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Electronics in the Suburban and Light Urban Loop Networks

By A. J. CIESIELKA and D. C. DOUGLAS

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Loop electronics has been used in the Bell System on a permanent basis primarily in the rural environment where loops are very long and relief costs are very high. The increasing costs of cable and the decreasing costs of electronics point to increasing application of electronics in suburban and light urban areas. To understand the economics of the permanent use of loop electronics in these new areas, detailed studies have been made of 36 actual telephone company routes. This paper reports the major results of these studies for two basic types of electronic systems, range extension and pair gain. These studies indicate that the use of loop electronics can result in significant cost savings in the suburban and light urban loop networks. Mathematical models are developed based on the detailed results of the application studies to permit generalization of pair gain system applicability.

I. INTRODUCTION

Loops are the facilities between the local central office and the subscriber. For the most part, the individual customer's loop is a pair of metallic conductors. The cost of the loop network has risen as a result of both material and labor inflation. Labor costs directly affect the cost of the loop plant because labor composes roughly 75 percent of the total expenditures in the loop plant.[1]

While loop construction costs have increased with inflation, the cost of electronic alternatives has actually decreased. Two basic types of electronic alternatives to standard cable relief (Fig. 1) are range extension and pair gain systems. Range extension permits the design of the loop plant with finer gauged cables, which reduces cable relief costs. The range extender is an electronic device that extends the central office supervisory range and provides voice frequency gain to compensate for the higher loss of the cable plant.

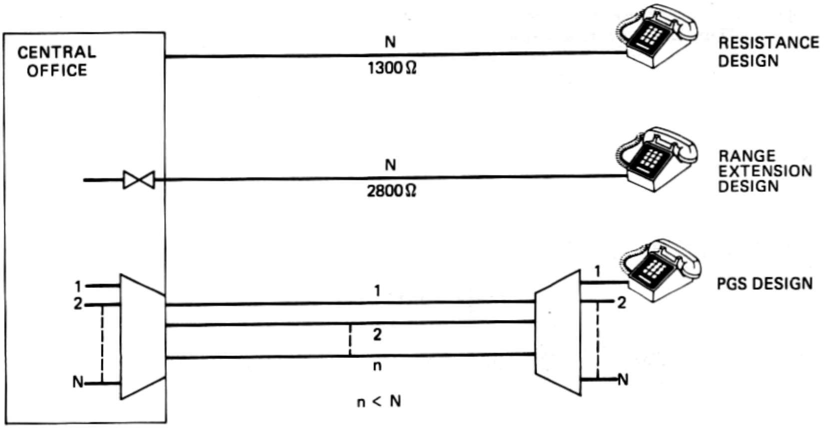

Fig. 1—Loop alternatives.

Pair gain systems (PGS) reduce the need for new physical facilities by electronically deriving more loops over existing pairs. Pair gain system techniques include multiplexing, as used in carrier systems such as the *SLC*™-8 system,[2] concentration as used in the LSS,[3] and a combination of the two techniques—carrier/concentrators as used in the *SLM*™[4] system and in the No. 10A RSS[5] system.

Loop electronics have been used in the Bell System on a permanent basis primarily in the rural environment where loops are very long (greater than 30 kft) and relief costs are very high. Because the trends of cable and electronics costs point to increasing application of electronics, a major effort has been under way to understand the economics of the permanent use of loop electronics in suburban and light urban areas. This effort has focused on detailed application studies of actual telephone company loop networks. Further, models have been developed to generalize the knowledge gained in the application studies. The purpose of this paper is to report on the results of the application studies and status of the models to date.

The application study economics reported here are based on the use of pair gain for message telephone service only. Additional economic benefits are expected from new services and special services applications. The economic impact of pair gain on these other services is presently under study.

II. THE LOOP NETWORK

In a typical wire center, four main routes provide service to customers. Each route consists of two basic elements, the feeder network and the distribution network. The feeder network begins at the central office and consists of individual feeder sections as shown diagrammatically in the example of Fig. 2. Sectionalization is a planning expedient that results in constant cross-section units that can be independently engineered for minimum cost relief. The distribution network is the network of cable between the customer premises and the feeder network. The distribution network is short (typically less than a kilometer) and usually of small cross section (typically up to a few hundred pairs). This part of the loop is usually sized for the ultimate demand. The studies reported in this paper only reflect the impact of loop electronics on the feeder network; the impact of loop electronics on distribution cable is presently under study.

At first glance, it seems that the economic impact of electronics on loops should be easy to evaluate. Unfortunately, the loop network is very complex and, as a result, the costs of relief are quite variable.

The feeder network is designed to match the demand along the route; it therefore tapers, since the total number of pairs at the central office must exceed the total demand on the whole route while the total number of pairs at, say, 10 kft from the office must only exceed the demand past 10 kft.

Further, the network is complex because the gauge of the cables varies. The transmission quality to individual customers is controlled by designing to a maximum conductor resistance. The basic design in the Bell System is called Resistance Design (RD). The primary design rule is that the maximum conductor resistance to any given customer must be less than 1300 ohms. This is accomplished by installing a number of different gauge cables on a route. Service to individual

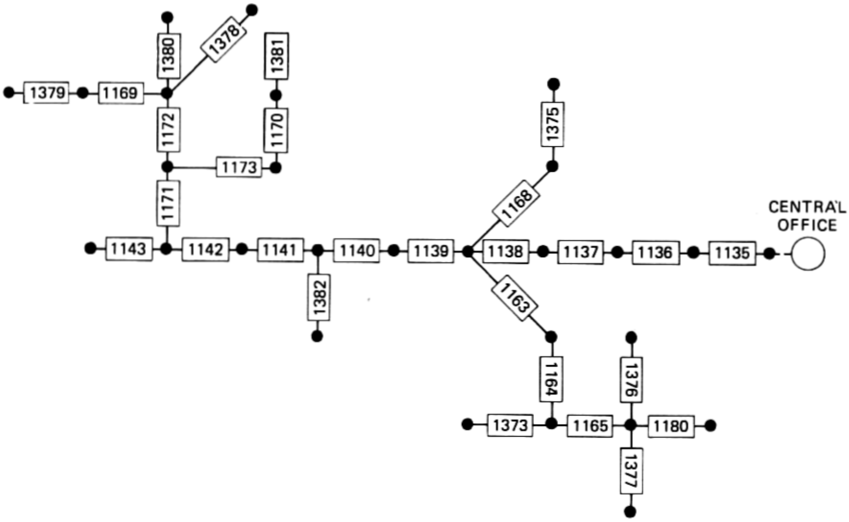

Fig. 2—Typical feeder route configuration.

customers is designed with a two-consecutive-gauge loop make-up, whose total resistance is less than 1300 ohms. As a result, it is quite likely that there will be more than one cable and more than one gauge in a given feeder section.

Various construction techniques are employed in the feeder plant. Cables are placed underground in conduit (underground plant), buried directly in the ground (buried plant), or placed on telephone poles (aerial plant).

Because cables are manufactured in discrete sizes and because growth varies along a route, feeder routes are typically relieved in sections or groups of sections.

For all these reasons, the costs of feeder route relief per loop can be quite variable from route to route.

In summary, the costs of relieving an individual route are a function of:

(*i*) Route length.
(*ii*) Spare cable and structure.
(*iii*) Growth rate and distribution of growth.
(*iv*) Type of construction.
(*v*) Local costs.

III. ELECTRONICS ALTERNATIVES

3.1 Voice frequency range extension

Voice frequency range extension permits the design of the loop plant to 2800 ohms.[6] The plan evaluated in this paper is the Concentrated Range Extender with Gain (CREG) system, which was introduced in 1979 to the Bell System on Nos. 1, 1A, 2, and 2B ESS.

The main design rules are:

(*i*) All loops out to 1500 ohms require no range extension.

(*ii*) Loops beyond 1500 ohms must be terminated on range-extended terminations in the office.

(*iii*) All loops past 15 kft must be H88 loaded (as opposed to the 18-kft rule of resistance design).

The range extenders, as the CREG system name indicates, are provided on a concentrated basis; that is, the range extenders are behind the first stage of switching. This decreases the effective cost of range extension by the concentration ratio of the first stage of switching. Per-line range extenders have been used extensively in rural areas of the Bell System.[6] The CREG system drastically reduces the cost of range extension, which should make the system economical in suburban applications.

3.2 Pair gain systems

A number of pair gain systems have been evaluated in the studies described here. For simplicity, only one of the systems will be described. The block diagram of the system is shown in Fig. 3. The system, *SLC*-96, is a digital carrier/concentrator system. The basic system electrical module supports 96 lines. Two-to-one digital concentration is employed in the system, which results in 48 channels. These 48 channels are provided over two standard T1 digital lines (24 channels each). A spare T1 line is required to assure continuity of service if one of the T1 lines should fail.* Thus, the pair gain of the system is 90 (96 − 3T1 LINES × 2 PAIRS/T1 LINE) and the pair gain ratio is 15 (90 ÷ 6). The *SLC*-96 system was introduced in the Bell System in 1979.

The *SLC*-96 system was configured to meet the varied growth and environmental needs of the loop plant. Accordingly, two physical versions of the system are being made available. A cabinet version of *SLC*-96 provides service to 96 customers from a self-contained closure suitable for pedestal or pole mounting (Fig. 4). A mini-hut version of *SLC*-96 permits the stacking of 10 *SLC*-96 systems to provide service to 960 customers for applications requiring large capacities (Fig. 5).

IV. STUDY METHODOLOGY

Section II described the varied nature of the loop plant and the associated costs of relief. To understand the application of electronics in a variety of situations, detailed application studies of relief alternatives for a subset of Bell System routes were performed. The detailed results of the application studies are covered in Section VI. These detailed results are used to generate mathematical models to permit generalization of the results. The models are described in Section VII.

4.1 Route sample

There are approximately 40,000 routes in the Bell System. Detailed analysis of the large number of routes required for a scientific sample without an automated program is prohibitive. However, an attempt was made to select routes for analysis that have a spread in characteristics that affect the cost of relief.

A total of 36 routes were used for detailed analysis. Twenty-eight of these routes were complete samples of all the routes from seven Bell System wire centers. In addition, eight routes are from wire centers where less than all the routes were included in the sample. Figure 6 is a scatter plot of the 36 routes; the abscissa is the maximum length of the route and the ordinate is the growth rate (in lines/year) of the route. The routes selected for this analysis are primarily light urban or suburban in nature (modest length and high growth) because this is expected to be the primary source of loop electronics applications.

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* Also, additional pairs (not shown in Fig. 3) are required for maintenance purposes, but these can be shared among the systems at an RT site, and in a multisystem application would typically amount to less than one additional pair per system.

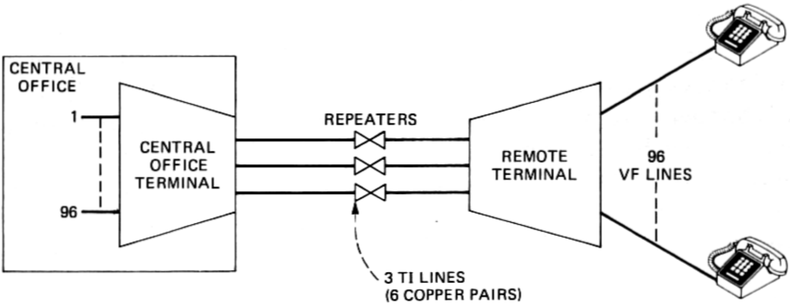

Fig. 3—*SLC*™-96 pair gain system.

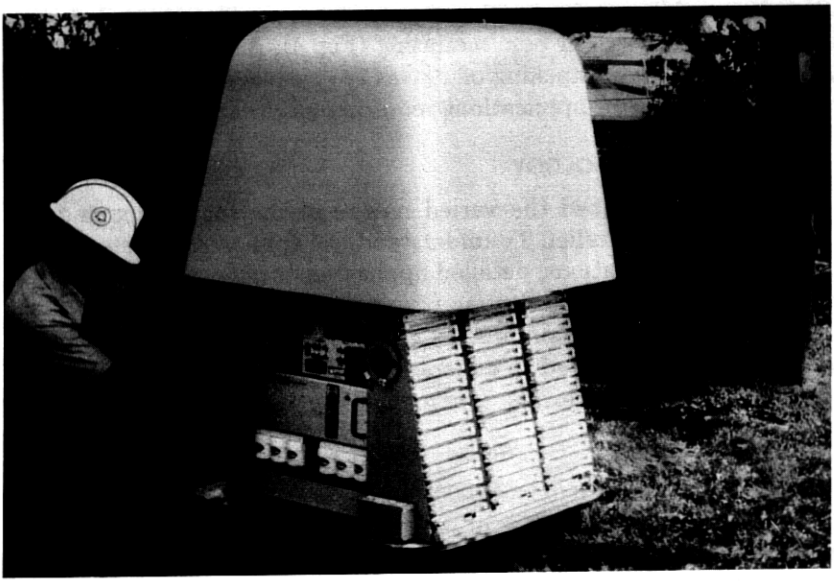

Fig. 4—Cabinet remote terminal.

However, one rural wire center has been included in the analysis to better understand the effects of low growth rates and long loops and to determine the applicability of the methodology developed here for use on rural routes. No heavy urban routes (i.e., center city) are included in the sample; these are high-density short loops that clearly cost less than present-generation pair gain alternatives.

4.2 Detailed application studies

The detailed application studies employed a methodology comparable to the method that telephone companies use today to plan cable relief on suburban routes.

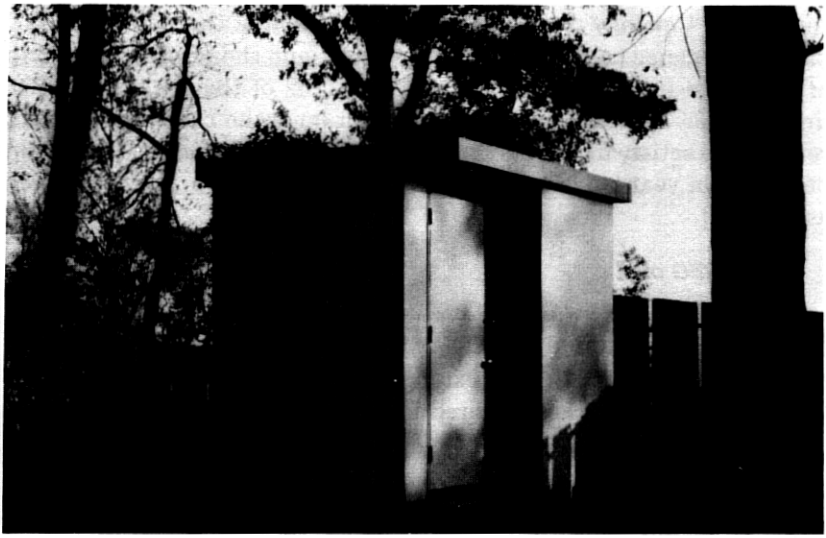

Fig. 5—Mini-hut remote terminal.

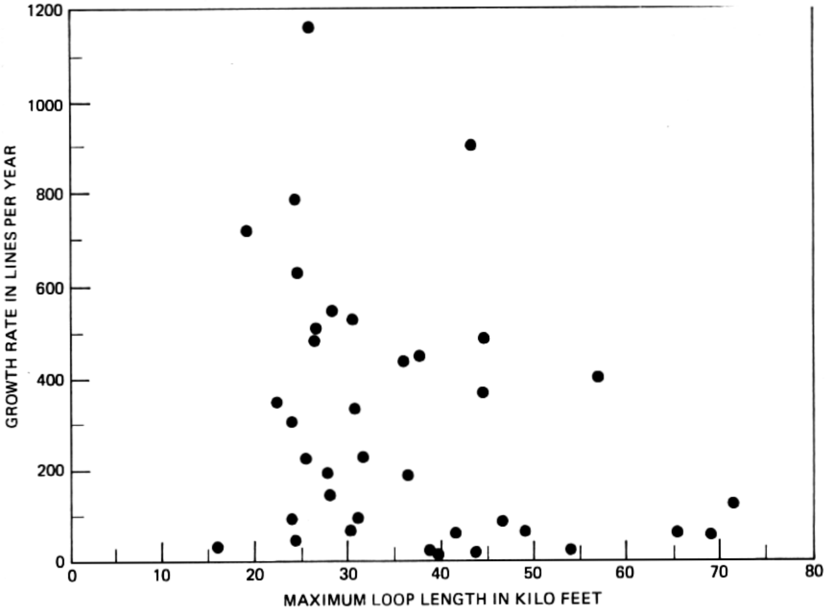

Fig. 6—Route sample (36 routes).

4.2.1 Base cable plan (resistance design)

Costs and material required without electronics were obtained using EFRAP (Exchange Feeder Route Analysis Program).[7] The inputs to the program are local costs and economic factors, the basic makeup of the route (e.g., topology and lengths), the condition of the route (e.g., 600 pairs of 22-gauge cable are in Section 1102), and the forecasted growth of the route (e.g., the growth in Section 1102 of the route will be 15 lines in year 10). The output of the program is the cable and conduit required to satisfy future needs (e.g., a 2700-pair, 26-gauge cable should be placed in year 3 in Section 1102) as well as the PWAC costs (defined as $\mathrm{PWAC}_{\mathrm{RD}}$).

4.2.2 CREG plan

The CREG plan PWAC ($\mathrm{PWAC}_{\mathrm{RD+CREG}}$) was obtained by re-running EFRAP with a maximum conductor range of 2800 ohms and manually calculating the cost of range extenders and additional load coils (LC) (additional load coils are required for growth loops between 15 and 18 kft):

$$\mathrm{PWAC}_{\mathrm{RD+CREG}} = \mathrm{PWAC}_{\mathrm{RD/CREG}} + \mathrm{PWAC}_{\mathrm{CREG}} + \mathrm{PWAC}_{\mathrm{LC}}.$$

The output of EFRAP for the CREG plan (i.e., cost represented by $\mathrm{PWAC}_{\mathrm{RD/CREG}}$) demonstrates a drastic reduction in coarse gauge relief requirements and a modest reduction in conduit requirements. Conduit relief is deferred in some cases because CREG permits the use of large, fine-gauge cables (e.g., 2700 pairs of 26 gauge) instead of multiple cables of varying gauges (e.g., 1800 pairs of 26 gauge and 900 pairs of 24 gauge). The range extender requirement and associated costs were calculated based on the assumed need for range extension on all growth lines past 18 kft (1500 ohms of 26-gauge cable). This is a conservative assumption, since some growth loops past 18 kft do not require range extenders, i.e., those that are less than 1500 ohms, resulting from a composition of mostly existing coarse gauge cable and a small amount of fine gauge cable. However, the simple assumption is reasonable when one considers the administrative advantages of requiring all growth loops past 18 kft to be terminated on range extension equipment.

4.2.3 Pair gain system plan

The detailed study of the use of pair gain systems is not as straightforward. The placement of pair gain systems on a route has an effect on the entire route. In addition, many pair gain system topologies can be employed, as described in Section V. As a result, the pair gain system results are not optimum. Nevertheless, experience with the following method indicates that the results in terms of savings are reasonable, considering the uncertainties of the inputs (e.g., relief costs, forecasts). The following steps describe the pair gain system methodology:

(*i*) Trial PGS remote terminal (RT) sites on the route and their installation dates are determined.

(*ii*) The EFRAP data are modified to reflect the use of PGSs (or the decreased use of cable) on the feeder route and new cable costs are calculated with EFRAP ($\mathrm{PWAC}_{RD/PGS}$).

(*iii*) The costs associated with electronics (PWAC_{PGS}) are calculated and manually added to the PGS cable cost to determine the total PWAC cost of the PGS plan.

(*iv*) The resulting cost of the PGS plan is:

$$\mathrm{PWAC}_{RD+PGS} = \mathrm{PWAC}_{RD/PGS} + \mathrm{PWAC}_{PGS}.$$

These steps are repeated to find a near-optimum solution. During the course of the study, various methodologies were developed and used to reduce the amount of trial and error required.

V. PAIR GAIN SYSTEM DEPLOYMENT STRATEGIES

As mentioned, a large number of pair gain system topologies and deployment strategies are possible. The following deployment strategies were evaluated in the application studies:

1. Commitment (Fig. 7):
    - A. Growth only—*only growth* lines are assigned to an RT (physical and derived lines mixed).
    - B. Growth plus existing—*all* growth and working lines in the area beyond the RT are assigned to it (physical and derived lines not mixed). By transferring existing customers to the PGS, the same cable saving effect (as growth only) can be achieved with fewer remote terminal sites.
2. Gauging (Fig. 8):
    - A. Regauging—all cables placed beyond an RT and terminated on it are gauged as if the RT were a central office. This reduction in conductor size saves copper costs and is possible because the PGS provides a low-loss path to the CO.
    - B. No regauging—all cables placed beyond an RT are gauged using 1300-ohm resistance design as if they terminated on the CO. Regauging may not be desirable for administrative reasons or because of the reduction in flexibility with regard to future rearrangements.
3. Network configuration (Fig. 9):
    - A. Nonseries sites—trunk pairs from one RT site *do not* pass another RT site on their path back to the CO. These sites tend to be large because all pair gain lines must be committed to one site.
    - B. Series sites—trunk pairs from one RT site pass another RT site

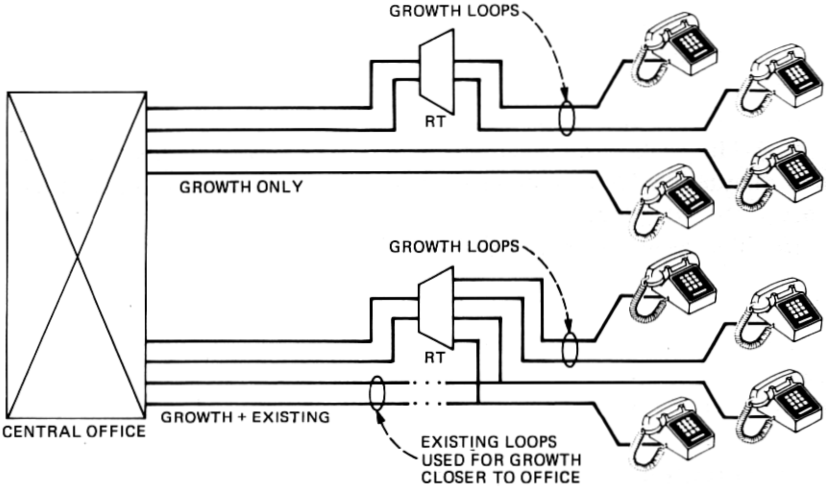

Fig. 7—PGS commitment strategies.

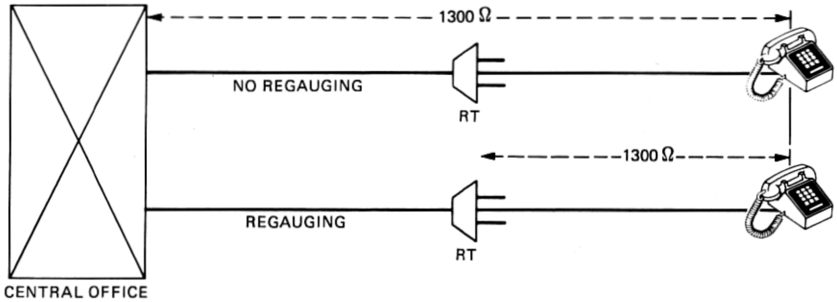

Fig. 8—PGS gauging strategies.

on their path back to the CO. These sites are distributed along the feeder network and are smaller than nonseries sites.

The RT sites and installation dates selected in the analysis (growth or growth plus existing) are not necessarily optimum. Additional savings can be obtained through use of a phased commitment strategy. In such a strategy, just enough customers would be assigned to PGSs so that the placement of additional cable and support structure on a route could be deferred.

VI. RESULTS OF APPLICATION STUDIES

The results of the detailed application studies indicate that electronics can be economically employed on a permanent basis in the suburban and light urban loop networks. The electronic alternatives will be

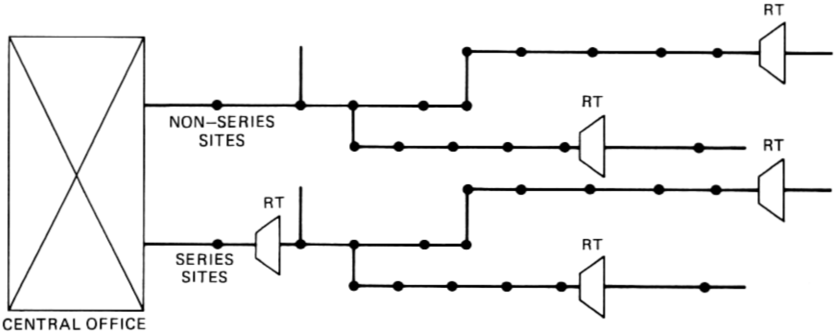

Fig. 9—PGS network configurations.

compared both individually and collectively to the base solution of resistance design.

6.1 CREG results

The detailed application study results shown in Fig. 10 indicate that the CREG system is very attractive compared to resistance design. CREG was more economic than resistance design on 27 out of 36 routes. This assumes, of course, that all the routes are served by switching machines that are compatible with CREG.* The total CREG PWAC savings compared to resistance design was 12 percent, which amounts to a substantial reduction in the outside plant construction budget.

6.2 Pair gain system results

The pair gain system results shown in Fig. 11 are also very encouraging. For simplicity, all the results discussed in this section are for growth-plus-existing, regauging, and nonseries sites. The *SLC*-96 system was attractive on 14 out of the 36 routes and saved 9 percent referenced to 36 routes and 17 percent referenced to the routes that were attractive. This savings level, on a Bell System company-wide basis, corresponds to over $100 million per year.

6.3 CREG and pair gain system results

The CREG results indicate that CREG is generally attractive on light urban and suburban routes (i.e., routes with lengths greater than 20 kft). CREG primarily reduces gauge requirements, while pair gain systems greatly reduce cable and conduit requirements. Although pair gain can provide a greater reduction in cable costs than CREG, the pair gain electronics (*SLC*-96) costs are also greater, resulting in comparable net savings.

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* CREG is being made available in Nos. 1, 1A, 2, and 2B ESS machines only.

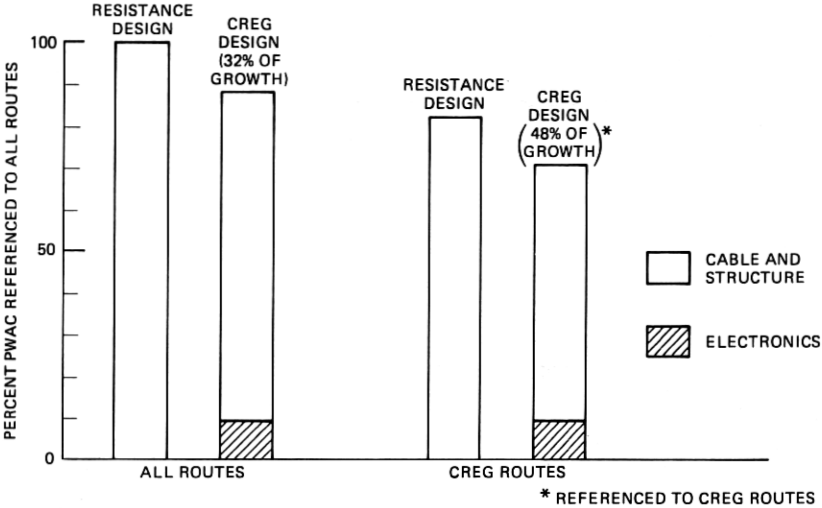

Fig. 10—CREG results (36 routes).

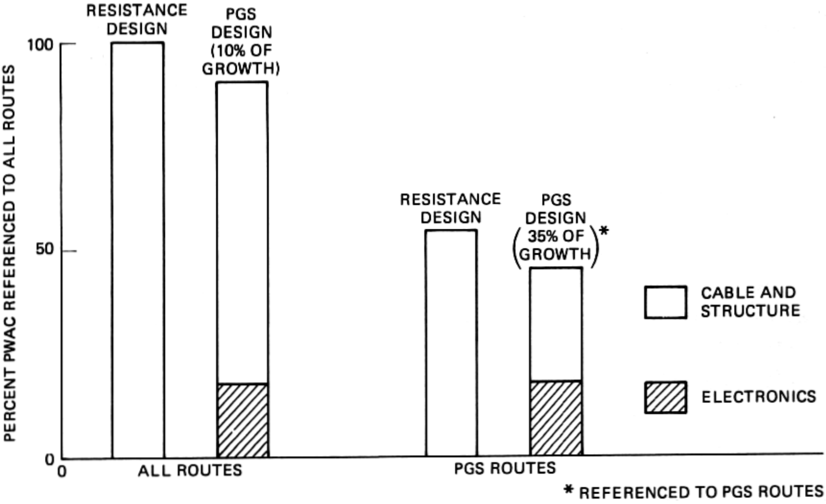

Fig. 11—PGS results (36 routes).

A question of considerable interest is whether or not the use of CREG precludes the use of pair gain. Accordingly, five routes that were attractive for pair gain systems were selected for CREG/pair gain system analysis. In this analysis, CREG was applied first and pair gain was then employed to further reduce costs by selectively displacing both cable and CREG. Pair gain and CREG were used on the same route but not on the same loops. The results of the analysis (Fig. 12) indicate that on three of the routes the use of both CREG and PGSs was more attractive than the use of CREG alone. On two of the routes, the CREG plan was preferable to either the PGS plan or the CREG/PGS plan. In summary, the use of CREG somewhat reduces the attractiveness of pair gain due to the lower cost of facility relief, yet pair gain systems are still attractive on some routes for feeder and conduit relief.

6.4 Pair gain system deployment strategy comparison

It is not possible to provide a final comparison between deployment strategies due to the facts that:

(*i*) The pair gain system placements and timings are not necessarily optimum.

(*ii*) Other deployment strategies may be more optimum than those studied (e.g., placing a number of lines on pair gain systems between growth and growth-plus-existing).

(*iii*) Operations costs (e.g., rearrangements needed to use pairs gained) not accounted for in the studies may vary greatly with various deployment strategies and pair gain systems.

In spite of the unknowns, the following observations can be made based on a comparison of 15 of the routes.

(*i*) A growth-plus-existing strategy tends to result in greater savings than a growth-only strategy (Fig. 13). This advantage could disappear if the operational costs are significantly greater for the growth-plus-existing strategy.

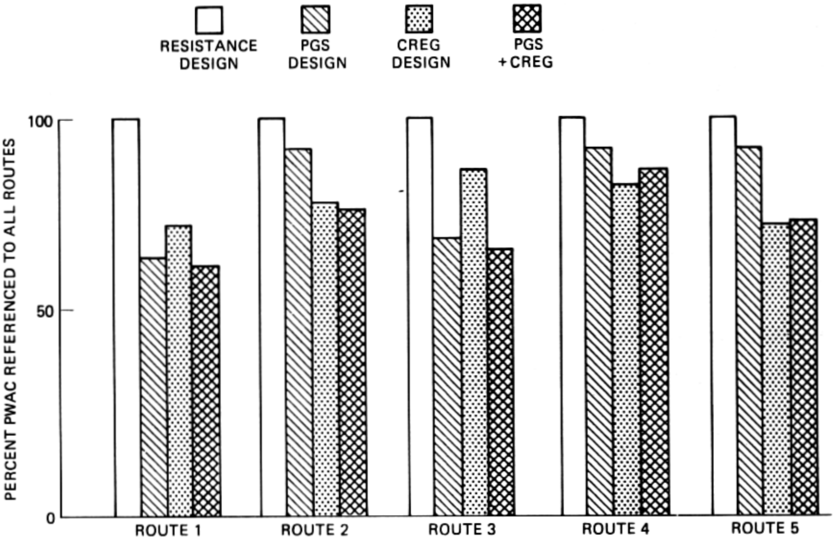

Fig. 12—CREG/PGS results

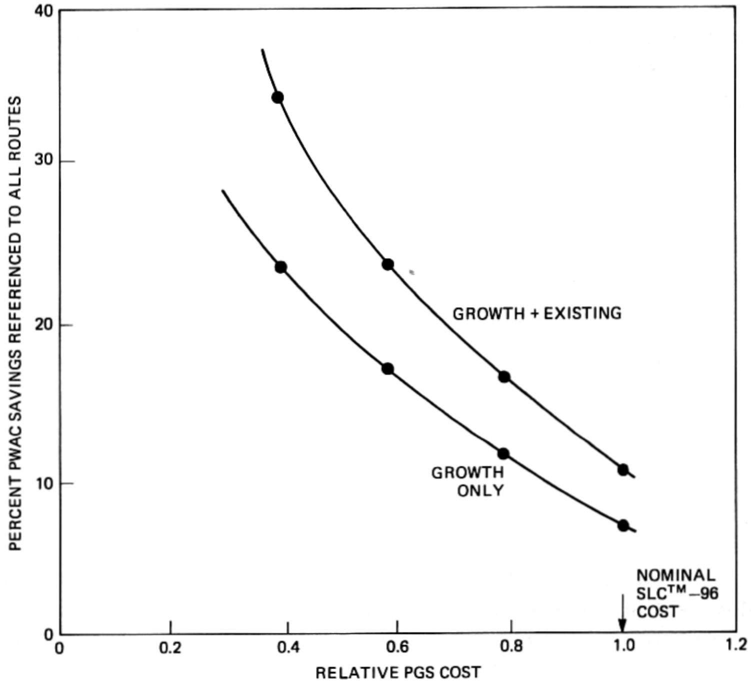

Fig. 13—*SLC*™-96 (no regauging, nonseries sites).

(*ii*) Regauging of the feeder route beyond the remote terminal (RT) appears to increase the PWAC savings on winning routes by only 10 percent (Fig. 14). Offsetting this advantage are penalties that remain to be evaluated. Since regauged cables may not be capable of providing service except from the RT, these cables may be inefficiently used if the telephone company is forced to revert to serving the customers from the CO. Special services that cannot be served via pair gain may result in additional gauge requirements and wire cable sheaths. Finally, all records and engineering would need to be reorganized around RT areas, incurring additional initial costs and ongoing inefficiency. A practical compromise may be to regauge for 2800 ohms from the CO, which would be compatible with either CREG or pair gain systems.

6.5 Main application study findings

The main findings of the detailed application studies are:

(*i*) CREG is generally applicable in light urban and suburban wire centers (light urban and suburban being defined as having routes greater than 20 kft).

(*ii*) Pair gain systems have the potential of substantial savings on some routes. The results are extremely variable even for routes that superficially appear similar.

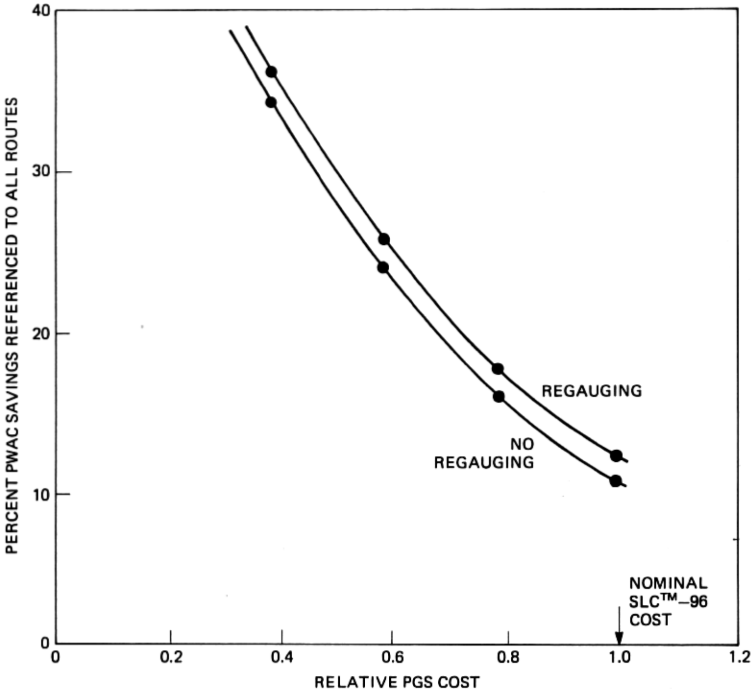

Fig. 14—*SLC*™-96 (growth plus existing nonseries sites).

(*iii*) CREG and pair gain systems can be economically deployed on the same routes (but not the same loops) in some cases.

(*iv*) The pair gain system results are a strong function of the installed cost of the system (Fig. 15). Studies have indicated that the added cost of pair gain systems on a digital central office are less than half the cost of adding pair gain systems to an electromechanical central office.[8] The incremental pair gain system savings significantly increase the overall benefits of digital central offices.

To realize the savings potential for pair gain systems, the following steps are being taken:

(*i*) CREG and pair gain system guidelines and study tools are being developed for light urban and suburban areas to permit the telephone companies to quickly identify applications and provide the basis for making detailed economic comparisons.

(*ii*) The total operational impact of a large number of pair gain systems is being evaluated to determine the total economics of a pair gain intensive suburban plant.

VII. GENERALIZED MODELS OF PAIR GAIN SYSTEM RESULTS

As described in Section VI, the pair gain system results were extremely variable from route to route. To understand the reasons pair

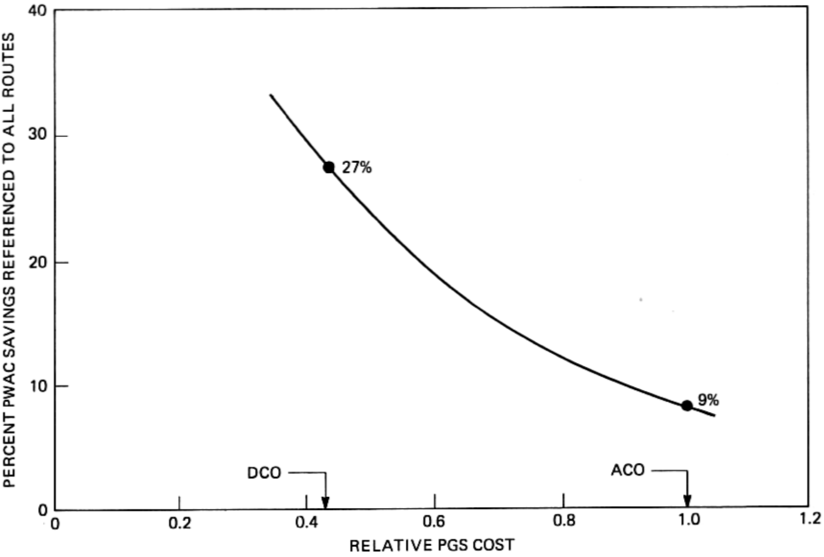

Fig. 15—Savings on study routes (36 routes).

gain systems results vary widely and to generalize the results of the detailed application studies, mathematical models were derived to relate basic wire center characteristics to the economic results. The goal of these studies was to predict pair gain system penetration (*P*) and the resulting pair gain system savings (*S*) from wire center parameters:

P = penetration, average pair gain growth as a fraction of total assigned pair growth in the wire center
S = PWAC savings as a fraction of total PWAC for cable and structure (without pair gain) in the wire center

or, mathematically,

$$P = G_{\text{PGS}}/G_{\text{RD}}, \tag{1}$$

where

G_{PGS} = average yearly growth in assigned pairs gained*
G_{RD} = average yearly assigned pair growth in the wire center*

and

$$S = \frac{\text{PWAC}_{\text{RD}} - (\text{PWAC}_{\text{RD/PGS}} + \text{PWAC}_{\text{PGS}})}{\text{PWAC}_{\text{RD}}}. \tag{2}$$

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* Twenty-year averages were used in this analysis.

In the following sections, key parameters strongly related to PGS attractiveness are described, and a general consistency relationship between S and P is derived as a function of these parameters. These general results are then applied to the switching center study data to develop specific penetration and savings models.

7.1 Cable relief versus pair gain system relief

As described in Section II, the cost of normal cable and conduit relief varies greatly due to individual route characteristics such as route length, spare cable and structure, growth rate distribution, and type of construction. All these variations are reflected in the total PWAC cost of cable relief. PGS lines are incorporated selectively as an alternative to pairs that would incur greater PWAC. Thus, in comparing cable relief to pair gain system relief, it is not surprising that the most meaningful single wire center characteristic that has been found is the effective PWAC cost per incremental assigned pair. This measure of total cost applies to both cable and pair gain systems relief and is defined as a levelized PWAC, distributed by incremental growth lines:

$$\text{PGS: } C_{\text{PGS}} = \frac{\text{PWAC}_{\text{PGS}}}{PW\{g_{\text{PGS}}\}_i}, \tag{3}$$

$$\text{Cable: } C_{\text{RD}} = \frac{\text{PWAC}_{\text{RD}}}{PW\{g_{\text{RD}}\}_i}, \tag{4}$$

where

PWAC_{PGS} = PWAC cost of pair gain, i.e., the electronic part of the pair gain solution
PWAC_{RD} = PWAC cost of normal cable relief, without pair gain
g_{PGS} = Yearly growth in assigned pairs gained
g_{RD} = Yearly growth in total assigned pairs in the wire center
$PW\{\ \}_i$ = Present worth for an infinite period at cost-of-money, i
= $(1 + i)/i$ for a constant growth rate.

PWAC costs must be used to reflect both capital and operating costs. Levelization per added working line is useful, in part, because this tends to eliminate PGS penetration as a variable in cost comparisons.

The above PWAC parameters are evaluated for an infinite period by application study programs which account for inflation, variations in costs, and variations in growth rates. For constant growth rate, annual charge rate, and effective capital cost per line, these parameters can be written as

$$\text{PWAC} = (\text{IFC})(G)(ac)\left(\frac{1 + I}{I^2}\right), \tag{5}$$

where

IFC = installed first cost per line in year-0 dollars
G = average yearly assigned pair growth
ac = annual charge rate, including inflation,

I = convenience rate $= \dfrac{i - \gamma}{1 + \gamma}$
i = cost of money
γ = inflation rate.

Although economic factors and growth rates are not constant in general, variations at the wire center level are usually sufficiently small that the use of equivalent constant amounts is reasonable and greatly simplifies the development of models.

A parameter (r) indicative of PGS attractiveness in a central office is the composite value of $C_{\text{PGS}}/C_{\text{RD}}$ for the entire feeder plant of the office.

$$r = \frac{C_{\text{PGS}}}{C_{\text{RD}}}. \tag{6}$$

A simple, practical expression for r can be obtained by letting growth and annual charge rates be constant:

$$r = \frac{\text{PWAC}_{\text{PGS}}/G_{\text{PGS}}}{\text{PWAC}_{\text{RD}}/G_{\text{RD}}}. \tag{7}$$

Or, substituting for PWAC quantities from eq. (5),

$$r = \frac{\text{IFC}_{\text{PGS}}ac_{\text{PGS}}((1 + I_{\text{PGS}})/I_{\text{PGS}}^2)}{\text{IFC}_{\text{RD}}ac_{\text{RD}}((1 + I_{\text{RD}})/I_{\text{RD}}^2)}, \tag{8}$$

where

IFC_{RD} = effective installed first cost per added assigned pair for relief without pair gain.
IFC_{PGS} = effective installed first cost of electronics per added assigned pair gained by PGS.

IFC_{RD} can be estimated from EFRAP results or from actual and budgeted construction expenditures. From EFRAP PWAC, assuming constant growth (or using a constant equivalent),

$$\text{IFC}_{\text{RD}} = \frac{\text{PWAC}_{\text{RD}}}{G_{\text{RD}}ac_{\text{RD}}((1 + I_{\text{RD}})/I_{\text{RD}}^2)}. \tag{9}$$

IFC_{PGS} can be expressed as a function of PGS capital cost at 100-percent fill and an average fill factor (μ_{PGS}):

$$\text{IFC}_{\text{PGS}} = \frac{\text{IFC}_{\text{PGS}}(100\%\ \text{fill})}{\mu_{\text{PGS}}}, \tag{10}$$

where

$$\mu_{\mathrm{PGS}} = \frac{\mathrm{IFC}_{\mathrm{PGS}}(100\%\ \mathrm{fill})\, ac_{\mathrm{PGS}} G_{\mathrm{PGS}}((1 + I_{\mathrm{PGS}})/I^2_{\mathrm{PGS}})}{\mathrm{PWAC}_{\mathrm{PGS}}}. \tag{11}$$

In most *SLC*-96 applications studied, light urban and suburban, $\mu_{\mathrm{PGS}} \approx 1$ and $\mathrm{IFC}_{\mathrm{PGS}}$ is simply the PGS cost per pair gained at 100-percent fill. In rural applications, μ_{PGS} can be significantly less than 1. Additional modeling work is required to provide a means of estimating μ_{PGS} for these cases.

7.2 Consistency requirement, S vs P

Pair gain impact modeling is greatly facilitated by recognizing that savings and penetration are not independent parameters. Treating penetration as a continuous variable, consistency between penetration and savings requires that

$$dS = \frac{\partial S}{\partial P}\, dP + \frac{\partial S}{\partial C_{\mathrm{PGS}}}\, dC_{\mathrm{PGS}}. \tag{12}$$

This can be simplified by noting that at the penetration P which gives maximum savings, S,

$$\frac{\partial S}{\partial P} = 0. \tag{13}$$

Also, at constant penetration, from eq. (2),

$$\frac{\partial S}{\partial C_{\mathrm{PGS}}} = -\left(\frac{1}{\mathrm{PWAC}_{\mathrm{RD}}}\right) \frac{\partial \mathrm{PWAC}_{\mathrm{PGS}}}{\partial C_{\mathrm{PGS}}}. \tag{14}$$

Substituting for PWAC quantities from eqs. (3) and (4) and noting that $P = G_{\mathrm{PGS}}/G_{\mathrm{RD}}$, we obtain

$$\frac{\partial S}{\partial C_{\mathrm{PGS}}} = -\frac{P}{C_{\mathrm{RD}}}. \tag{15}$$

Equation (12) becomes

$$dS = -\frac{P}{C_{\mathrm{RD}}}\, dC_{\mathrm{PGS}}. \tag{16}$$

Alternatively, from eq. (6),

$$\frac{dC_{\mathrm{PGS}}}{C_{\mathrm{RD}}} = dr \tag{17}$$

and

$$dS = -P\, dr. \tag{18}$$

Thus, if S is known, P can be found as the derivative with respect to r; or if P is known, S is given by

$$S_1 = S_0 - \int_0^{r_1} P\,dr. \tag{19}$$

Note that S_0, savings at $C_{\text{PGS}} = 0$, can be a function of C_{RD} but not C_{PGS}. Ideally, with complete flexibility of PGS deployment,

$$P(C_{\text{PGS}} = 0) = 1$$

and S_0 would be equal to 1 except for additional pairs needed for carrier lines.

7.3 Penetration

An empirical expression for penetration is obtained by fitting a curve to the application studies data for P versus r as shown in Fig. 16. Data points are shown for seven wire centers and two PGS prices, corresponding to *SLC*-96 with an analog CO switch and *SLC*-96 integrated with a digital switch. All the data used for model development were derived for "nonseries" RT sites and a "growth-plus-existing" pair commitment strategy.

The following expression for P is obtained by fitting a straight line to the data:

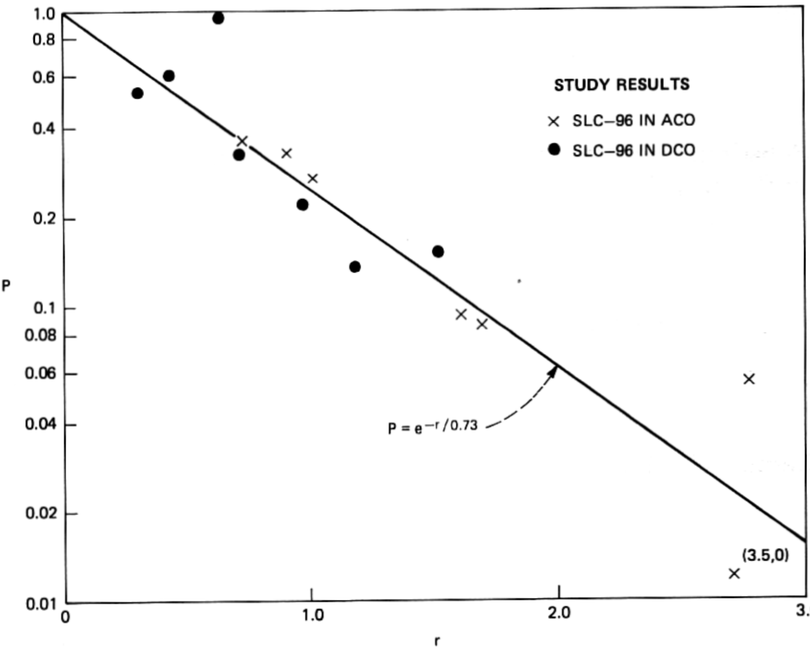

Fig. 16—Penetration.

$$P = \exp\left[-\frac{r}{0.73}\right]. \tag{20}$$

7.4 Savings

From eqs. (19) and (20), the expression for savings is

$$S = S_0 + 0.73(P - 1). \tag{21}$$

To complete the model, S_0 must be determined. S_0 is independent of PGS cost but can be a function of route characteristics and RT deployment strategy. For example, S_0 will be higher on routes where the growth is distributed more toward the end of the route. Also, a series site strategy will give $S_0 \approx 1$, whereas a nonseries strategy will result in S_0 significantly less than 1. That is, with a nonseries site strategy there will be relief jobs beyond the RT site that cannot be deferred.

If the models were exact, consistency between S and P would require that $S_0 = 0.73$ (for $P = S = 0$). However, the models are not exact and a better fit to savings data can be obtained by letting S_0 be a function of C_{RD}. S_0 can be estimated from the savings data and the model, eq. (21), i.e.,

$$S_0 = S_{\text{actual}} - 0.73\left(\exp\left\{-\frac{r_{\text{actual}}}{0.73}\right\} - 1\right). \tag{22}$$

Figure 17 gives the results as a function of IFC_{RD_0}, which is proportional

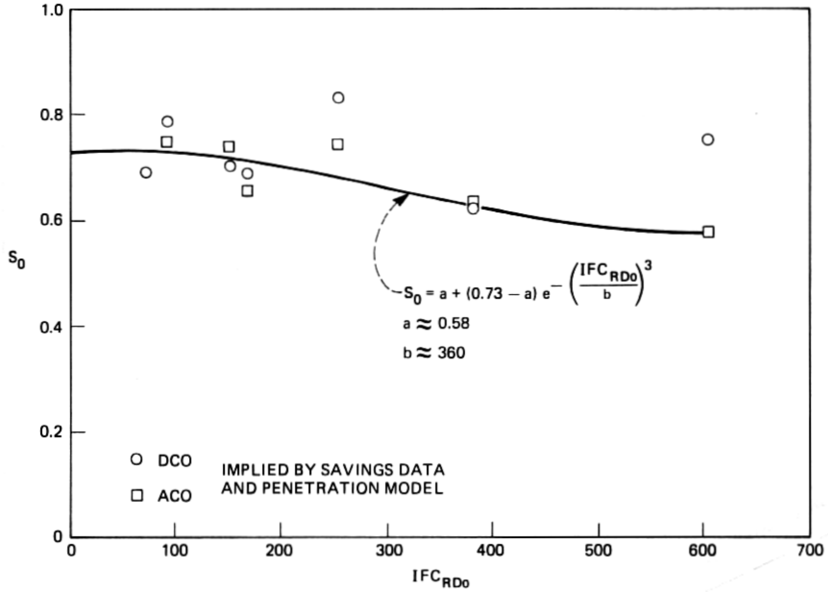

Fig. 17—Savings for 0-cost PGS.

to C_{RD} but is more easily interpreted, since it is roughly equivalent to average capital cost per added assigned pair.

$$\mathrm{IFC}_{\mathrm{RD}_0} = \frac{\mathrm{PWAC}_{\mathrm{RD}}}{G_{\mathrm{RD}}ac_{\mathrm{RD}_0}((1 + I_{\mathrm{RD}_0})/I^2_{\mathrm{RD}_0})} \tag{23}$$

where

$$ac_{\mathrm{RD}_0}\left[\frac{1 + I_{\mathrm{RD}_0}}{I^2_{\mathrm{RD}_0}}\right] = 40.$$

The data indicate that $S_0 \approx 0.73$ for a wide range of average cost per pair ($\mathrm{IFC}_{\mathrm{RD}_0}$ or C_{RD}), but suggest that S_0 decreases slightly as $\mathrm{IFC}_{\mathrm{RD}}$ increases; i.e., for rural wire centers. This apparent variation in S_0 may be a result of the deployment strategies used (nonseries sites and growth-plus-existing pair commitment) coupled with relative distributions of growth and existing lines. The data are inadequate for estimating this variation reliably. However, a lower limit of $S_0 = 0.5$ to 0.6 seems reasonable, if not conservative.

A function which approximately fits the data, with the assumed variation with $\mathrm{IFC}_{\mathrm{RD}_0}$, is

$$S_0 = a + (0.73 - a)e^{-(\mathrm{IFC}_{\mathrm{RD}_0}/b)^3}, \tag{24}$$

where

$$a \approx 0.58$$

$$b \approx 360.$$

These simple models give a good first-order estimate of pair gain impact. However, the underlying mechanisms relating P and S are clearly too complicated to be accounted for completely in models this simple. For various reasons, S cannot be expected to follow exactly the above relationship with P. For example, neither gauge savings nor the effect of undeferred cable relief beyond the RT site is accounted for directly. Further, the present models are developed for an infinite range of r as an approximation to finite r-range wire centers.

7.5 Effect of improved PGS application strategies

Nonseries RT sites and growth-plus-existing pair commitment have been used for the present initial development of PG impact models. The impact of pair gain can obviously be improved to some extent by allowing series sites and by using a growth-plus-part-of-existing-pair commitment strategy.

PGS penetration may be relatively insensitive to these changes, although savings will increase. In going from nonseries to series sites, we tend to distribute the sites along the route, thereby deferring more cable jobs, but not necessarily increasing the overall penetration. The overall penetration on a route tends to be limited to the growth beyond a point where the loops are long enough to cost more than a pair gain line. As r in the penetration model approaches 0, penetration approaches 1 in any case. Also, for large r, pair gain sites tend to be near the end of the route where the best strategy is likely to be close to nonseries, growth plus existing.

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