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TELBECC—A Computational Method and Computer Program for Analyzing Telephone Building Energy Consumption and Control

By P. B. GRIMADO*

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Telephone Building Energy Consumption and Control (TELBECC) program has been developed to accurately and efficiently analyze environmental control and energy use in telephone company buildings. The program simulates various operational plans to determine the relative energy and cost savings. By analyzing the operation of the heating, ventilation, and air conditioning system as it regulates a changing environment, TELBECC calculates the heating and cooling load, dry-bulb temperature, and relative humidity in the building. The user specifies the building's dry-bulb temperature limits, which are the control variables for the program analysis. The simplified computational procedure of the program incorporates a recursive scheme using time series to perform the necessary calculations. The results of the computations can be obtained for different periods: the quarter hour, hour, day, or month. Energy consumption and control in several equipment buildings located in three different geographical areas have been analyzed by TELBECC. Analysis and comparison of the resulting data demonstrate the advantages of the program.

I. INTRODUCTION

An ambitious energy-cost savings program has been instituted to reduce energy use in telephone company buildings. In recent years, telephone companies have saved energy mainly by redesigning and

* Bell Laboratories.

retrofitting buildings to operate and maintain environmental control equipment at peak performance, to turn out unneeded lights, to reduce heating and cooling losses, and the like. Further energy and cost savings, although requiring additional capital investment, could be achieved through modification of environmental control systems, purchase of sophisticated microprocessors for more efficient control of building Heating, Ventilating, and Air conditioning (HVAC) systems, and installation of alternative energy sources, such as solar power and wind power. Before adopting any conservation plans that require appreciable capital investment, however, we should make a thorough economic evaluation. Such an evaluation can be carried out by correlating the changing operating characteristics of a building with selected energy conservation plans. This procedure would enable us to pinpoint the most economical operating strategies.

There are several commercially available computer programs to perform this type of analysis, such as DOE II, ESP, and BLAST;[1,2] most, however, are proprietary. These complex programs can analyze any of a broad spectrum of commercial, industrial, and residential buildings. But, because of their versatility, they require large computer systems, extensive data preparation, and high costs. The use of energy in the majority of telephone company equipment buildings, which are small, single-story structures varying in area from 1500 to 10,000 square feet, can be best evaluated by a more focussed computer program.

This paper describes a new computational method and computer program called Telephone Building Energy Consumption and Control (TELBECC). This program simulates building operations and quickly evaluates numerous energy conservation plans and cost-saving strategies under variable weather conditions [according to standard hourly Test Reference Year (TRY) weather data[3]]. The program can evaluate energy consumption for intermittent or proportional HVAC plant operation, economy cycle operation, enthalpy cycle operation, and wideband temperature operation with no heating or cooling between preset room temperature limits. Also, the program can calculate the optimum building orientation and U factor (heat transmission characteristics) of the outside walls and roof, chiller and heater plant size, dry-bulb temperature and Relative Humidity (RH) variations, and quantity of water required to maintain 20-percent minimum RH during economy cycle operations.

II. PROGRAM DESCRIPTION

We can derive the heating or cooling load in an enclosed building from the following considerations:

1. Conduction of heat through the building walls and roof.

2. Permeation of outside air through the building envelope.
3. Internal heat generation from equipment, lights, and people.
4. Direct solar radiation through windows and skylights (fenestration).

Since most operating company equipment buildings have few windows, the program does not consider item 4 in the present version.

A Constant-Air-Volume (CAV) supply fan system typically controls the air temperature of a building. The building engineer normally sizes the fan system using the elementary steady-state heat balance, which takes into account the internal heat loads, outside air temperatures, and solar radiation in conjunction with the U factor of the building envelope. In general, this conservative approach produces fan systems that are oversized and therefore inefficient. To find a smaller, and perhaps more efficient, fan capacity, a TELBECC user selects different fan capacities for analysis by the program. The program generates data on the space temperatures, relative humidity, peak heating and chiller loads, and the hours of system operation for the different fan capacities that can be used to find an optimum air supply fan system.

For comfort, a limit is imposed on the difference between HVAC supply and return air temperatures. For cooling, this temperature difference is −20 degrees F; for heating, +40 degrees F. These default values may be overridden by the user. With an environmental dry-bulb temperature standard specified, the program computes the required operation of the HVAC.

The user can specify one of two basic ways to operate the HVAC: intermittent operation or proportional control. With intermittent operation, the HVAC does not supply any heating or cooling when the dry-bulb air temperature is within the wideband temperature range. Reaching or exceeding either wideband temperature limit activates the HVAC. The HVAC stays on and does not deactivate until the dry-bulb air temperature reaches 3 degrees F above the lower limit of the wideband temperature range for heating and 3 degrees F below the upper limit of the wideband temperature range for cooling. The TELBECC user can reset the numerical values of the throttling range if a different range is appropriate. The proportional control plan operates by continuously adjusting the supply and return air temperature difference in increments of 1 degree F to satisfy the instantaneous building heating or cooling load. This plan follows the building load to closely track the lower and upper limits of the wideband temperature range with essentially no throttling. When selecting a dual or extended wideband temperature standard (that is, one with different wideband limits for occupied and unoccupied times), the HVAC activates before occupancy in order to reach the preset temperature standard.

TELBECC calculates the heat added or removed by the HVAC system in controlling the dry-bulb air temperature every quarter hour. In particular, when cooling is required, the sensible and latent loads on the chiller plant are simultaneously computed by incorporating any of three standard methods of fan system operation:

1. Conventional operation, which is chiller operation with no economizer.
2. Chiller operation with a dry-bulb economy cycle.
3. Enthalpy cycle.

In conventional operation, the minimum quantity of outside air needed for ventilation is circulated. This mode is also used as a benchmark for the program. The dry-bulb economy cycle uses outside air for cooling whenever the outside dry-bulb air temperature falls below the maximum value. The default value is 55 degrees F, but the user can reset the value. The enthalpy cycle checks the enthalpy of the inside air and the outside air. If the outside air enthalpy is lower, 100 percent outside air is circulated to reduce the load on the chiller regardless of the relative humidity. Otherwise, only the minimum quantity of outside air required for ventilation is circulated.

System control is based on dry-bulb air temperature and is not predicated on maintaining a particular value of relative humidity. Nevertheless, the program computes changes in relative humidity for the three methods of fan-system operation discussed above. The program summarizes the variation in relative humidity for the time period chosen by giving the number of hours the relative humidity is less than 10 percent, between 10 and 15 percent, between 15 and 20 percent, between 20 and 55 percent, between 55 and 60 percent, and greater than 60 percent.

In addition, since dry-bulb economy cycle operation generally calls for bringing in winter air with low humidity, the program calculates the quantity of water required for humidification. The operating company minimum standard of 20 percent RH in the inside air for dry-bulb economy cycle operation in winter is the basis for calculating the amount of water added to the air.

III. TRANSIENT HEAT CONDUCTION THROUGH THE BUILDING ENVELOPE

Weather conditions influence the heating and cooling load of a building by heat conduction through the structural and decorative materials of the exterior walls and roof, as well as by permeation of outside air and direct absorption of solar radiation through window areas. Since, as previously mentioned, most operating company equipment buildings have few windows, only heat conduction and permeation are treated in the computer program. The program must account for the heat storage effects of the structure, as well as the daily and seasonal variation of the outside air temperature and solar radiation. We can account for these influences on the building by considering the building envelope elements as one-dimensional flat slabs or plates. We then obtain a solution to a partial differential equation with time-dependent boundary conditions. A classical analytical solution of this equation[4] produces a set of equations that require an inordinate quantity of computational effort and time, rendering the whole idea of performing the analysis impractical and uneconomical. However, the analytical solution can be recast into a simpler, though effective, computational scheme with a method first introduced by Mitalas and Stephenson,[5] which is ideally suited to calculation by computer.

The inside-wall and roof-surface temperature $T_{\mathrm{BE}}(t)$ and the air temperature of the building $T_{\mathrm{a}}(t)$, which are dependent on time, determine the environmental load due to convection.* $T_{\mathrm{BE}}(t)$ is represented in the form of the following time series:

$$T_{\mathrm{BE}_t} = -\sum_{i=1}^{m-1} b_i T_{\mathrm{BE}_{t-i}} + \sum_{i=1}^{m} a_i T_{\mathrm{O}_{t-i-1}} + \sum_{i=1}^{m} a_i' T_{\mathrm{a}_{t-i-1}}, \qquad (1)$$

where

t = current time,
T_{BE_t} = inside-wall temperature of the building at the current time,
$T_{\mathrm{BE}_{t-i}}$ = inside-wall temperature of the building i time units prior to t,
$T_{\mathrm{O}_{t-i}}$ = outside sol-air temperature[6] i time units prior to t, and
$T_{\mathrm{a}_{t-i}}$ = air temperture of the building i time units prior to t.

For hourly temperature calculations, the number of terms, m, will rarely exceed 5, and for quarter-hour calculations, m will generally be less than 15. The recursive properties of the calculation make it extremely efficient and economical, especially when the operating characteristics of the building may need to be tracked for an entire calendar year. The coefficients b_i, a_i, and a_i' in eq. (1) are determined from the thermophysical properties of the structure. Only six values are needed to uniquely specify these coefficients: wall thickness, wall U factor, wall-weight density, effective heat-transfer coefficient of the inside- and outside-wall surface, and the time interval between successive calculations. The appendix presents the mathematical procedure to evaluate these coefficients. Figures 1 and 2 show the mathematical and physical models for deriving the coefficients.

We can validate this simplified computational approach by compar-

* Radiative interchange between inside building-wall surfaces is not included in the present version of the program.

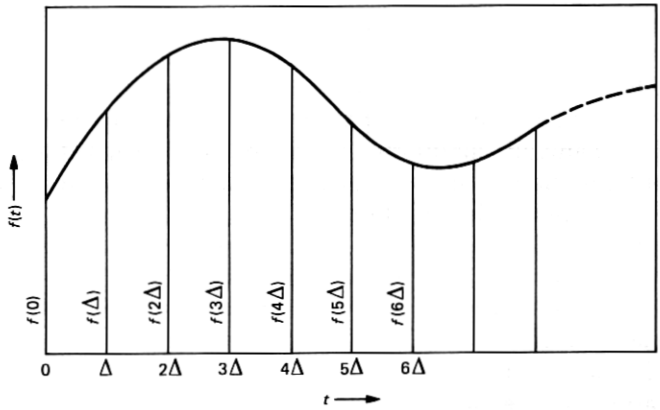

Fig. 1—Discrete and continuous functions.

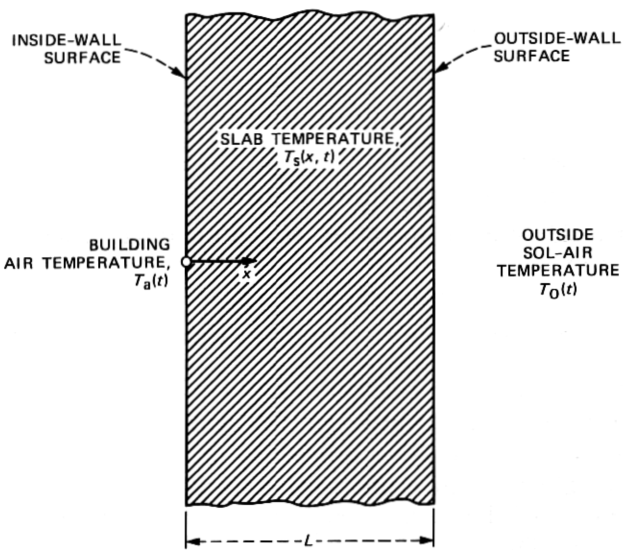

Fig. 2—Homogeneous flat slab.

ing it with an exact solution given in the literature.[7] We can see this in Figs. 3 and 4 for several values of the inside-wall dimensionless convective heat-transfer parameter $B_i \equiv h_i L/k$ and two limiting values of the outside-wall convective heat-transfer parameter $B_O \equiv h_O L/k$. In Fig. 3 the outside-wall convective heat transfer parameter $B_O = 0$; i.e., the surface $x = L$ is insulated. In Fig. 4 the solution corresponds to B_O approaching ∞; i.e., the surface $x = L$ is maintained at a constant temperature. The initial and boundary conditions are indicated in the figures. The solid curves represent the exact condition given in Ref. 7,

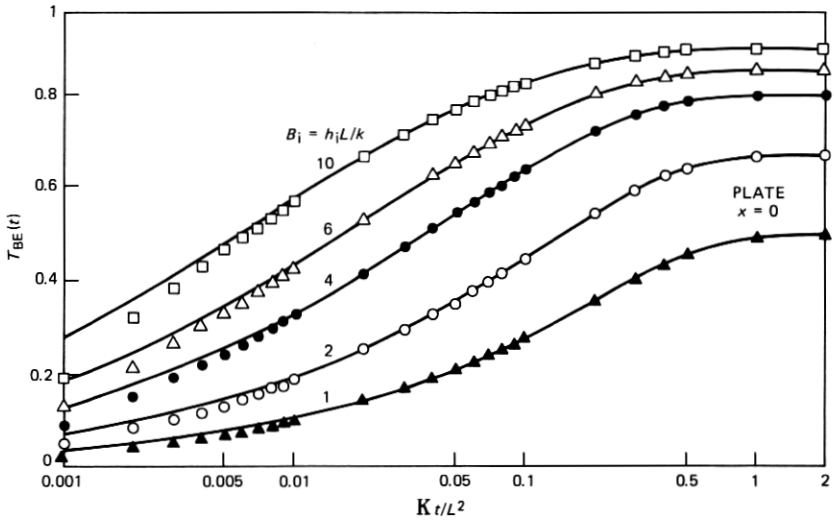

Fig. 3—Temperature response of front face of plate, $0 \leq x \leq L$, with back face $0 \leq x \leq L$, with back face $x = L$ maintained at $T_0 = 0$ degree F after sudden exposure to uniform-temperature convective environment $T_a = 1$ degree F at $x = 0$. Sampling interval Δ is one min.

and the dots represent the numerical values computed by the time series in eq. (1). The sampling interval for this example is $\Delta = 1$ min. or, in terms of dimensionless time, $k\Delta/L^2 = 0.001$. We can see that after some time has elapsed the exact solution and the time-series solution match identically. For the problem considered here at $t = 0$, the ambient convected temperature T_a is suddenly raised from $T_a = 0$ to $T_a = 1$; i.e., the boundary condition is a step function. However, since the development of the time series assumes a linear variation between time intervals, as stated in the appendix, the solution resembles an initial ramp followed by a constant value, as shown in Fig. 5. Once the effect of the initial ramp input diminishes after about 10 sampling intervals, the solution coincides with the exact solution. This characteristic of the time series is not a problem here, since instantaneous changes of air temperatures inside and outside the building do not occur.

IV. CALCULATION OF BUILDING AIR TEMPERATURE AND ENERGY USE

The air temperature of the building is obtained through the following equation for the heat balance within the building:

$$q_{\mathrm{air}}(t) = q_{\mathrm{equipt}}(t) + q_{\mathrm{lights}}(t) + q_{\mathrm{people}}(t) + q_{\mathrm{infiltration}}(t) + q_{\mathrm{walls}}(t) + q_{\mathrm{HVAC}}(t), \quad (2)$$

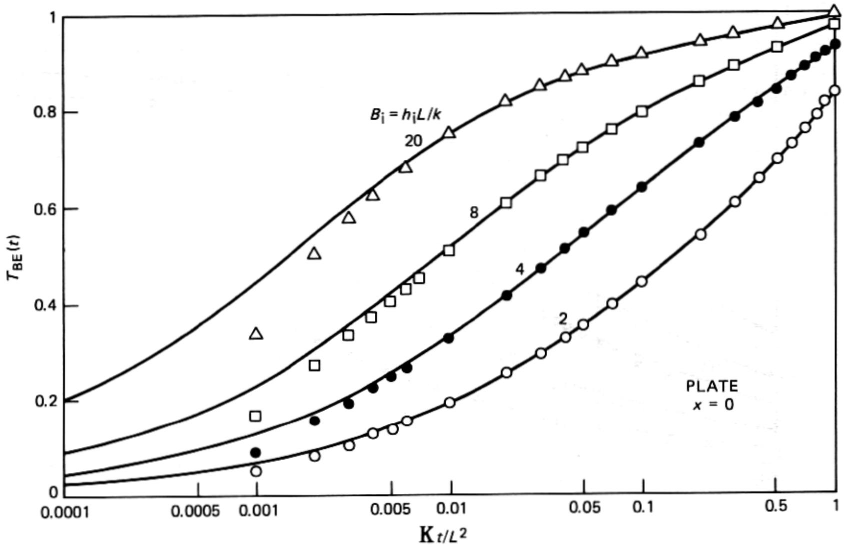

Fig. 4—Temperature response of front face of plate $0 \leq x \leq L$, with insulated back face $x = L$ after sudden exposure to uniform-temperature convective environment T_a = 1 degree F at $x = 0$. Sampling interval Δ in one min.

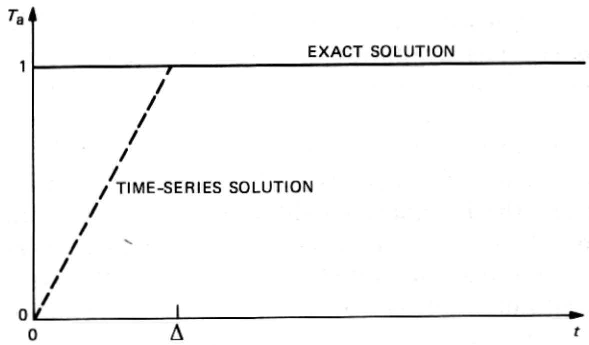

Fig. 5—Convection temperature environment at inside boundary surface $x = 0$.

where $q_{air}(t)$ (Btu/hr) represents the sensible thermal-energy convection rate of the inside air, resulting in a change in the overall dry-bulb air temperature. The other terms on the right represent the rate at which heat is convected to the air from the following: equipment heat dissipation; lighting; people; inadvertent infiltration of outside air; the exterior walls, floor, and roof of the building; and the building's HVAC control system.

To evaluate the air temperature $T_a(t)$ from eq. (2), each term is expressed as the difference in temperatures between the air and the heat-convecting medium. We can determine $T_a(t)$ from the following differential equation:

$$\rho c v \frac{dT_a(t)}{dt} = H_{EL}A_{EL}[T_{EL}(t) - T_a(t)] + \rho c m[T_O(t) - T_a(t)] + H_{BE}A_{BE}[T_{BE}(t) - T_a(t)] + \rho c Q[DT_{SP}(t)], \quad (3)$$

where

$T_a(t)$ = average building dry-bulb air temperature;
$T_{EL}(t)$ = combined average temperature of the equipment, lights, and people;
$T_O(t)$ = outside dry-bulb air temperature;
$T_{BE}(t)$ = inside surface temperature of the building envelope;
$DT_{SP}(t)$ = difference between the air-supply temperature and the air temperature of the building;
ρ = air density;
c = air specific heat;
v = volume of the building space;
H_{EL} = heat-transfer coefficient between the air and equipment;
A_{EL} = average surface area of equipment;
m = rate at which outside air infiltrates the building;
H_{BE} = heat-transfer coefficient between the building envelope and air;
A_{BE} = surface area of the building envelope; and
Q = air-supply rate of the HVAC system fan units.

Only one of the temperatures in eq. (3) is presumed known: the outside-air dry-bulb temperature, $T_O(t)$. The other four temperatures—$T_a(t)$, $T_{EL}(t)$, $T_{BE}(t)$, and $DT_{SP}(t)$—are coupled; therefore, additional equations are needed for their resolution.

We can consolidate our terms into two other equations. When we combined the heat gain generated by the equipment, lighting, and people into a single term for heat dissipation per unit of building floor area $W(t)$(W/ft^2), one additional equation can be written as

$$C_{EL} \frac{dT_{EL}(t)}{dt} = 3.41\ A_f W(t) + H_{EL}A_{EL}[T_a(t) - T_{EL}(t)], \quad (4)$$

where

C_{EL} = heat capacity of the equipment, lights, and people;
A_f = building floor area; and
$W(t)$ = combined heat dissipation of the equipment, lights, and people per unit of building floor area.

A third equation coupling the building envelope temperature $T_{BE}(t)$ and the building air temperature $T_a(t)$ is needed. A likely equation would be the time series given by eq. (1). But before this can be applied, both eqs. (3) and (4) must also be recast in the form of time series. This is easily done by using the properties of the z transform and the procedure already delineated in the appendix. The time-series solution of eqs. (3) and (4) assumes the form

$$T_{a_t} = sT_{a_{t-1}} + \left(\frac{\Delta\tau + s - 1}{\Delta t^2}\right)[\tau_{EL}T_{EL} + \tau_O T_O + \tau_{BE}T_{BE} + \tau_{SP}DT_{SP}]_t$$

$$+ \left(\frac{1 - s(\Delta\tau + 1)}{\Delta\tau^2}\right)[\tau_{EL}T_{EL} + \tau_O T_O + \tau_{BE}T_{BE} + \tau_{SP}DT_{SP}]_{t-1}, \quad (5)$$

and

$$T_{EL_t} = \hat{s}T_{EL_{t-1}} + \frac{(\Delta\hat{\tau} + \hat{s} - 1)}{\Delta\hat{\tau}^2}[\hat{\tau}_w W + \hat{\tau}_a T_a]_t$$

$$+ \frac{[1 - \hat{s}(\Delta\hat{\tau} + 1)]}{\Delta\hat{\tau}^2}[\hat{\tau}_w W + \hat{\tau}_a T_a]_{t-1}, \quad (6)$$

where

Δ = sampling time interval,
$\tau_{EL} = H_{EL}A_{EL}/\rho c v$,
$\tau_O = m/v$,
$\tau_{BE} = H_{BE}A_{BE}/\rho c v$,
$\tau_{SP} = Q/v$,
$\tau = \tau_{EL} + \tau_O + \tau_{BE} + \tau_{SP}$,
$s = \text{EXP}(-\Delta\tau)$,
$\hat{\tau}_w = A_f/C_{EL}$,
$\hat{\tau}_a = H_{EL}A_{EL}/C_{EL}$,
$\hat{\tau} = \hat{\tau}_w + \hat{t}_a$, and
$\hat{s} = \text{EXP}(-\Delta\hat{\tau})$,

and the subscripts t and $t-1$ indicate that the temperature is evaluated at times $t = n\Delta$ and $t = (n-1)\Delta$.

The calculation of the humidity ratio in the space is also formulated in terms of a first-order linear differential equation in time similar to eqs. (3) and (4). This equation is also recast in the form of a time series [see eqs. (5) and (6)]. Knowing the humidity ratio and the dry-bulb air temperature, we can find the relative humidity by employing standard psychometric formulae.[6]

As previously noted in Section II, the system of eqs. (1), (5), and (6) permits controlling the dry-bulb air temperature in two basically different ways, intermittent operation and proportional control, and determines the hours of operation. We use these equations for intermittent HVAC operation where we assume that the HVAC system holds the supply and return air-temperature difference constant at $DT_{SP} = -20$ degrees F whenever cooling is required and at $DT_{SP} = 40$ degrees F whenever heating is required. The hours of operation, and hence the total quantity of heat removed or added to satisfy the imposed dry-bulb temperature standard, can then be determined. For the proportional control plan, eqs. (1), (5), and (6) are also used to calculate not only the hours of HVAC operation but also the numerical value of the supply and return air-temperature difference $DT_{SP}(t)$, which in general varies continuously for this mode of control. The variation in time of the numerical value of $DT_{SP}(t)$ is determined by just satisfying the instantaneous building environmental load. When the HVAC system is activated, the proportional control plan closely follows the lower limit (for heating) or the upper limit (for cooling) of the building wideband temperature range.

Once $DT_{SP}(t)$ and the hours of operation are known, the program calculates the heat added or removed from the building by the HVAC system during every quarter hour and for whatever other period is of interest, e.g., monthly. As a corollary, we can estimate the environmental control system energy use, assuming the following HVAC system characteristics:

1. For chiller operation, a constant Coefficient Of Performance (COP) supplied by the user, together with the quantity of heat removed from the building air, characterizes its energy requirements.

Table I—Equipment buildings analyzed

	Energy Consumption and Control			
	Wideband Temperature (°F)			
Case	Occupied Times	Unoccupied Times	Control	Geographic Location
1	65–80	65–80	Intermittent	New York City
2	65–80	65–80	Proportional	New York City
3	65–80	60–85	Intermittent	New York City
4	65–80	65–80	Intermittent	New Orleans
5	65–80	65–80	Intermittent	Phoenix

Building Parameters

Factor	Parameter
Size (L × W × H)	60 ft × 40 ft × 13 ft
Average heat transmission	U = 0.25 Btu/hr – ft^2
Occupancy time	8 a.m. to 6 p.m.
Fan support rate	7400 CFM
Ventilation capacity	150 CFM
Static fan pressure	2 in. of water
Internal heat load	15W/ft^2
Economy cycle temperature limit	≤65 degrees F

Table IIa—Intermittent control of building space air temperature—New York City, case 1 in Table I

Month	Degree-Days		Space Temp		Max Load (tons)		Total Load (MBtu)		Number of Hours			Heating (kWh Elect)	Cooling (kWh)		Water (gal) to Maintain 20% Min RH
	Heat	Cool	Min	Max	Heat	Cool	Heat	Cool	Heat	Cool	Econ		No Econ	Econ	
Jan	858	0	76.1	80.5	0.0	13.3	0.0	49.2	0	319	314	0.0	5291.2	1229.3	1352.7
Feb	797	0	76.3	80.5	0.0	13.3	0.0	44.2	0	256	281	0.0	4754.4	1114.7	1143.5
Mar	703	0	76.5	80.5	0.0	13.3	0.0	56.1	0	361	356	0.0	6021.7	1391.1	1026.2
Apr	358	0	76.6	80.6	0.0	13.6	0.0	65.1	0	414	334	0.0	6967.9	2582.9	321.7
May	118	38	76.6	80.6	0.0	13.9	0.0	77.5	0	486	179	0.0	8271.2	5901.2	101.2
Jun	23	147	76.6	80.9	0.0	14.2	0.0	83.1	0	512	36	0.0	8833.2	8348.4	0.0
Jul	0	325	76.7	80.8	0.0	14.2	0.0	92.6	0	563	0	0.0	9819.7	9819.7	0.0
Aug	0	268	76.7	81.1	0.0	14.2	0.0	90.4	0	550	0	0.0	9590.8	9587.5	0.0
Sep	32	116	76.8	80.9	0.0	14.1	0.0	80.1	0	495	42	0.0	8521.6	7960.9	20.2
Oct	192	21	76.4	80.6	0.0	13.9	0.0	71.7	0	450	207	0.0	7659.2	4928.8	79.3
Nov	625	0	76.1	80.6	0.0	13.6	0.0	53.7	0	344	306	0.0	5755.8	1764.3	1124.1
Dec	793	0	76.4	80.5	0.0	13.4	0.0	50.2	0	323	292	0.0	5385.9	1593.2	1245.5
Totals	4499	915	76.1	81.1	0.0	14.2	0.0	814.0	0	5107	2351	0.0	86873.0	56222.0	6414.0

Notes: Fan supply rate = 7400 CFM, ventilation = 150 CFM, wideband temperature limits for occupied and unoccupied times = 65° to 80°F, economizer temperature limit = 65°F, time period = 1 to 365 days, total hours = 8760.

2. For fan operation, fan power is calculated from the following equation:

$$\text{Fan power (kW)} = 2.487 \times 10^{-4} \times Q \times \Delta P, \tag{7}$$

where

$Q(CFM)$ = air flow delivered by the fan; and
ΔP = static pressure head of the fan in inches of water.

By multiplying the fan power by the total hours of fan operation, we can obtain the total energy use (kWh).

3. Humidification costs are based on supplying energy at the rate of 1000 Btu per pound of water added to the supply air stream. Costs are derived from the unit cost of energy, such as electricity ($/kWh), natural gas ($/1000 ft^3), and fuel oil ($/gal), which is supplied by the user. An 80 percent efficiency rate is assumed for these energy sources.

4. Heating costs are similarly calculated by the unit cost. An 80 percent efficiency rate is also used in these calculations.

V. ILLUSTRATIVE EXAMPLES OF ENERGY CONSUMPTION AND CONTROL

Different geographic loctions of equipment buildings, dual or extended wideband temperature limits, and the method of HVAC control (intermittent or porportional) are considered in several variations. Table I gives this information along with some of the more salient building parameters. The results of the calculation are summarized by month in Tables II through VI for the cases specified in Table I. We assume here that a conventional cooling system, consisting of a chiller

Table IIb—Intermittent control of building space air temperature—New York City, case 1 in Table I

Number of Hours at Specified Relative Humidity (No Humidity Control)	<10%	10–15%	15–20%	20–55%	55–60%	>60%
Conv (no econ)	697.00	1090.00	1086.75	5886.25	0.0	0.0
Economy	815.25	1048.75	1029.75	5866.25	0.0	0.0
Enthalpy	697.50	1101.00	1144.00	5817.50	0.0	0.0

Estimated Operating Cost for Cooling at $0.10/kWh for Electricity (Chiller COP = 3.50)		
Conv (no econ)	$9687 for 86870 kWh	(Fans = 18669 kWh, chiller = 68201 kWh)
Economy	$8622 for 56221 kWh	(Fans = 18669 kWh, chiller = 37552 kWh)
Enthalpy	$7534 for 75342 kWh	(Fans = 18669 kWh, chiller = 56673 kWh)

Estimated Operating Cost for Humidification (20% min) and Heating at $0.10/kWh for Electricity	
Humidification	$1959 for 19580 kWh
Heating	$0 for 0 kWh

Notes: Min space temp occurred on day 300, max space temp occurred on day 213, max cooling load occurred on day 224.

Table IIIa—Proportional control of building space air temperature—New York City, case 2 in Table I

Month	Degree-Days		Space Temp		Max Load (tons)		Total Load (MBtu)		Number of Hours			Heating (kWh Elect)	Cooling (kWh)		Water (gal) to Maintain 20% Min RH
	Heat	Cool	Min	Max	Heat	Cool	Heat	Cool	Heat	Cool	Econ		No Econ	Econ	
Jan	8.58	0	78.9	80.0	0.0	7.8	0.0	44.7	0	744	741	0.0	6465.4	2734.1	4261.7
Feb	797	0	79.9	80.0	0.0	8.3	0.0	40.4	0	672	667	0.0	5839.7	2487.0	3737.9
Mar	703	0	79.9	80.0	0.0	8.5	0.0	52.6	0	744	744	0.0	7126.1	2719.8	2885.7
Apr	358	0	79.9	80.0	0.0	10.0	0.0	62.3	0	720	652	0.0	7855.6	3182.1	837.9
May	118	38	79.9	80.0	0.0	11.4	0.0	75.4	0	744	434	0.0	9034.2	5461.4	262.5
Jun	23	147	79.9	80.0	0.0	12.2	0.0	81.4	0	720	139	0.0	9451.8	8252.0	0.0
Jul	0	325	79.9	80.0	0.0	13.0	0.0	91.0	0	744	0	0.0	10342.3	10342.3	0.0
Aug	0	268	79.9	80.0	0.0	12.3	0.0	88.9	0	744	10	0.0	10172.1	10080.5	0.0
Sep	32	116	79.9	80.0	0.0	11.4	0.0	78.2	0	720	135	0.0	9187.3	8078.6	76.3
Oct	192	21	79.9	80.0	0.0	11.2	0.0	69.2	0	744	514	0.0	8520.7	4672.8	241.7
Nov	625	0	79.9	80.0	0.0	8.6	0.0	50.1	0	720	696	0.0	6828.1	2797.4	3312.4
Dec	793	0	79.9	80.0	0.0	8.5	0.0	46.1	0	744	731	0.0	6584.7	2806.8	4105.5
Totals	4499	915	79.9	80.0	0.0	13.0	0.0	780.3	0	8960	5466	0.0	97408.0	63615.0	19721.0

Notes: Fan supply rate = 7400 CFM, ventilation = 150 CFM, wideband temperature limits for occupied and unoccupied times = 65° to 80°F, economizer temperature limit = 65°F, time period = 1 to 365 days, total hours = 8760.

and air-handling unit, provides cooling. By comparing the different cases, we find some interesting results.

Case 1 differs from case 2 in that the HVAC is intermittently controlled in case 1, but proportionally controlled in case 2. We see from Tables IIb and IIIb, for example, that the maximum cooling loads for both cases occur close in time [August 12 (day 224) and July 31 (day 212)]. However, the maximum cooling load of 13 tons for the proportional control plan (Table IIIa), which compares favorably with the load of 14.2 tons for the intermittent control plan (Table IIa), reduces the required size of the chiller plant by 9 percent. We would expect such a reduction from using a control sequence that follows the load closely and minimally overshoots the dry-bulb air temperature. Also, a control plan that matches the fan capacity to the load would compare favorably in energy use with on-off fan operation.

We can see in Table IIIa (in the column labeled "Space Temp") that the proportional control plan regulates the temperature to within one-tenth of a degree of the wideband temperature limit for the entire calendar year. For the economy cycle operation, the yearly electrical use of case 1 in Table IIb is 56,221 kWh, and that of case 2 in Table IIIb is 63,614 kWh. The chiller energy consumption for case 1, 37,522 kWh, is larger than that for use 2, 31,591 kWh. The proportional control plan, which modulates the air-supply temperature, requires the fan to run continuously at maximum power for the entire year. This maximum use of the fan creates larger overall energy requirements in spite of lower chiller energy use. However, a variable-air-volume system that modulates the fan supply rate to match loads should decrease the required fan power and significantly reduce total energy use.

Table IIIb—Proportional control of building space air temperature—New York City, case 2 in Table I

	Number of Hours at Specified Relative Humidity (No Humidity Control)					
	<10%	10–15%	15–20%	20–55%	55–60%	>60%
Conv (No Econ)	869.75	1090.75	1118.75	4393.25	1011.25	276.50
Economy	911.25	1092.50	1110.50	4354.00	1015.00	276.75
Enthalpy	869.75	1095.50	1134.50	4337.75	938.75	383.75

	Estimated Operating Cost for Cooling at $0.10/kWh for Electricity (Chiller COP = 3.50)	
Conv (no econ)	$9741 for 97400 kWh	(Fans = 32023 kWh, chiller = 65383 kWh)
Economy	$6361 for 63614 kWh	(Fans = 32023 kWh, chiller = 31591 kWh
Enthalpy	$8930 for 88296 kWh	(Fans = 32023 kWh, chiller = 56273 kWh)

	Estimated Operating Cost for Humidification (20% min) and Heating at $0.10/kWh for Electricity
Humidification	$6022 for 60219 kWh
Heating	$0 for 0 kWh

Notes: Min space temp occurred on day 100, max space temp occurred on day 10, max cooling load occurred on day 212.

Table IVa—Intermittent control of building space air temperature—New York City, case 3 in Table I

Month	Degree-Days		Space Temp		Max Load (tons)		Total Load (MBtu)		Number of Hours			Heating (kWh Elect)	Cooling (kWh)		Water (gal) to Maintian 20% Min RH
	Heat	Cool	Min	Max	Heat	Cool	Heat	Cool	Heat	Cool	Econ		No Econ	Econ	
Jan	858	0	74.7	85.4	0.0	13.2	0.0	46.8	0	304	299	0.0	5033.1	1171.1	1590.2
Feb	797	0	75.1	85.4	0.0	13.3	0.0	42.2	0	274	268	0.0	4534.1	1074.7	1341.3
Mar	703	0	76.0	85.4	0.0	13.3	0.0	53.8	0	347	343	0.0	5776.8	1326.7	1302.5
Apr	358	0	76.3	85.6	0.0	13.6	0.0	62.8	0	400	326	0.0	6725.0	2452.0	446.6
May	118	38	76.3	85.6	0.0	13.9	0.0	75.4	0	473	225	0.0	8047.1	5073.1	158.9
Jun	23	147	76.3	85.6	0.0	14.1	0.0	81.3	0	502	70	0.0	8643.7	7711.2	0.0
Jul	0	325	76.3	85.7	0.0	14.2	0.0	90.7	0	553	0	0.0	9625.4	9625.4	0.0
Aug	0	268	76.2	85.8	0.0	14.2	0.0	88.6	0	541	3	0.0	9406.7	9356.9	0.0
Sep	32	116	75.7	85.6	0.0	14.1	0.0	78.3	0	485	72	0.0	8336.3	7384.6	45.6
Oct	192	21	76.0	85.5	0.0	13.9	0.0	69.5	0	437	245	0.0	7422.9	4189.6	148.0
Nov	625	0	75.1	85.5	0.0	13.6	0.0	51.7	0	332	307	0.0	5545.3	1556.7	1332.1
Dec	793	0	75.5	85.4	0.0	13.4	0.0	48.0	0	310	285	0.0	5152.6	1457.1	1429.6
Totals	4499	915	74.7	85.8	0.0	14.2	0.0	789.0	0	4962	2447	0.0	84249.0	52379.0	7795.0

Notes: Fan supply rate = 7400 CFM, ventilation = 150 CFM, wideband temperature limits for occupied times = 65° to 80°F, wideband temperature limits for unoccupied times = 60° to 85°F, economizer temperature limit = 65°F, time period = 1 to 365 days, total hours = 8760.

Tables IVa and IVb display monthly energy use for case 3 of Table I. Differing from case 1, this plan imposes dual wideband temperature limits. The wideband temperature limits for unoccupied times increase to 60 degrees F and 85 degrees F. Tables IVa and IVb show that this simple change with economy cycle operation reduces annual cooling energy use by 7 percent, from 56,222 kWh to 52,379 kWh. We can attribute this saving mainly to the lower chiller energy requirement, from 37,552 kWh to 34,237 kWh, and to a lesser extent to the smaller fan energy requirements, from 18,669 kWh to 18,141 kWh.

Tables V and VI show the results of the simulation for the buildings located in New Orleans and Phoenix, cases 4 and 5 of Table I. We can see that the maximum chiller load, 14.4 tons, is the same for these diverse locations. The total energy required in these buildings for economy cycle cooling is also nearly equal, 85,027 kWh and 85,123 kWh. Since the cooling load includes both sensible and latent energy, we can surmise from the degree-day totals that the dominant load on the system for Phoenix is sensible heat, and on that for New Orleans latent heat. The distribution of the relative humidity and the costs for humidification shown in Tables Vb and VIb tends to support these observations.

We can see the advantages in running the TELBECC program to compare different control plans. For example, although the intermittent control plan appears to use less energy overall, the proportional control plan actually reduces the size of the chiller plant by 9 percent in the same locale under similar conditions. The higher overall costs can be attributed to continuous operation of the fan, which, if operated to match the load, would consume much less power and make the

Table IVb—Intermittent control of building space air temperature—New York City, case 3 in Table I

Number of Hours at Specified Relative Humidity (No Humidity Control)						
	<10%	10–15%	15–20%	20–55%	55–60%	>60%
Conv (No Econ)	857.50	1205.75	1213.00	5483.75	0.0	0.0
Economy	976.75	1115.75	1138.25	5529.25	0.0	0.0
Enthalpy	857.50	1241.00	1212.00	5449.50	0.0	0.0

Estimated Operating Cost for Cooling at $0.10/kWh for Electricity (Chiller COP = 3.50)		
Conv (no econ)	$8425 for 84247 kWh	(Fans = 18141 kWh, chiller = 66106 kWh)
Economy	$5238 for 52378 kWh	(Fans = 18141 kWh, chiller = 34237 kWh)
Enthalpy	$7278 for 72776 kWh	(Fans = 18141 kWh, chiller = 54635 kWh)

Estimated Operating Cost for Humidification (20% min) and Heating at $0.10/kWh for Electricity	
Humidification	$2380 for 23802 kWh
Heating	$0 for 0 kWh

Notes: Min space temp occurred on day 22, max space temp occurred on day 224, max cooling load occurred on day 224.

Table Va—Intermittent control of building space air temperature—New Orleans, case 4 in Table I

Month	Degree-Days		Space Temp		Max Load (tons)		Total Load (MBtu)		Number of Hours			Heating (kWh Elect)	Cooling (kWh)		Water (gal) to Maintain 20% Min RH
	Heat	Cool	Min	Max	Heat	Cool	Heat	Cool	Heat	Cool	Econ		No Econ	Econ	
Jan	502	0	76.4	80.6	0.0	13.5	0.0	60.8	0	388	341	0.0	6518.0	2044.9	386.5
Feb	465	0	76.4	80.6	0.0	13.8	0.0	55.9	0	356	287	0.0	5984.1	2231.2	865.2
Mar	204	8	76.6	80.7	0.0	14.0	0.0	72.8	0	458	223	0.0	7776.9	4836.3	74.3
Apr	21	133	76.6	80.9	0.0	14.2	0.0	81.1	0	499	36	0.0	8624.8	8136.8	0.2
May	0	296	76.7	81.1	0.0	14.3	0.0	91.7	0	555	2	0.0	9716.7	9690.2	0.0
Jun	0	459	76.8	81.1	0.0	14.4	0.0	95.7	0	570	0	0.0	10103.2	10103.2	0.0
Jul	0	457	76.8	81.1	0.0	14.4	0.0	98.3	0	583	0	0.0	10369.4	10369.4	0.0
Aug	0	477	76.7	80.8	0.0	14.4	0.0	98.6	0	586	0	0.0	10402.5	10402.5	0.0
Sep	0	397	76.8	81.0	0.0	14.4	0.0	92.8	0	553	0	0.0	9797.1	9797.1	0.0
Oct	25	147	76.7	80.9	0.0	14.1	0.0	83.9	0	516	31	0.0	8915.4	8503.2	0.0
Nov	123	62	76.6	80.9	0.0	14.2	0.0	73.5	0	456	141	0.0	7824.2	5953.5	83.7
Dec	383	1	76.1	80.6	0.0	13.9	0.0	64.6		409	300	0.0	6909.0	2960.9	228.1
Totals	1723	2437	76.1	81.1	0.0	14.4	0.0	969.7	0	5934	1364	0.0	102941.0	85029.0	1638.0

Notes: Fan supply rate = 7400 CFM, ventilation = 150 CFM, wideband temperature limits for occupied and unoccupied times = 65° to 80°F, economizer temperature limit = 65°F, time period = 1 to 365 days, total hours = 8760.

Table Vb—Intermittent control of building space air temperature—New Orleans, case 4 in Table I

Number of Hours at Specified Relative Humidity (No Humidity Control)						
	<10%	10–15%	15–20%	20–55%	55–60%	>60%
Conv (No Econ)	14.25	395.50	441.25	7909.00	0.0	0.0
Economy	60.0	383.00	474.75	7842.25	0.0	0.0
Enthalpy	14.25	419.00	455.25	7871.50	0.0	0.0

Estimated Operating Cost for Cooling at $0.10/kWh for Electricity (Chiller COP = 3.50)		
Conv (no econ)	$10294 for 102939 kWh	(Fans = 21693 kWh, chiller = 81246 kWh)
Economy	$8503 for 85027 kWh	(Fans = 21693 kWh, chiller = 63334 kWh)
Enthalpy	$9295 for 92949 kWh	(Fans = 21693 kWh, chiller = 71256 kWh)

Estimated Operating Cost for Humidification (20% min) and Heating at $0.10/kWh for Electricity	
Humidification	$500 for 5002 kWh
Heating	$ 0 for 0 kWh

Notes: Min space temp occurred on day 345, max space temp occurred on day 147, max cooling load occurred on day 165.

proportional control plan much more attractive. We can conclude that TELBECC has great potential for pinpointing significant energy reductions and cost savings before a building's HVAC system is purchased.

VI. CONCLUSIONS

The TELBECC program analyzes more efficiently and quickly than any method used heretofore the possible telephone building environmental energy use and control options. To pinpoint the most economical energy-conservation plan, the program analyzes multiple plans at minimal cost and with minimal expenditure of time. The program calculates the energy consumed every quarter hour by the HVAC in regulating the environment under changing weather conditions. It computes the required energy from the physical characteristics of the building envelope, such as the U factor, internal heat generation, geographic location, orientation of the building, and the dry-bulb temperature standard. In order to make it feasible to calculate by computer, we employ a simplified recursive computation procedure using time series. For each of the illustrative problems in Tables II through VI, the procedure produced monthly projections; yet it took less than 40 seconds to calculate results on an IBM/3033 computer. From the examples, we see the advantages and disadvantages of both the intermittent and proportional control plans, as well as the significant savings obtained from increasing the range of the dual or

Table VIa—Intermittent control of building space air temperature—Phoenix, Ariz., case 5 in Table I

	Degree-Days		Space Temp		Max Load (tons)		Total Load (MBtu)		Number of Hours			Heating (kWh	Cooling (kWh)		Water (gal) to Maintain 20%
Month	Heat	Cool	Min	Max	Heat	Cool	Heat	Cool	Heat	Cool	Econ	Elect)	No Econ	Econ	Min RH
Jan	432	0	76.3	80.6	0.0	13.5	0.0	62.8	0	401	300	0.0	6724.2	2812.0	1132.0
Feb	257	1	76.5	80.7	0.0	13.6	0.0	61.5	0	390	249	0.0	6578.6	3313.1	455.0
Mar	139	12	76.6	80.7	0.0	13.5	0.0	74.0	0	468	212	0.0	7908.0	5125.0	1705.1
Apr	38	122	76.6	81.1	0.0	13.7	0.0	79.6	0	497	104	0.0	8486.9	7105.4	336.4
May	16	403	76.7	81.2	0.0	13.9	0.0	92.1	0	570	38	0.0	9807.0	9301.8	64.9
Jun	0	560	76.7	81.1	0.0	14.1	0.0	95.8	0	589	2	0.0	10181.0	10148.0	256.0
Jul	0	847	76.9	81.2	0.0	14.4	0.0	109.5	0	657	0	0.0	11574.5	11574.5	0.0
Aug	0	679	76.7	81.2	0.0	14.4	0.0	104.1	0	626	0	0.0	11016.5	11016.5	0.0
Sep	0	521	76.8	80.8	0.0	14.2	0.0	94.4	0	574	0	0.0	10010.3	10010.3	0.0
Oct	14	201	76.6	81.0	0.0	14.0	0.0	84.5	0	525	86	0.0	9003.7	7860.2	12.0
Nov	196	0	76.5	80.6	0.0	13.6	0.0	67.8	0	428	236	0.0	7243.1	4138.2	161.3
Dec	413	0	76.5	80.6	0.0	13.4	0.0	63.0	0	401	307	0.0	6742.1	2719.4	458.0
Totals	1505	3346	76.3	81.2	0.0	14.4	0.0	989.0	0	6130	1536	0.0	105276.0	85124.0	4582.0

Notes: Fan supply rate = 7400 CFM, ventilation = 150 CFM, wideband temperature limits for occupied and unoccupied times = 65° to 80°F, economizer temperature limit = 65°F, time period = 1 to 365 days, total hours = 8760.

Table VIb—Intermittent control of building space air temperature—Phoenix, Ariz., case 5 in Table I

Number of Hours at Specified Relative Humidity (No Humidity Control)	<10%	10–15%	15–20%	20–55%	55–60%	>60%
Conv (No Econ)	168.50	611.50	1430.00	6550.00	0.0	0.0
Economy	199.00	627.00	1432.25	6501.75	0.0	0.0
Enthalpy	185.50	648.50	1375.00	6551.00	0.0	0.0

Estimated Operating Cost for Cooling at $0.10/kWh for Electricity (Chiller COP = 3.50)		
Conv (no econ)	$10527 for 105274 kWh	(Fans = 22412 kWh, chiller = 82862 kWh)
Economy	$8512 for 85123 kWh	(Fans = 22412 kWh, chiller = 62711 kWh)
Enthalpy	$9205 for 92054 kWh	(Fans = 22412 kWh, chiller = 69642 kWh)

Estimated Operating Cost for Humdification (20% min) and Heating at $0.10/kWh for Electricity	
Humidification	$1399 for 13991 kWh
Heating	$0 for 0 kWh

Notes: Min space temp occurred on day 18, max space temp occurred on day 225, max cooling load occurred on day 204.

extended wideband temperature limits fcr unoccupied times. In the larger view, we can understand how TELBECC can significantly contribute toward the operating companies' energy-conservation plan for future savings.

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APPENDIX

Calculating Inside-Wall Temperature Time Series Coefficients b_i, a_i, a'_i of Eq. (1)

The basis for computing the time-series coefficients is the z transform,[8] a discrete function transformation. This transformation is applied to time functions sampled at regular intervals of time. The z transform has the same role in discrete systems that the Laplace transform has in continuous systems analysis.

Let us consider a continuous function of time $f(t)$. When the function is sampled at regular intervals Δ, the output consists of a train of pulses, as illustrated in Fig. 1. We defined the z transform of this output as a polynomial in powers of z^{-1} in the following:

$$f(0) + f(\Delta)z^{-1} + f(2\Delta)z^{-2} + f(3\Delta)z^{-3} + \cdots . \tag{8}$$

The successive outputs of the sampler are the coefficients of the successive powers of z^{-1} in the z transform.

A linear system is characterized when its response to an elementary input (such as a pulse, a unit step, or, as will be adopted here, a unit ramp) is ascertained. This is nothing more than obtaining a transfer function of the system. If both input and output of the system are expressed in terms of their z transforms, the ratio of output/input is the z transform of the system. If we assume that such a transfer function, $G(z)$, can be found and that it can be expressed as the quotient of two polynomials in z^{-1}, then

$$G(z) = \frac{N(z)}{D(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots a_j z^{-j}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_p z^{-p}}. \tag{9}$$

It follows that the z-transform of the output $O(z)$ resulting from an arbitrary input $I(z)$ is represented by

$$O(z) = G(z)I(z) \quad \text{or} \tag{10}$$

$$O(z)D(z) = N(z)I(z). \tag{11}$$

Since both sides of (11) are polynomials, the coefficients of the various powers of z^{-1} must be the same on both sides of the equation. If, say, the coefficients of z^{-n} are equated, eq. (11) yields

$$\begin{aligned} b_0 O_n = a_0 I_n + a_1 I_{n-1} + a_2 I_{n-2} + \cdots + a_j I_{n-j} \\ - [b_1 O_{n-1} + b_2 O_{n-2} + \cdots + b_p O_{n-p}]. \end{aligned} \tag{12}$$

The subscript n on O and I indicates the value of the function at $t = n\Delta$; i.e., $O_n \equiv O(n\Delta)$, the coefficient of z^{-n} in the z transform of $O(z)$. This expression relates the output at any time $t = n\Delta$ to the input at that time and the input and output at earlier times. The coefficients $a_0, \cdots a_j$ and $b_0, \cdots b_p$ contain all the characteristics of the system. With the properties of the z transform described above, a method for determining the z transform or time-series coefficients for the inside building wall temperature follows.

If we consider the outside building walls and roof structure as homogeneous flat slabs (Fig. 2), the temperature in the slab adheres to the following equations:

$$\kappa \frac{\delta^2 T_s}{\delta x^2} = \frac{\delta T_s}{\delta t},$$

$$T_s(x, 0) = 0,$$

$$k \frac{\delta T_s}{\delta x}(L, t) = -h_O[T_s(L, t) - T_O(t)],$$

$$k \frac{\delta T_s}{\delta x}(0, t) = h_i[T_s(0, t) - T_a(t)], \tag{13}$$

where

$T_s(x, t)$ (°F) = temperature in the slab,
L(ft) = slab thickness,
k(Btu/hr − ft − °F) = thermal conductivity,
$k \equiv \kappa/\rho c$(ft^2/hr) = diffusivity,
ρc(Btu/ft^3) = volumetric heat capacity,
h_O(Btu/hr − ft^2 − °F) = outside-wall heat transfer coefficient,
h_i(Btu/hr − ft^2 − °F) = inside-wall heat transfer coefficient,
$T_O(t)$ = outside sol-air temperature, and
$T_a(t)$ = inside building air temperature.

It is convenient to use the Laplace transform

$$\bar{T}_s(x, p) = \int_0^\infty T_s(x, t)e^{-pt}dt$$

to eliminate the independent time variable t in eq. (13). Then the solution for the inside wall surface ($x = 0$) temperature in terms of the transform parameter p assumes the form:

$$\bar{T}_s(0, p) = \frac{h_i[kq \cosh(qL) + h_O \sinh(qL)]\bar{T}_a(p) + h_O kq \bar{T}_O(p)}{h_i[kq \cosh(qL) + h_O \sinh(qL)] + kq[kq \sinh(qL) + h_O \cosh(qL)]}, \tag{14}$$

where $q = (p/\kappa)^{1/2}$.

Letting $T_a(t)$ and $T_O(t)$ be unit ramp functions and inverting eq. (14) back to the real-time domain by using standard residue theory in the complex plane, the solution for $T_s(0, t)$, the temperature of the inside surface, is expressed as

$$T_s(0, t) = T_s^{(1)}(0, t) + T_s^{(2)}(0, t), \tag{15}$$

where $T_s^{(1)}(0, t)$ is the portion of the solution dependent on the outside sol-air temperature, and $T_s^{(2)}(0, t)$ the part dependent on the building space-air temperature. These temperatures are explicitly:

$$T_s^{(1)}(0, t) = B_O \left[\frac{1}{B_i + B_O B_i + B_O} \left(t - \frac{L^2}{6\kappa} \frac{(3B_i + B_O B_i + 3B_O + 6)}{(B_i + B_O B_i + B_O)} \right) - \frac{2L^2}{\kappa} \sum_{n=1}^{\infty} \frac{e^{-\alpha_n^2 \kappa t/L^2}}{\alpha_n^2 \{[(B_i + B_O B_i + B_i) - \alpha_n^2] \cdot \cos \alpha_n - \alpha_n (2 + B_O + B_i) \sin \alpha_n\}} \right], \tag{16}$$

and

$$T_s^{(2)}(0, t) = B_i \left[\frac{1}{B_i + B_O B_i + B_O} \left(\frac{L^2}{6\kappa} (3 + B_O) - \frac{L^2}{6\kappa} \frac{(1 + B_O)(3B_i + B_i B_O + 3B_O + 6)}{(B_i + B_O B_i + B_O)} + (1 + B_O) t \right) - \frac{2L^2}{\kappa} \sum_{n=1}^{\infty} \frac{[\alpha_n \cos \alpha_n + B_O \sin \alpha_n] e^{-\alpha_n^2 \kappa t/L^2}}{\sigma_n^3 [(B_i + B_i B_O + B_O - \alpha_n^2] \cdot \cos \alpha_n - \alpha_n [2 + B_O + B_i] \sin \alpha_n} \right], \tag{17}$$

where $B_O = \frac{h_O L}{k}$, $B_i = \frac{h_i L}{k}$, and α_n are roots of the transcendental equation

$$\cot \alpha_n = \frac{\alpha_n^2 - B_O B_i}{\alpha_n (B_O + B_i)}, \qquad n = 1, 2, \cdots .$$

Equations (15) through (17) contain all the ingredients for forming the z-transform transfer functions for the inside-wall surface temperature. These are obtained by forming the ratio of output/input z transforms as per eq. (9):

$$G^{(1)}(z) = \frac{T_s^{(1)}(0, z)}{T_O(z)} \quad \text{and} \quad G^{(2)}(z) = \frac{T_s^{(2)}(0, z)}{T_a(z)}. \tag{18}$$

$T_O(t)$ and $T_a(t)$ were taken as unit ramp functions, and therefore their z transform from Ref. 8 is given as

$$T_O(z) = T_a(z) = \frac{\Delta}{z(1 - z^{-1})^2}. \tag{19}$$

The sampling interval is Δ. The use of the input ramp function amounts to linear interpolation between the discrete values given by the z-transform coefficients.

The z transforms of both $T_s^{(1)}(0, z)$ and $T_s^{(2)}(0, z)$ are similar in form and, with the aid of the table of z transforms given in Ref. 8, can be expressed as

$$T_s^{(1,2)}(0, z) = \frac{A^{(1,2)}}{1 - z^{-1}} + \frac{B^{(1,2)}\Delta}{z(1 - z^{-1})^2} + \sum_{j=1}^{\infty} \frac{C_j^{(1,2)}}{1 - s_j z^{-1}}, \tag{20}$$

where

$$A^{(1)} = \frac{-B_O L^2}{6\kappa(B_i + B_O B_i + B_O)^2}(3B_i + B_O B_i + 3B_O + 6),$$

$$B^{(1)} = \frac{B_O}{B_i + B_O B_i + B_O},$$

$$C_j^{(1)} = -\frac{2L^2 B_O}{\kappa}\left(\frac{1}{\alpha_j^2\{[(B_i + B_O B_i + B_O) - \alpha_j^2] \cdot \cos\alpha_j - \alpha_j(2 + B_O + B_i)\sin\alpha_j\}}\right),$$

$$A^{(2)} = \frac{B_i}{B_i + B_O B_i + B_O}\left(\frac{L^2}{6\kappa}(3 + B_O) - \frac{L^2}{6\kappa}\,\frac{(1 + B_O)(3B_i + B_i B_O + 3B_O + 6)}{(B_i + B_O B_i + B_O)}\right),$$

$$B^{(2)} = \frac{B_i(1 + B_O)}{B_i + B_O B_i + B_O},$$

$$C_j^{(2)} = -\frac{2B_i L^2}{\kappa}\left(\frac{\alpha_j \cos\alpha_j + B_O \sin\alpha_j}{\alpha_j^3[(B_i + B_i B_O + B_O - \alpha_j^2)] \cdot \cos\alpha_j - \alpha_j(2 + B_O + B_i)\sin\alpha_j}\right), \quad \text{and}$$

$$s_j = e^{-\alpha_j^2 \Delta\kappa/L^2}$$

Equation (18) can now be expressed in the form of a ratio of polynomials in z^{-1}:

$$G^{(1,2)}(z) = \frac{N^{(1,2)}(z)}{D(z)}, \tag{21}$$

where, from the results of eqs. (19) and (20),

$$N^{(1,2)}(z) = \left(A^{(1,2)}\frac{z(1 - z^{-1})}{\Delta} + B^{(1,2)}\right)\prod_{j=1}^{\infty}(1 - s_j z^{-1}) + \frac{z(1 - z^{-1})^2}{\Delta}\sum_{n=1}^{\infty} C_n^{(1,2)} \prod_{j=n}^{\infty}(1 - s_j s^{-1}),$$

$$D(z) = \prod_{j=1}^{\infty}(1 - s_j z^{-1}).$$

Equation (21) is the form of eq. (9); consequently, the b_i coefficients that are derived, as in eq. (12), from the coefficients of the polynomial $D(z)$ can be generated by a recursive scheme given as

$$b_O = 1,\ b_i^{(1)} = 0,\ b_i^{(n+1)} = b_i^{(n)} - s_{i-n+1}b_{i-1}^{(n)}\ n = 1, \cdots N \qquad (22)$$

The number of s_j terms needed to obtain the desired degree of accuracy for the b_i coefficients is indicated by the index $n = N$, which in most instances should not exceed 20.

The a_i and a_i' coefficients in eq. (1) came from $N^{(1)}(z)$ and $N^{(2)}(z)$, respectively, by expanding these functions into polynomials in powers of z^{-1}; i.e.,

$$N^{(1)}(z) = a_1z^{-1} + a_2z^{-2} + \cdots$$

$$N^{(2)}(z) = a_1'z^{-1} + a_2'z^{-2} + \cdots$$

The desired coefficients are sorted out.

AUTHOR

Philip B. Grimado, B.S. (Civil Engineering), 1961, City University of New York; M.S. (Applied Mechanics), 1962, Columbia University; Ph.D. (Applied Mechanics), 1968, Columbia University; Bell Laboratories, 1968—. Mr. Grimado's responsibilities include vulnerability studies of antiballistic missile systems, fire protection studies involving fire risk analyses, heat-transfer calculations, development of standard fire testing methods for operating company equipment, and development of algorithms for optimum control of building environmental equipment. He is currently engaged in developing computer software for automatic generation of optimum layout configurations of administrative office space.