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Economic Evaluation of Loop Feeder Relief Alternatives

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The loop feeder network is the arterial part of the transmission network between telephone subscribers and their local central office. As subscriber demand grows or otherwise changes, the feeder network must be expanded or reconfigured to keep up with the demand. This expansion process, known as feeder relief, requires careful planning and evaluation to ensure that feeder facilities are provided where and when they are needed at the lowest possible cost. This paper identifies a number of feeder relief alternatives and presents a method for computing the present worth of expenditures (PWE) associated with a given alternative. This method enables one to compare relief alternatives and approach an optimal relief plan. The method uses mathematical models of loop cost, published previously, and standard economic computations.

I. INTRODUCTION

The loop network, which is the transmission network between the local central office and the telephone subscribers, consists of a *feeder network* and a *distribution network*. The feeder network originates at the central office and fans out in a tree-like fashion into the area served by the central office. The distribution network branches from points along the feeder network and extends to individual streets and rights of way to reach the subscribers' locations. The boundary between feeder and distribution is not always well defined. Often, however, the feeder and distribution networks are joined by a *feeder-distribution interface*.

Both the feeder and the distribution networks are *facility networks.* Together, they must supply the transmission facilities, which are usually called *pairs,** required to serve the subscribers. The feeder and distribution networks differ, however, in the way they are expanded to meet growing or otherwise changing subscriber requirements. The capacity of the feeder network is expanded in steps to meet growing pair requirements.[2] The distribution network, however, is built to provide the capacity ultimately required in one step.[3] This difference follows from a number of factors which are not discussed in this paper.† The concern here is, rather, that the feeder network requires periodic *relief,* i.e., addition or reconfiguration of facilities.

This paper presents and discusses new methods associated with planning feeder relief. Much of what has been previously written about feeder relief has been concerned with *sizing* feeder capacity additions.[2,4] This paper considers additional issues of feeder relief planning, specifically:

(*i*) How to identify and evaluate relief plans which make optimum use of existing (i.e., previously placed) feeder facilities.

(*ii*) How to evaluate temporizing measures, such as clearing defective feeder pairs.

(*iii*) How to determine when to place feeder additions.

The goal of feeder relief planning methods is to help the operating company engineer develop a feeder relief plan which provides feeder facilities where and when they are needed at the lowest possible cost.

The methods described in this paper are built around a *feeder route‡ model* presented in Section II. The general composition of a feeder relief plan is outlined in Section III. Section IV presents the total cost model which serves as the basis for comparing relief plans and provides some examples to illustrate applications of the methods.

II. FEEDER ROUTE MODEL

The feeder route model represents both the feeder network and the area it serves.

2.1 Feeder network model

The feeder network is modeled as a graph. The links of this graph are called feeder sections, or *sections,* and represent the feeder facilities between two points in the real network. The nodes represent points of

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* The term *pair* is derived from the pair of wires generally used for loop transmission. However, the meaning of pair will be generalized here to include pairs derived from pair-gain systems (Ref. 1).

† See Refs. 2 and 3.

‡ The area served by a single central office is generally divided into three to five smaller areas called routes.

access to the feeder network. One node represents the central office. Figure 1 shows how the feeder network model might look for a simple feeder route. Figure 2 shows, in the format of operating company cable records, the actual facilities which might be represented by one of the sections in Fig. 1. In this case, the nodes correspond to manholes which provide access to the underground feeder network. Access is accomplished through lateral cables, or *laterals*, which are spliced to the feeder cables.

Feeder sections represent the capacity of the feeder network, i.e., how many feeder pairs are available between two nodes. Feeder

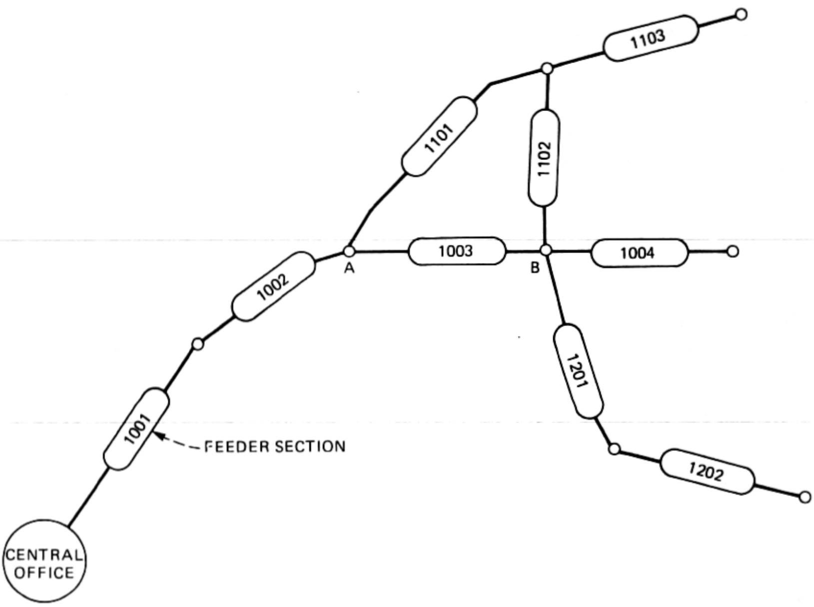

Fig. 1—Model of a simple feeder network.

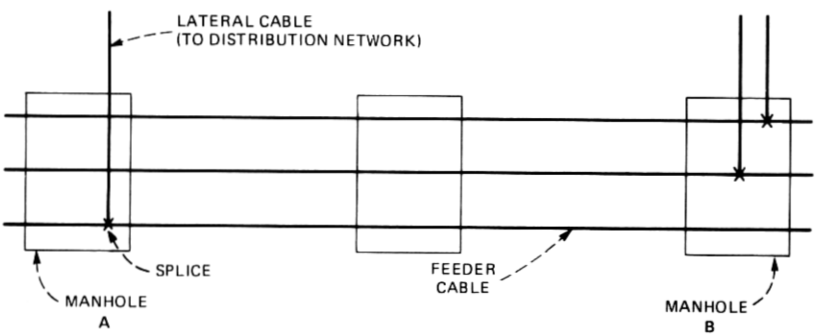

Fig. 2—Feeder facilities in a feeder section.

capacity can be increased by adding pairs to one or more feeder sections. Such additions can be accomplished by the placement of new cable or pair-gain systems.

2.2 Area model

The area served by the feeder network is partitioned into *allocation areas*. An allocation area represents a geographic unit containing from about 500 to 2000 telephone subscribers. Allocation area boundaries often correspond to physical or political boundaries. Ideally, an allocation area should represent a unit which is homogeneous with respect to housing type, growth pattern, and telephone service requirements.

Allocation areas represent the demand for feeder pairs. This demand includes requirements for existing subscribers and growth. Additional pairs may be required because of high subscriber turnover.*

Each allocation area is associated with a number of feeder pairs which make up the *pair-group* for that allocation area. A pair group, then, is a "pipeline" between the central office and an allocation area. A feeder pair may belong to, at most, one pair group. Some feeder pairs may belong to no pair group; these are called *unallocated* pairs.

2.3 Complete route model

Figure 3 shows how the feeder route model looks when allocation areas are added to the feeder network model of Fig. 1. Figure 4 is a more realistic map of the six allocation areas shown in Fig. 3.

The relationship among allocation areas, pair groups, and feeder sections is an important aspect of the route model. Consider Allocation Area 1002 in Fig. 3. The associated pair group leaves Allocation Area 1002 and enters the feeder network at node A. From node A, it continues through Feeder Sections 1002 and 1001 to the central office. In general, a pair group will traverse one or more feeder sections between its allocation area and the central office. It follows, then, that a feeder section may have one or more pair groups feeding through it (Feeder Section 1001 hosts all six pair groups). The capacity of a feeder section can be no less than the sum of the sizes of the pair groups which feed through it.

Feeder relief planning is defined here, in terms of the route model, to include two related processes:

Capacity planning—determining what the capacity of each feeder section should be.

Allocation—determining how many pairs belong in each pair group.

In general, both the section capacities and the pair group sizes are

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* For example, consider an apartment complex which is 50 percent occupied. Additional pairs beyond the minimum required can reduce the cost of constantly moving pairs around. See Ref. 7 for a related discussion.

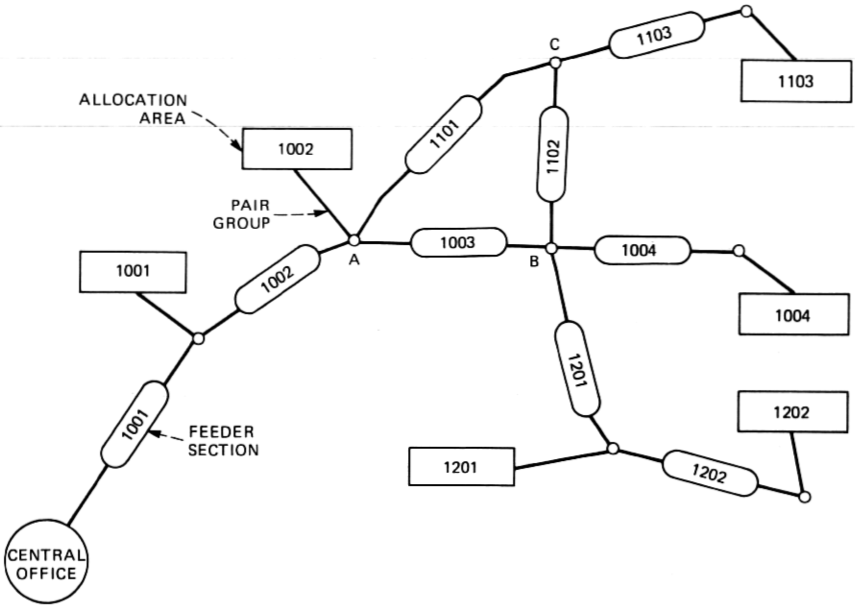

Fig. 3—Model of a simple feeder route.

time-varying. Therefore, a relief plan is not a "snapshot," but a projection of relief events over an interval of time or *study period.* Also, because of the relationship among pair groups and feeder sections, capacity planning and allocation cannot be done independently. In Sections III and IV, a practical approach to relief planning is developed.

2.4 Refining the model

The route model presented here is obviously not a completely accurate representation of a real route. There are, however, some straightforward extensions that can make the model more realistic. The extensions are mentioned but not discussed in detail.

First, it is not always possible to organize a route so that each allocation area is clearly associated with one feeder section as in Fig. 2. Second, four different gauges of cable are in use in the Bell System loop network, and different allocation areas may require different gauges. These complexities can be handled with the framework outlined in Sections 2.1 to 2.3.

It should be noted that this route model is a *planning model.* The relief plan is not a detailed work order, but a more general picture of how the route will evolve through time. A more complex route model, which may be required for other tasks, could be very inefficient from the planners' point of view.

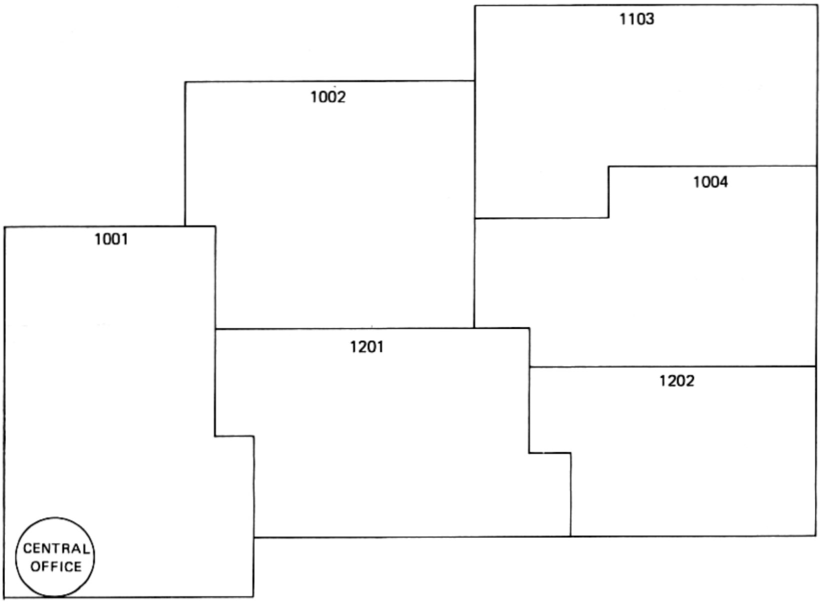

Fig. 4—Allocation area map.

III. FEEDER RELIEF ALTERNATIVES

Recall that the objective of feeder relief planning is to (*i*) provide feeder facilities where and when they are needed (*ii*) at the lowest possible cost. It would appear that there are many, many relief alternatives which satisfy (*i*), and the problem is which of these satisfies (*ii*). One practical approach to this problem is to somehow select a small number of alternatives (e.g., 10 or fewer) which satisfy (*i*) and hope that the lowest cost plan among these is the best possible plan (or at least a *good* plan).

In this section, three kinds of relief alternatives are discussed:

(*i*) Add capacity to one or more feeder sections.

(*ii*) Vary the size of one or more pair groups within the limits imposed by existing section capacities.

(*iii*) Combine the above alternatives.

Throughout this section, it is assumed that a relief plan is being developed because one or more allocation areas is:

(*i*) approaching a shortage condition because of subscriber growth or

(*ii*) experiencing high operating cost because of subscriber movement.

The second of the above "relief triggers" reflects the fact that, even with no growth, a lack of feeder can generate unacceptably high cost.

See Refs. 5 and 6 for more detail. In either case, the allocation areas affected are referred to here as *problem* allocation areas.

3.1 Capacity addition

One sure way to provide feeder relief is to add capacity to one or more feeder sections and allocate additional pairs to the problem allocation areas. In fact, if the total pair requirements among the allocation areas which feed through a given section exceed the capacity of that section, then capacity *must* be added.

It will be assumed that a capacity addition *project* is defined and will be considered as a feeder relief alternative. The project definition includes the feeder sections to which capacity will be added, the size (in pairs) of the addition to each section, and the cost of the capacity. The time at which the capacity addition will occur may also be specified or left as a variable to be determined.

Methods for determining the size of feeder capacity additions for routes with growing demand are discussed in Ref. 2. These methods are based on minimizing the total present worth of feeder cable investments for each feeder section. In practice, capacity addition alternatives reflect other considerations such as the use of pair gain rather than cable[7] and the benefit of adding capacity to more than one section at the same time. Methods for dealing with these considerations are not discussed here.

The timing of capacity additions is considered in some detail in Section IV. Clearly, pairs must be added before a shortage occurs. But when does a shortage occur and how much in advance of the date of shortage should capacity be added?

In one sense, a shortage occurs as soon as even one allocation area exhausts the capacity of its pair group. On the other hand, as discussed in Section 3.2, it is often possible to eliminate a shortage in an allocation area by making better use of existing capacity. Thus, the time at which a shortage occurs depends upon how existing capacity is used. In addition, the *network operating cost*, a component of the total cost, rises sharply as the times of shortages approach. Thus, it is generally less costly to place new capacity some time before the actual shortage occurs.

3.2 Reallocation

As has been suggested, another relief alternative is to add pairs to the problem allocation areas subject to the constraint of existing section capacities. These pairs have to come from either the pool of unallocated pairs or another allocation area which has a surplus of pairs.[8,9]

In practice, a reallocation is generally accomplished by changing the way feeder pairs are spliced. For example, Fig. 5 shows how 200 pairs are moved from one allocation area (AA 1002) to another (AA 1001). In Fig. 5, cable pairs are identified by a cable number (01) and a pair number (from 1 to 1200). Initially, 600 pairs (01, 1-600) are allocated to AA 1002. The reallocation is accomplished by changing the splice in Manhole 1 so that 200 pairs (01, 601-800) which were formerly spliced through to Manhole 2 and AA 1002 are spliced to the lateral cable which feeds AA 1001. Note that no cable is physically removed from the network. Rather, the 200 pairs from AA 1002 which are disconnected at Manhole 1 become "dead" pairs, i.e., pairs which do not extend to the central office.

Often, much more complex operations, including the placement of short lengths of cable, are required to accomplish a reallocation. There is no simple way to model the cost of a reallocation as, say, a function of the number of pairs reallocated. For this reason, it will be assumed that any reallocations included in the relief plan are defined by the planner. The definition must include the number of pairs reallocated, the "from" and "to" allocation areas, and the cost of the reallocation.

3.3 Other relief alternatives

A few additional alternatives for providing additional pairs to an allocation area are mentioned here. There are, of course, many others.

3.3.1 Working pair transfers

It is possible to reduce the number of working pairs (i.e., pairs that are serving current subscribers) in one allocation area and increase the number in another. This has an effect similar to that of reallocation.

According to the route model, a pair may belong to only one allocation area. In practice, however, a given complement of pairs* may physically appear in more than one allocation area. Such a complement, which is said to be "in multiple," may serve current subscribers in more than one allocation area. If the complement is small (say, 25 or 50 pairs), it is generally associated with only one allocation area under the assumption that the multiple appearance will ultimately be eliminated.

Suppose a complement which is in multiple is associated with a given allocation area (A) but serves some current subscribers in another allocation area (B). According to the route model, since these subscribers are served by pair group A, they are effectively in allocation area A. However, if these subscribers are transferred to pairs in pair group B, they are effectively in allocation area B (where they belong). This is how working pairs can be moved between allocation areas.

* Here a complement means some multiple of 25 pairs with the same cable number and consecutive pair numbers (e.g., 01, 51-150).

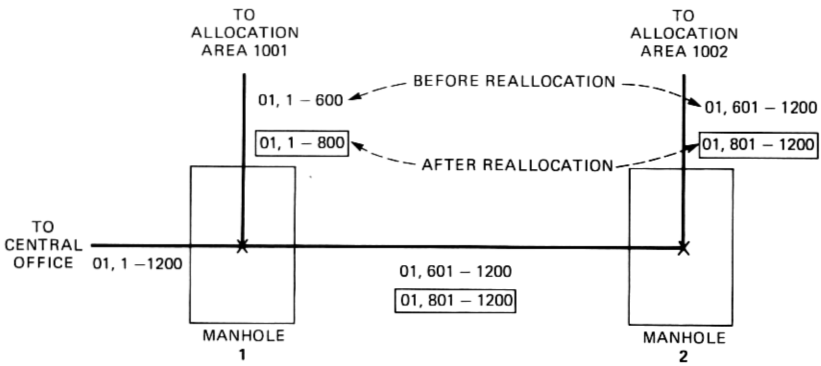

Fig. 5—Reallocation example.

Note that this kind of working pair transfer is a key step toward eliminating multiple.

As in the case of reallocation, it is assumed that, for each working pair transfer alternative, the planner specifies the number of pairs transferred, the "from" and "to" allocation areas, and the cost of the transfer.

3.3.2 Clearing defective pairs

Some pairs associated with an allocation area may be defective. If this is so, the number of pairs available to serve subscribers in the allocation area will be increased if defective pairs are "cleared." Thus, clearing defective pairs is another relief alternative.

It is assumed that, for each clear defective pair alternative, the planner specifies the number of pairs made available, the allocation area, and the cost.

3.4 Combined alternatives

Generally, a relief plan includes one or more of the alternatives discussed in Sections 3.1 to 3.3. For example, suppose one or two allocation areas in a route are approaching a shortage condition but others have large surpluses. Then a reallocation to "balance" the route may be appropriate. However, if feeder requirements continue to grow, a capacity addition may eventually be required. Nevertheless, the reallocation can still be a cost-effective interim step, particularly if it significantly defers the need for an expensive capacity addition. This reallocation-followed-by-capacity addition is an example of a combined alternative.

Clearly, more complex relief plans, involving several reallocations, working pair transfers, capacity additions, etc., can be defined by the planner. Moreover, by using feeder allocation methods such as those described in Refs. 8 and 9, along with some experience and judgment, a planner may come up with several candidate relief plans. In the next section, the method for accomplishing an economic evaluation of these plans is discussed.

IV. ECONOMIC EVALUATION

This section discusses the economic evaluation of a given relief plan. This evaluation would presumably be carried out for a number of alternative plans of the type discussed in Section III.

4.1 Economic criterion

The cost of a relief plan will be defined as the present worth of expenditures (PWE) associated with the plan. The PWE is a single dollar amount associated with a cash flow. Methods for computing PWE are discussed in Ref. 10. For convenience, simplified expressions for PWE will be used here. The use of more complex expressions does not significantly affect the economic evaluation process.

For a sequence of expenditures $\{X_i\}$ such that expenditure X_i occurs at time T_i, the PWE is defined here as

$$\text{PWE} = \sum X_i e^{-rT_i}, \tag{1}$$

where e^{-rt} is the present worth at time 0 of a dollar spent at time t (r is called the convenience rate). In the present application, the X_i may represent various expenditures associated with capacity additions (capital recovery, etc.) as well as expenses associated with network changes.

It will be convenient to represent some expenditures associated with a relief plan as continuous cash flows. Thus, the total PWE may be written as

$$\text{PWE} = \sum X_i e^{-rT_i} + \int x(t) e^{-rt}\, dt, \tag{2}$$

where $x(t)$ is the continuous component of the total cash flow. A continuous cash flow will be used to model network operating costs, as discussed in Section 4.2.

4.2 PWE of a relief plan

As discussed in Section III, a relief plan may include capacity additions, network changes (e.g., reallocation), or both. These actions require expenditures and therefore contribute to the PWE of the relief plan. As stated in Section III, it is assumed that the *amounts* of these expenditures are determined in the process of developing the relief alternative. For now, it is further assumed that the *times* of these expenditures are also determined. Given the amounts and times of the expenditures, the PWE for capacity additions and network changes can be determined in a straightforward manner:

$$\text{PWE}(CA + NC) = \sum_{i=1}^{N_{CA}} P_{CA}^i e^{-rT_{CA}^i} + \sum_{i=1}^{N_{NC}} P_{NC}^i e^{-rT_{NC}^i}, \tag{3}$$

where N_{CA} is the total number of capacity addition projects, P^i_{CA} and T^i_{CA} are the amount and time of the ith capacity addition expenditure ($i = 1, 2, \cdots, N_{CA}$), and N_{NC}, P^i_{NC}, and T^i_{NC} are similar quantities for network changes.

If eq. (3) is to truly represent PWE, the P^i_{CA} and P^i_{NC} have to be defined rather carefully. Consider a capacity addition, specifically, a cable placement. The associated P^i_{CA} is defined as the total present worth of all expenditures associated with the cable and its placement assuming the cable is placed at time $t = 0$. P^i_{CA} includes the present worth of capital recovery, taxes, maintenance, and eventual replacement. If the cable is placed at $t = T^i_{CA}$ rather than $t = 0$, the present worth changes by the factor $e^{-rT^i_{CA}}$. If all capacity additions and network changes are treated this way, eq. (3) yields the desired PWE.

As a practical matter, it may be difficult to determine the true values of the P^i_{CA} and P^i_{NC}. This is because of the uncertainty of the impact of future technology on the relief plan. This is not a serious problem here, however, because most of the emphasis is on varying the near-term parameters of the relief plan.

The *total* PWE of a relief plan includes an additional component, operating cost. Operating cost is the day-to-day cost of providing service through the loop network. It includes the cost of service-order-related activities of various operating company forces.

Reference 5 describes a mathematical model that can be used to estimate operating cost by allocation area. This model includes the effects of several allocation area parameters; in particular, those that may be varied by feeder relief. For the purposes of this paper, the PWE of operating cost in a given allocation area is defined as

$$\text{PWE}_{OC} = \int_0^\infty f[y(t), w(t), d(t)]e^{-rt}\, dt, \tag{4}$$

where $y(t)$, $w(t)$, and $d(t)$ are the numbers of available, working, and defective pairs, respectively, in the allocation area and f is the operating cost.

The total PWE now becomes

$$\begin{aligned} \text{PWE}(CA + NC + OC) = &\sum_{i=1}^{N_{CA}} P^i_{CA} e^{-rT^i_{CA}} \\ &+ \sum_{i=1}^{N_{NC}} P^i_{NC} e^{-rT^i_{NC}} \\ &+ \sum_{i=1}^{N_{AA}} \int_0^\infty f_i[y_i(t), w_i(t), d_i(t)]e^{-rt}\, dt, \end{aligned} \tag{5}$$

where N_{AA} is the number of allocation areas in the feeder route.

Although eq. (5) is the desired expression for the PWE of a relief plan, it obscures the interdependence among capacity additions, network changes, and operating cost. This interdependence is perhaps best illustrated by the examples in the following subsections.

4.2.1 Example: capacity addition

Consider a (very) simple route containing one feeder section (FS 1001) which feeds two allocation areas (AA 1001 and AA 1002). FS 1001 currently contains 1000 pairs; 500 pairs are allocated to AA 1001 and 500 to AA 1002. There are no defective pairs.

The numbers of working pairs by allocation area are given as

$$w_1(t) = \min[425 + 75t, 1250]$$

$$w_2(t) = \min[375 + 25t, 650],$$

where t is in years and the subscripts 1 and 2 correspond to AA 1001 and AA 1002. Note that the $w_i(t)$ increase linearly to plateau values.

The operating cost functions for the two allocation areas are assumed to be

$$f_i = \lambda_i C^i_{BLK} \left[\frac{w_i(t)}{y_i(t)} \right]^{\alpha_i}, \tag{6}$$

where λ_i is the subscriber arrival rate, y_i = number of available pairs, C^i_{BLK} is the blockage cost, and α_i is the effective access group size for the ith allocation area. For a further explanation of eq. (6), the reader is referred to Ref. 5.

For this example:

$$\lambda_1 = 150, \quad \lambda_2 = 50 \text{ arrivals/yr}$$

$$C^1_{BLK} = C^2_{BLK} = \$50$$

$$\alpha_1 = \alpha_2 = 10$$

$$r = 0.1.$$

The relief plan consists of placing a 900-pair cable at time $t = 1$ year (this is the time when AA 1001 exhausts) and increasing the allocations to 1250 pairs to AA 1001 and 650 pairs to AA 1002 (a "balanced" allocation, as defined in Ref. 8). The amount of the expenditure associated with this capacity addition is \$60,000.

Given the initial allocation and the relief plan, the numbers of available pairs become:

$$y_1(t) = \begin{cases} 500 & 0 \le t < 1 \\ 1250 & t \ge 1 \end{cases}$$

$$y_2(t) = \begin{cases} 500 & 0 \le t < 1 \\ 650 & t \ge 1. \end{cases}$$

At this point, the interdependence between capacity addition and operating cost is evident. The capacity addition provides pairs that are used to increase (or at least vary) the numbers of pairs available in the allocation areas. This in turn varies the pattern of operating cost.

A critical assumption here is that the new capacity is allocated and *made available to serve subscribers*. Merely adding capacity to one or more feeder sections has no effect on operating costs.

The total PWE for the relief plan can now be computed in a straightforward manner. The resulting amount is $97,900.

4.2.2 Example: reallocation and capacity addition

This example illustrates another relief alternative for the same route used in Section 4.2.1. This alternative consists of two steps:

(*i*) Reallocate 75 pairs from AA 1002 to AA 1001 at time $t = T_{NC} =$ 1 year ($P_{NC} = \$300$).

(*ii*) Place a 900-pair cable and increase the allocations to 1250 pairs to AA 1001 and 650 pairs to AA 1002 at time $t = T_{CA} = 2$ years ($P_{CA} = \$60{,}000$)

This alternative uses a reallocation to eliminate the shortage in AA 1001 at $t = 1$ year and defers the capacity addition to $t = 2$ years.

For this relief alternative, the numbers of available pairs by allocation area become,

$$y_i(t) = \begin{cases} 500 & 0 \le t < 1 \\ 575 & 1 \le t < 2 \\ 1250 & t \ge 2 \end{cases}$$

$$y_2(t) = \begin{cases} 500 & 0 \le t < 1 \\ 425 & 1 \le t < 2 \\ 650 & t \ge 2 \end{cases}$$

and the resulting PWE is $97,800. Thus, on the basis of the PWE criterion, this alternative is better* than the one evaluated in Section 4.2.1.

4.3 Optimum timing of relief

The total PWE given by eq. (5) can be used as a criterion for optimizing a relief plan with respect to one or more parameters. In this section, eq. (5) serves as the basis for optimum timing of relief, i.e., determining when to place capacity additions. To simplify the discussion, it will be assumed that only one capacity addition is included in the relief plan (i.e., $N_{CA} = 1$).

* In these examples, the numerical PWE differences among alternatives are smaller than they tend to be in practice. The main purpose of the examples is to illustrate that trade-offs among alternatives exist.

4.3.1 Optimal timing of a capacity addition

As illustrated in Section 4.2, a capacity addition affects operating cost by varying the feeder allocation — the y_i. Generally, the net result of a capacity addition is a sharp reduction in operating cost. The sooner capacity is added, the sooner the operating cost reduction occurs and therefore the lower the component of the PWE due to operating cost. On the other hand, the component of the PWE due to capacity addition clearly increases when the time of the addition is advanced. The optimum time of capacity addition is the one which minimizes the total cost given by eq. (5). If

$$N_{CA} = 1,\ N_{NC} = 0,\ N_{AA} = 2 \text{ and } d_i(t) \equiv 0\ \forall i,$$

eq. (5) becomes

$$\text{PWE} = P_{CA}e^{-rT_{CA}} + \sum_{i=1}^{2} \int_0^{\infty} f_i[y_i(t, T_{CA}), w_i(t)]e^{-rt}\,dt, \tag{7}$$

where the dependence of the y_i on T_{CA} has been explicitly noted. For this case, the relief timing problem is to find the $T_{CA} \geq 0$ which minimizes the PWE given by eq. (7). This is a simple, one-dimensional minimization problem.

For f_i given by eq. (6), the optimum T_{CA} may be such that $w_i(t) > y_i(t)$ for some i and t. Since this result is unacceptable (i.e., all subscribers must be served), the optimization must be subject to the constraint

$$w_i(t) \leq y_i(t),\ \forall i,\ t. \tag{8}$$

4.3.2 Example

In the example in Section 4.2.1, the relief plan consisted of a single capacity addition at time $t = 1$ year (the time at which AA 1001 exhausts its allocation). In this example, the optimum value of T_{CA} will be determined.

For a given T_{CA}, the y_i are

$$y_1(t, T_{CA}) = \begin{cases} 500 & 0 \leq t < T_{CA} \\ 1200 & t \geq T_{CA} \end{cases}$$

$$y_2(t, T_{CA}) = \begin{cases} 500 & 0 \leq t < T_{CA} \\ 650 & t \geq T_{CA}. \end{cases}$$

The optimal time $\hat{T}_{CA}$ must satisfy $0 \leq \hat{T}_{CA} \leq 1$ year; otherwise, eq. (8) is violated. A straightforward search shows that $\hat{T}_{CA} \approx 0.83$ year and the corresponding PWE is \$97,800.

In this example, there is economic justification for placing and allocating additional capacity prior to the actual exhaust date.

4.3.3 Optimal timing with network changes

In this section, it is assumed that $N_{CA} = 1$ and $N_{NC} > 0$. The object is still, however, to minimize PWE with respect to T_{CA}.

The PWE equation now becomes

$$\text{PWE} = P_{CA}e^{-rT_{CA}} + \sum_{i=1}^{N_{NC}} P_{NC}^{i}e^{-rT_{NC}^{i}}$$

$$+ \sum_{i=1}^{N_{AA}} \int_{0}^{\infty} f_i[y_i(t, T_{CA}), w_i(t)]e^{-rt}\, dt. \tag{9}$$

The impact of network changes on the optimization problem looks minimal at first—just an additional constant term in the PWE equation. However, the impact of network changes on the y_i, w_i, etc., creates some complexities that have to be considered more carefully.

First, assume $T_{NC}^{1} \leq T_{NC}^{2} \leq \cdots \leq T_{NC}^{N_{NC}} \leq T_{CA}$. Then the y_i and w_i can be represented as piecewise constant functions* with jumps at $t = T_{NC}^{i}$, $i = 1, 2, \cdots, N_{NC}$, and $t = T_{CA}$. In this case, the computation of PWE is straightforward.

If, on the other hand, a network change which includes reallocation occurs after $t = T_{CA}$, there may be a conflict. In such a case, the allocation prescribed as part of the capacity addition project is assumed to override the network change. Further, the cost of any network change which affects the y_i and for which $T_{NC}^{i} \geq T_{CA}$ is deleted from the total PWE. This convention introduces discontinuities in the PWE as a function of T_{CA}. The effect of these discontinuities is illustrated in the next example.

4.3.4 Example

In this example, the optimal value of T_{CA} for the example in Section 4.2.2 is determined.

The y_i are given by

$$y_1(t, T_{CA}) = \begin{cases} 500 & t < T_{NC} \text{ and } t < T_{CA} \\ 575 & t \geq T_{NC} \text{ and } t < T_{CA} \\ 1250 & t \geq T_{CA} \end{cases}$$

$$y_2(t, T_{CA}) = \begin{cases} 500 & t < T_{NC} \text{ and } t < T_{CA} \\ 425 & t \geq T_{NC} \text{ and } t < T_{CA} \\ 650 & t \geq T_{CA}. \end{cases}$$

Note that, if $T_{NC} \geq T_{CA}$, the y_i jump directly from the starting values

* If there are defective pairs, the d_i can be represented this way also.

(500, 500) to the final values (1250, 650). This is in keeping with the convention stated in Section 4.3.3. The convention also dictates that the PWE only include the \$300 reallocation cost at $t = 1$ year if $T_{CA} > 1$ year.

In this example, the optimal time $\hat{T}_{CA}$ is 1.58 years and the corresponding PWE is \$97,300. Since the reallocation occurs at $t = 1$ year, it is included in the optimal plan.

If the cost of the network change is increased to $P_{NC} = \$900$, the result is $\hat{T}_{CA} = 0.83$ year and the corresponding PWE is \$97,800. In this case, the network change which is evidently not cost-effective is excluded, and the result is identical to that obtained in Section 4.3.2.

Further variation of P_{NC} reveals that, for $P_{NC} <$ about \$865, the network change is cost-effective and $\hat{T}_{CA} \approx 1.5$ years. Otherwise, the optimal plan is to go ahead and add capacity at $t = 0.83$ year.

4.4 Discussion

This section has shown how a feeder relief plan can be economically evaluated and, in one sense, optimized.

In real applications, the size of the problems (i.e., numbers of feeder sections and allocation areas) is much larger than the examples used here. Moreover, there are generally several capacity addition projects in a relief plan. In spite of these and other complexities, the methods presented here have been successfully applied to real feeder routes. To deal with real routes, however, some further conventions are necessary.

First, it is generally not possible to obtain all capacity addition projects, complete with allocations, for an infinite planning horizon. Thus, the capacity addition sequence is truncated at $N_{CA} = \sim 3$. Now to compute the operating cost interval, it is necessary to somehow extrapolate the f_i. One possible convention is to establish an "end-of-study" date T_s and fix the f_i at their values at $t = T_s$ for $t > T_s$. Preferably, T_s should be before the last capacity addition is used up.

Optimization of the T^i_{CA} becomes difficult for $N_{CA} > 1$. One way around this is to fix T^i_{CA} at nominal values for $i > 1$ and optimize T^1_{CA}, the time of the first capacity addition. This convention is useful in practice because nominal values are easy to obtain and the time of first capacity addition is of most interest.

Finally, it should be pointed out that, in practice, network changes are considered only in the near term, generally prior to the first capacity addition.

V. SUMMARY

This paper has outlined a method for planning feeder relief in the loop network. The method includes three basic steps:

(*i*) Establish a route model.

(*ii*) Identify relief alternatives.

(*iii*) Evaluate identified alternatives.

In practice, the first step would be carried out once as a "long-range planning" effort. This long-range planning effort would establish a correspondence between the real feeder route and a model like the one presented in Section II. Presumably, the structure of the model would change relatively infrequently. Of course, the planner would have to keep the model parameters (e.g., section capacities) as well as other required data (e.g., pair usage, forecasts) up to date.

The next two steps are carried out periodically as feeder shortages or other feeder problems develop.

The second step relies most heavily on the planner's experience and professional judgment. Although certain tools, such as graphic displays, can help, the planner must ultimately identify the "feasible set" of relief alternatives.

The last step is a straightforward application of economic computation and optimization. This step is most amenable to computer implementation, say as a time-share, interactive program.

This feeder relief planning method is currently being introduced in the Bell System operating companies. Obviously, *much* more material than that presented here is necessary to establish the use of this method in the "real world." This material includes documentation, training, and computer aids. Nevertheless, the operating company engineer who uses this method will follow the same basic steps of identification and evaluation of alternatives as presented here.

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