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Satellite Phased Arrays: Use of Imaging Reflectors with Spatial Filtering in the Focal Plane to Reduce Grating Lobes

By C. DRAGONE and M. J. GANS

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The imaging reflector arrangement described in this paper forms a very compact antenna design suitable for generating a scanning fan beam for a 12/14-GHz synchronous satellite communicating with points located in the continental United States. A magnified image of a small array is obtained using a Gregorian arrangement of two paraboloids. A filter, placed in the focal plane of the main reflector, eliminates undesirable field components due to the grating lobes of the small array. Because of the filter, the illumination over the main aperture is a smoothed version of the array illumination. Thus, grating lobes are greatly reduced. By properly adjusting the excitation of the various array elements, an antenna with very low side lobes is obtained.

I. INTRODUCTION

Phased arrays will be needed[1] in satellites that form rapidly movable beams to communicate efficiently with ground stations in the U.S.A. Large antenna apertures will be needed to produce narrow beams. Thus, in a previous article[2] a large image of a small array was obtained, using an arrangement of confocal reflectors. The large image was formed at the aperture of the main reflector, and the magnification was chosen so that the image had the same dimensions as the main reflector, thus illuminating efficiently its aperture and essentially eliminating spillover.

However, since the aperture distribution in that arrangement was the image of the array distribution, the antenna far field was a replica of the array far field and, therefore, it contained the array grating lobes. To reduce them, we propose here use of a filter placed in the focal plane of the main reflector, as shown in Fig. 1. The filter is designed to eliminate the field components caused in the focal plane

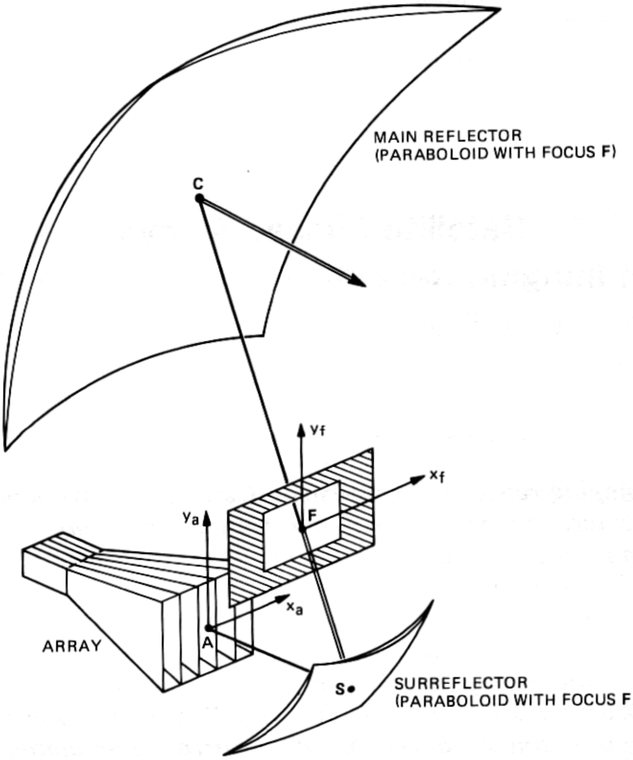

Fig. 1—A Gregorian arrangement combined with an array. The subreflector forms, over the main reflector, a magnified image of the array aperture. By use of a filter in the focal plane, grating lobes can be suppressed.

by the grating lobes of the array. The field distribution over the main reflector aperture is then a smoothed version of the array distribution and, as a consequence, grating lobes in the far field are virtually absent. An important property of the aperture distribution, in addition to being well-behaved, is that it can be varied by changing the excitation of the array elements. Thus, by properly optimizing the array excitation, excellent performance in side lobes can be achieved efficiently. This is shown in Section III where patterns obtained with a Chebyshev distribution using five elements with filtering are discussed.

Over the field of view defined by the filter, the antenna beam can also be scanned efficiently, with negligible grating lobes. An application of current interest, a satellite antenna using a linear array at 11.8 and 14.25 GHz, is also discussed.

II. ANALYSIS

In this paper, we wish to use the arrangement[2-4] of Fig. 1 to design an antenna with very low radiation in the side lobes. It is assumed that radiation from the array is confined to the vicinity of the central ray, which in Fig. 1 proceeds from the center A of the array, is reflected by the subreflector at S, passes through the focus F, and is then reflected by the main reflector at C. To determine propagation in the vicinity of this ray, we use Fresnel's diffraction formula[5] and replace the two reflectors by two thin lenses, as shown in Fig. 2. Let a sphere with center at C be drawn through F, and let the array be polarized in the x-direction. It is assumed that the subreflector is large enough so that it intercepts entirely the incident energy. As a consequence, the field distribution

$$\psi_f = \psi_f(x_f, y_f)\mathbf{i}_x \tag{1}$$

over the focal sphere is a faithful reproduction[5] of the array far field. To obtain over the main reflector the image of the array distribution, the distance of C from the subreflector must be chosen according to the lens formula

$$\frac{1}{|CS|} + \frac{1}{|SA|} = \frac{1}{f_1}, \tag{2}$$

where f_1 is the focal length $f_1 = |FS|$ of the subreflector. The magnification is then given by

$$M = \frac{f_2}{f_1}, \tag{3}$$

where f_2 is the main reflector focal length, $f_2 = |FC|$. This magnification specifies the dimensions of the main reflector, since its aperture is assumed to be the image of the array aperture. Because of this assumption, the main reflector intercepts entirely the incident field and, therefore, the antenna far field ψ_∞ is the image of ψ_f.* More precisely, consider at a great distance $d_\infty \simeq \infty$ from C, a sphere centered at C and let x_∞, y_∞ be the x, y-coordinates of a point P_∞ on this sphere. The coordinates x_f, y_f of the corresponding point P on the focal sphere are

$$x_f = \frac{-x_\infty}{d_\infty} f_2, \qquad y_f = -\frac{y_\infty}{d_\infty} f_2 \tag{4}$$

and therefore

$$\psi_\infty(x_\infty, y_\infty) = \frac{f_2}{d_\infty} \psi_f\left(-\frac{x_\infty}{d_\infty} f_2, -\frac{y_\infty}{d_\infty} f_2\right)\mathbf{i}_x, \tag{5}$$

ignoring the difference in phase arising because of the distance, d_∞ +

* Therefore, the far field can be determined using the laws of geometrical optics as shown by eq. (5).

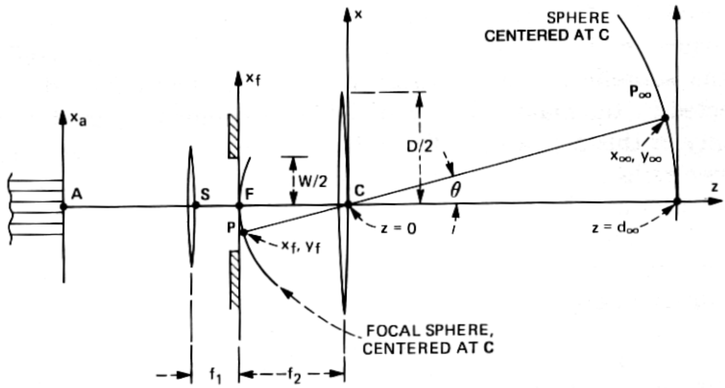

Fig. 2—The two reflectors in Fig. 1 can be replaced by two thin lenses, since it is assumed that the field of Fig. 1 is confined to the immediate vicinity of the central ray.

f_2, of P_∞ from P. If $\psi_a(x_a, y_a)\mathbf{i}_x$ is the field over the array aperture, ψ_f is related to the Fourier transform of ψ_a,

$$\psi_f(x_f, y_f) = \frac{1}{\lambda f_1} \iint_{-\infty}^{\infty} \psi_a(x_a, y_a) e^{+jk(x_a x_f + y_a y_f)/f_1}\, dx_a\, dy_a, \tag{6}$$

ignoring the difference in phase due to the optical path from A to F. Notice eq. (6) is valid provided the center C of the focal surface is the image of A.

We now insert in the focal surface a stop* with aperture A_f. Then ψ_f is given by eq. (6) only inside A_f, and

$$\psi_f = 0 \quad \text{outside} \quad A_f. \tag{7}$$

Because of this, we cannot assume any more that the main reflector intercepts entirely the incident field. Thus, to account for the finite aperture A_p of the main reflector, eq. (5) must be replaced with the convolution

$$\psi_\infty(x_\infty, y_\infty) = \frac{1}{\lambda^2 d_\infty f_2} \cdot \iint_{A_f} \psi_f(-x_f, -y_f) P\left(\frac{x_\infty f_2}{d_\infty} - x_f, \frac{y_\infty f_2}{d_\infty} - y_f\right) dx_f\, dy_f, \tag{8}$$

where $P(v_x, v_y)$ is the Fourier transform of A_p,

* Consideration is restricted here to the simplest form of filtering; i.e., by stops. Apodizing screens and phase plates can be used as more general forms of filtering for pattern synthesis.

$$P(x_f, y_f) = \iint_{A_p} e^{-j(2\pi/\lambda f_2)(x_f x + y_f y)}\, dx\, dy. \tag{9}$$

In the two examples of the following section, ψ_a is the product of two functions,

$$\psi_a = f(x_a) g(y_a) \mathbf{i}_x,$$

and both the filter and the main reflector have rectangular apertures given for $y = 0$ by the intervals

$$-\frac{D}{2} < x < \frac{D}{2} \tag{10}$$

and

$$-\frac{W}{2} < x < \frac{W}{2}. \tag{11}$$

Then from eqs. (6) to (9), one finds that ψ_∞ is the product of two functions,

$$\psi_\infty(x_\infty, y_\infty) = \frac{1}{\lambda^2 d_\infty f_2} f_\infty(x_\infty) g_\infty(y_\infty), \tag{12}$$

where

$$f_\infty(x_\infty) = \int_{-(W/2)}^{W/2} U(x_f) S\left(\frac{x_\infty f_2}{d_\infty} - x_f\right) dx_f, \tag{13}$$

U being given by the Fourier transform of $f_a(x_a)$,

$$U(x_f) = \frac{1}{\sqrt{\lambda f_1}} \int_{-(D/2M)}^{D/2M} f_a(x_a) e^{+jk(x_a x_f/f_1)}\, dx_a \tag{14}$$

and

$$S(x_f) = D\frac{\sin(\pi x_f D/\lambda f_2)}{(\pi x_f D/\lambda f_2)}. \tag{15}$$

Similar expressions can be written for g_∞. In the following section, only f_∞ is discussed, since the same considerations apply to both f_∞ and g_∞.

III. AN EXAMPLE

Consideration is restricted to the principal plane $y_\infty = 0$. Let θ be the angle specifying in Fig. 2 the direction of P_∞,

$$\theta \simeq \frac{x_\infty}{d_\infty}, \tag{16}$$

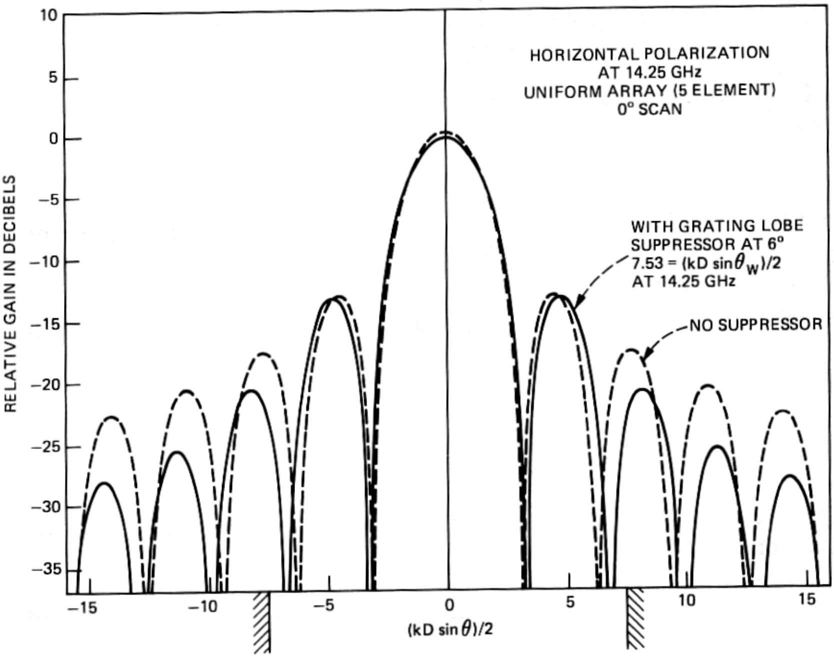

Fig. 3—Patterns in the principal plane $y = 0$ of the far field of Fig. 2, for polarization in the x-direction and $|a_1| = \cdots = |a_5|$. Notice, in the filtered pattern, the lower side lobes for $\theta > \theta_f$.

and let $\theta = \pm\theta_w$ be the values of θ corresponding to the two edges of the filter,

$$\theta_w = \frac{W}{2f_2}. \tag{17}$$

To illustrate the effect of filtering, we consider a linear array of five elements with rectangular apertures arranged as shown in Fig. 1. Let the nth element be illuminated by the TE_{10}-mode with amplitude a_n, and assume for the filter aperture*

$$\frac{kD}{2}\sin\theta_w = 7.53. \tag{18}$$

Figures 3 and 4 show the effect of this filter on the far field, when all the elements are polarized in the x-direction and have the same amplitude

$$|a_1| = |a_2| = \cdots = |a_5|. \tag{19}$$

* The value of θ_w was chosen to reduce grating lobes without excessive gain degradation under the conditions of Fig. 4.

In Fig. 3 it is assumed that all the elements are in phase, and therefore the main beam is centered at $\theta = 0$. There are no grating lobes, in this case, and the effect of the filter is a decrease of all the side lobes for $\theta > \theta_f$. In Fig. 4, the main beam is displaced from the axis $\theta = 0$ by the angle of scan θ_s corresponding to

$$\frac{kD}{2} \sin \theta_s = 4.22. \tag{20}$$

Notice the grating lobe appearing in the pattern without filtering is greatly reduced by the filter. Similar results are obtained for polarization in the y-direction. Suppose, for instance, the array is required to transmit at 11.8 GHz with polarization in the y-direction, and receive at 14.25 GHz with polarization in the x-direction. Then, at 11.8 GHz, from eq. (18) one has, for the same θ_w,

$$\frac{kD}{2} \sin \theta_w = 7.53 \cdot \frac{11.8}{14.25} = 6.24,$$

and one obtains for polarization in the y-direction the patterns of Figs. 5 and 6. Notice in Fig. 5 the grating lobe due to the nonuniform illumination of each array element for $y = 0$.

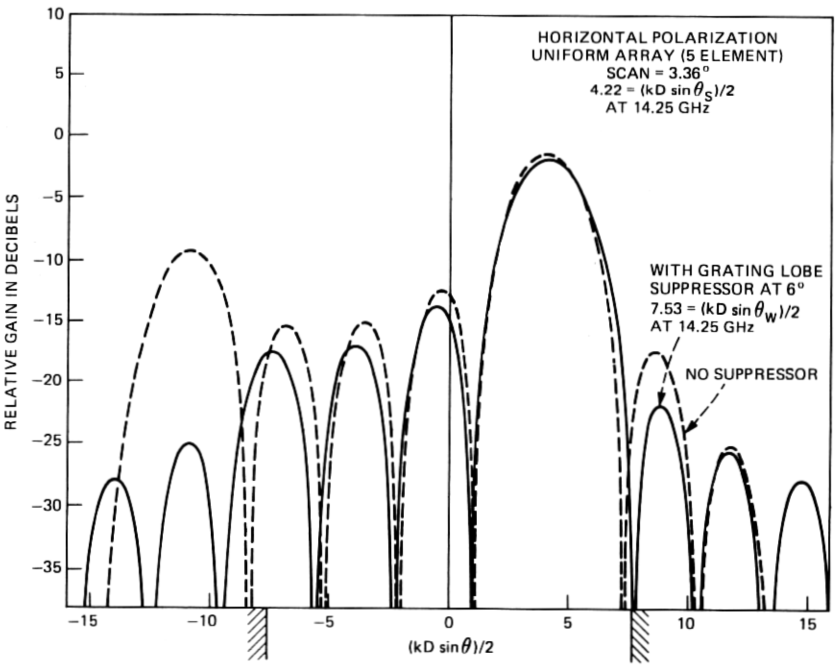

Fig. 4—Patterns for $\theta_s \neq 0$, assuming the same conditions as in Fig. 3. Notice the grating lobe arising without filter is effectively suppressed by the filter.

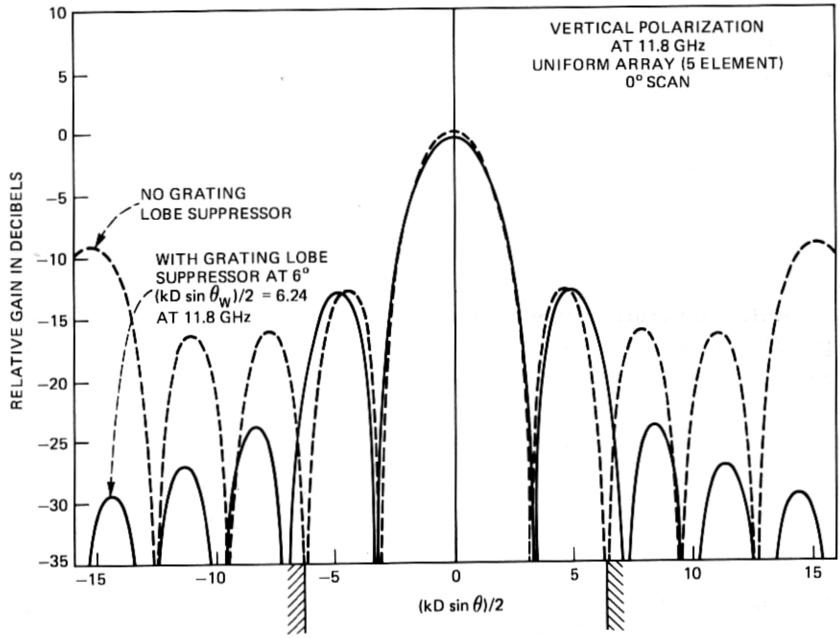

Fig. 5—Patterns obtained for polarization in the y-direction, assuming the same array distribution as in Fig. 3. Notice the grating lobes, due to the nonuniform illumination of each array element, are suppressed by the filter.

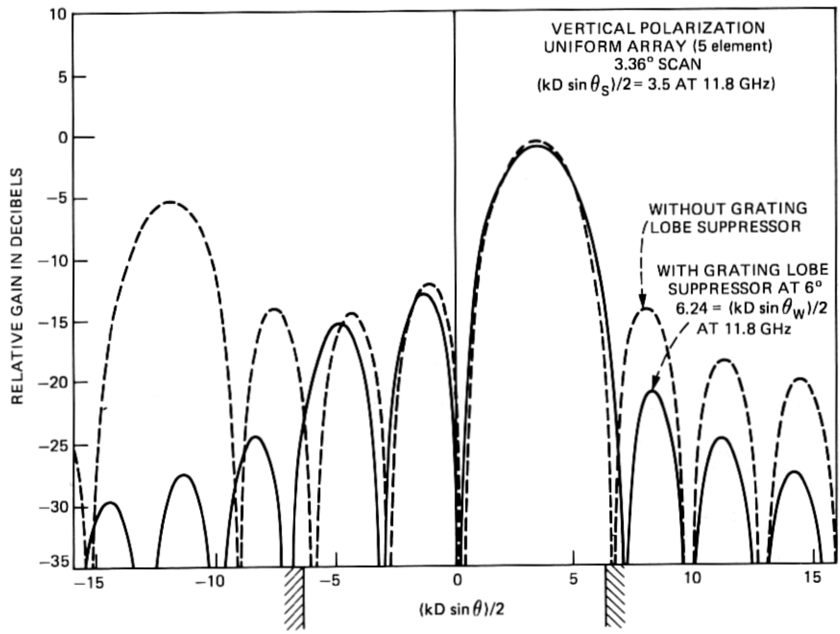

Fig. 6—Patterns for polarization in the y-direction, assuming all other conditions as in Fig. 4.

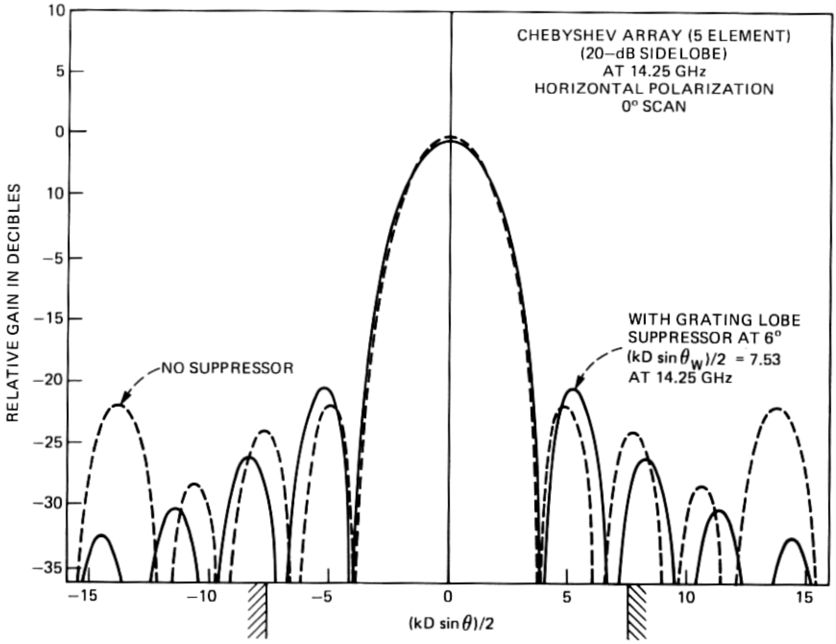

Fig. 7—The only difference between these patterns and those of Fig. 3 is that now $|a_n|$ are given by the Chebyshev distribution of eq. (21).

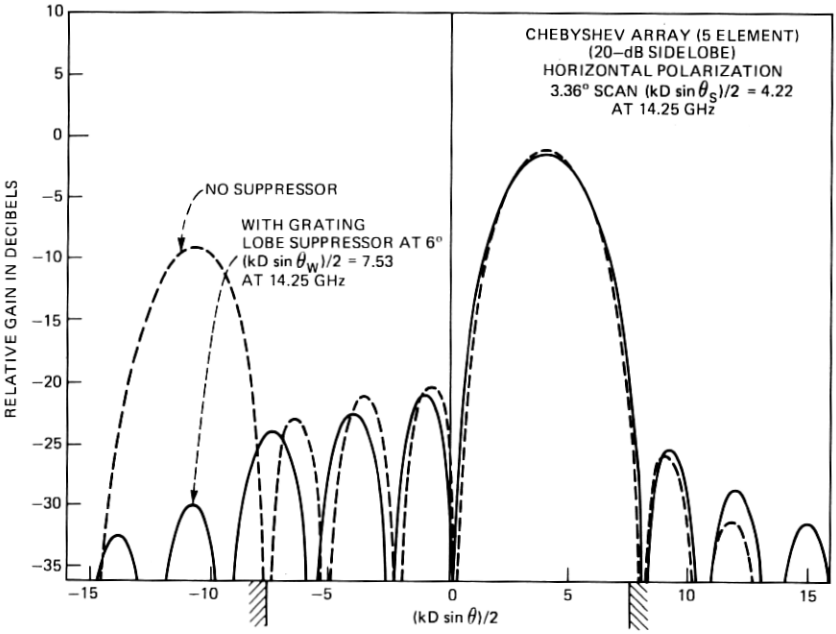

Fig. 8—Patterns obtained for $\theta_s \neq 0$ with Chebyshev distribution polarized in the x-direction.

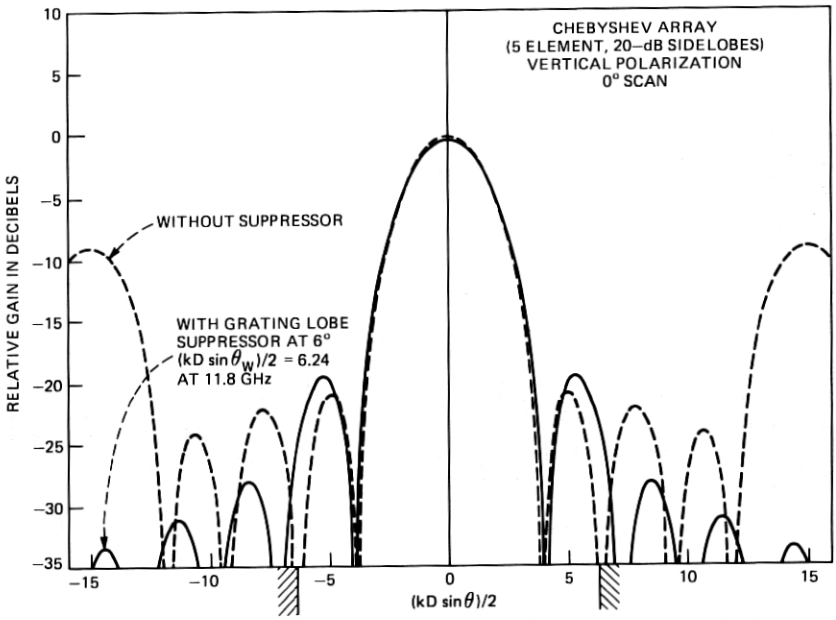

Fig. 9—Patterns obtained for $\theta_s = 0$ with Chebyshev distribution polarized in the y-direction.

To reduce the peaks of the side lobes of order 1, 2, and 3 we now choose for the amplitudes a_n the Chebyshev distribution, designed for 20-dB side lobes,

$$\left.\begin{aligned} a_1 = a_5 &= 1.3988 \\ a_2 = a_4 &= 2.2500 \\ a_3 &= 2.7024 \end{aligned}\right\} . \tag{21}$$

The resulting patterns for the two polarizations are shown in Figs. 7 to 10. In the filtered patterns, all the side lobes are now very low, appreciably lower than in the unfiltered patterns of Figs. 3 and 5.

An application of current interest is a synchronous satellite antenna with a movable beam required to illuminate at 11.8 GHz a narrow strip of the U.S.A., as shown in Fig. 11. The illuminated area covers the entire width of the U.S.A., from north to south. From east to west, only one-tenth of the U.S.A. is illuminated and a linear array must be used to direct the beam to any desired location of the U.S.A. Since the beamwidth is about one-tenth the field of view, the number N of array elements must be greater than 10. A possible antenna design is shown in Fig. 12, using four identical arrays, with four elements each combined

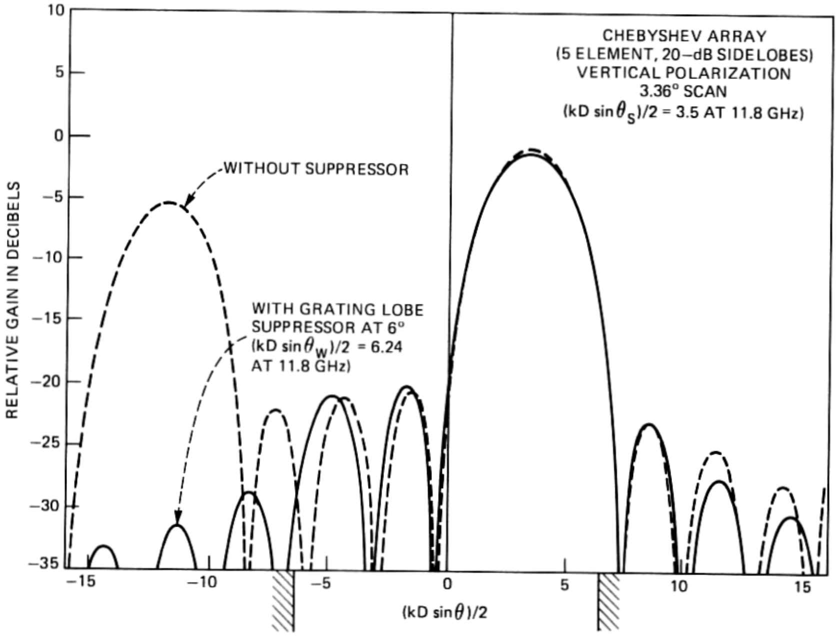

Fig. 10—Patterns obtained for $\theta_s \neq 0$ with Chebyshev distribution polarized in the y-direction.

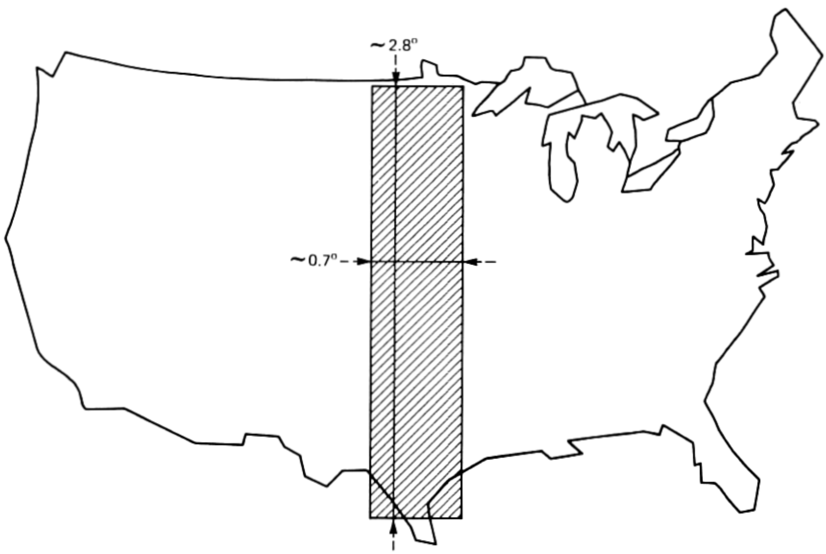

Fig. 11—The satellite antenna of Fig. 12 is required to illuminate at 11.8 GHz a narrow strip of the U.S.A., roughly of 0.7° × 2.8°.

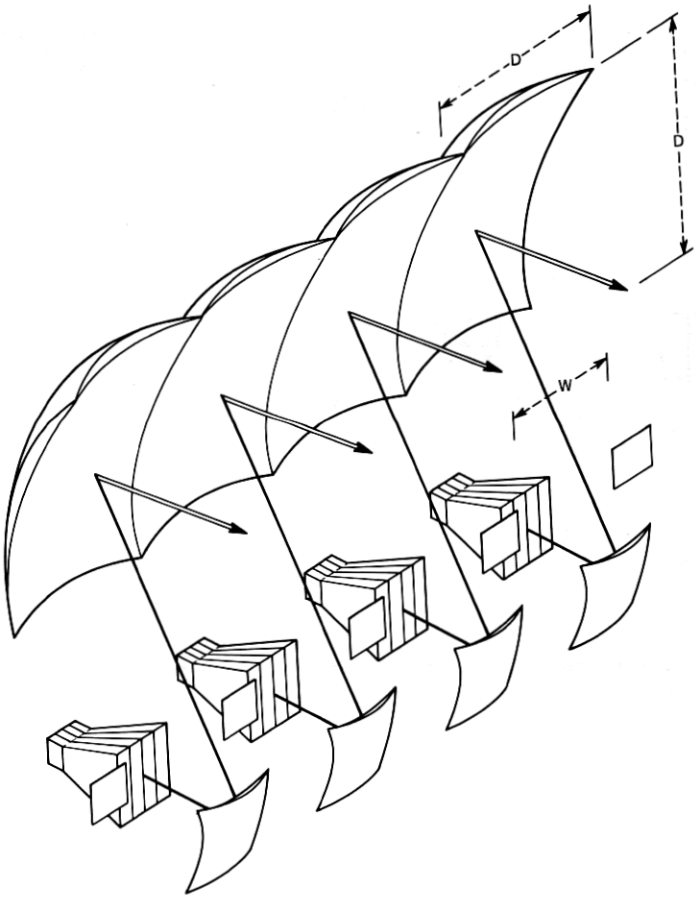

Fig. 12—To obtain the beam of Fig. 11, four identical Gregorian arrangements are combined with four arrays. Notice the total number of array elements is 16. For each arrangement, $D = 19\lambda$ at 11.8 GHz.

with a Gregorian arrangement and a filter with

$$\frac{W}{2f_2} = \tan 6°.$$

The antenna receives horizontal polarization (in the x-direction) at 14.25 GHz and, at 11.8 GHz, transmits vertical polarization. Strong grating lobes arising without filtering are substantially reduced by this filtering, with only a small reduction (<0.4 dB) in beam gain.

IV. CONCLUSIONS

We have shown that grating lobes in the antenna far field can be reduced appreciably by use of a properly chosen filter in the focal plane, as shown in Fig. 1. In general, the effectiveness of the filter can be shown to depend on the number N of array elements. If this number is large, the grating lobes will occur far from the main beam, and therefore they can be efficiently eliminated by a properly designed filter without appreciably affecting the main beam. By properly optimizing the values of a_n in such an antenna, excellent performance in side lobes will be obtained. In the example discussed in Section III, $N = 5$. In this case, for each array the grating lobes occur close to the main beam, and therefore they cannot be reduced appreciably without affecting somewhat the main beam. It was shown, nevertheless, that appreciable reductions are possible, even for $N = 5$ with little gain degradation; e.g., in Fig. 9 grating lobes were reduced by 24 dB with only 0.2-dB reduction in gain, in addition to the loss in gain due to Chebyshev taper.

In a phased array, it is often desirable to minimize the level of some of the side lobes by properly optimizing the element amplitudes a_n. Without filtering, such an optimization will result in general in increased grating lobes, as in Figs. 7 to 10 for a Chebyshev distribution. With filtering, however, considerable reduction in level for all side lobes can be achieved, as shown in Figs. 7 to 10.

Notice that aberrations, which increase for larger-scan angles, magnifications, and diameter to focal length ratios, have been neglected throughout this paper.

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