

Blocking Distributions for Trunk Network Administration

By S. R. NEAL

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Administrative control of the Bell System telephone network requires demand-servicing procedures that maintain network service in a cost-effective manner and an administrative measurement plan that determines whether the network provisioning process is providing a proper balance between network service and network utilization. The traffic measurements available are busy-season, busy-hour, trunk-group usage and average blocking. Statistical variation of these measurements can lead to an incorrect assessment of network performance and an unstable network provisioning process, resulting in costly excess reserve capacity in the network. To minimize sensitivity to volatile measurements, administrative measurement bands have been employed to define a new demand-servicing procedure and a new trunk service measurement plan for use within the Bell System. This paper derives the probability distribution function for measured blocking, which is required to construct the measurement bands, and outlines the new Bell System administrative procedures. The distribution has a discrete component, the probability of no blocking, and a continuous component. The last function is well approximated by a two-parameter beta distribution. The resulting measurement bands are a strong function of trunk group size and a weak function of traffic characteristics. Small trunk groups (fewer than 10 circuits) are much more volatile than large groups. Large groups, properly sized for 0.01 blocking, should not experience more than a 0.03 average blocking during the 20-day busy season busy hour (BSBH), although a properly sized two-circuit group may experience as much as 0.08 average BSBH blocking.

I. INTRODUCTION

The traffic measurements available for network administration are also used to estimate the number of trunks required to meet the

network engineering objective: an average blocking of 0.01 on the final trunk groups and a minimum-cost configuration for the high-usage trunk groups.

The statistical variation of the traffic measurements during the 20-day busy season causes volatility in the trunks-required estimates. This can lead to an incorrect assessment of network performance and an unstable network provisioning process. Such problems eventually result in unnecessary excess reserve capacity in the trunk and facility network, which can exceed 10 percent of that actually required. Accordingly, a stable provisioning process is important to minimize the reserve capacity while maintaining network service at the objective level.

To minimize the sensitivity of the provisioning process to volatile measurements, a concept of administrative measurement bands has been used recently to define a new demand-servicing procedure and a new trunk service measurement plan for use within the Bell System. The demand-servicing procedure provides specific guidelines for detecting and correcting service problems. The trunk service measurement plan provides a measure of the balance between network service and network utilization.

The probability distribution functions for the measured average blocking, \hat{B} , and estimated trunks required, \hat{c} , are required to define the administrative measurement bands. The distribution for \hat{c} is provided in Refs. 1 and 2. The purpose of this paper is to establish the distribution of \hat{B} and outline the new Bell System administrative procedures.

The distribution for \hat{B} has a discrete component, the probability of no blocking, and a continuous component. The continuous component is well approximated by a two-parameter beta distribution.

The distribution for \hat{B} and the previously derived distribution for \hat{c} were used to obtain probability intervals, called measurement bands, for both \hat{B} and \hat{c} . These bands have been used to establish a new demand-servicing policy and a new trunk service measurement plan for the Bell System.

The demand-servicing procedure provides specific guidelines for detecting and correcting service problems. For the larger final trunk groups, demand servicing is not initiated unless \hat{B} exceeds 0.03. For the smaller groups, the threshold is a function of trunk-group size, and increases as the number of circuits decreases.

The trunk-service measurement plan is based on a comparison of the network-sample distribution of \hat{B} and \hat{c} , with the corresponding theoretical probability distributions. The deviation between the sample and theoretical distributions is measured and reduced to a single performance index which is a balanced measure of both service and utilization.

The probability distribution of \hat{B} is derived in Section II. Section III gives numerical results which illustrate the statistical properties of \hat{B} , and Section IV discusses the major application of the results for network administration.

II. THE DISTRIBUTION FUNCTION FOR AVERAGE BLOCKING

In the Bell System, the trunk-group blocking probability for final trunk groups is estimated by measuring the fraction of blocked calls in each of the time-consistent busy hours of the 20 consecutive business days in the busy season and then averaging the 20 measurements to obtain the busy-season, busy-hour, average blocking. The average blocking can be highly variable. The variation is a function of the length and number of measurement intervals, the traffic characteristics (load, peakedness, and day-to-day load variation), and trunk group size. In this section, we establish an approximation for the probability distribution for average blocking which is adequate for engineering purposes.

2.1 Mathematical model

We consider a service system with c servers having exponentially distributed service times and serving traffic under a blocked-calls-cleared service discipline. The mean service time is the time unit. The system is observed during n disjoint measurement intervals $\mathcal{J}_1, \dots, \mathcal{J}_n$, each of length T . During \mathcal{J}_j , the interarrival times are independent and identically distributed (i.i.d.) with mean $1/\alpha_j$. The peakedness z of the offered traffic is constant over all the intervals.* The system is in statistical equilibrium during each interval, and the initial point of each interval is a stationary (random) point for the arrival process. The loads $\alpha_1, \dots, \alpha_n$ are independent and identically distributed according to the distribution function $\Gamma(\alpha | \bar{\alpha}, v_d)$ with mean $\bar{\alpha}$ and variance v_d (the day-to-day source-load variance).

We use $A_j(T)$ and $O_j(T)$, respectively, to denote the number of arrivals (call attempts) and the number of calls which were blocked (the overflows) during \mathcal{J}_j ; the (measured) observed blocking is $B_j = O_j(T)/A_j(T)$. The sample average of the observed blocking is

$$\bar{B}_n = \frac{1}{n} \sum_{j=1}^n B_j.$$

In this section, we derive an approximation for the probability distribution function for \bar{B}_n .

* It is known that the peakedness of the traffic overflowing typical high usage trunk groups is relatively insensitive to fluctuation in the day-to-day loads offered to those groups (see Ref. 4).

The distribution can be expressed as a sum of two components; for $0 \leq b \leq 1$,

$$P\{\bar{B}_n \leq b\} = P\{\bar{B}_n = 0\} + P\{\bar{B}_n > 0\}P\{\bar{B}_n \leq b | \bar{B}_n > 0\}.$$

In the next subsection, we derive an approximation for the discrete component $P\{\bar{B}_n = 0\}$ and obtain the relations necessary for the computation of the conditional distribution.

2.1.1 The discrete component

Since $\alpha_1, \dots, \alpha_n$ are i.i.d., B_1, \dots, B_n are also i.i.d. Thus,

$$P\{\bar{B}_n = 0\} = (P\{B_1 = 0\})^n. \quad (1)$$

Since $B_1 = 0$ if and only if there are no overflows in \mathcal{J}_1 , we need to determine the probability distribution function G_c for the length of time to the first overflow after the initial point of \mathcal{J}_1 . After G_c is determined, we then have (setting $\alpha_1 = \alpha$ for notational simplicity)

$$\begin{aligned} P\{B_1 = 0\} &= \int_0^\infty P\{B_1 = 0 | \alpha\} d\Gamma(\alpha | \bar{\alpha}, \nu_d) \\ &= \int_0^\infty [1 - G_c(T | \alpha)] d\Gamma(\alpha | \bar{\alpha}, \nu_d). \end{aligned} \quad (2)$$

If H_c is the conditional distribution for interoverflow times given the offered load α , then G_c is the corresponding excess-time distribution;⁵ i.e.,

$$G_c(T | \alpha) = \alpha_0 \int_0^T [1 - H_c(t | \alpha)] dt, \quad (3)$$

where α_0 is the mean overflow load corresponding to the offered load α . Hence we need to specify the interoverflow distribution H_c .

The Laplace-Stieltjes transform of H_c is easily obtained from Ref. 5. However, the analytical inversion of the transform is not practical. If necessary, one could do the inversion numerically. However, for teletraffic applications, it is more efficient to approximate the interoverflow distribution as a mixture of exponentials using the Interrupted Poisson Process (IPP)⁶ with a three-moment match. The approximation is

$$H_c(t | \alpha) \approx 1 - k_1 e^{-r_1 t} - k_2 e^{-r_2 t} \quad (4)$$

where the parameters k_1 , k_2 , r_1 and r_2 are functions of the load α_0 and the peakedness z_0 of the overflow traffic. The required relations are given in Ref. 6.

It now follows from eqs. (3) and (4) that

$$1 - G_c(T|\alpha) \approx 1 - \alpha_0 \left\{ \frac{k_1}{r_1} (1 - e^{-r_1 T}) + \frac{k_2}{r_2} (1 - e^{-r_2 T}) \right\}.$$

From Ref. 6, we know that

$$\frac{k_1}{r_1} + \frac{k_2}{r_2} = \frac{1}{\alpha_0},$$

so

$$1 - G_c(T|\alpha) \approx \alpha_0 \left(\frac{k_1}{r_1} e^{-r_1 T} + \frac{k_2}{r_2} e^{-r_2 T} \right). \quad (5)$$

Combining eqs. (1), (2), and (5), we have

$$P\{\bar{B}_n = 0\} = \left[\int_0^\infty \alpha_0 \left(\frac{k_1}{r_1} e^{-r_1 T} + \frac{k_2}{r_2} e^{-r_2 T} \right) d\Gamma(\alpha|\bar{\alpha}, \nu_d) \right]^n, \quad (6)$$

where it is understood that α_0 , k_1 , r_1 , and r_2 are functions of α , z , and c .

The distribution function $\Gamma(\alpha|\bar{\alpha}, \nu_d)$ of the daily loads is assumed to be a gamma distribution.² Following Ref. 3, we assume that the source-load variance ν_d is of the form

$$\nu_d = 0.13(\bar{\alpha})^\phi - \frac{2\bar{\alpha}z}{T}, \quad (7)$$

where ϕ is an adjustable parameter. For Bell System applications, the standard values for ϕ are 1.5 (low day-to-day variation), 1.7 (medium), and 1.84 (high). To illustrate the results, we assume throughout this study that the mean holding time is 225 seconds and that the measurement interval is one hour.* Consequently, the relative measurement time is $T = 16$. Results for other cases can be generated easily. From (6), it follows that the discrete component decreases as T increases. More generally, the spread of the distribution of \bar{B}_n will decrease as T increases. The integration in (6) can be carried out numerically using a 51-point compound Simpson's rule.

2.1.2 The continuous component

Using a simulation, we found that the conditional distribution $P\{\bar{B}_n \leq b | \bar{B}_n > 0\}$ could be well approximated over the regions of engineering interest by a beta distribution.^{†7} We call its mean $\mu_\beta = E(\bar{B}_n | \bar{B}_n > 0)$ and its variance $\sigma_\beta^2 = \text{Var}(\bar{B}_n | \bar{B}_n > 0)$.

* By numerical experimentation, we found that the results were not very sensitive to moderate deviations in mean holding time. A period of 225 seconds is a Bell System average holding time.

† We first tried a lognormal distribution. However, further experimentation indicated that a beta distribution would provide a better fit, particularly near the origin.

Parameters. It is straightforward to show that

$$E(\bar{B}_n | \bar{B}_n > 0) = \frac{E(\bar{B}_n)}{P\{\bar{B}_n > 0\}} \quad (8)$$

and, using the analog of (8) for higher moments than the first,

$$\text{Var}(\bar{B}_n | \bar{B}_n > 0) = \frac{\text{Var}(\bar{B}_n)}{P\{\bar{B}_n > 0\}} - E^2(\bar{B}_n | \bar{B}_n > 0)P\{\bar{B}_n = 0\}. \quad (9)$$

Hence, only the ordinary moments of \bar{B}_n need be specified. An approximation for the mean was given in Ref. 3; namely,

$$E(\bar{B}_n) = \int_0^\infty E(B_1 | \alpha) d\Gamma(\alpha | \bar{\alpha}, \nu_d) \quad (10)$$

where

$$E(B_1 | \alpha) \approx B(c, \alpha, z) \left\{ 1 + \frac{\text{Var} A(T)}{E^2 A(T)} - \frac{\text{Cov}[A(T), O(T)]}{E^2 A(T)} \right\} \quad (11)$$

and $B(c, \alpha, z)$ is the trunk group call-congestion corresponding to the load α and peakedness z . Algorithms for the computation of the (conditional) moments of $A(T)$ and $O(T)$ (given α and z) are given in Ref. 5.

Since B_1, \dots, B_n are i.i.d.,

$$\text{Var}(\bar{B}_n) = \frac{1}{n} \text{Var}(B_1). \quad (12)$$

Using the methods described in Ref. 2, $\text{Var}(B_1)$ can be expressed as

$$\begin{aligned} \text{Var}(B_1) &= \frac{E(\text{Var}(B_1 | \alpha))}{\alpha} + \frac{\text{Var}(E(B_1 | \alpha))}{\alpha} \\ &= \int_0^\infty \text{Var}(B_1 | \alpha) d\Gamma(\alpha | \bar{\alpha}, \nu_d) \\ &\quad + \int_0^\infty E^2(B_1 | \alpha) d\Gamma(\alpha | \bar{\alpha}, \nu_d) - E^2(B_1), \end{aligned} \quad (13)$$

where $\text{Var}(B_1 | \alpha)$ is computed using the algorithms derived in Ref. 5.

Computation. As noted above and illustrated below, simulated data have shown that the conditional distribution $P\{\bar{B}_n \leq b | \bar{B}_n > 0\}$ is well approximated by a beta distribution $I_b(\delta, \eta)$. That is, for $0 \leq b \leq 1$,

$$P\{\bar{B}_n \leq b | \bar{B}_n > 0\} \approx I_b(\delta, \eta) = \frac{\int_0^b x^{\delta-1}(1-x)^{\eta-1} dx}{\int_0^1 x^{\delta-1}(1-x)^{\eta-1} dx}, \quad (14)$$

where (see Ref. 7, p. 930) δ and η are related to μ_β and σ_β^2 (defined above) by

$$\mu_\beta = \frac{\delta}{\delta + \eta} \quad \text{and} \quad \sigma_\beta^2 = \frac{\delta\eta}{(\delta + \eta)^2(\delta + \eta + 1)}.$$

The corresponding inverse relations are

$$\delta = (1 - \mu_\beta) \left(\frac{\mu_\beta^2}{\sigma_\beta^2} \right) - \mu_\beta \quad (15)$$

and

$$\eta = (1 - \mu_\beta) \left((1 - \mu_\beta) \left(\frac{\mu_\beta}{\sigma_\beta^2} \right) - 1 \right). \quad (16)$$

The integral in the denominator of (14) is the beta function $\mathcal{B}(\delta, \eta)$ and is related to the gamma function $\Gamma(x)$ as follows (see Ref. 7, page 258):

$$\mathcal{B}(\delta, \eta) = \int_0^1 x^{\delta-1}(1-x)^{\eta-1} dx \quad (17)$$

$$= \frac{\Gamma(\delta)\Gamma(\eta)}{\Gamma(\delta + \eta)}. \quad (18)$$

To cover the range of engineering interest, it is necessary to use logarithms to carry out the computation. Standard programs are available for computing the logarithm of the gamma function.

2.2 Comments

Collecting the preceding results from Section II, we have

$$P\{\bar{B}_n \leq b\} = P\{\bar{B}_n = 0\} + P\{\bar{B}_n > 0\}P\{\bar{B}_n \leq b | \bar{B}_n > 0\}.$$

The approximations for the individual terms are as follows: From Section 2.1.1,

$$P\{\bar{B}_n = 0\} = \left[\int_0^\infty \alpha_0 \left(\frac{k_1}{r_1} e^{-r_1 T} + \frac{k_2}{r_2} e^{-r_2 T} \right) d\Gamma(\alpha | \bar{\alpha}, \nu_d) \right]^n,$$

where α_0 , k_1 , k_2 , r_1 and r_2 are functions of α , z , and c as given in Ref. 6. From Section 2.1.2,

$$P\{\bar{B}_n \leq b | \bar{B}_n > 0\} \approx I_b(\delta, \eta) \\ = \frac{\Gamma(\delta + \eta)}{\Gamma(\delta)\Gamma(\eta)} \int_0^b x^{\delta-1}(1-x)^{\eta-1} dx,$$

where δ and η are related to the conditional mean and variance of \bar{B}_n as given in eqs. (15) and (16). The integrations are carried out using a 51-point compound Simpson's rule.

2.3 Quantiles

The p -quantile of a continuous distribution F is the value ξ_p such that $F(\xi_p) = p$; i.e., $\xi_p = F^{-1}(p)$, where F^{-1} denotes the inverse function for F .

From the preceding section, we have

$$P\{\bar{B}_n \leq b\} \approx P\{\bar{B}_n = 0\} + P\{\bar{B}_n > 0\}I_b(\delta, \eta).$$

Using this result, the quantiles of the distribution are easily expressed in terms of the quantiles of the beta distribution. That is, for a given p between $P\{\bar{B}_n = 0\}$ and 1, the value ξ_p such that

$$P\{\bar{B}_n \leq \xi_p\} = p$$

is given approximately by

$$\xi_p \approx I_{q(p)}^{-1}(\delta, \eta) \quad (19)$$

where

$$q(p) = \frac{p - P\{\bar{B}_n = 0\}}{P\{\bar{B}_n > 0\}}$$

and $I_q^{-1}(\delta, \eta)$ denotes the q -quantile of the beta distribution (of which I^{-1} is the inverse). Standard programs are available for computing the beta distribution and its quantiles.⁸

III. NUMERICAL RESULTS

In this section we present a few examples of the results which have been obtained from the approximations developed in Section II. We consider the discrete component and the cumulative distribution of B_n , the corresponding probability density function, and a set of probability intervals. Applications of the results are described in Section IV.

3.1 The discrete component

The discrete component, $P\{\bar{B}_n = 0\}$, has several potential engineering applications in addition to being a fundamental part of the distribution of \bar{B}_n . Accordingly, in this subsection we verify the accuracy of the approximation for $P\{\bar{B}_n = 0\}$ and illustrate the general characteristics which should be useful in future applications.

Approximation (6) was computed for a wide range of system parameters and compared with estimates obtained from a random-walk simulation of a trunk group satisfying the assumptions given in Section

2.1. Typical results are shown in Fig. 1, where a graph of $P\{B_1 = 0\}$ versus trunk group size c is shown for trunk groups engineered at a mean blocking of 0.01. The three curves correspond to $z = 1$ and low day-to-day variation ($\phi = 1.5$), $z = 2$ and medium variation ($\phi = 1.7$), and $z = 4$ and high variation ($\phi = 1.84$).

The agreement between the simulation and the approximation is quite good for $z = 1$ and 2. For $z = 4$, the approximation is a little low, but even in this case (which is relatively extreme for most traffic applications), the relative difference is less than 7 percent and should be adequate for engineering applications. (The approximation was not tested for $z > 4$.) Similar accuracy was obtained in the other cases tested, particularly when $P\{B_1 = 0\}$ was on the order of 0.1 or larger (i.e., low-blocking applications). The accuracy is statistically more difficult to check for smaller values. However, when $P\{B_1 = 0\}$ is much less than 0.1, it has very little influence on the distribution of \bar{B}_n . Also, notice that, as n increases, $P\{B_n = 0\}$ decreases rapidly; i.e., it has very little engineering significance for n larger than 10.

Figure 2 shows $P\{B_1 = 0\}$ versus the load α for $c = 40$ trunks and several values of peakedness (no day-to-day variation). As the figure shows, $P\{B_1 = 0\}$ is a monotone decreasing function of both α and z when no day-to-day variation is present. Similar results were also

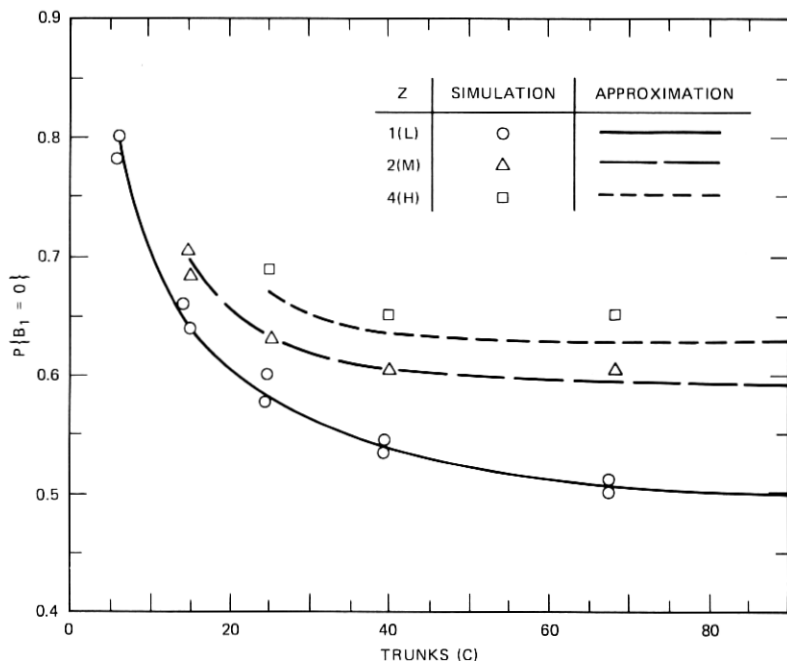


Fig. 1— $P\{B_1 = 0\}$ vs trunk-group size.

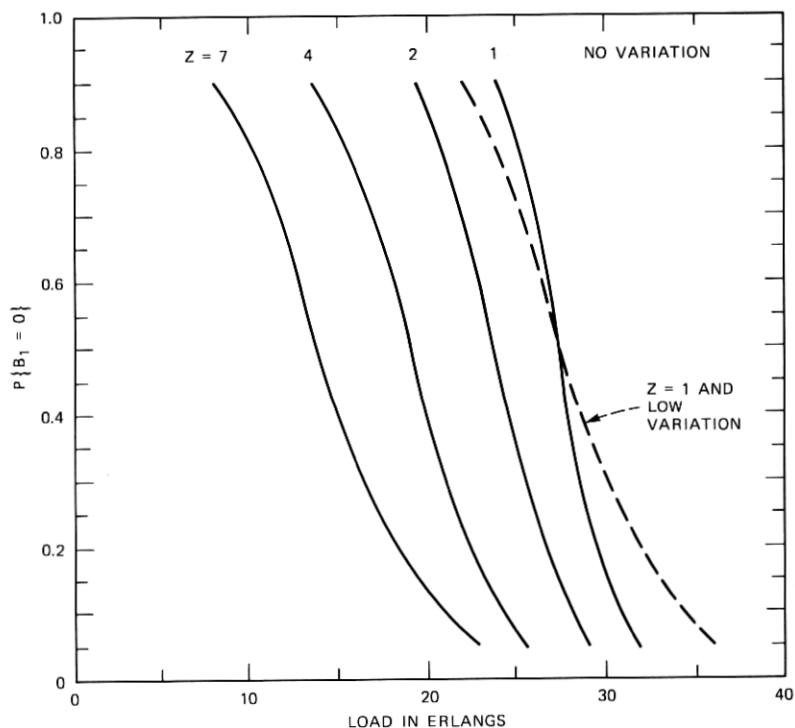


Fig. 2— $P\{B_1 = 0\}$ vs load for $C = 40$ trunks.

obtained for any fixed level of variation. Note that the separation of the curves introduces the possibility of estimating α and z by using measurements of $P\{B_1 = 0\}$ and usage.

The effect of day-to-day variation for the case $z = 1$ is also illustrated in Fig. 2. Day-to-day load variation decreases $P\{B_1 = 0\}$ at lower loads and increases it at higher loads. At a high mean load, day-to-day variation increases the likelihood of a low load and therefore increases $P\{B_1 = 0\}$. At a low mean load, day-to-day variation increases the probability of a relatively larger load and thereby decreases $P\{B_1 = 0\}$. The difference can be considerable, and day-to-day variation should be taken into account when computing the discrete component.

3.2 The cumulative distribution

The approximation for the cumulative distribution

$$P\{\bar{B}_n \leq b\} \approx P\{\bar{B}_n = 0\} + P\{\bar{B}_n > 0\} I_b(\delta, \eta), \quad 0 \leq b \leq 1,$$

(derived in Section II) was computed for several system parameters and compared with estimates of the distribution obtained from the

simulation. Typical results are shown in Fig. 3 for $c = 40$ trunks, $z = 1$, $\phi = 1.5$ (low variation), and $n = 1, 5$, and 20 days. The approximation compares quite well with the simulated data.

A number of additional cases were generated to determine the range of validity of the approximation. We considered $c = 3, 5, 10, 40$, and 100, $z = 1$ with low day-to-day variation and $z = 4$ with high variation. For all combinations of load and peakedness, the accuracy of the approximation increases with c and should be adequate for engineering applications with $c \geq 10$. For the smaller values of c , $c \leq 5$, the approximation did not agree as well with the simulated data. In those cases, a simulation is probably the most appropriate tool to estimate the distribution.

3.3 The probability density

Since the distribution of \bar{B}_n has a discontinuity at zero, the probability density function is not defined at zero in the usual sense; i.e., the

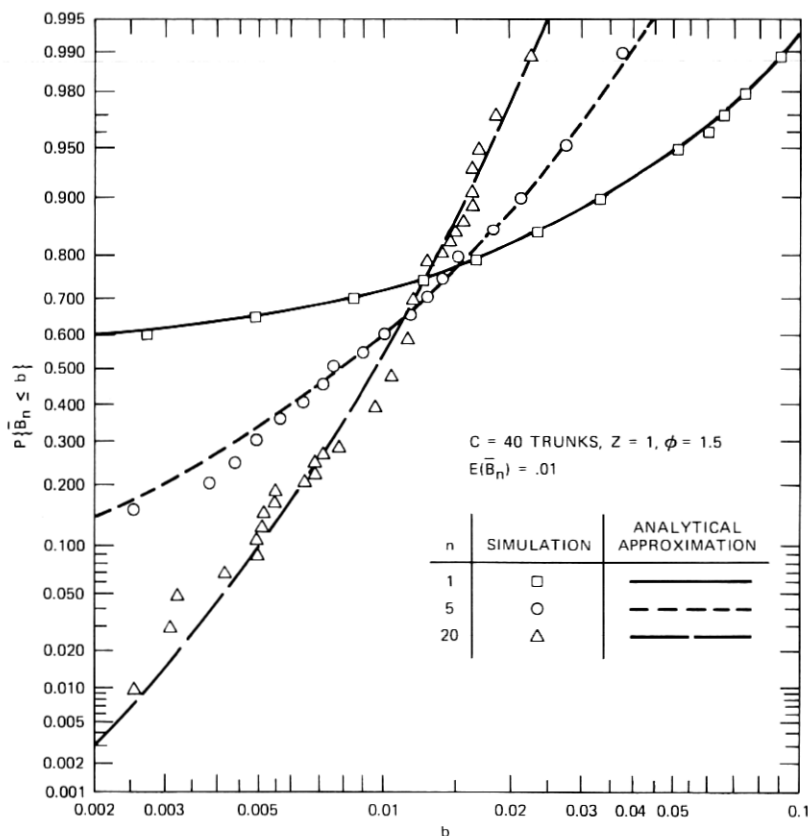


Fig. 3—Distribution of observed blocking.

density function contains an impulse at $b = 0$.⁹ For $b > 0$, an approximation for the probability density is obtained by differentiating the approximate distribution to obtain

$$f(b) = \frac{d}{db} P\{\bar{B}_n \leq b\} = P\{\bar{B}_n > 0\} \frac{b^{\delta-1}(1-b)^{\eta-1}}{\mathcal{B}(\delta, \eta)},$$

where δ , η and \mathcal{B} were defined in (15) to (17).

The number of hours of data has a strong influence on the shape of the density function. Figure 4 illustrates the cases $n = 1, 5$, and 20. For each case, $c = 30$, $z = 1$, the day-to-day variation is low, and the average blocking is 0.01. Notice that the mode of each density is located to the left of the mean value 0.01. This is probably caused by the nonlinear effect on trunk-group blocking of a variable load.

IV. ADMINISTRATIVE MEASUREMENT BANDS

The statistical variation of the traffic measurements during the 20-day busy season causes volatility in the measured average blocking

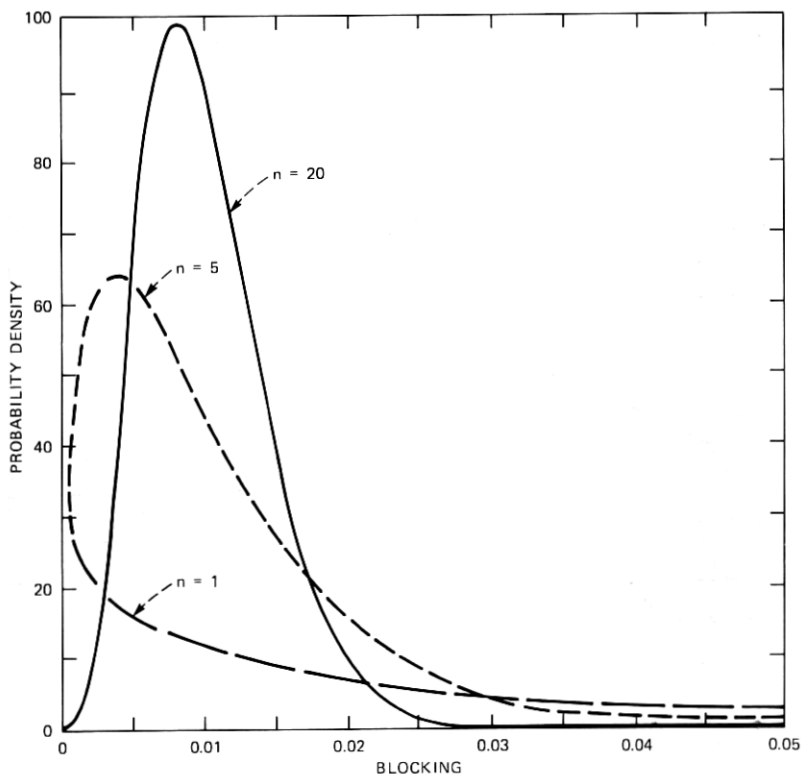


Fig. 4—Probability density for observed blocking.

and in the trunks-required estimates. Such volatility can cause unnecessary demand servicing which will result in excess reserve trunk capacity. Unless the provisioning process is stabilized, this excess capacity can eventually exceed 10 percent of the actual required trunks.

A concept of administrative measurement bands has been used to define a stable demand servicing procedure and trunk administrative measurement plan. In this section, we describe the measurement bands and summarize their planned use in the Bell System provisioning process.

4.1 Measured blocking

The numerical results presented in Section III show that when a trunk group is properly engineered for an average blocking of 0.01, the observed average blocking may deviate considerably from 0.01. In fact, the probability density function for \bar{B}_n is skewed to the right sufficiently that the observed blocking will fall below the 0.01 average about 60 percent of the time. Consequently, if a naive observer took a single look at a properly engineered network, he might conclude that many of the probability-engineered trunk groups were overprovided (observed average blocking somewhat less than 0.01). On the other hand, nearly 40 percent of the groups would experience blocking in excess of 0.01 and thereby be of concern to the trunk engineer who sized the groups for an 0.01 objective.

To account properly for the volatility of measured blocking, intervals of acceptable and anticipated measured average blocking have been calculated for Bell System network administration. The resulting intervals are illustrated in Fig. 5.

The analytical approximation was used to generate the curves for $c \geq 7$. A simulation was used to provide the results for $2 \leq c \leq 6$. For $c < 5$, the analytical approximation is not adequate for our purposes. Moreover, a simulation is inexpensive and effective for the small trunk-group sizes.

The upper and lower curves correspond to the theoretical 0.99- and 0.01-quantiles, respectively. However, simulation results have shown that these theoretical quantiles are the effective end points of the distribution. The lower curve vanishes for $c < 50$ because the smaller groups can be properly sized and still, with probability greater than 0.01, experience no busy-season, busy-hour blocking. The two central curves bound an 80-percent probability interval. That is, 80 percent of all the average-blocking measurements should lie within the interval if the network is properly sized.

The curves are fairly constant for $c > 10$. However, they increase substantially for the smaller groups, illustrating the statistical volatility of traffic measurements on the smaller groups.

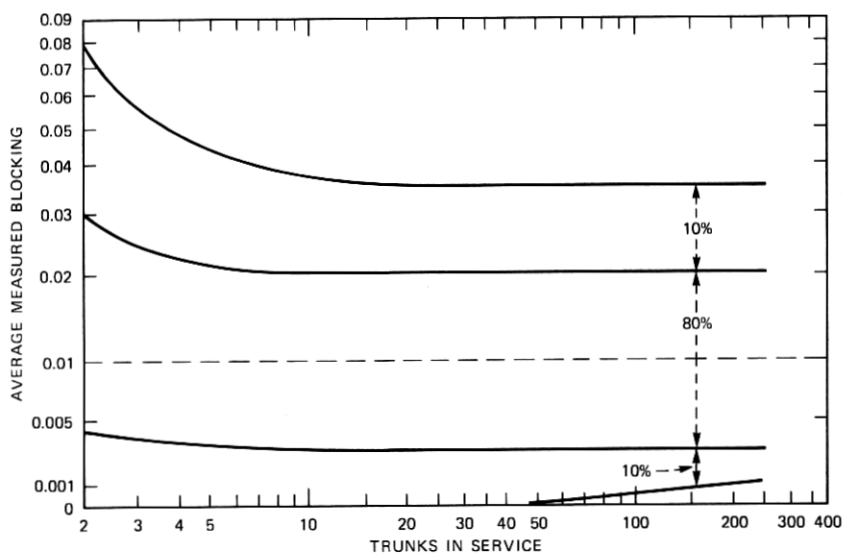


Fig. 5—Trunk service measurement plan, average blocking measurement bands.

4.2 Trunks required

The distribution function for estimates of trunks-required is provided in Refs. 1 and 2. It has been used to construct administrative bands that are analogous to those for average blocking. The results for low day-to-day variation are given in Fig. 6.

The vertical axis represents acceptable deviations in the estimates. For example, if a 10-trunk group were properly sized, then the estimates should fall between 9 and 11 about 80 percent of the time. The results shown must be modified for medium and high day-to-day variation; the bands increase in width as the level of variation increases.

4.3 Network administration

4.3.1 Demand servicing

Trunk network administration is composed of two major operations, demand servicing and planned servicing. Planned servicing uses the forecasts of demand in the next busy season to decide how many and where circuits are required to insure adequate network service. Demand servicing refers to the use of trunk group measurements to detect and correct current service problems. The measurement bands for \hat{B} and \hat{c} have been used to establish a new demand servicing policy which is planned to be used in the Bell System.¹⁰

Based on the new policy, a network cluster is assumed to be providing adequate service provided the average blocking for the final

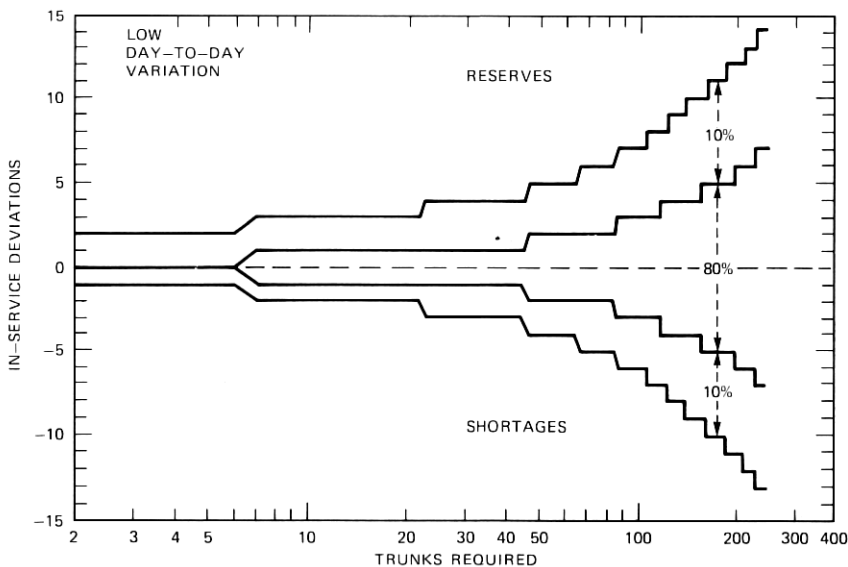


Fig. 6—Trunk service measurement plan, trunks required measurement bands.

trunk group does not exceed the upper curve. If the blocking exceeds the threshold, then the trunks-required administrative bands are used to determine cost-effective trunk additions which will restore network service.

4.3.2 Measurement plan

A new Trunk Administrative Measurement Plan was recently issued for use within the Bell System. The trunking accuracy component of the plan, called the Trunk Service Measurement Plan (TSMP), uses the measurement bands for \hat{B} and \hat{c} to measure the balance between network service and network utilization.

The TSMP compares the network-sample distribution of B and c with the theoretical distribution; a 5-point comparison is made using the 0-10-80-10-0 intervals illustrated in Figs. 5 and 6. The deviation between the sample and theoretical distributions is measured and reduced to a single performance index which is a measure of the balance between network service and utilization.

V. ACKNOWLEDGMENTS

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