2 d\psi. \tag{24}$$

The above differential equations are meaningful only if the coefficients C_1, C_2, C_3, and C_4 can be expressed in terms of the cable strain ϵ_c and twist ψ. This can be accomplished in two steps. First, apply the regression analysis to the tensile and torsional test data of dominating cable components to obtain least-squares-fit polynominals, which approximate the stress versus strain and shearing-stress versus shearing-strain curves. By differentiating the polynominals, we obtain the tensile moduli and the moduli of rigidity of the cable components in terms of axial strain and twist:

$$\begin{aligned} E_{si} &= E_{si}(\epsilon_{si}) \\ E_c &= E_c(\epsilon_c) \\ G_{si} &= G_{si}(\psi_{si}) \\ G_c &= G_c(\psi), \end{aligned} \tag{25}$$

where ϵ_{si} and ψ_{si} are the strain and twist of the stranded member in the direction of its own axis. Second, transform the independent variables ϵ_{si} and ψ_{si} in (25) into the cable strain ϵ_c and twist ψ using the following relations:[3]

$$\epsilon_{si} = |\sin^2\alpha_i - \nu_i^*\cos^2\alpha_i|\epsilon_c - |l_i\cos^2\alpha_i|\psi \tag{26}$$

$$\psi_{si} = \psi \sin \alpha_i, \tag{27}$$

where l_i is the lay length of the ith stranded layer. Substitution of the above equations into (23) and (24) yields

$$dT_t = C_3|\epsilon_c, \psi|d\epsilon_c - C_4|\epsilon_c, \psi|d\psi \tag{28}$$

and

$$dM_t = C_1|\epsilon_c, \psi|d\epsilon_c - C_2|\epsilon_c, \psi|d\psi. \tag{29}$$

For a cable with its ends twist-restrained, $\psi = 0$, and eqs. (28) and (29) become

$$T_t = \int C_3(\epsilon_c)d\epsilon_c \tag{30}$$

and

$$M_t = \int C_1(\epsilon_c)d\epsilon_c. \tag{31}$$

Since $C_1(\epsilon_c)$ and $C_2(\epsilon_c)$ are polynominals, the above equations can readily be integrated. For a cable with its ends free to rotate, $M_t = 0$, and eqs. (28) and (29) become

$$T_t = \int \left| C_3(\epsilon_c, \psi) - \frac{C_1(\epsilon_c, \psi)C_4(\epsilon_c, \psi)}{C_2(\epsilon_c, \psi)} \right| d\epsilon_c \tag{32}$$

and

$$\frac{d\psi}{d\epsilon_c} = \frac{C_1(\epsilon_c, \psi)}{C_2(\epsilon_c, \psi)}. \tag{33}$$

In this case, the differential eq. (33) should be solved first to obtain a relation between ψ and ϵ_c. This relation is applied to eq. (32) to eliminate ψ. Then, eq. (32) can be integrated to give a relation between T_t and ϵ_c.

IV. APPLICATION

The recovery operation induces higher tensions in undersea cable than any other handling operation. During a steady-state recovery, the cable tension is highest at the overboard sheave, where the cable twist is practically zero. Therefore, only the case of twist restraint, $\psi = 0$, will be studied here. The method is applied to an experimental undersea lightguide cable to characterize its tensile and torsional properties. The cable structure and component dimensions are shown in Fig. 2. The cable consists of a lightguide core protected by two layers of high-strength steel-strand wires, a copper conductor, and a low-density polyethylene (LDPE) jacket for high-voltage insulation and environmental protection. The lightguide core consists of a copperplated-steel

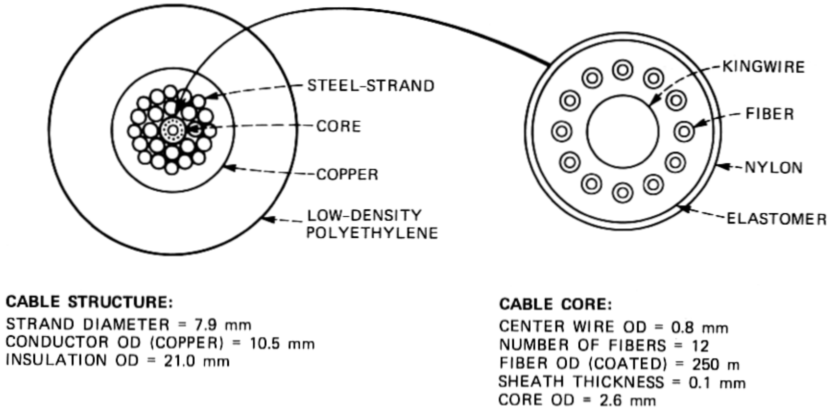

Fig. 2—Undersea cable design.

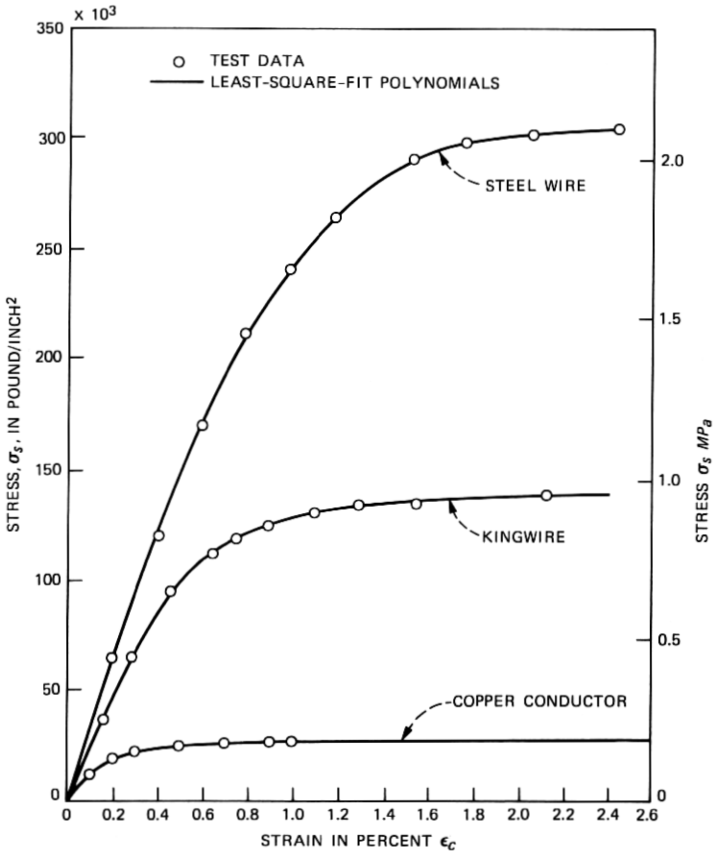

Fig. 3—Tensile testing results and the least-squares-fit polynomials of the high-strength steel-wire, copper sheath, and kingwire.

conductor called the kingwire, up to 12 helically wound fibers embedded in an elastomer material, and a thin outer nylon sheath.

From the cable structure, the metallic components, i.e., the steel wires, the copper conductor, and the kingwire, will carry most of the tensile load. Several tensile tests were conducted on individual steel wires of different sizes and on the kingwire. The test results of a representative 1.6-mm-diameter steel wire and a 0.81-mm-diameter kingwire are shown in Fig. 3. A thorough experimental study on the tensile behavior of the copper conductor was previously done in an unpublished work by R. C. Mondello of Bell Laboratories. His result on newly made copper conductors is also shown in Fig. 3. The nonlinear behavior of each component is evident at high load. Application of the regression analysis to each set of test data in Fig. 3 yields a least-squares-fit polynominal of sixth degree, approximating the stress versus strain relation for each material. The polynomials with the stresses σ_s in MP_a and ϵ_c in mm/mm are given below.

4.1 Steel wire

The polynomial for the steel wires is

$$\begin{aligned}\sigma_s &= 0.663587607686 + 214542.346398\,\epsilon_s + 3198478.98934\,\epsilon_s^2 \\ &\quad - 1617068166.73\,\epsilon_s^3 + 111346531878\,\epsilon_s^4 \\ &\quad - 3.45797334075 \times 10^{12}\,\epsilon_s^5 + 4.21128175626 \times 10^{13}\,\epsilon_s^6\ (MP_a), \end{aligned} \tag{34}$$

where the standard error for estimate "CHI" is CHI = 5.523 (MP_a).

4.2 Kingwire

The polynomial for the kingwire is

$$\begin{aligned}\sigma_s &= 1.15682945607 + 162675.763595\,\epsilon_s \\ &\quad + 7250437.88515\,\epsilon_s^2 - 4367711589.93\,\epsilon_s^3 \\ &\quad + 459662005052\,\epsilon_s^4 - 2.03575240495 \times 10^{13}\,\epsilon_s^5 \\ &\quad + 3.3234268429 \times 10^{14}\,\epsilon_s^6\ (MP_a), \end{aligned} \tag{35}$$

where the standard error for estimate "CHI" is CHI = 5.871 (MP_a).

4.3 Copper conductor

The polynomial for the copper conductor is

$$\begin{aligned}\sigma_s &= 0.299934161316 + 110226.149362\,\epsilon_s \\ &\quad - 28633487.4233\,\epsilon_s^2 + 3792414452.98\,\epsilon_s^3 \\ &\quad - 264983411326\,\epsilon_s^4 + 9.25874279974 \times 10^{12}\,\epsilon_s^5 \\ &\quad - 1.27105877316 \times 10^{14}\,\epsilon_s^6\ (MP_a), \end{aligned} \tag{36}$$

where the standard error for estimate "CHI" is CHI = 1.780 (MP_a).

Table I—Mechanical properties of cable components

Cable Components	Tensile Modulus, $E\,(GP_a)$	Shear Modulus, $C(GP_a)$	No. of Strand Members, N	Helix Radius, r (mm)	Helix Angle, α (degree)	Cross-Sectional Area (mm^2)	Polar Moment of Inertia (mm^4)
Kingwire	158.58	61.02	—	—	—	16.58×10^{-2}	2.14×10^{-2}
Elastomer	0.05	0.03	—	—	—	3.87	2.19
Glass fiber	71.71	31.51	12	0.79	88.8	0.95×10^{-2}	7.2×10^{-6}
Nylon	1.24	0.44	—	—	—	26.32×10^{-2}	0.68
1st layer steel wires (1.6-mm dia.)	206.84	79.29	8	2.08	86.8	2	0.32
2nd layer steel wires (1.48-mm dia.)	206.84	79.29	8	3.27	83.9	1.74	0.24
2nd layer steel wires (1.135-mm dia.)	206.84	79.29	8	3.51	83.9	1.03	0.16
Copper sheath	117.21	44.13	—	—	—	12.64	116.54
LDPE	0.24	0.08	—	—	—	2.82×10^{2}	9.24×10^{3}

The tensile moduli $E(\epsilon_s)$'s of each component can be obtained by taking the derivative of the corresponding polynomial

$$E(\epsilon_s) = \frac{d\sigma_s}{d\epsilon_s}. \tag{37}$$

For the stranded-steel wires and fibers, the independent variables ϵ_s can be transformed to ϵ_c using eq. (28); for the copper conductor and kingwire, ϵ_s can be directly replaced by ϵ_c since their axial direction coincides with that of the cable. Substitutions of the tensile moduli of the components into eq. (15) and (17) give

$$C_1(\epsilon_c) = |\sin^2\alpha_f - \nu_f^*\cos^2\alpha_f|n_f E_f A_f r_f \cos\alpha_f + \sum_{i=1}^{4} |\sin^2\alpha_i - \nu_i^*\sin^2\alpha_i\| n_i E_{si}(\epsilon_c) A_{si} r_{si}\cos\alpha_i|$$

$$C_3(\epsilon_c) = E_k(\epsilon_c)A_k + E_e A_e + E_n A_n + E_c(\epsilon_c)A_c + E_p A_p + |\sin^2\alpha_f - \nu_f^*\cos^2\alpha_f\| n_f E_f A_f \sin\alpha_f|, + \sum_{i=1}^{4} |\sin^2\alpha_i - \nu_i^*\cos^2\alpha_i\| n_i E_{si}(\epsilon_c) A_{si}\sin\alpha_i|,$$

where the subscripts k, e, n, c, p, and f refer to the kingwire, elastomer, nylon, copper, LDPE, and fibers, respectively. The tensile moduli of the fibers, elastomer, nylon, and LDPE are all assumed constant and are listed in Table I. Substitution of the above expressions for $C_1(\epsilon_c)$ and $C_3(\epsilon_c)$ into eqs. (29) and (30) and integrating give the tensile and torsional properties of the cable under twist-restraint condition. These functions are calculated and the results are shown as solid lines in Figs. 4 and 5. The dashed lines in these figures are the results using constant-elastic moduli listed in Table I.

V. EXPERIMENTAL RESULTS

Tensile tests were conducted for five cable specimens, 100.5-meter-length each. The cable ends of the cable specimen were potted into tapered-plug epoxy terminations. The cable specimen with the terminations was installed in the tensile test bed in a straight line configuration with one end fixed to the anchor and another end attached to the hydraulic ram. Both ends were twist-restrained. The cable tension was increased in steps from 8.9 to 53.4 kN, then in steps from 4.5 to the maximum load of 62.3 kN. The cable tension and moment were measured by a load cell. The cable elongation was measured by using a 10-turn, 10-kΩ linear potentiometer. All data were monitored and displayed in digital form on a control panel and then recorded.

The experimental results are also shown in Figs. 4 and 5. The

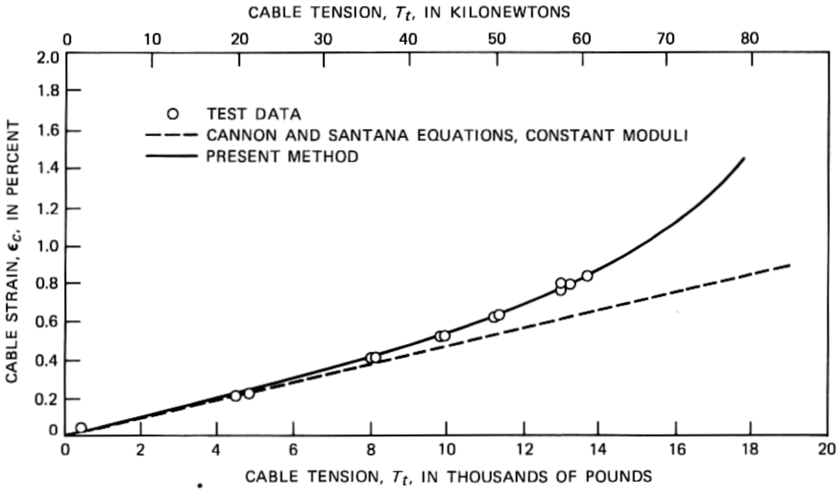

Fig. 4—Comparison of theoretical and experimental results for cable strain versus cable tension.

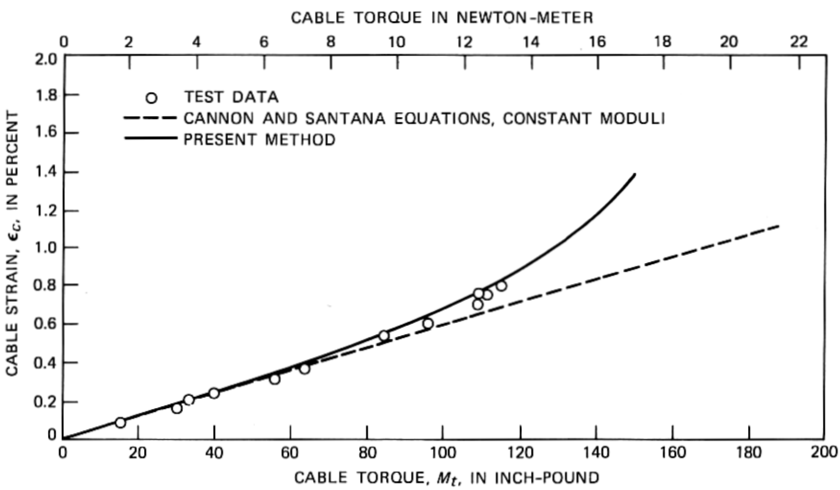

Fig. 5—Comparison of theoretical and experimental results for cable strain versus cable torque.

calculated results from the present method and the experimental results are in excellent agreement.

VI. CONCLUSION

A method has been formulated to evaluate the mechanical properties of cables containing stranded-strength members, in both linear elastic and nonlinear plastic regions. Application of the method to the experimental undersea lightguide cable shows excellent agreement to the tensile test results of the cable. This method can be applied to future cable design to evaluate the cable breaking strength, cable tension versus strain, cable moment versus strain, and cable twist versus strain in both linear elastic and nonlinear plastic regions.

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