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A Class of Lossless, Reciprocal Anti-Sidetone Networks for Telephone Sets

By D. R. MEANS

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Anti-sidetone networks are used in telephone sets to interconnect the transmitter, receiver, and telephone line. Most passive networks used in modern telephone instruments belong to the class of lossless, reciprocal networks known as maximum output networks. This paper defines a larger class of lossless, reciprocal networks suitable for use as anti-sidetone networks in telephone sets and describes the properties of the networks belonging to this class. The class includes maximum output networks, such as those conventionally used for this application, as well as certain nonmaximum output networks. The latter, although they do not exhibit maximum power transfer characteristics at all network ports, have the same transmit and receive power transfer ratios as the maximum output networks. The network presently used in the Trimline® *telephone set is an example of a nonmaximum output network belonging to this class. The use of such a network provides a convenient method of biasing the* LEDs *which are used to illuminate the dial in this telephone set. However, the power transfer characteristics between the transmitter, receiver, and telephone line ports of this network are the same as those of a conventional maximum output network.*

I. INTRODUCTION

In a classic paper,[1] Campbell and Foster defined a class of networks suitable for use as anti-sidetone networks in telephone sets. Campbell and Foster called these networks maximum output networks. Such networks have been used as anti-sidetone networks in telephone sets for many years.[2–4] As we show in this paper, however, the properties which make maximum output networks suitable for this application are properties of certain nonmaximum output networks as well. Therefore, maximum output networks are but a subset of a larger class of anti-sidetone networks.

The network presently used in the *Trimline®* telephone set is an example of a nonmaximum output network belonging to this larger class. In this telephone set, LEDs (light-emitting diodes) illuminate the dial. The LEDs are powered from the telephone line through an additional winding on the network transformer. This eliminates the need for ac isolation of the LED dc biasing circuit, as would have been required if a conventional maximum output network had been used. Instead, the dc biasing circuit, being one of the windings on the network transformer, is an integral part of the network itself.

If a conventional maximum output network had been used in this application, an isolated dc biasing circuit for the LEDs would have been required. The impedance of this circuit would have had to be high enough to prevent its loading the network. Although this could have been accomplished using an inductor, an inductor large enough to be effective at voice frequencies would have been undesirable in terms of both cost and physical size. Therefore, the use of a nonmaximum output network both lowered the cost of the telephone set and simplified the physical design.

Before the network used in the *Trimline* telephone set is described, the general properties of this class of networks are examined. Because the definition of this class of networks is based on certain properties of maximum output networks, maximum output networks are considered first.

II. MAXIMUM OUTPUT NETWORKS

A maximum output network can be defined as any lossless, reciprocal network having the property that maximum output power is obtained from a source in any of the network terminations. Campbell and Foster showed that such a network must have an even number of terminations. Since the network in a telephone set must have at least three terminations, namely, the transmitter, the receiver, and the telephone line, Campbell and Foster considered networks having four terminations as shown in Fig. 1, in which T, R, and L represent the transmitter, the receiver, and the telephone line, and N represents the required additional termination.

Among the properties of maximum output networks having four terminations is the property that the power transfer ratio between any pair of terminations is identical to the power transfer ratio between the other pair of terminations, where the power transfer ratio between two terminations is defined as the ratio of the power dissipated in one termination to the power available from a source in the other termination. Therefore, for a maximum output network having four terminations there are only three unique power transfer ratios. However, as we will show, one of them must be zero and their sum must be unity.

Therefore, there are only two unique nonzero power transfer ratios and only one of the two can be chosen independently.

For example, let one pair between which there is no power transfer be T and R, as required of an anti-sidetone network. Then there is also no power transfer between L and N. Furthermore, the power transfer ratio between T and L and the power transfer ratio between R and N must be equal and the power transfer ratio between L and R and the power transfer ratio between T and N must also be equal. This is illustrated in Fig. 2, in which α and β represent the two nonzero power transfer ratios, which can be called the transmit and receive power transfer ratios, respectively.

Campbell and Foster called the termination pairs between which there is no power transfer conjugate pairs and they called a network having two such pairs biconjugate. A maximum output network having four terminations is necessarily biconjugate.

The proofs of the preceding statements regarding the properties of maximum output networks are based on scattering matrix theory. Let the scattering matrix of a lossless, passive, four-port network be

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}. \tag{1}$$

If all the ports are simultaneously matched so that maximum output power can be obtained from a source in any termination, the four main diagonal elements of the matrix are zero, i.e., $s_{ij} = 0$ for $i = j$. Furthermore, the scattering matrix of any lossless network is unitary,[5] i.e.,

$$S^{*T}S = I, \tag{2}$$

where the asterisk denotes "complex conjugate."

Equation (2) can be expanded as

$$\sum_k s^*_{kp} s_{kq} = \delta_{pq}, \tag{3}$$

where

$$\delta_{pq} = \begin{matrix} 1, & p = q \\ 0, & p \neq q. \end{matrix}$$

The four equations obtained with $p = q$ and with $s_{ij} = 0$ for $i = j$ are

$$\left.\begin{aligned} |s_{21}|^2 + |s_{31}|^2 + |s_{41}|^2 = 1 \\ |s_{12}|^2 + |s_{32}|^2 + |s_{42}|^2 = 1 \\ |s_{13}|^2 + |s_{23}|^2 + |s_{43}|^2 = 1 \\ |s_{14}|^2 + |s_{24}|^2 + |s_{34}|^2 = 1 \end{aligned}\right\}. \tag{4}$$

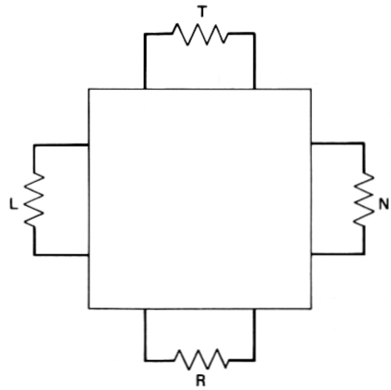

Fig. 1—Maximum output network having four terminations.

Since $|s_{ij}|^2$ is the ratio of the power dissipated in termination i to the power available from a source in termination j, eqs. (4) are simply statements of the fact that all the power available from a source in any termination is delivered to the remaining three terminations. If the network is reciprocal, $|s_{ij}|^2 = |s_{ji}|^2$ and eqs. (4) reduce to

$$\left.\begin{aligned} |s_{21}|^2 + |s_{31}|^2 + |s_{41}|^2 = 1 \\ |s_{21}|^2 + |s_{32}|^2 + |s_{42}|^2 = 1 \\ |s_{31}|^2 + |s_{32}|^2 + |s_{43}|^2 = 1 \\ |s_{41}|^2 + |s_{42}|^2 + |s_{43}|^2 = 1 \end{aligned}\right\}. \tag{5}$$

Equations (5) can be satisfied only if

$$\left.\begin{aligned} |s_{21}|^2 = |s_{43}|^2 \\ |s_{31}|^2 = |s_{42}|^2 \\ |s_{41}|^2 = |s_{32}|^2 \end{aligned}\right\}. \tag{6}$$

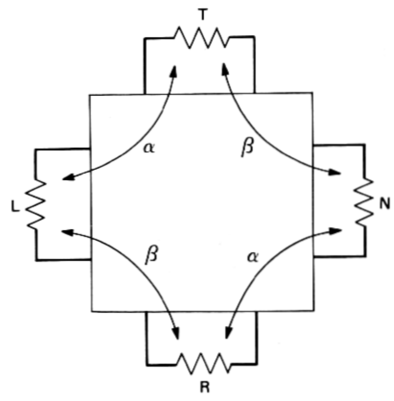

Fig. 2—Power transfer ratios between the terminations of a maximum output network.

Therefore, the power transfer ratio between any pair of terminations is equal to the power transfer ratio between the other pair of terminations. Consequently, there can be but three unique power transfer ratios and their sum, according to eqs. (5), is unity.

The proof that at least one of the three power transfer ratios must be zero is as follows. If the network is reciprocal, then the scattering matrix is symmetric,[5] i.e., $s_{ij} = s_{ji}$. Then three of the remaining six equations of the form of (3) are

$$\left.\begin{aligned} s_{31}^* s_{32} &= -s_{41}^* s_{42} \\ s_{21} s_{32}^* &= -s_{41} s_{34}^* \\ s_{21}^* s_{31} &= -s_{42}^* s_{34} \end{aligned}\right\}, \tag{7}$$

and their product is

$$|s_{31}s_{32}s_{21}|^2 = -|s_{42}s_{41}s_{34}|^2. \tag{8}$$

With eq. (6), this becomes

$$|s_{31}s_{32}s_{34}|^2 = -|s_{31}s_{32}s_{34}|^2. \tag{9}$$

Therefore, at least one of the three power transfer ratios must be zero. The practical case is that in which only one of the three is zero. If two are zero, the network decomposes into two uncoupled subnetworks.

Since one of the three power transfer ratios is zero, the sum of the other two is unity. In the telephone set application, the three power transfer ratios are those between the transmitter and the telephone line, between the telephone line and the receiver and between the transmitter and the receiver. If the last is zero, the sum of the other two is unity, i.e., the sum of the transmit and the receive power transfer ratios is unity.

Other properties of maximum output networks are considered in Ref. 1. However, the definition of a more general class of networks suitable for use in the telephone set application can be based on the properties already considered.

III. A MORE GENERAL CLASS OF NETWORKS

Although maximum output networks possess properties that make them suitable for use as anti-sidetone networks in telephone sets, they are not the only networks which possess those properties. As is shown, maximum power transfer is necessary only at the transmitter, receiver, and telephone line ports of the network, whereas maximum output networks require maximum power transfer at a fourth port as well. The removal of this restriction admits other networks to the class of those suitable for use in this application. In fact, lossless, reciprocal nonmaximum output networks exist such that the power transfer ratios involving three of their terminations are identical to those of a maximum output network connecting the same three terminations and the required fourth termination. Therefore, a more general class of anti-sidetone networks can be defined as the class that includes any lossless, reciprocal network having the same power transfer ratios between pairs of three of its terminations as a maximum output network connecting the same three terminations and the required fourth termination. The properties of this class of networks are now considered.

The three terminations of interest will be denoted by T, R, and L in accordance with the previous notation. Let T and R be conjugate as required of an anti-sidetone network. It follows from the definition of this class of networks that, since the transmit and receive power transfer ratios must be the same as those of a maximum output network, their sum must be unity. However, no network having only three terminations can satisfy these requirements. There must be, in addition to T, R, and L, at least one other termination.

This can be proved as follows. If termination 1 corresponds to T, termination 2 corresponds to L and termination 3 corresponds to R, and if T and R are conjugate, the scattering matrix is

$$S = \begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & s_{23} \\ 0 & s_{32} & s_{33} \end{bmatrix}. \tag{10}$$

If the network is lossless, S must be unitary. One of the equations of the form of eq. (3) which must be satisfied is found to be

$$s_{21}^{*} s_{23} = 0. \tag{11}$$

However, neither s_{21} nor s_{23} can be zero, since $|s_{21}|^2$ is the transmit power transfer ratio and, if the network is reciprocal, $|s_{23}|^2 = |s_{32}|^2$, which is the receive power transfer ratio. Therefore, the requirements cannot be satisfied by a lossless, reciprocal network having only three terminations. It is observed that, in this proof or in any of those pertaining to maximum output networks, there is no loss of generality if the network is required to be lossless since the resistive components in any lossy network can simply be considered to be terminations of a lossless network.

Although there must be at least one additional termination, there can be more than one. In fact, there can be any number as long as they are all conjugate to L. If, again, termination 1 corresponds to T, termination 2 corresponds to L and termination 3 corresponds to R, then, if there are n total terminations,

$$|s_{12}|^2 + |s_{22}|^2 + |s_{32}|^2 + \cdots + |s_{n2}|^2 = 1,$$

which is simply a statement of the fact that the power available from a source in L is equal to the sum of what is reflected back into the source and what is delivered to the other terminations. However, $|s_{12}|^2$ and $|s_{32}|^2$ are the transmit and receive power transfer ratios, respectively, whose sum must be unity. Therefore, the remaining terms in eq. (12) must be zero. This means that no power is reflected back into the source and that each remaining termination must be conjugate to L.

It follows that there must be maximum power transfer at the transmitter port and at the receiver port as well as at the telephone line port, since two of the remaining equations which must be satisfied for the scattering matrix of the network to be unitary now reduce to

$$s_{11}^* s_{12} = 0 \tag{13}$$

and

$$s_{32}^* s_{33} = 0. \tag{14}$$

Because neither s_{12} nor s_{32} can be zero, s_{11} and s_{33} must both be zero.

Such a network need not be a maximum output network. If the total number of terminations is odd, the network cannot be a maximum output network. In this case, although the network is matched at the ports connecting T, L, and R, it will not be matched at the remaining ports. A nonmaximum output network such as this is presently used in the *Trimline* telephone set. This example, which constitutes a proof of the existence of such networks, is now considered.

IV. THE *TRIMLINE* NETWORK

The network presently used in the *Trimline* telephone set is an example of a nonmaximum output network belonging to the class defined in the preceding section. This network differs from the Campbell and Foster type of network used previously in *Trimline* telephone sets as well as other Bell System telephone sets in that it is a five-port network. The fifth port is provided for connection of the LEDs. Because the total number of terminations is odd, the network cannot be a maximum output network. Yet it has the same transmit and receive power transfer ratios as the conventional maximum output network.

The basic structure of the network which was used in *Trimline* telephone sets prior to the incorporation of LED dial illumination and which is also used in other Bell System telephone set designs is shown in Fig. 3. This is a network consisting of a three-winding transformer connecting the four terminations, which are labeled T, L, R, and N in accordance with the previous notation. The transformer turns ratios are m and n.

As can be determined from straightforward circuit analysis, the

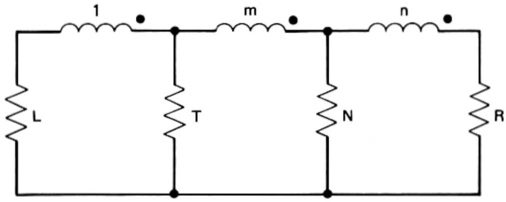

Fig. 3—Basic structure of conventional network.

biconjugacy conditions are

$$N = Lmn/(1 + m + n) \tag{15}$$

and

$$R = Tn(1 + m + n)/m. \tag{16}$$

The transmit and receive voltage transfer ratios are then

$$G_T = L/[T(1 + m)/m + L(m + n)/(1 + m + n)] \tag{17}$$

and

$$G_R = R/[L(m + n) + R(1 + m)/n], \tag{18}$$

respectively. Maximum output conditions are obtained if

$$T = Lm(m + n)/[(1 + m)(1 + m + n)] \tag{19}$$

or, equivalently, if

$$R = Ln(m + n)/(1 + m). \tag{20}$$

Equations (19) and (20) can be obtained by observing that maximum power transfer results when the two terms in the denominator of the right-hand side of either eq. (17) or eq. (18) are equal.

The basic structure of the network presently used in the *Trimline* telephone set is shown in Fig. 4. This network consists of a four-winding transformer connecting the five terminations, which are la-

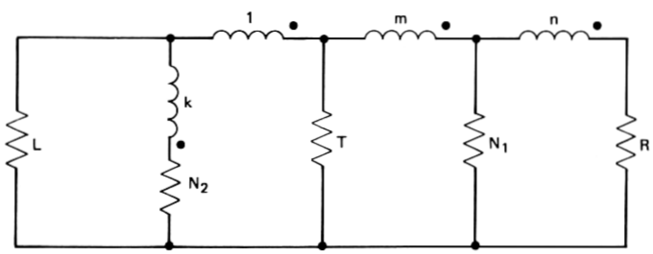

Fig. 4—Basic structure of LED *Trimline*® network.

beled T, L, R, N_1, and N_2. The transformer turns ratios are labeled m, n, and k. The letters T, L, and R again represent the transmitter, telephone line, and receiver, while N_1 and N_2 represent the additional terminations, N_2 being the LEDs, which are biased through transformer winding k.

As can again be determined from straightforward circuit analysis, the expressions corresponding to eqs. (15) through (18) are, respectively,

$$N_1 = mnLN_2/[(1 + m + n)N_2 + (1 - k)(1 + m + n - k)L] \quad (21)$$

$$R = n(1 + m + n)TN_2/[mN_2 + k(1 + m - k)T] \quad (22)$$

$$G_T = L/[L(m + n)/(1 + m + n) + T(1 + m)/m + TL(1 + m + n - k)(1 + m - k)/m(1 + m + n)N_2] \quad (23)$$

and

$$G_R = R/[L(m + n) + R(1 + m)/n + LR(1 - k)(1 + m - k)/nN_2]. \quad (24)$$

It can be seen that, if $k = 1 + m$, eqs. (23) and (24) reduce to eqs. (17) and (18), respectively, i.e., the expressions for the transmit and receive voltage transfer ratios for the LED network become identical to those of the conventional network.

A further consequence of setting k equal to $1 + m$ is that eq. (22) reduces to eq. (16). However, eq. (21) does not reduce to eq. (15). It reduces instead to

$$N_1N_2/(N_1 + N_2) = mnL/(1 + m + n). \quad (25)$$

Insight can be gained if it is observed that, if $k = 1 + m$, the voltage across N_2 must be equal to the voltage across N_1, which can be proved by summing the voltages around the path consisting of N_1, N_2, and transformer windings 1, m, and k. Then, if N_1 is conjugate to L, so is N_2.

It can be concluded that, if $k = 1 + m$, this network, although it is not a maximum output network, is equivalent to a maximum output network with $N = N_1N_2/(N_1 + N_2)$. As a practical note, it is observed that, although the impedance of the LEDs themselves, with the range of bias currents used, is only a few ohms, the resistance of transformer winding k can be considered to be part of N_2. This winding resistance is large enough so that (25) can be satisfied with N_1 greater than zero.

Of course, infinite sidetone rejection would require a perfect impedance balance, which is not possible in practice. In fact, the sidetone rejection is limited by variations in telephone line impedance since telephone line impedance depends on the type and length of the line to which the telephone set is connected. However, some sidetone is desirable as feedback to the customer. For the efficiencies of the transducers used in the *Trimline* telephone set, approximately 30 dB of sidetone rejection is required of the network to produce acceptable sidetone levels. The sidetone voltage transfer ratio near balance and with $k = 1 + m$ is given by

$$G_s = [R(1 + m + n)/Lmn(m + n)][mnL/(1 + m + n) - N_1N_2/(N_1 + N_2)]/[L(m + n)/(1 + m + n) + T(1 + m)/m] \quad (26)$$

which, of course, is zero if eq. (25) is satisfied. Equation (26) is identical to the corresponding equation for the conventional network if $N = N_1N_2/(N_1 + N_2)$.

It can be concluded that this network is an example of a nonmaximum output network belonging to the class of networks considered in this paper. The use of such a network in the *Trimline* telephone set provided a convenient means of biasing the LEDs without sacrificing transmission performance or requiring extra components to provide ac isolation for the LED dc biasing circuit.

V. SUMMARY

A class of networks suitable for use as anti-sidetone networks in telephone sets has been defined, and the properties of the networks belonging to this class have been described. This class includes maximum output networks as well as certain nonmaximum output networks. The networks belonging to this class are simultaneously matched at the ports connecting the transmitter, the telephone line, and the receiver, but not necessarily at the ports connecting the remaining terminations, of which there must be at least one, but can be any number as long as they are all conjugate to the telephone line. These networks then exhibit the same transmit and receive power transfer ratios as do conventional maximum output anti-sidetone networks.

A nonmaximum output network belonging to this class is presently used in the *Trimline*® telephone set. An analysis of this network was presented. This example is proof of the existence as well as proof of the practical applicability of such networks.

REFERENCES

1. G. A. Campbell and R. M. Foster, "Maximum Output Networks for Telephone Substation and Repeater Circuits," Trans. AIEE, *39* (1920), pp. 231–280.
2. A. H. Inglis, "Transmission Features of the New Telephone Sets," B.S.T.J., *17,* No. 3 (July 1938), pp. 358–380.
3. A. H. Inglis and W. L. Tuffnell, "An Improved Telephone Set," B.S.T.J., *30,* No. 2 (April 1951), pp. 239–270.
4. A. F. Bennett, "An Improved Circuit for the Telephone Set," B.S.T.J., *32,* No. 3 (May 1953), pp. 611–626.
5. H. J. Carlin, "The Scattering Matrix in Network Theory," IRE Trans. on Circuit Theory, *CT-3,* No. 2 (June 1956), pp. 88–97.